Analytic Modeling of The Cocoon in sGRB Jets

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Special thanks to:

Kyohei Kawaguchi

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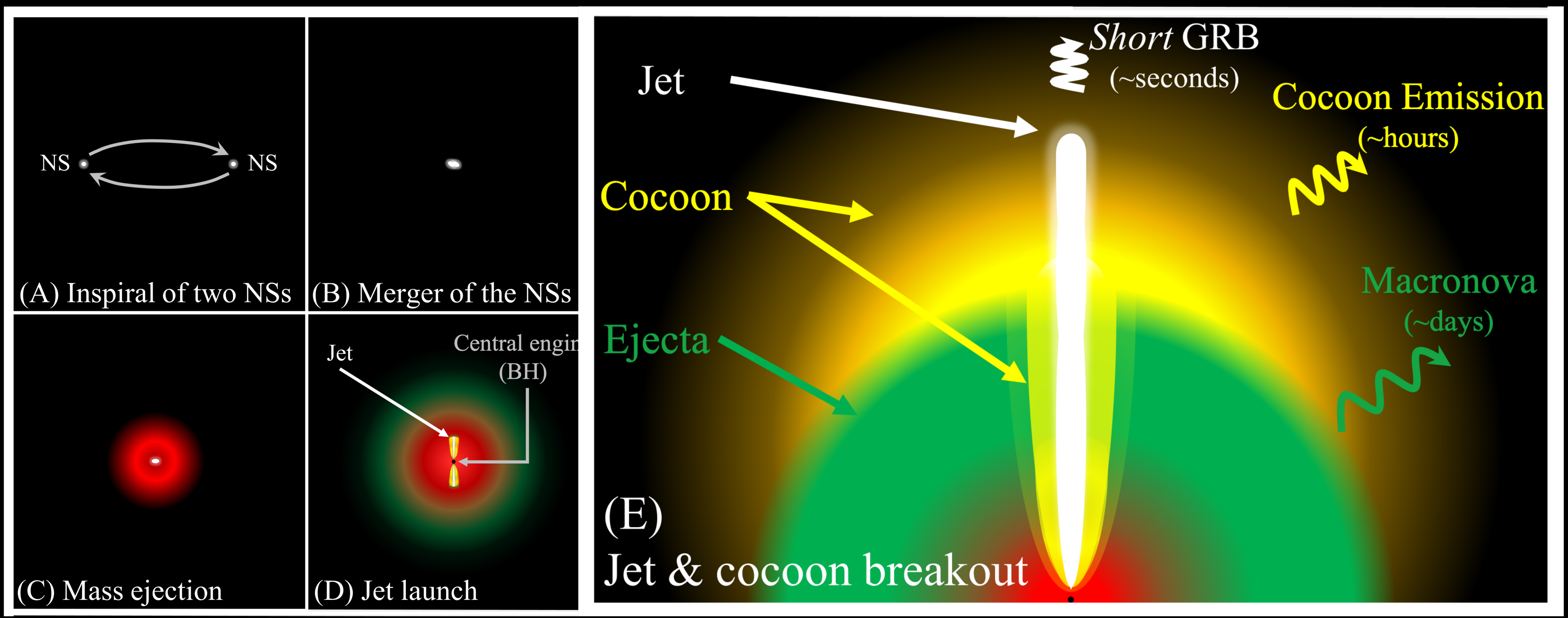
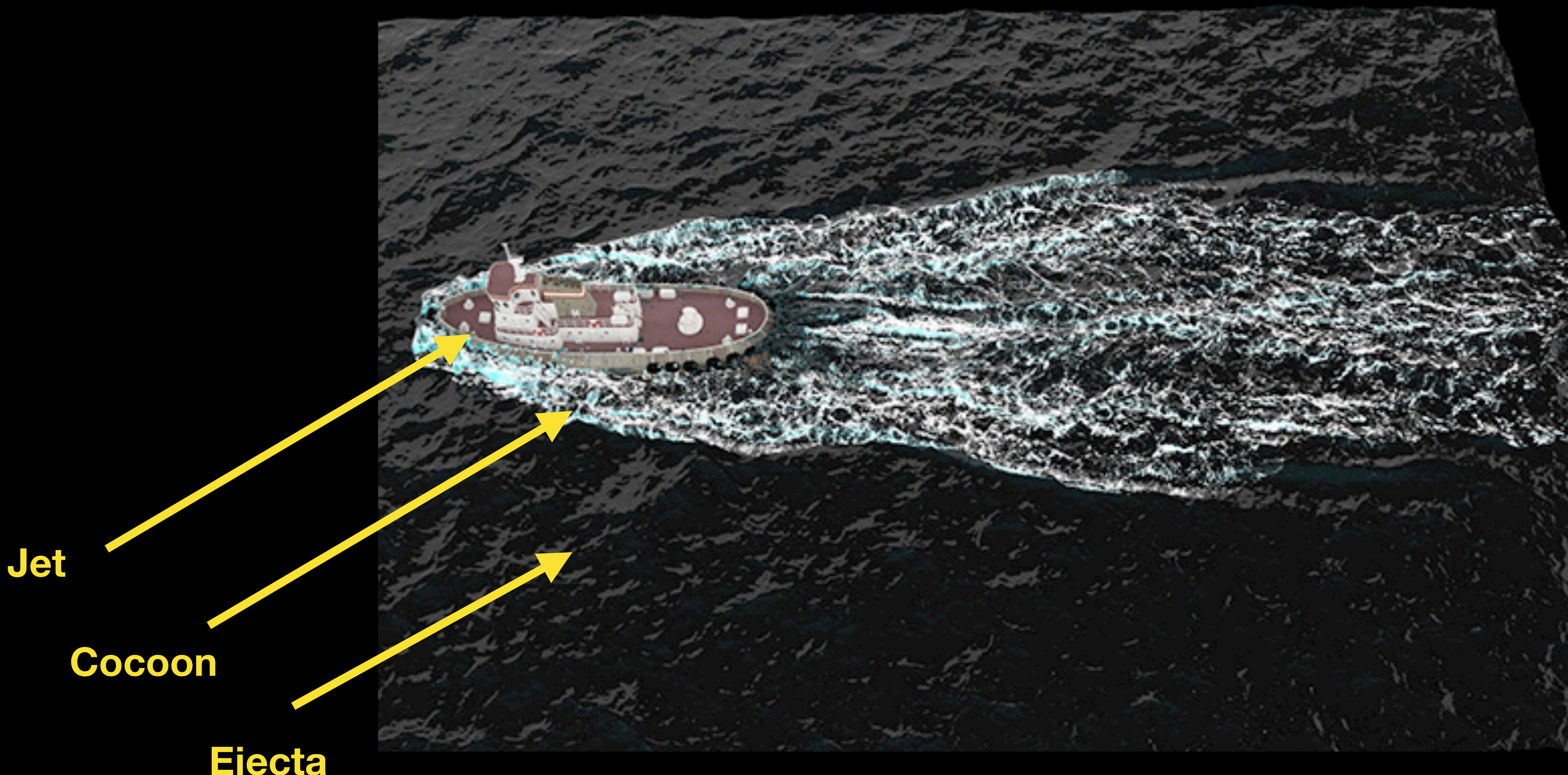
Extreme Outflows in Astrophysical Transients - August 24th 2021

NS mergers & sGRBs



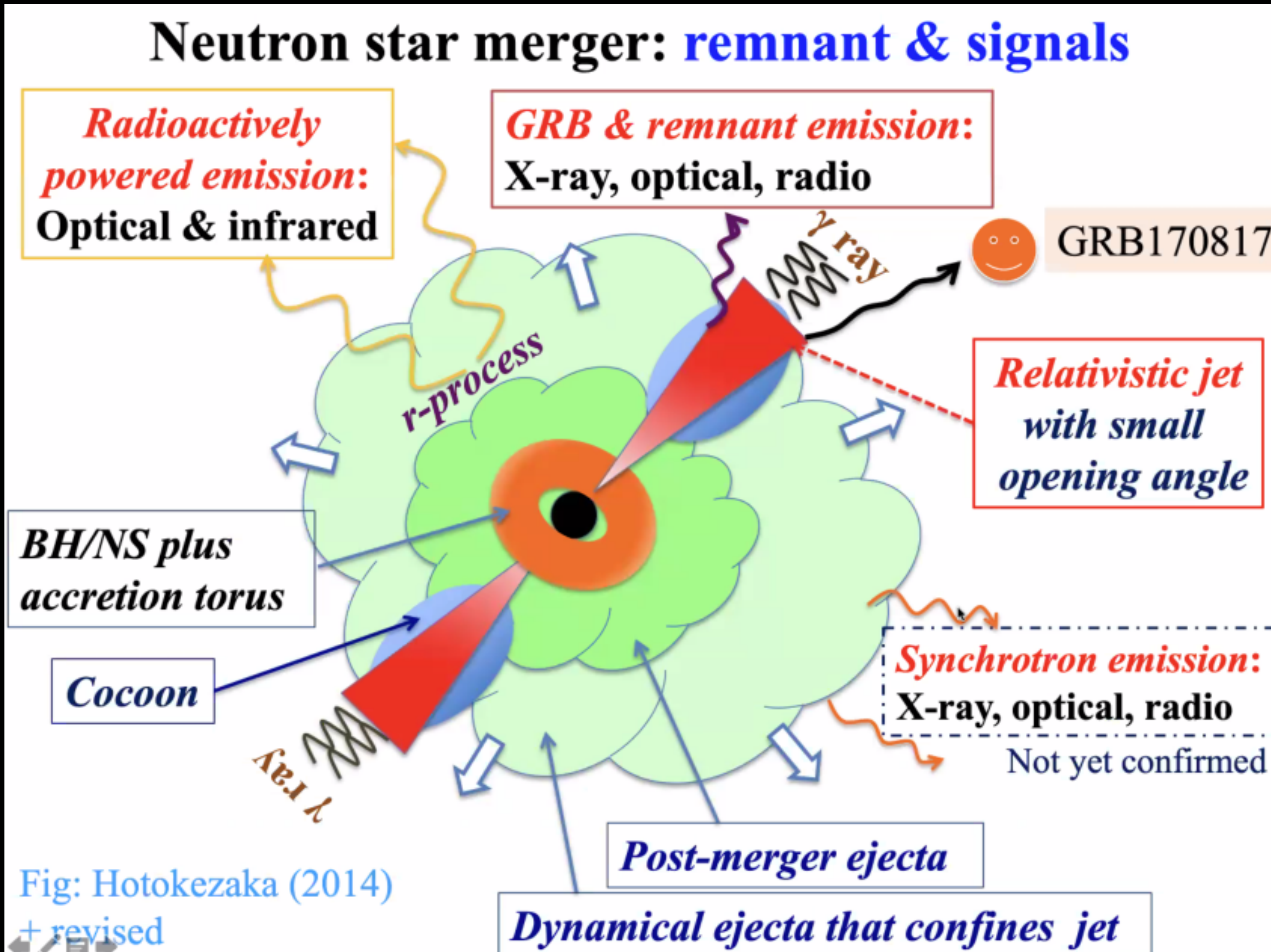
Credit: HH+20

Analogy for the cocoon



Credit: DOSCH DESIGN

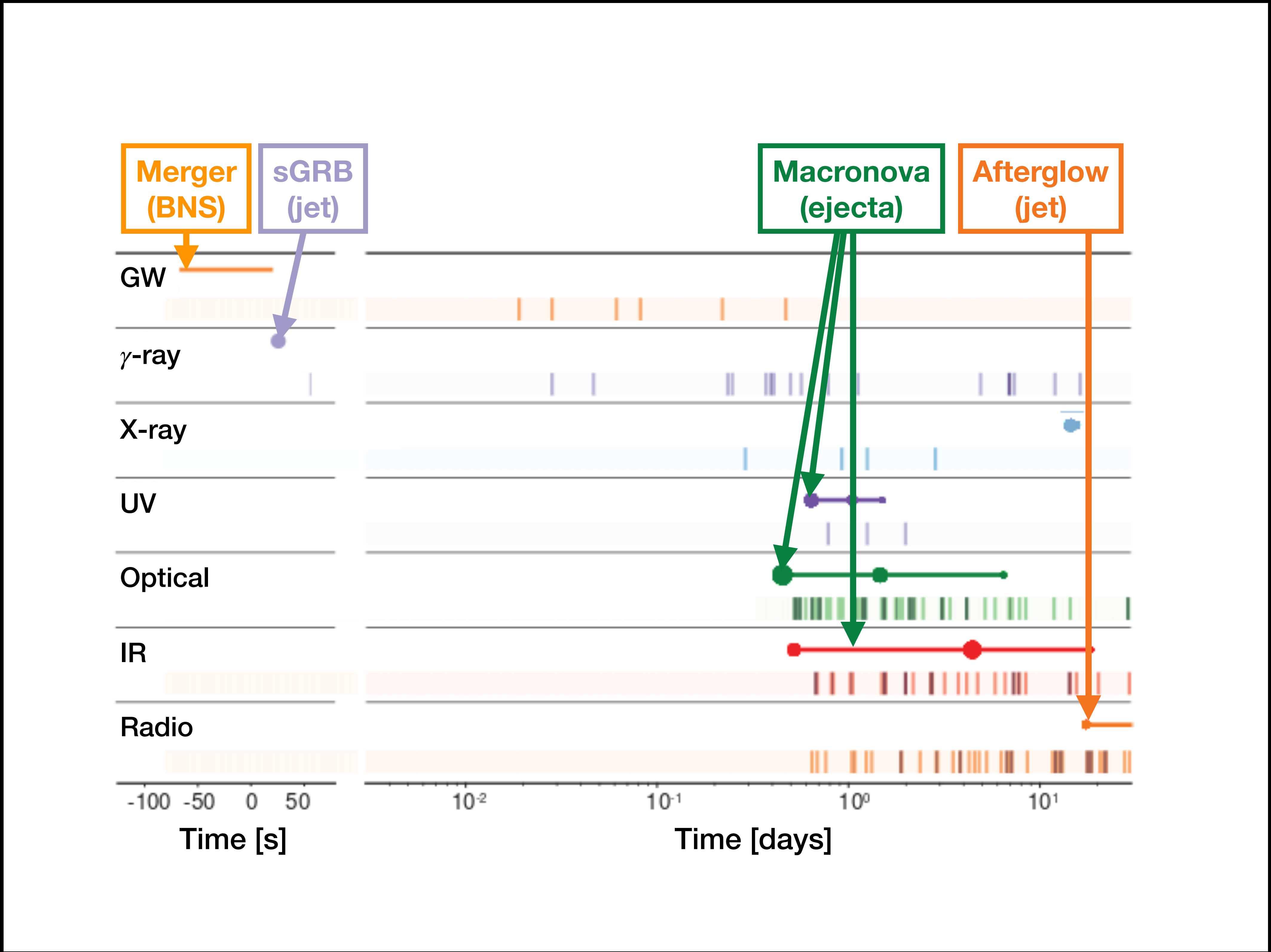
The Key Elements in NS mergers



Credit: Shibata & Hotokezaka

GW170817: Multi-messengers

Credit: LIGO



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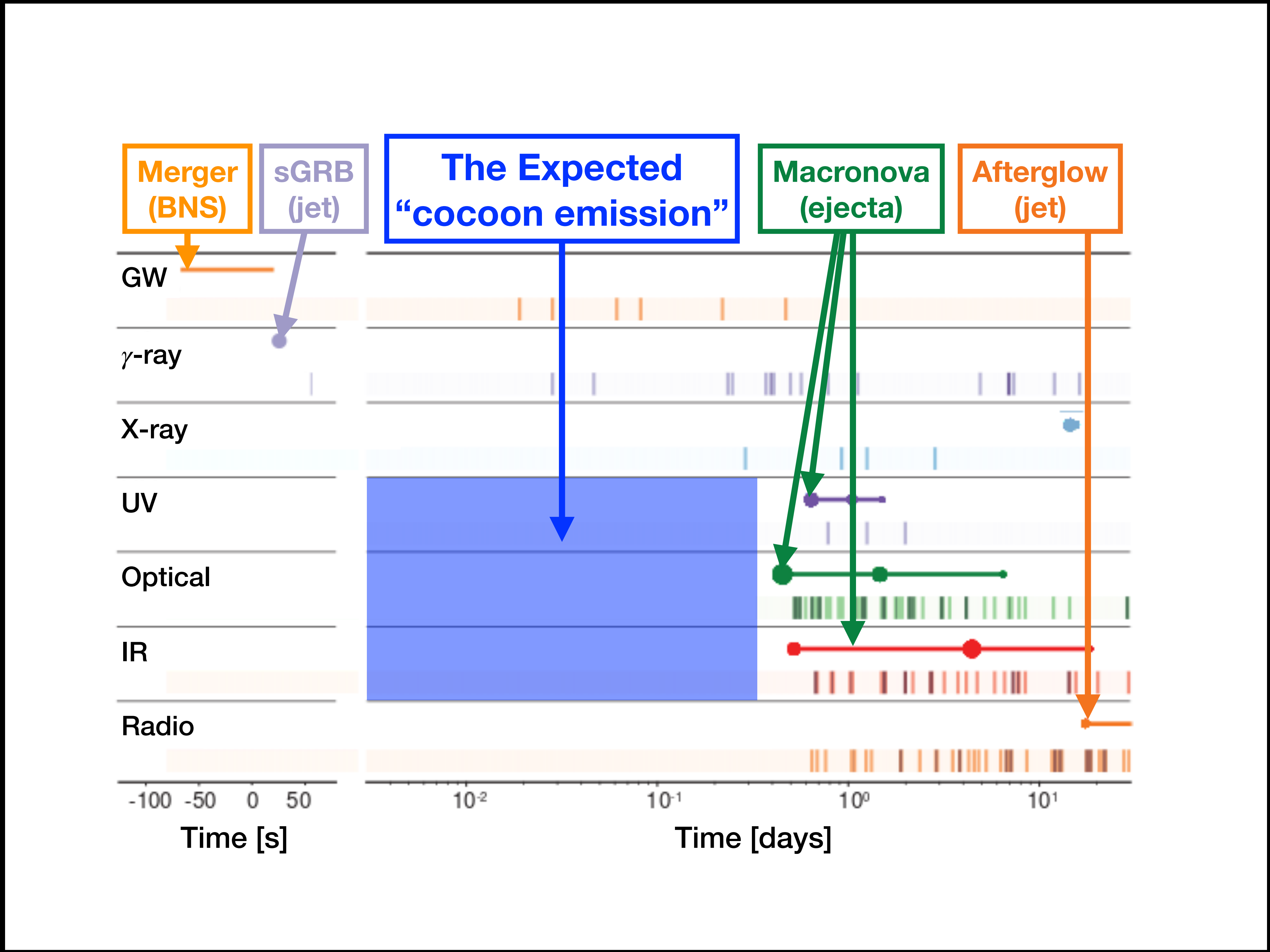
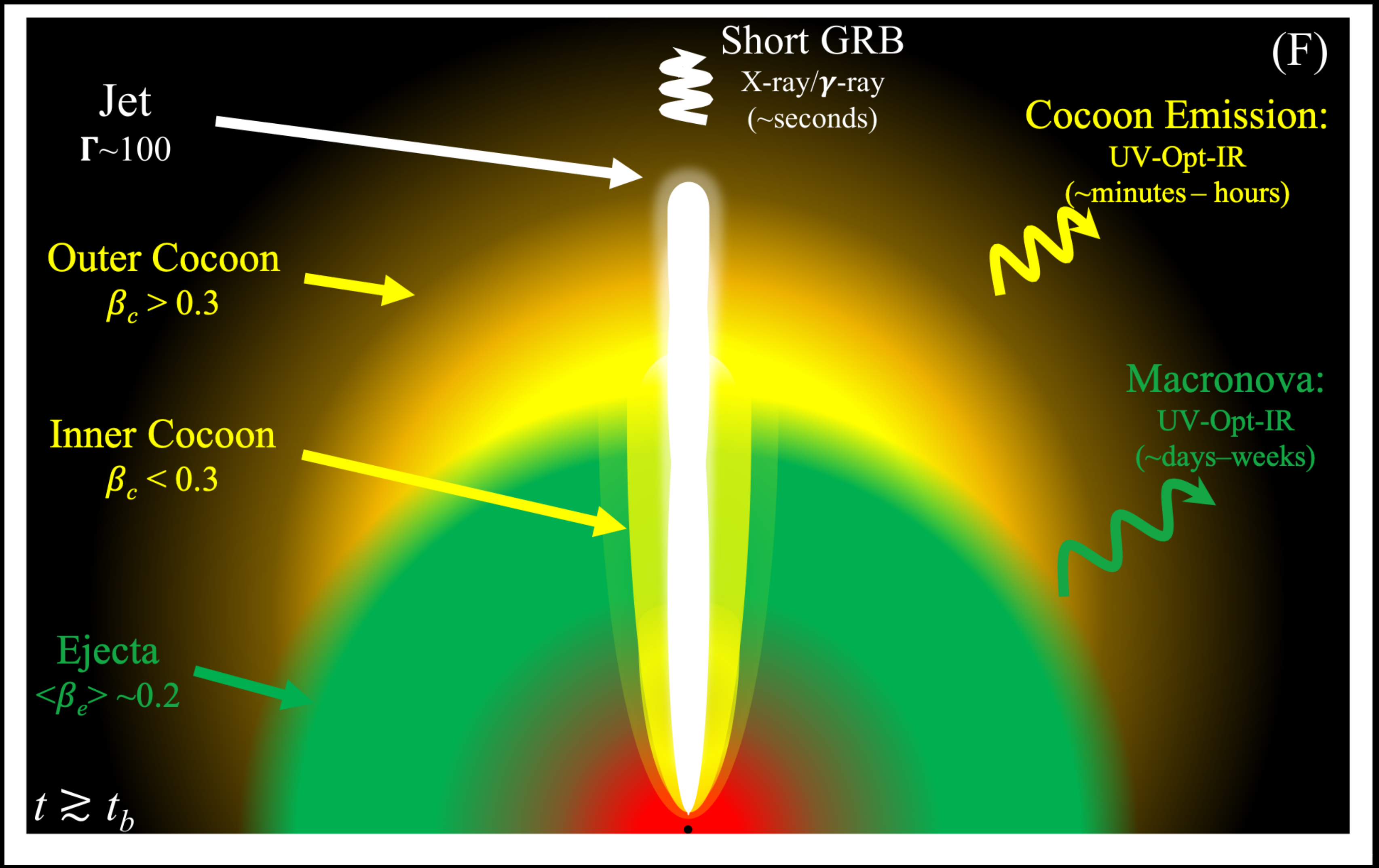


Why the cocoon?

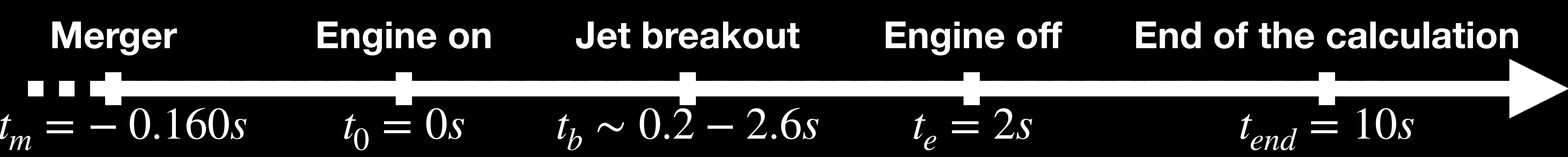
- An “**Extreme Outflow**” [$v > 0.3c$ & $10^{48} - 10^{49}$erg]
- Powers an “**astrophysical transient**”, & make an excellent EM counterpart to GWs [~Nova]
- A potential site for r-process nucleosynthesis [allow probing heating rate & opacity]

Motivation

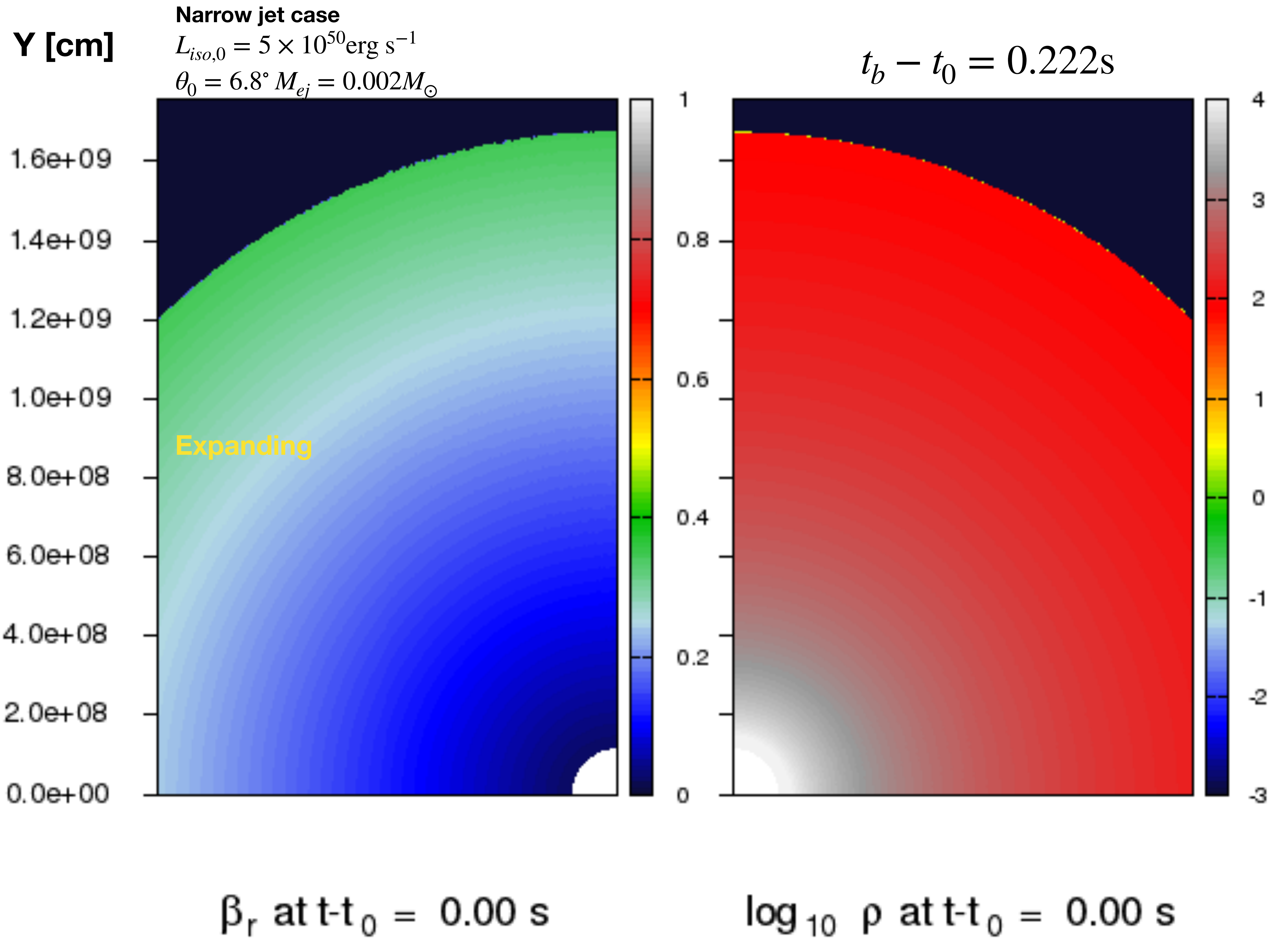
1. Modeling of the cocoon breakout [for the 1st time]
2. Estimate the cocoon imprint/emission
3. Use the cocoon to prob the central engine of sGRBs/r-process etc.



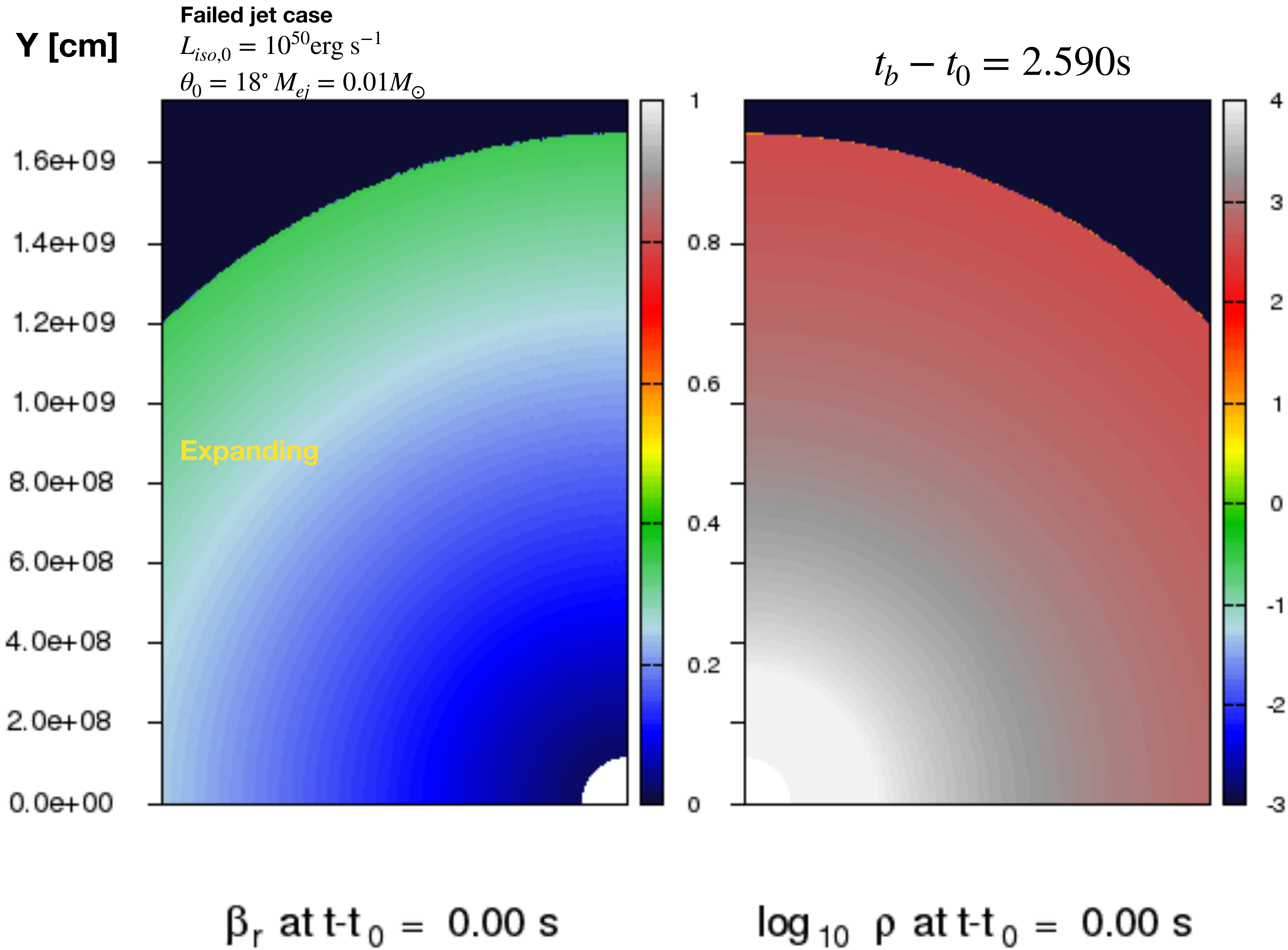
Hydrodynamical simulations (2D)



BNS merger [Short GRB] case



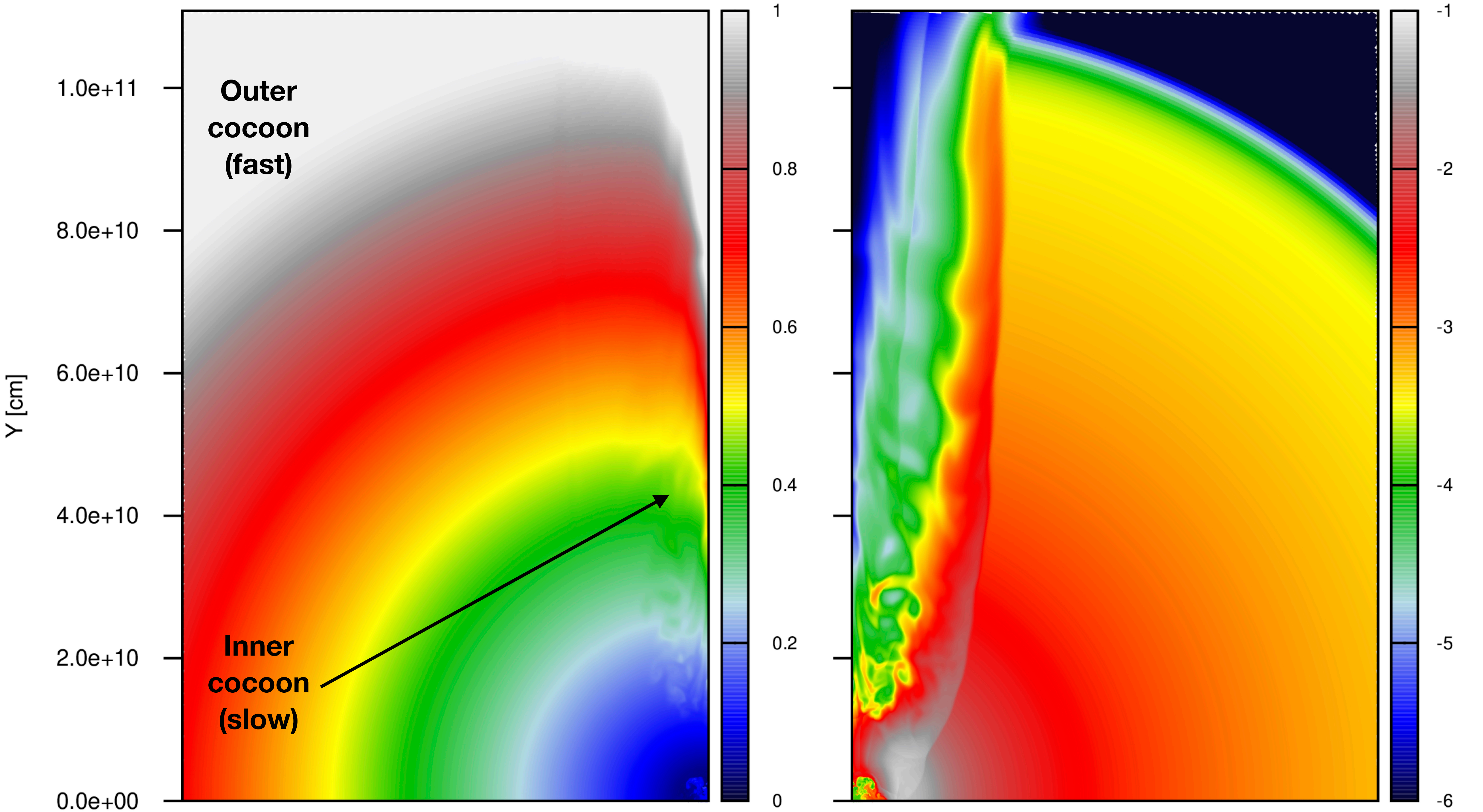
BNS merger [Short GRB] case



Outer Vs. Inner Cocoon

Narrow jet case
$L_{iso,0} = 5 \times 10^{50}$ erg s^{-1}
$\theta_0 = 6.8^\circ$ $M_{ej} = 0.002 M_\odot$

$t_b - t_0 = 0.222$s



(β_{inf}/β_m) at t-t_0 = 10.00 s

$\log_{10} \rho$ at t-t_0 = 10.00 s

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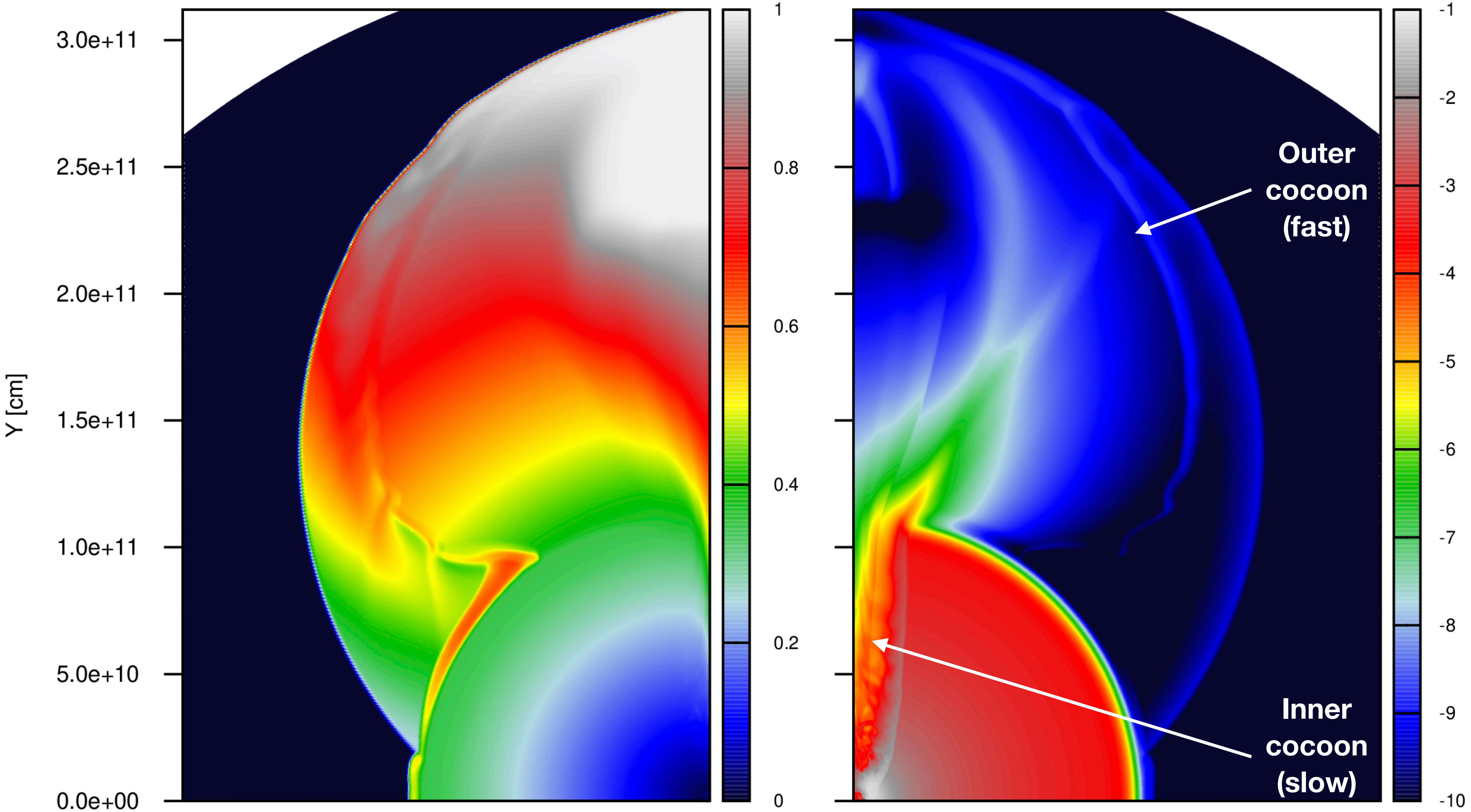

β_{inf} at $t-t_0$ = 10.00 s

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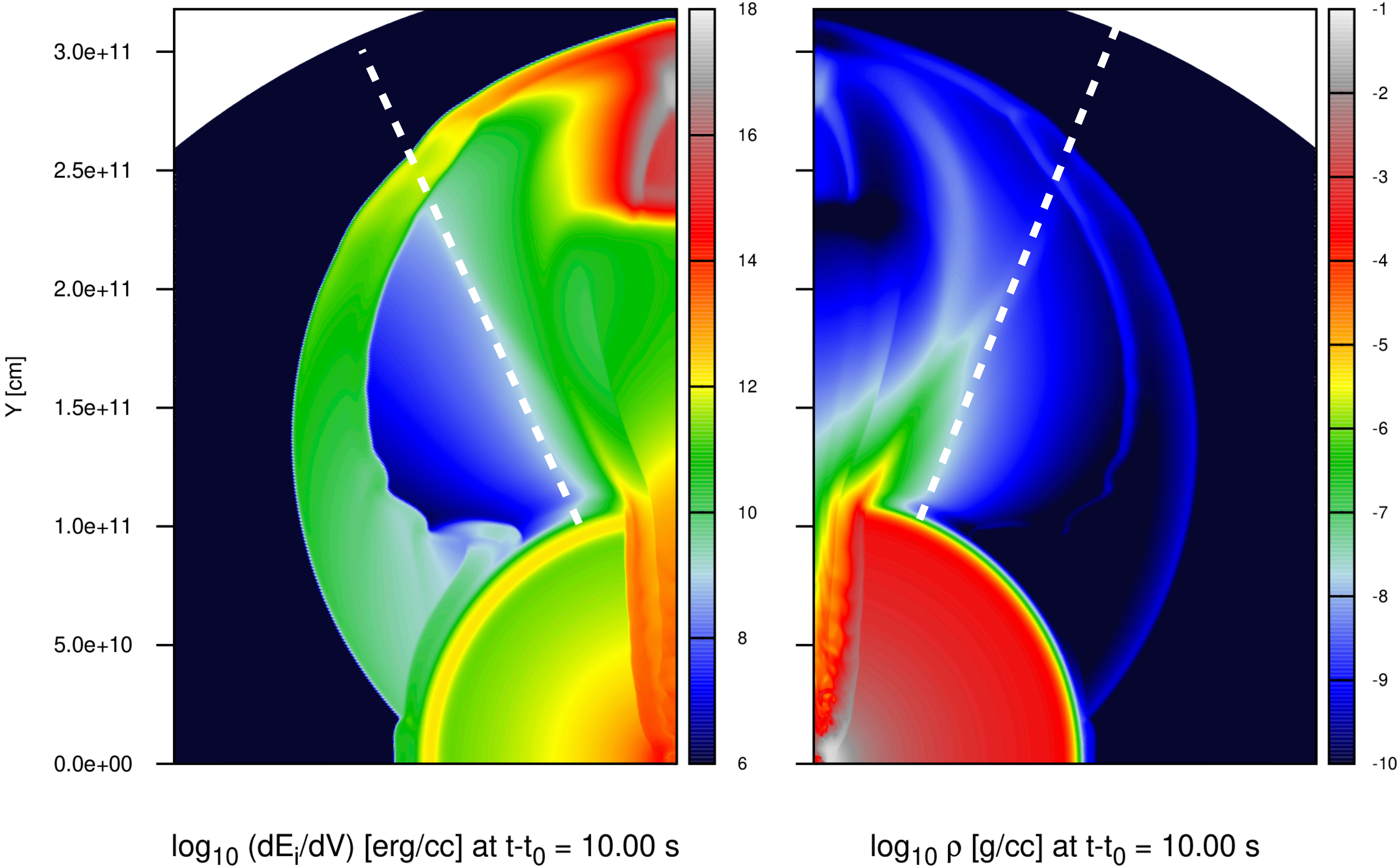


Key Points
[about the cocoon breakout]

- Homologous expansion $r \propto vt$
- Radiation pressure dominated $E_i = 3P_cV_c$ [P_c = Const.]
- Special case: $n = 2 \Rightarrow E_a \propto \int \frac{1}{2} v^2 \rho dV \propto V$ [const. energy density]
- Geometry: conical with angle θ_{OUT} [not spherical]
- Mass fraction: $M_{c,OUT}/M_c \sim 0.01$ [Very small]
- Density profile: $n_{OUT} \approx 10$ [successful jet] $n_{OUT} \approx 2$ [failed jet]

Modeling the cocoon breakout

Parameters of the jet & ejecta:

$L_j, \theta_0, M_a, n, v_m, t_0 - t_m$

Jet propagation:

t_b, r_b, r_c

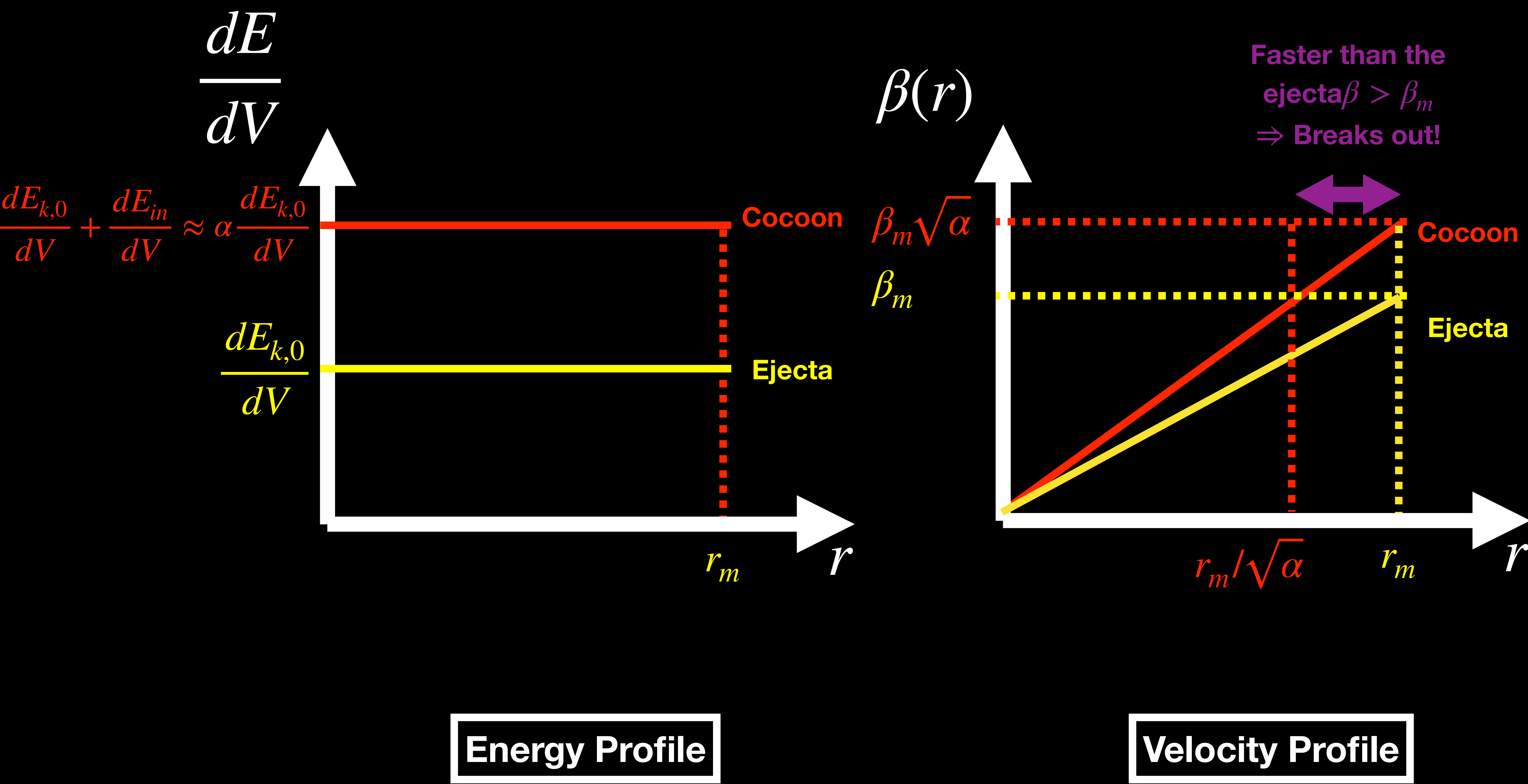
Fraction of the cocoon that breaks out:

$M_{c,OUT}, E_{c,OUT}, E_{c,i,OUT}$

Cocoon emission:

v_{ph}, L_{bl}, T_{BB}

Main Argument



Calculating α

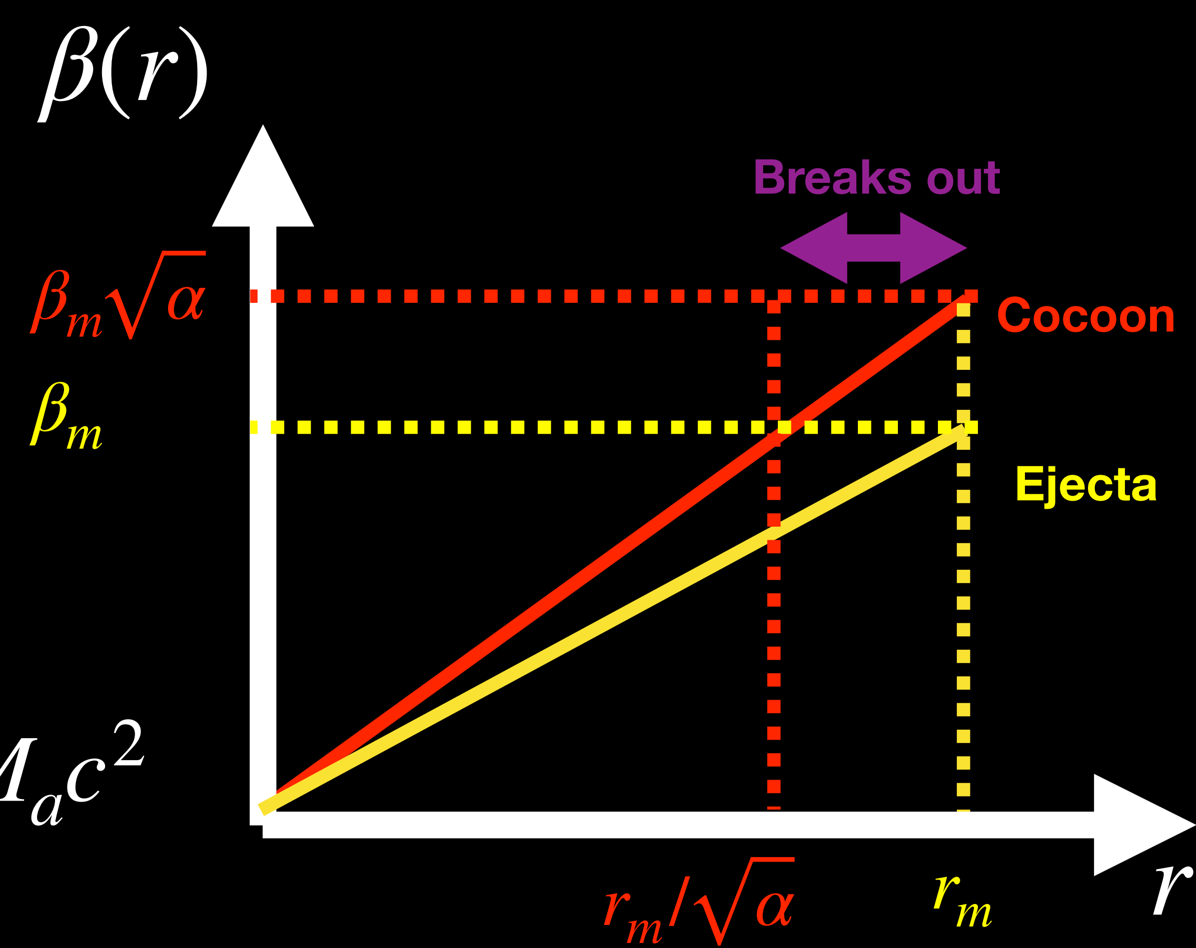
The cocoon energy:

$$E_c = E_{k,0} + E_{in}$$

From the expanding ejecta ($E_{k,0}$)

From the jet/engine (E_{in})

For $n = 2$: $E_{k,0} = \frac{V_c}{V_a} E_a \qquad E_a \approx \frac{1}{6} \beta_m^2 M_a c^2$



$$E_{in} = 2L_j (t_b - t_0)(1 - \langle \beta_h \rangle)$$

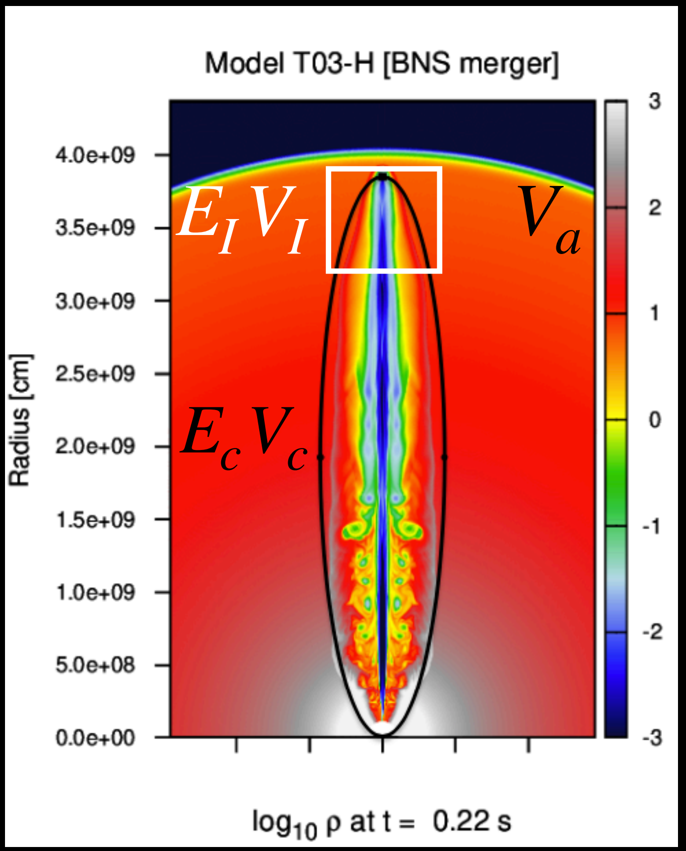
The key parameter is:

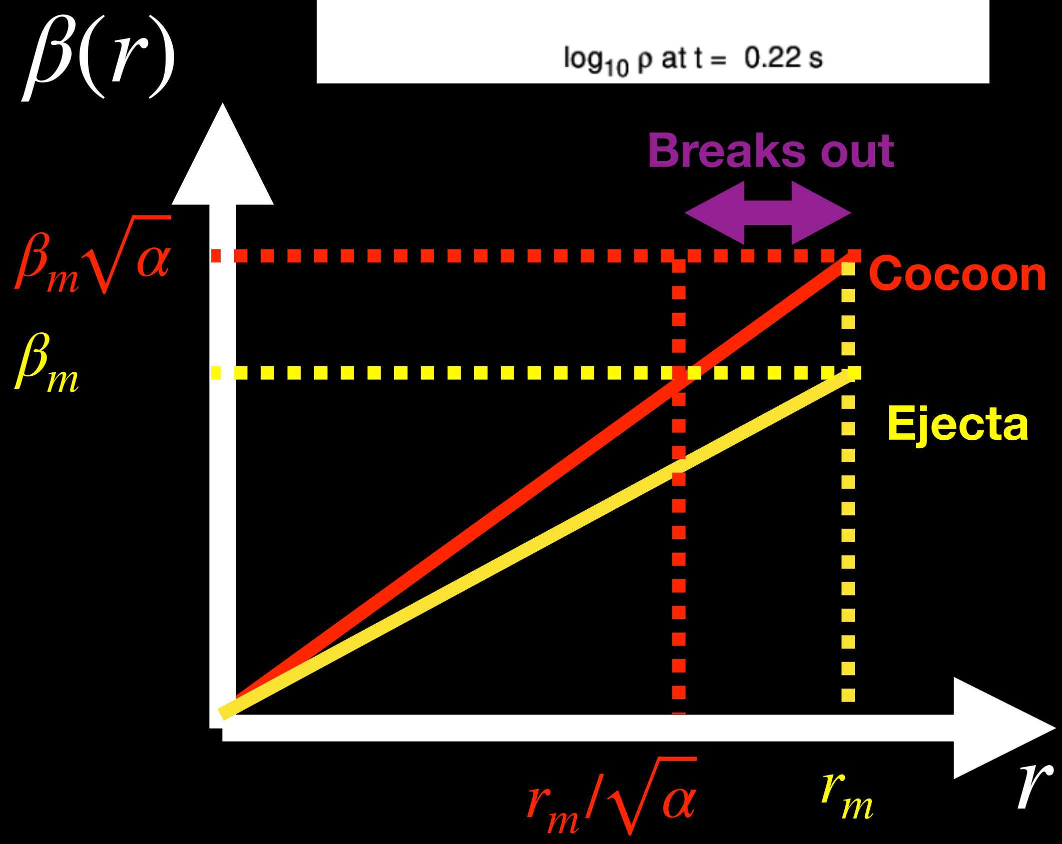
$$\alpha = E_c / E_{k,0} = 1 + E_{in}/E_{k,0} = 1 + \frac{12 L_j (t_b - t_0)(1 - \langle \beta_h \rangle)}{(r_c/r_b)^2 \beta_m^2 M_a c^2}$$

The cocoon breakout [I]

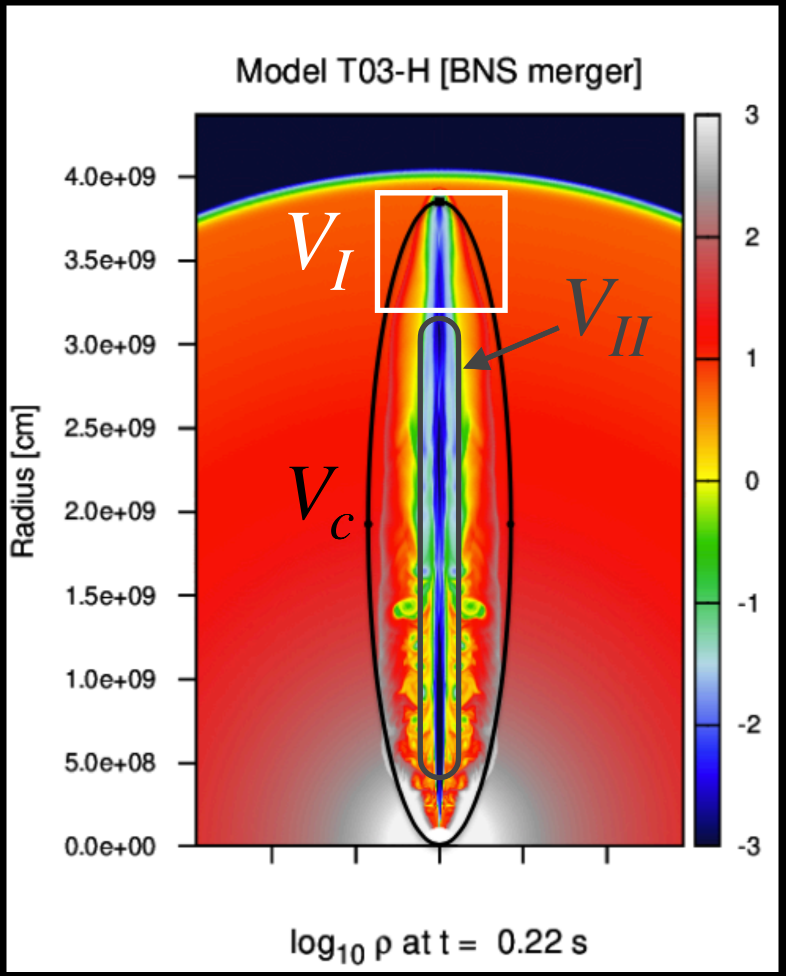
At the breakout time, the part of the cocoon that is faster than the ejecta occupies the following volume; which is the volume of the cap of the ellipsoid with a volume V_c that represent the cocoon volume:

$$\frac{V_I}{V_c} = \frac{E_I}{E_c} = \left(1 + \frac{2}{\alpha^{3/2}} - \frac{3}{\alpha}\right)$$





The cocoon breakout [II]



$$E_{c,i,OUT}/E_{c,OUT}[=f_{i,c,OUT}] \approx \beta E_{c,i}/E_c[=\beta f_{i,c}]$$

$$\frac{V_{II}}{V_c} = \frac{V_I}{V_c}\frac{2\alpha(\beta-1)}{2\alpha-\beta(\alpha-1)} \qquad \frac{E_{II}}{E_c} = \frac{(\beta-1)(\alpha-1)}{2\alpha-\beta(\alpha-1)}\frac{E_I}{E_c}$$

The cocoon breakout

The fraction of the total energy, and internal energy, of the cocoon that breaks out is:

$$\frac{E_{c,OUT}}{E_c} = \left(\frac{1+\alpha}{2\alpha-\beta(\alpha-1)}\right)\left(1+\frac{2}{\alpha^{3/2}}-\frac{3}{\alpha}\right)$$

$$\frac{E_{c,i,OUT}}{E_{c,i}} = \beta\left(\frac{1+\alpha}{2\alpha-\beta(\alpha-1)}\right)\left(1+\frac{2}{\alpha^{3/2}}-\frac{3}{\alpha}\right)$$

Calculating M_c analytically

$$M_c/2 = \int_{-a}^{a} dy \pi x^2 \rho \qquad (y/a)^2 + (x/b)^2 = 1$$

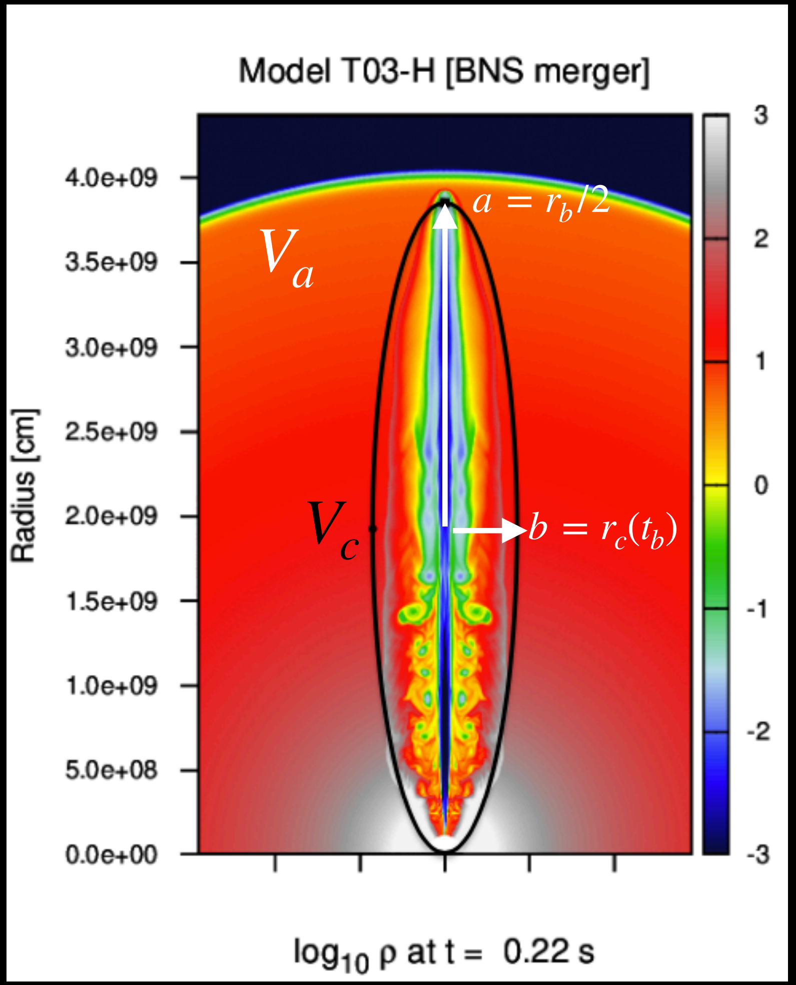
$$\rho = \rho_0 (r_0/r)^2 \qquad \rho_0 = \frac{M_a}{4\pi r_0^2} \frac{1}{r_{m,0} - r_0} \frac{r_{m,0}}{r_m}$$

$$M_c = 2\pi \rho_0 r_0^2 \int_{-a}^{a} \frac{dy}{1 + \left(\frac{y+a}{x}\right)^2}$$

$$a \gg b \Rightarrow \qquad M_c \approx 2\pi \rho_0 r_0^2 \int_{-a}^{a} dy \left(\frac{x}{y+a}\right)^2$$

$$\left(\frac{M_c}{M_a}\right) = \left\{ 2 \left[\ln\left(\frac{r_b}{r_0}\right) - 1 \right] \right\} \left(\frac{V_c}{V_a}\right)$$

$$r_b \approx c\beta_m t_b$$



Calculating $M_{c,OUT}$ analytically

$$M_c/2 = \int_{-a}^{a} dy \pi x^2 \rho \qquad (y/a)^2 + (x/b)^2 = 1$$

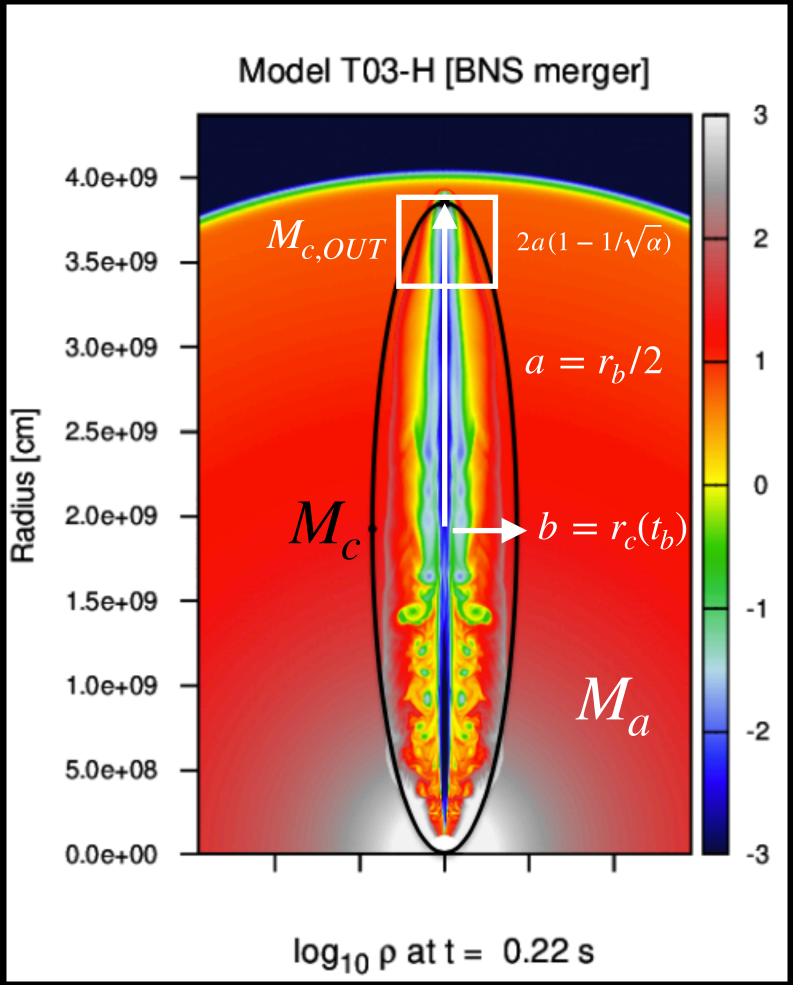
$$\rho = \rho_0 (r_0/r)^2 \qquad \rho_0 = \frac{M_a}{4\pi r_0^2} \frac{1}{r_{m,0} - r_0} \frac{r_{m,0}}{r_m}$$

$$M_{c,OUT} = 2\pi \rho_0 r_0^2 \int_{a(2/\sqrt{\alpha}-1)}^{a} \frac{dy}{1 + \left(\frac{y+a}{x}\right)^2}$$

$$a \gg b \Rightarrow \qquad M_{c,OUT} \approx 2\pi \rho_0 r_0^2 \int_{a(2/\sqrt{\alpha}-1)}^{a} dy \left(\frac{x}{y+a}\right)^2$$

$$\left(\frac{M_{c,OUT}}{M_a}\right) = \left\{ 2 \left[\frac{1}{\sqrt{\alpha}} - 1 + \ln\left(\sqrt{\alpha}\right) \right] \right\} \left(\frac{V_c}{V_a}\right)$$

$$\alpha = E_c/E_{k,0} = 1 + E_{in}/E_{k,0} = 1 + \frac{12 L_j (t_b - t_0)(1 - \langle \beta_h \rangle)}{(r_c/r_b)^2 \beta_m^2 M_a c^2}$$



Comparison with simulations

[at $t = t_b$]

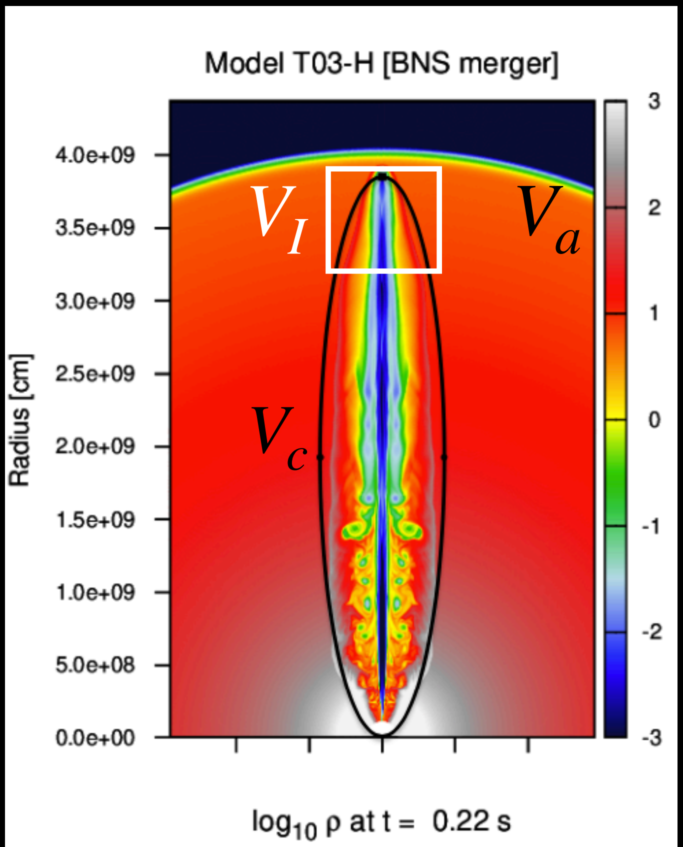
Jet Type		Breakout Time [s]	Alpha	Cocoon Energy [erg]	Outer cocoon energy [erg]	Outer cocoon mass [Msun]
Narrow	Analytic	0.203	1.469	8.90E+47	8.90E+46	8.12E-07
	Simulation	0.222	1.548	1.09E+48	9.44E+46	4.93E-07
Wide	Analytic	0.450	2.632	9.57E+48	5.71E+48	1.50E-05
	Simulation	0.412	2.138	1.11E+49	4.70E+48	1.83E-05
Failed	Analytic	3.162	1.548	2.78E+49	3.57E+48	1.67E-05
	Simulation	2.590	1.523	2.93E+49	7.20E+48	3.68E-05

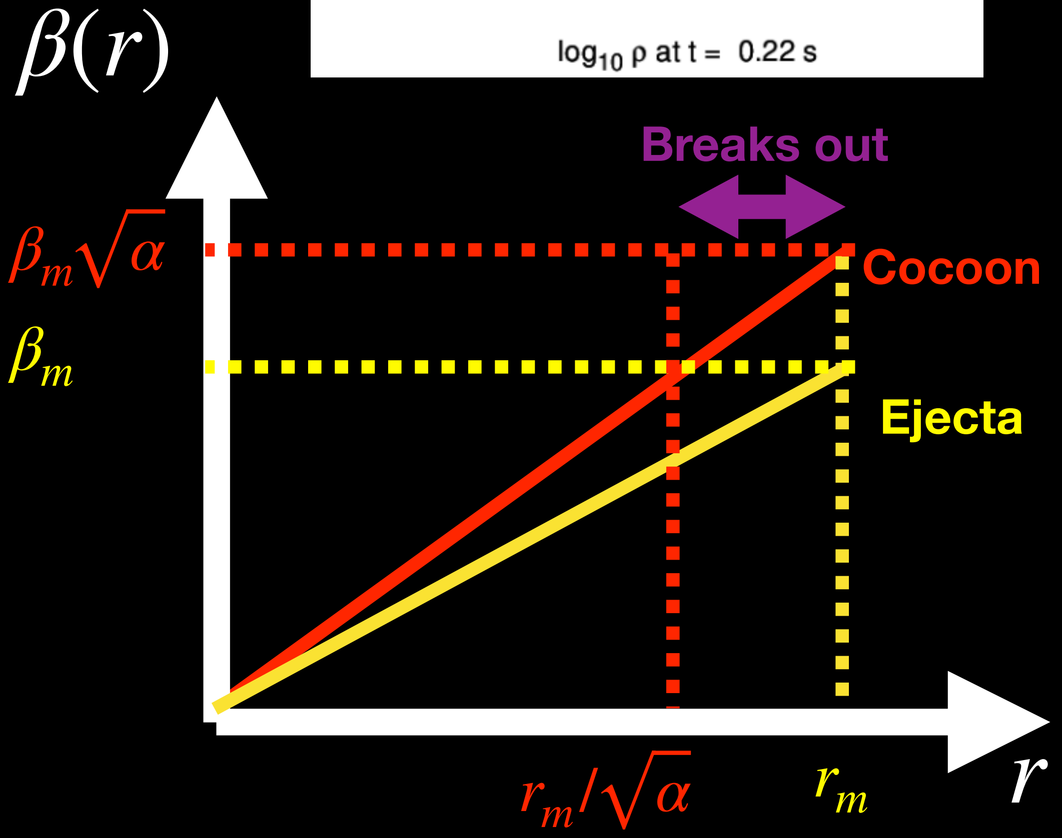
Why only a small fraction of the cocoon breaks out?

$$E_c = E_{k,0} \times \alpha[\alpha \sim 2]$$

$$\Rightarrow \beta_c \sim \beta_a \times \sqrt{2}$$

$$n = 2 \Rightarrow \langle \beta_a \rangle = \frac{1}{\sqrt{3}} \beta_m$$





Estimate the cocoon imprint [as an EM counterparts]

Density and velocity profiles

Homologous expansion:

$$\beta_{inf} \propto r \Rightarrow r \approx vt$$

The power-law density profile:

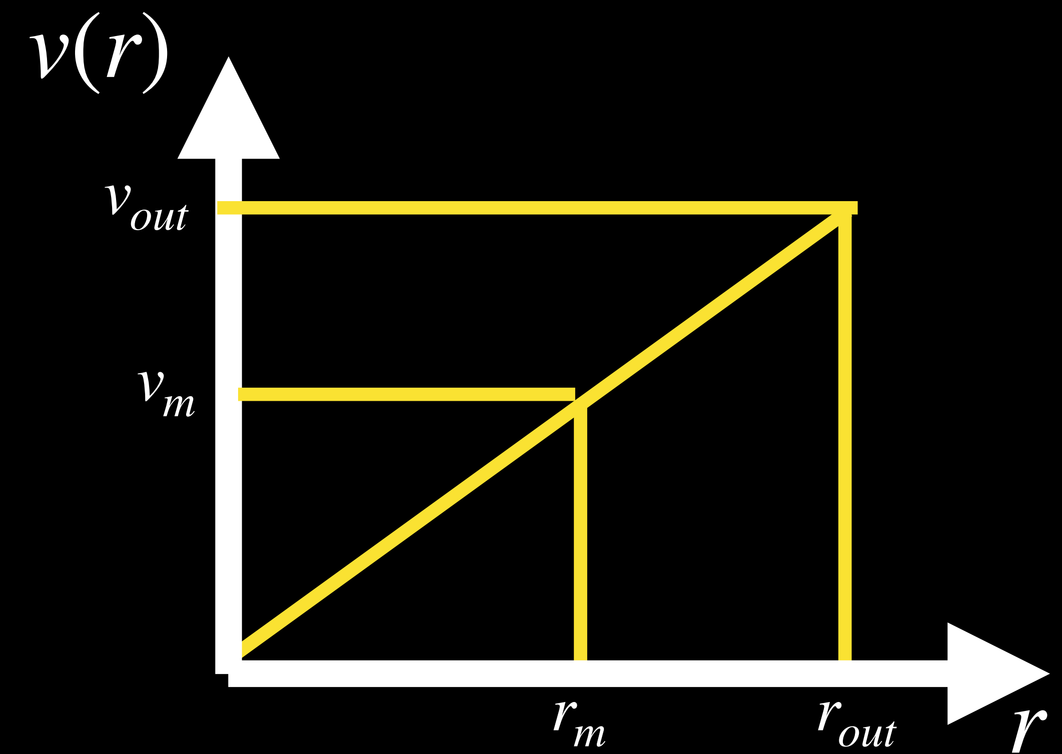
$$\rho_{\rm out}(r,t) = \left\{ \frac{(n-3)M_{c,out}v_{out}^{n-3}}{\Omega[(v_{out}/v_m)^{n-3}-1]} \right\} v^{-n}t^{-3}$$

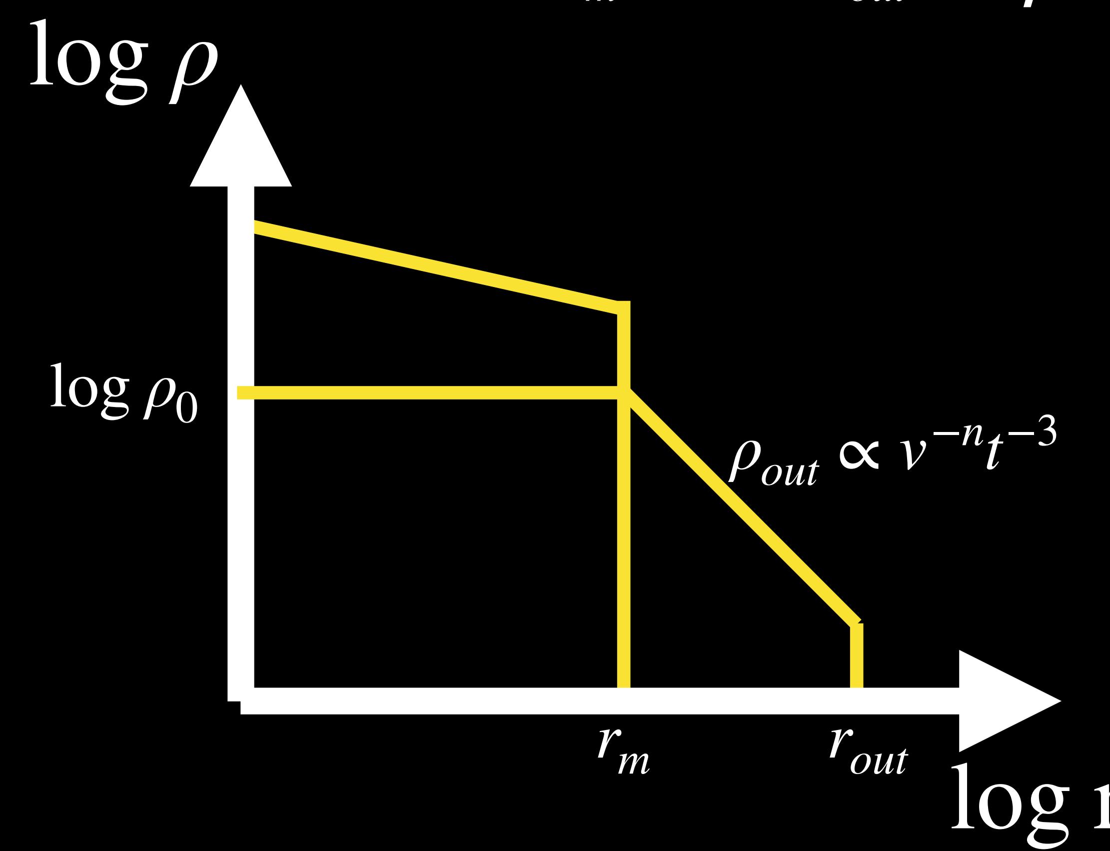
$$\Omega = 4\pi(1-\cos\theta_{out})$$

For the outer cocoon we get:

$n \approx 10$ **(successful jet case)** $M \propto r^{-10}r^3 \propto r^{-7}$

$n \approx 2$ **(failed jet case)** $M \propto r^{-2}r^3 \propto r$



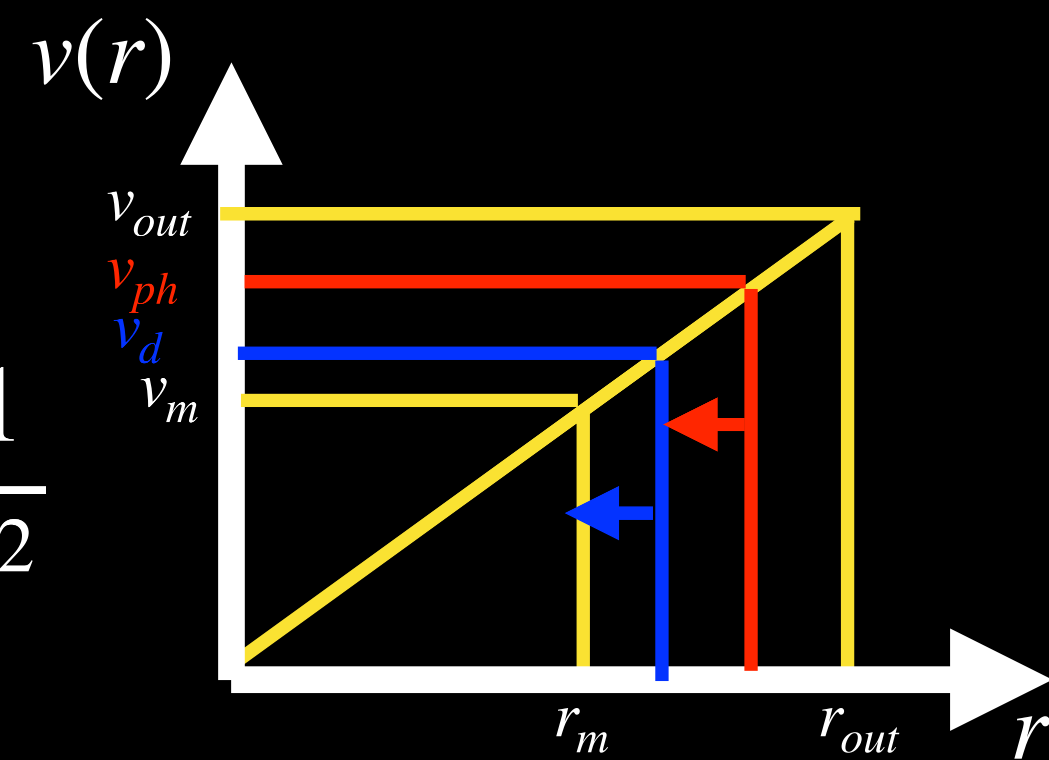


Optical depth

Optical depth:

$$\tau(r,t) = \int_{r_{out}}^{r} \kappa\rho(r,t)dr$$

$$\tau(r,t) = \frac{\kappa M_{c,out}}{\Omega}\frac{n-3}{n-1}\frac{v^{1-n} - v_{out}^{1-n}}{v_m^{3-n} - v_{out}^{3-n}}\frac{1}{t^2}$$

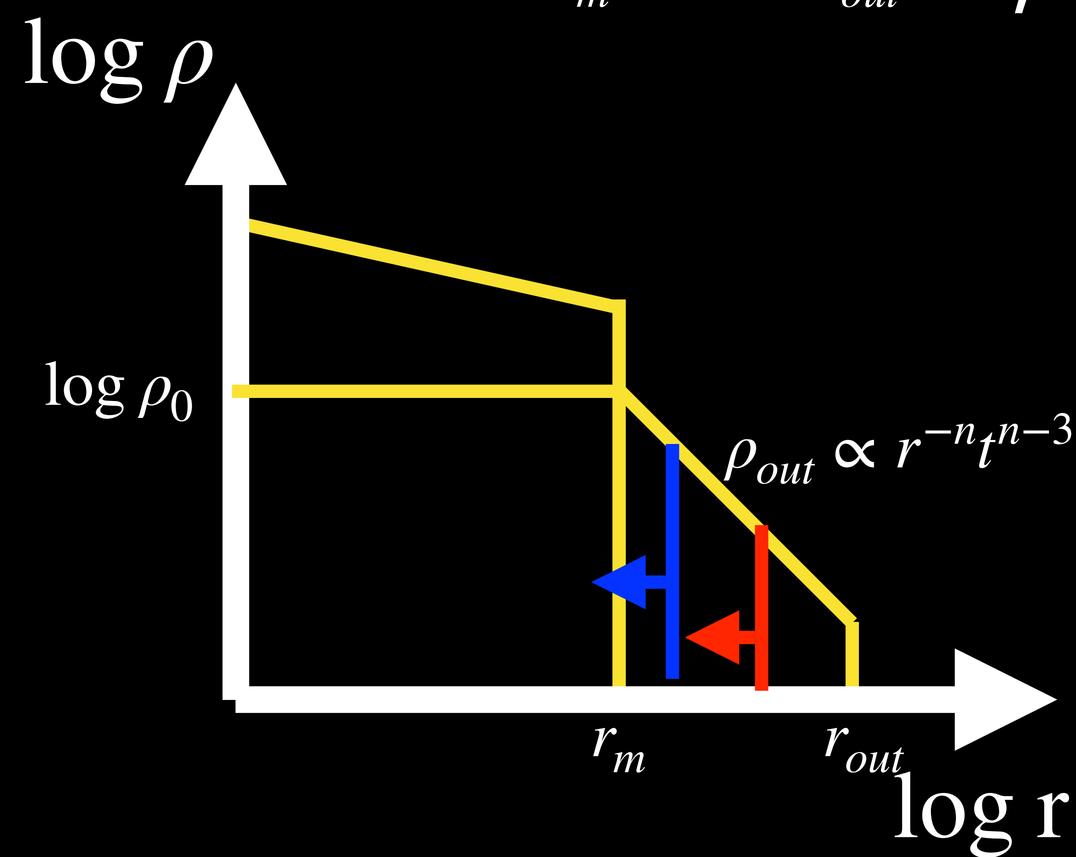


The photospheric radius is at [as in Piro+20]:

$$\tau(r,t) = \tau_{ph}[=2/3] \rightarrow r_{ph}(t)$$

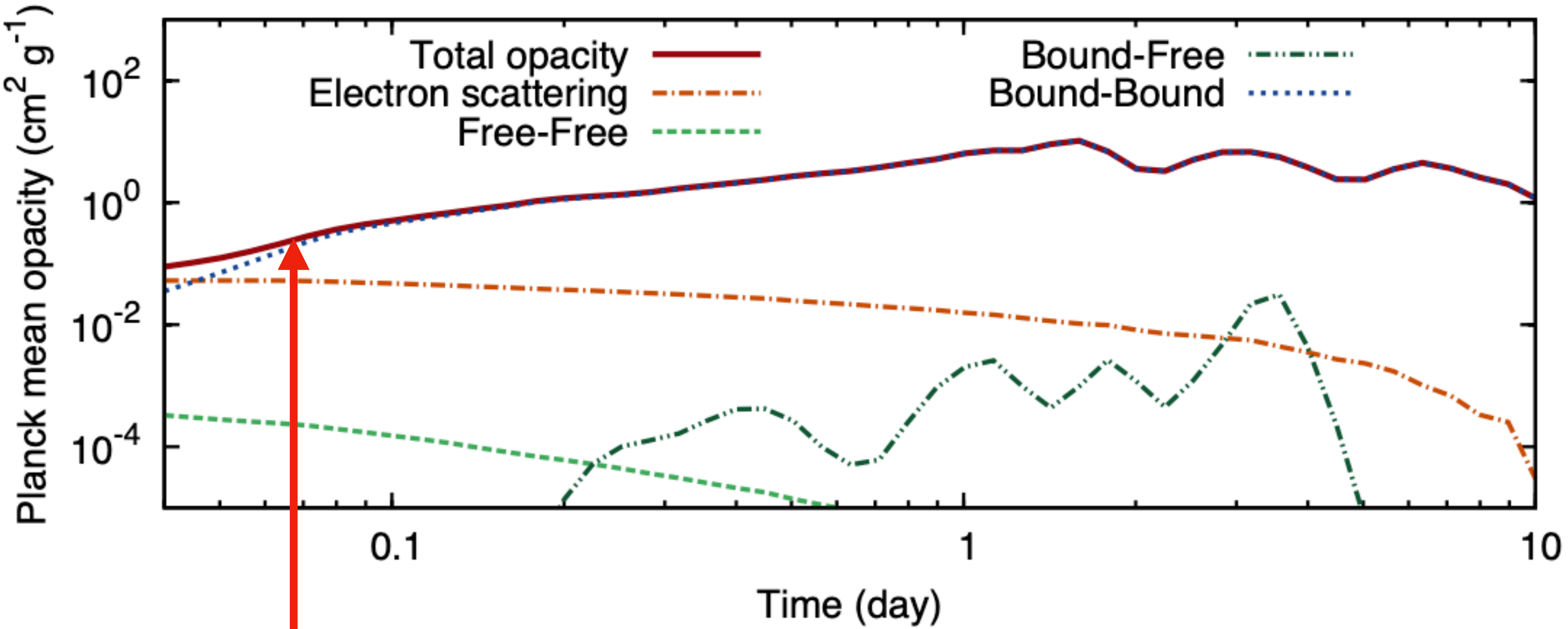
The diffusion radius is at:

$$\tau(r,t) \approx \frac{1}{d}\frac{c}{v_d}[d=1] \rightarrow r_d(t),\ L_{bl}(t)$$



Opacity [See Tanaka-san's Talk]

- κ at early times is not well determined.
- Below is κ for $Z = 20 - 56$ (Banerjee et al. 2020)
- Lanthanides?



$\kappa \sim 0.1 - 1$ cm^2 g^{-1}
@ ~ a few hours?

Credit: Banerjee et al. 2020

The photosphere

The photosphere reaches the ejecta/macronova at:

$$t(v_{ph} = v_m) = t_{\rm ph} = \left[\frac{\kappa M_{out}}{\Omega \tau_{ph} v_m^2} \left(\frac{n-3}{n-1} \right) \frac{1}{1-(v_m/v_{out})^{n-3}} \right]^{1/2}$$

$$v_{ph}(t) = \left[\left(\frac{t}{t_{ph}} \right)^2 + \left(\frac{v_m}{v_{out}} \right)^{n-1} \right]^{\frac{1}{n-1}} v_m \approx \left[\frac{t}{t_{ph}} \right]^{\frac{2}{n-1}} v_m$$

Diffusion velocity

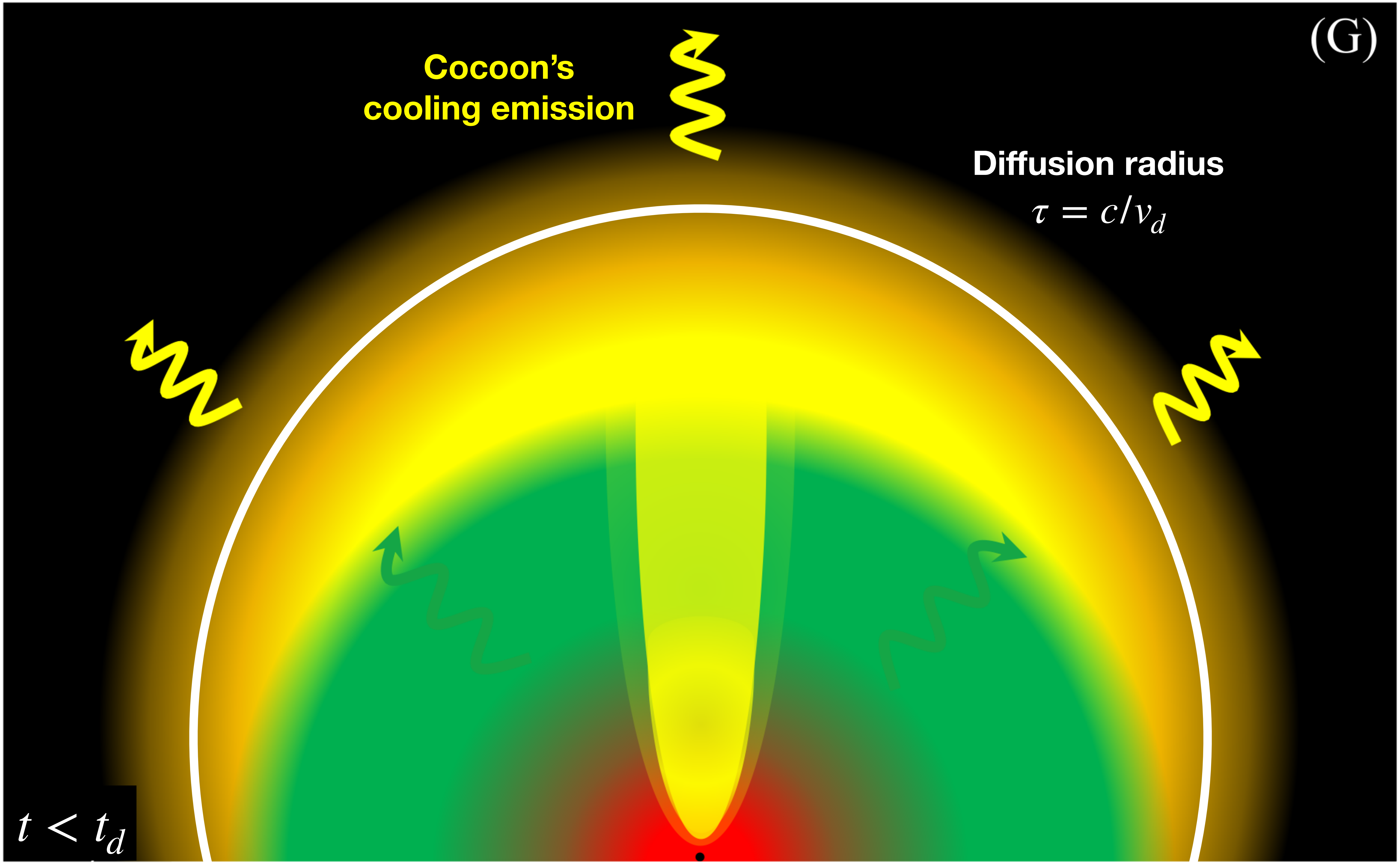
The diffusion radius reaches the ejecta/macronova at:

$$t(v_d = v_m) = t_\mathrm{d} \qquad\qquad v_d(t) = \left[t_d/t\right]^{\frac{2}{n-2}} v_m$$

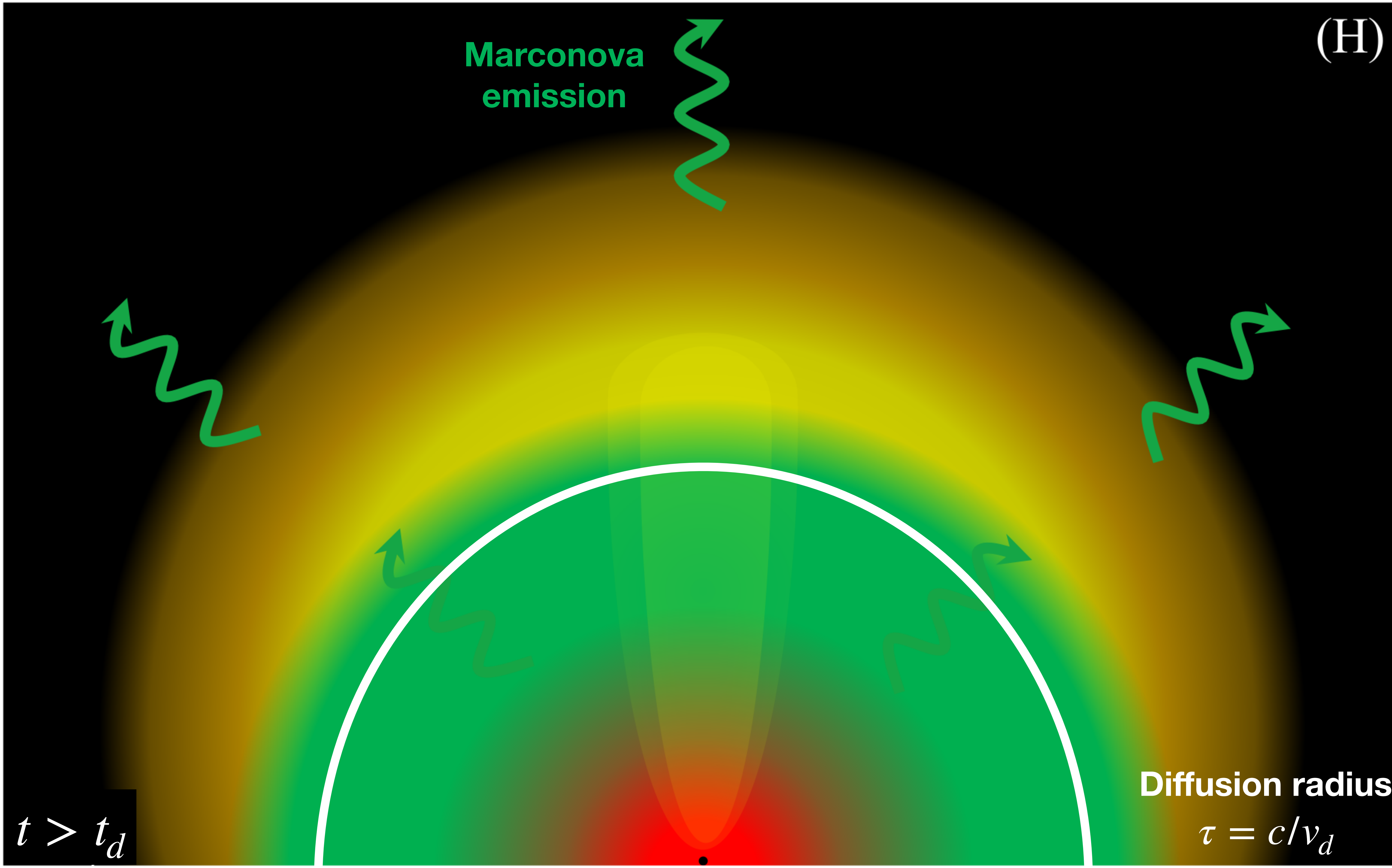
$$t_d = \left\{ \left(\frac{d\kappa M_{out}}{c\Omega v_m} \frac{n-3}{n-1} \right) \left[\frac{1-(v_d/v_{out})^{n-1}}{1-(v_m/v_{out})^{n-3}} \right] \right\}^{1/2}$$

$$t \gg t_1 \Rightarrow v_{out} \gg v_d \Rightarrow t_d \approx \left\{ \left(\frac{d\kappa M_{out}}{c\Omega v_m} \frac{n-3}{n-1} \right) \left[\frac{1}{1-(v_m/v_{out})^{n-3}} \right] \right\}^{1/2} \quad [n > 1]$$

Optically thick phase



Optically thin phase



Internal energy [Preliminary]

Over the outer cocoon, $E_i = 3PV$, hence the energy is equally distributed over the volume. With $E_{i,out}(t_1)$ being the total internal energy, the internal energy moving faster that v at $t = t_1$ can be written as:

$$E_{i,out}(>v, t_1) = E_{i,out}(t_1)\frac{V_{out}(>v, t_1)}{V_{out}(t_1)} = E_{i,out}(t_1)\left[\frac{1-(v/v_{out})^3}{1-(v_m/v_{out})^3}\right]$$

$$V_{out}(>v, t) = \frac{\Omega}{3}t^3(v_{out}^3 - v^3)$$

At $t > t_1$, taking into account the adiabatic expansion of the outer cocoon we get:

$$E_{i,out}(>v, t) = E_{i,out}(t_1)\left[\frac{1-(v/v_{out})^3}{1-(v_m/v_{out})^3}\right]\left[\frac{t_1}{t}\right]$$

Bolometric luminosity

$$L(t) \approx \frac{E_{i,out}(> v_d, t)}{t}$$

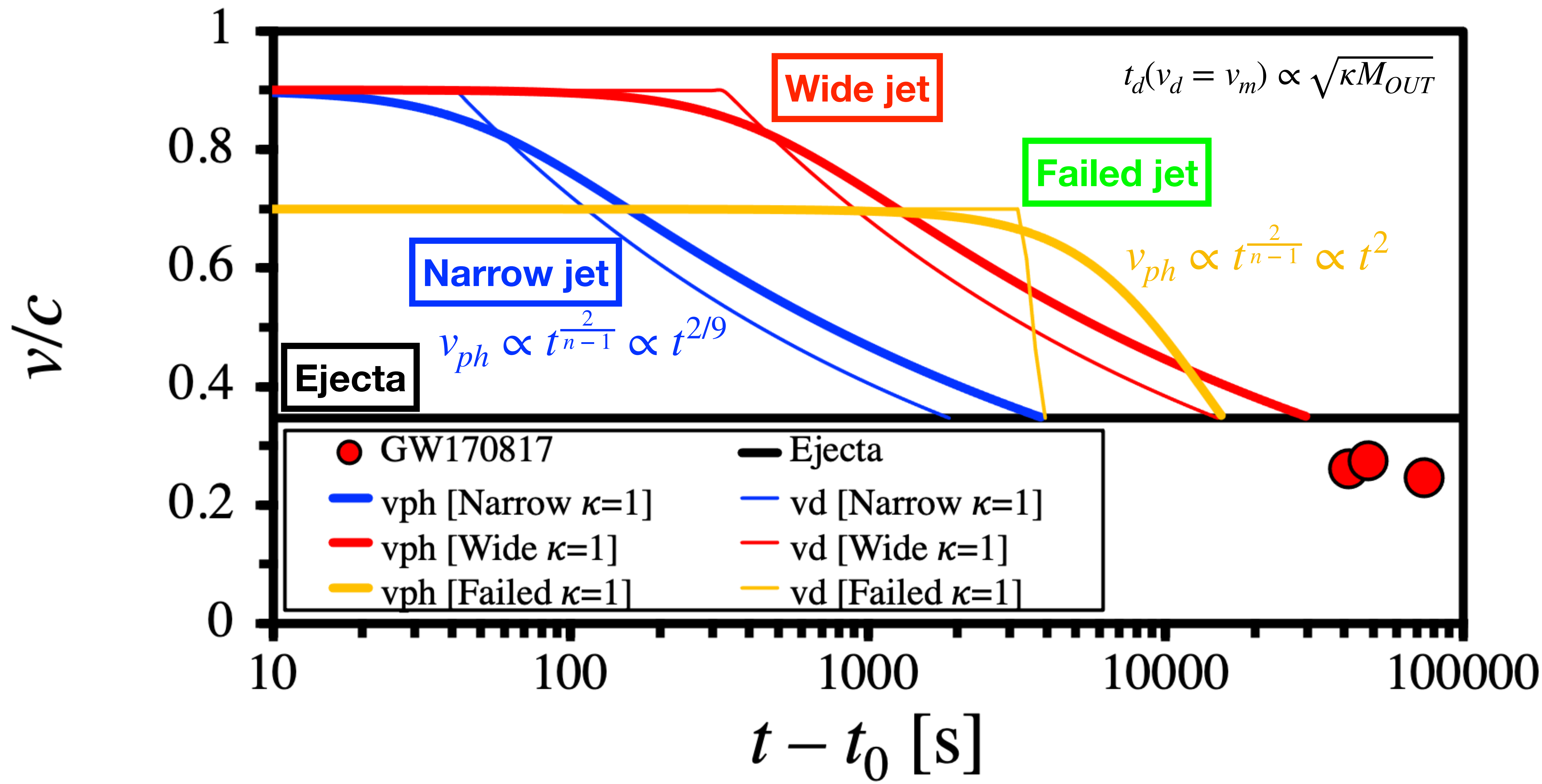
$$E_{i,out}(> v_d, t) = E_{i,out}(t_1) \left[\frac{1 - (v_d/v_{out})^3}{1 - (v_m/v_{out})^3} \right] \left[\frac{t_1}{t} \right]$$

$$v_d(t) = \left[t_d/t \right]^{\frac{2}{n-2}} v_{out}$$

$$t_d = \left\{ \left(\frac{p\kappa M_{out}}{c\Omega v_{out}} \frac{n-3}{n-1} \right) \left[\frac{1 - (v_d/v_{out})^{n-1}}{(v_{out}/v_m)^{n-3} - 1} \right] \right\}^{1/2} \approx \left\{ \left(\frac{p\kappa M_{out}}{c\Omega v_{out}} \frac{n-3}{n-1} \right) \left[\frac{1}{(v_{out}/v_m)^{n-3} - 1} \right] \right\}^{1/2} [n > 1]$$

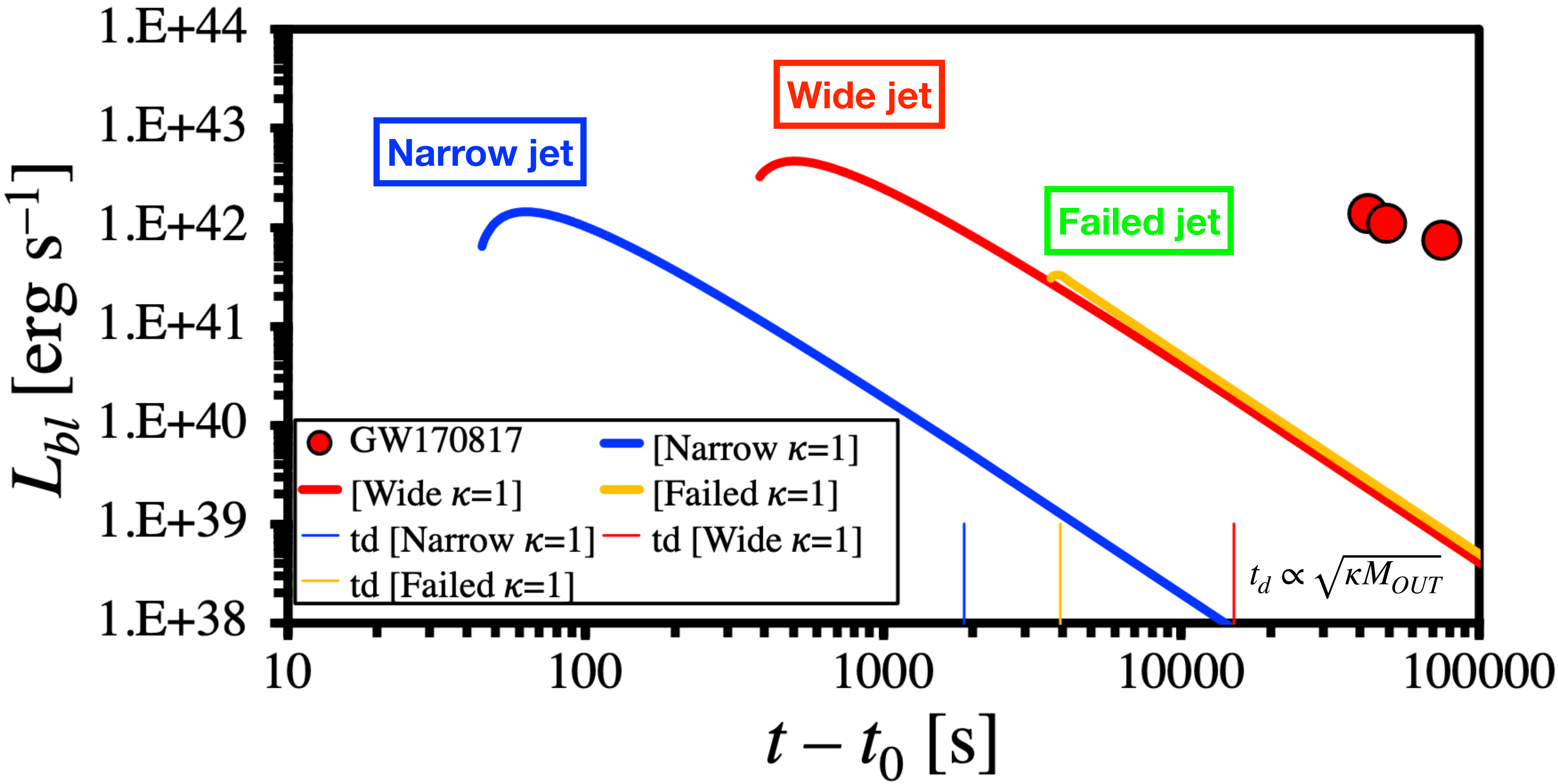
Cocoon Emission: Results

v_{ph} & v_d [$\kappa = 1 cm^2/g$]



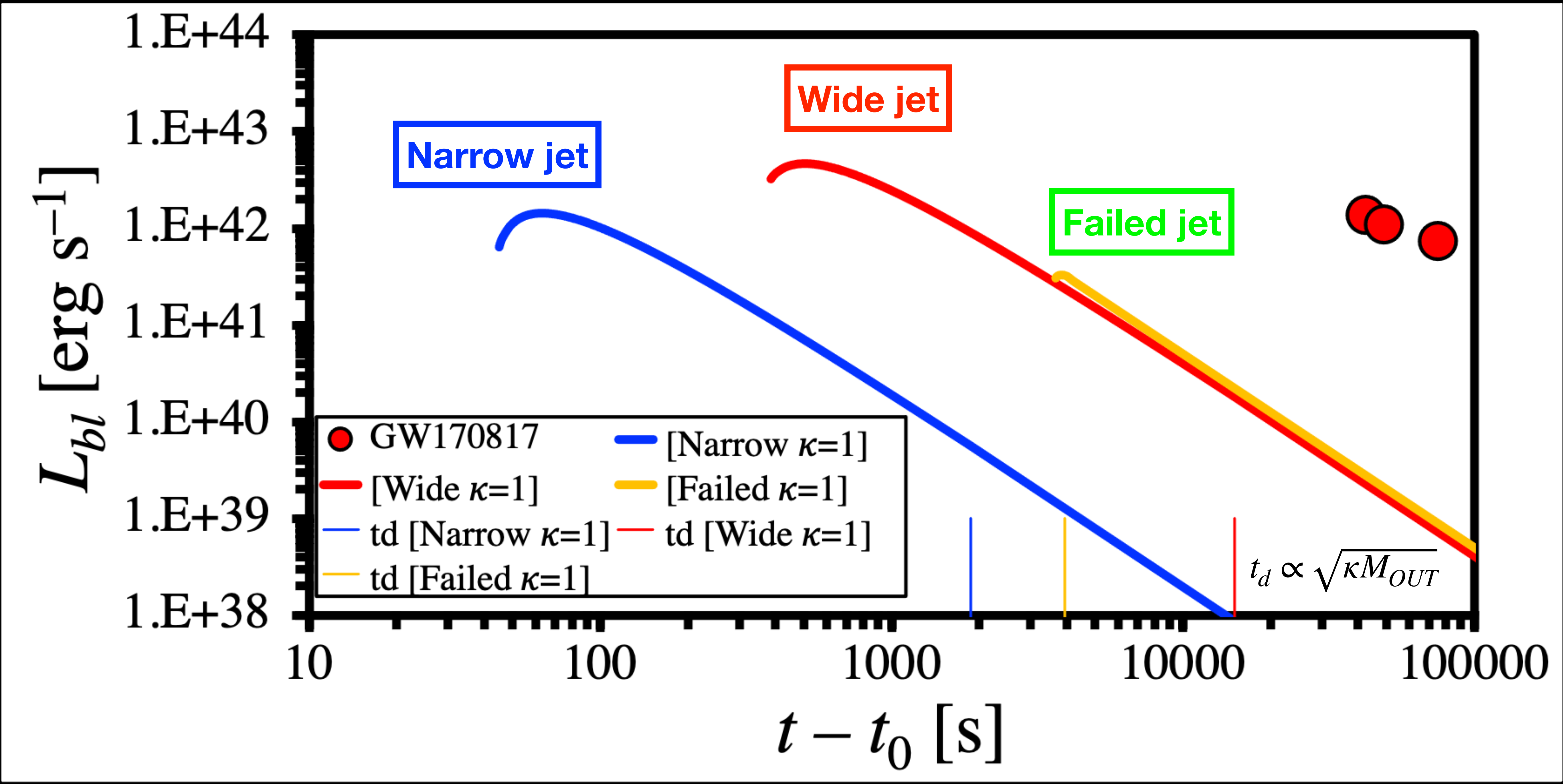
t=0 — 1m — 30m — 10h

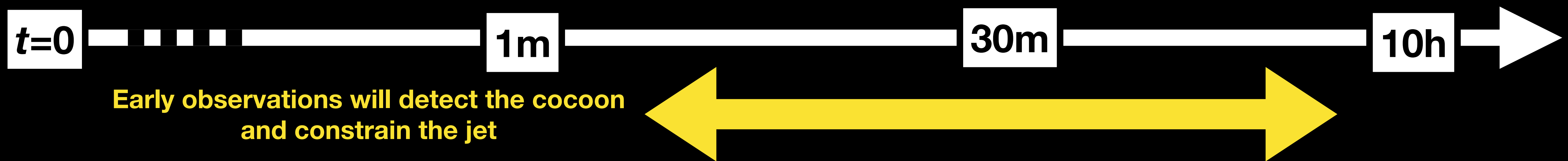
$L_{bl}\ [\kappa = 1 cm^2/g]$



t=0 — 1m — 30m — 10h

$L_{bl}\ [\kappa = 1 cm^2/g]$





Early observations will detect the cocoon and constrain the jet

Summary & Conclusion

Successful modeling of the cocoon breakout

based on the parameters of the engine/ejecta we can model the cocoon breakout and its emission

Toward a new astrophysical transient

the cocoon can power an EM transient; it can be detected at early times [~hour] and discriminated from the Macronova.

Not quite bright, but a useful EM counterpart

it is theoretically possible to constraint some of the jet and the ejecta's properties (jet type, opening angle, the ejecta mass, opacities & r-process, etc.)

Outlook

Take into account:

- r-process heating [see Kawaguchi-san's Talk]

$$L(t) \approx \frac{E_{\rm c,out}(> v_d) + E_{\rm r-process}(> v_d)}{t}$$

- Early opacities? [see Tanaka-san's Talk]
- Radiative transfer/reprocessed KN emission [see Kawaguchi-san's Talk]
- Cooling of the relativistic component of the cocoon, afterglow, etc.

Backup Slides