MHD Outflows

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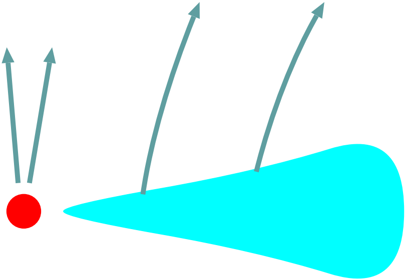
Outline

- Introduction
- MHD winds
  - Global $\boldsymbol{B}$ –Magnetocentrifugal Winds
  - Turbulent $\boldsymbol{B}$ –Fluctuation driven upflows
- Long-time evolution of disks with winds
- Local substructure – wind-induced instability: on-going research
- Cylindrical shearing box: on-going research

Winds & Outflows from Stars & Disks



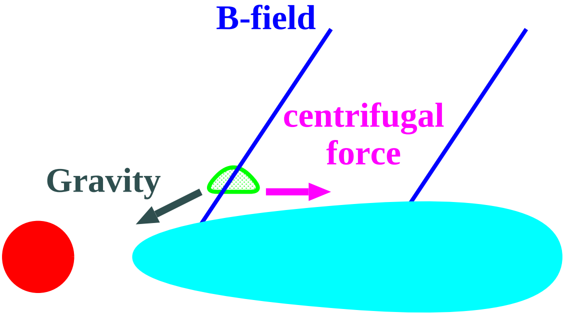
Outward Momentum Inputs against Gravity

- Gas Pressure
- Lorentz Force (MHD processes)
  - Coherent (Ordered) $\boldsymbol{B}$ field
  - Turbulent $\boldsymbol{B}$ field
- Radiation Pressure

Winds by Globally ordered B field

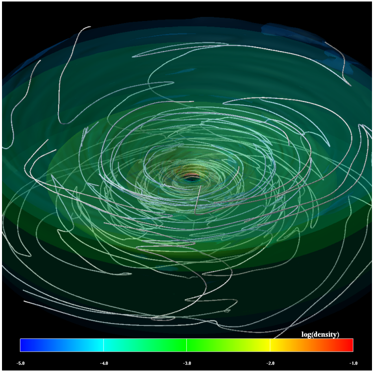
Magneto-centrifugal driven winds by global B field

(Blandford & Payne 1982; Pelletier & Pudritz 1992; Kudoh & Shibata 1998; Salmeron+ 2011; Gressel+ 2020

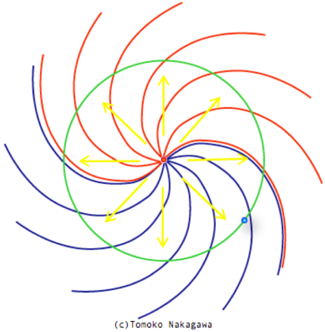


- Direct Mass Loss – Mass loading is another issue
- Angular Momentum Loss (Magnetic Braking)
  ⇒ Accretion

Magnetic Braking



Suzuki & Inutsuka (2014)



http://www.ice.tohtech.ac.jp/ñakagawa/outreach/spiralfield_0.htm

- Angular momentum flux: $4\pi r\rho v_\phi v_p - rB_\phi B_p$
- $l = rv_\phi - \frac{rB_\phi B_p}{4\pi\rho v_p}$ (=const. for $\partial_t = 0$)

  Magnetic lever arm: $\lambda = \frac{l}{R_0^2\Omega_0}$

Accretion-Wind Connection

Angular momentum balance under $\partial_t = 0$

$$\frac{d}{dr}(\dot{M}_{\rm acc} R^2 \Omega) - 2\pi \frac{d}{dr}\left(R^2 \overline{\rho W_{R\phi}}\right) - 2l \frac{d\dot{M}_{\rm wind}}{dr} = 0$$

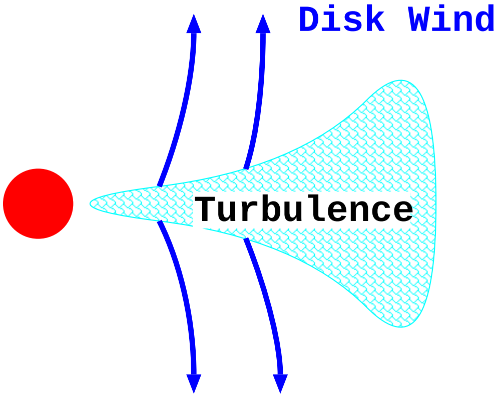
- $\dot{M}_{\rm acc} \equiv -2\pi R \int dz \rho v_R$
- $\dot{M}_{\rm wind} \equiv \int_0^R 2\pi R' dR' \rho v_z$
- $\overline{\rho W_{R\phi}} \equiv \int dz \left(\rho v_R \delta v_\phi - \frac{B_R B_\phi}{4\pi}\right)$

If the 2nd term is negligible (= wind-driven accretion)

$$\frac{\dot{M}_{\rm acc}}{\dot{M}_{\rm wind}} \sim \frac{l}{R_0^2 \Omega_0} = \lambda$$

Turbulent-B/Wave driven Upflows

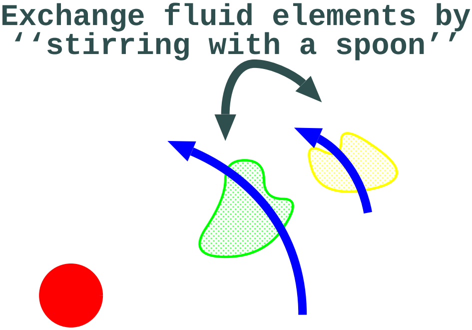
A possible mechanism
– Uplift by MHD turbulence and/or Alfvénic waves



Suzuki & Inutsuka 2009; Bai & Stone 2013; Fromang+ 2013; Lesur+ 2013

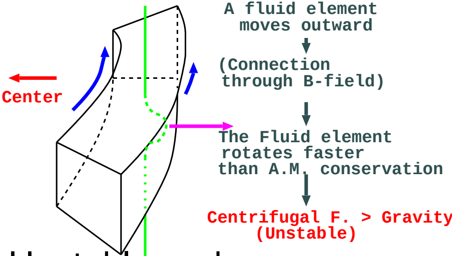
Turbulence in Accretion Discs

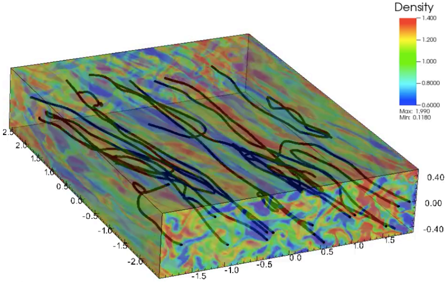
Turbulence ⇒ Macroscopic (effective) Viscosity



- Outward Transport of Angular Momentum
- Inward Accretion of Matters
- MRI (MagnetoRotational Instability)

Magneto-Rotational Instability (MRI)





Unstable under

- Weak B-fields
- (inner-fast) Differential Rotation

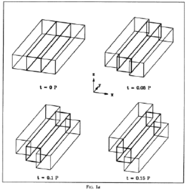
Velikov (1959); Chandrasekhar (1960); Balbus & Hawley (1991)

MHD in Local Shearing Box

- Local Cartesian coordinate with co-rotating with Ω_0. (neglect curvature)
- $x = r - r_0$; $y \leftrightarrow \phi$-direction
- Basic equations for Keplerian rotation ($\Omega_0 = \sqrt{GM/r^3}$)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0$$
$$\frac{\partial v_x}{\partial t} = -\frac{1}{\rho}\nabla_x\left(p + \frac{B^2}{8\pi}\right) + \frac{(B\cdot\nabla)B_x}{4\pi\rho} + 2\Omega_0 v_y + 3\Omega_0^2 x$$
$$\frac{\partial v_y}{\partial t} = -\frac{1}{\rho}\nabla_y\left(p + \frac{B^2}{8\pi}\right) + \frac{(B\cdot\nabla)B_y}{4\pi\rho} - 2\Omega_0 v_x$$
$$\frac{\partial v_z}{\partial t} = -\frac{1}{\rho}\nabla_z\left(p + \frac{B^2}{8\pi}\right) + \frac{(B\cdot\nabla)B_z}{4\pi\rho} - \Omega_0^2 z$$
$$\frac{\partial B}{\partial t} = \nabla \times (v \times B - \eta \nabla \times B)$$
$$\nabla \cdot B = 0$$

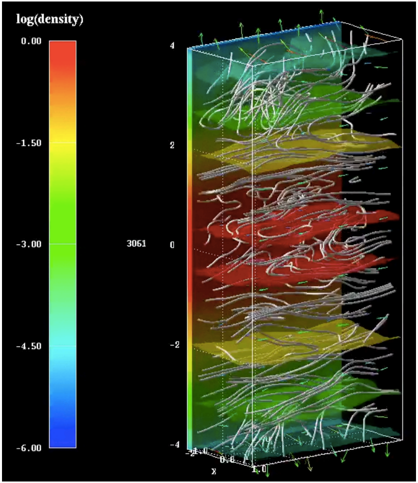
Hawley, Gammie, & Balbus 1995



- Any EoS / Energy Equation
- Steady-state solution
  - $B = (0, B_y, B_z)$ & $v = (0, -\frac{3}{2}\Omega_0 x, 0)$
  - $\rho = \rho_0 \exp(-z^2/H^2)$ ($H^2 \equiv 2c_s^2/\Omega_0^2$): hydrostatic eqlbrm.

Shearing Box with Vertical Stratification

Suzuki & Inutsuka 2009



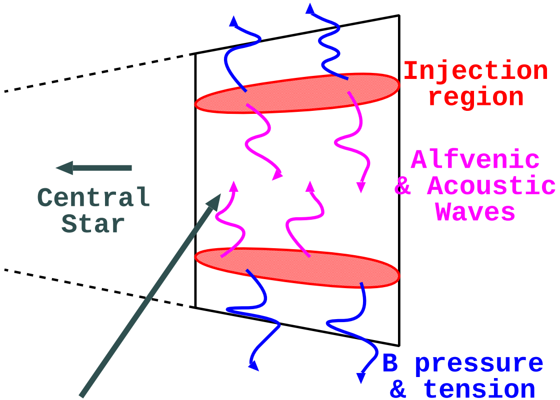
Upflows from top + bottom boundaries

Energy flux:

$$F_z = \rho v_z \left(\frac{1}{2}v^2 + \Phi + h\right) + v_z \frac{B_\perp^2}{4\pi} - \frac{B_z}{4\pi} v_\perp B_\perp$$

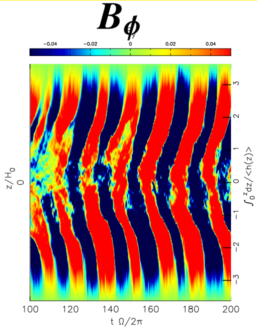
Both $\boldsymbol{B}$ pressure & tension are important.

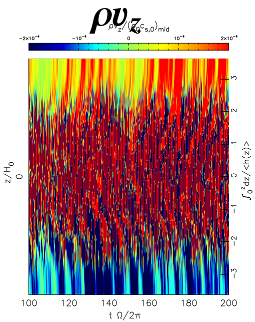
Characteristics of Turbulence



- Vertical outflows from Injection Regions at $z \approx \pm(1.5-2)H$ with $\beta \sim$1–10

Time dependency: $t - z$ diagrams



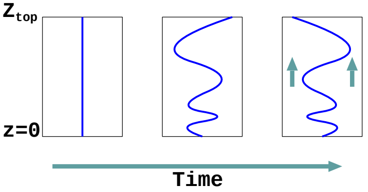


- quasi-periodic inversion of B_ϕ

  e.g. Davis et al.2010; Shi et al.2010

- The vertical outflows are also quasi-periodic.

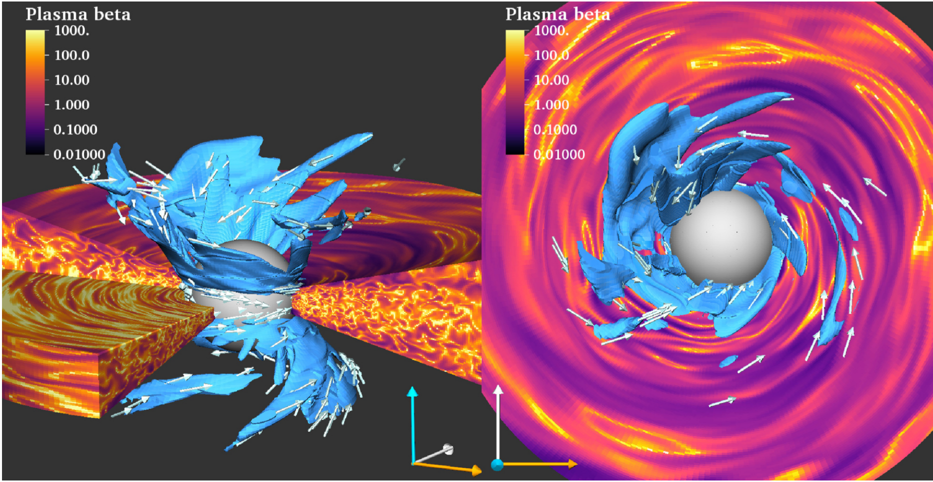
Upper half of the local box



Latter+ 2010

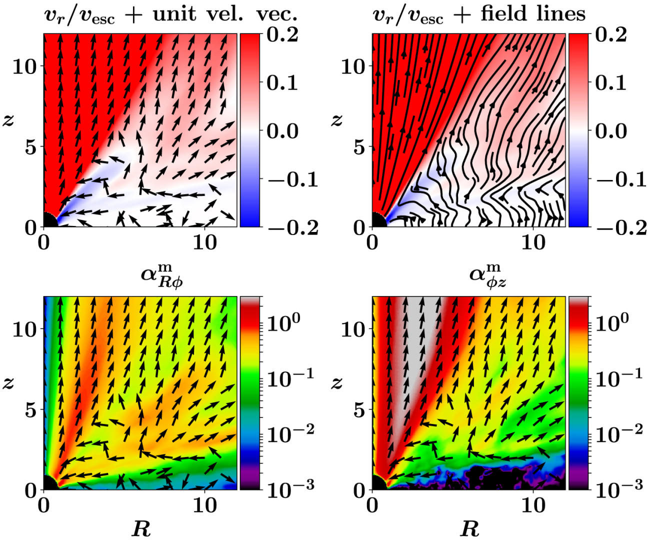
Turbulent driven failed winds

Takasao+ 2018



Turbulent driven failed winds

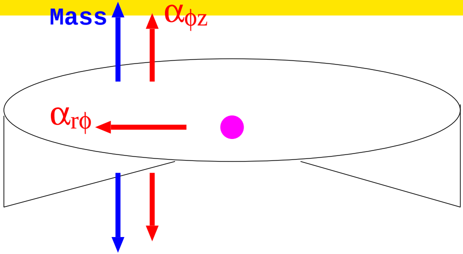
Takasao+ 2018



Turb.driven upflows

- ⇒ Funnel-wall accretion
- +Global $\boldsymbol{B}$ ⇒ Disk winds

Long-time Evolution of Disks –$t + r$ Model–



Suzuki, Ogihara, Morbidelli, Crida, & Guillot 2016; See also Hasegawa+ 2017

$$\frac{\partial \Sigma}{\partial t} - \frac{1}{r}\frac{\partial}{\partial r}\left[\frac{2}{r\Omega}\left\{\frac{\partial}{\partial r}(\Sigma r^2 \alpha_{r\phi} c_s^2) + r^2 \alpha_{\phi z}(\rho c_s^2)\right\}\right] + (\rho v_z)_{\rm w} = 0$$

$\Sigma(=\int \rho dz)$: Surface density; Ω: Keplerian freq.

α & $(\rho v_z)_{\rm w}$ $\Leftarrow$ Local Simulations

- Turbulent Viscosity: $\alpha_{r\phi} = (v_r \delta v_\phi - B_r B_\phi / 4\pi\rho)/c_s^2$
- Wind Torque: $\alpha_{\phi z} = (\delta v_\phi v_z - B_\phi B_z / 4\pi\rho)/c_s^2$ Bai 2013
- Mass Loss Rate: $(\rho v_z)_{\rm w}$

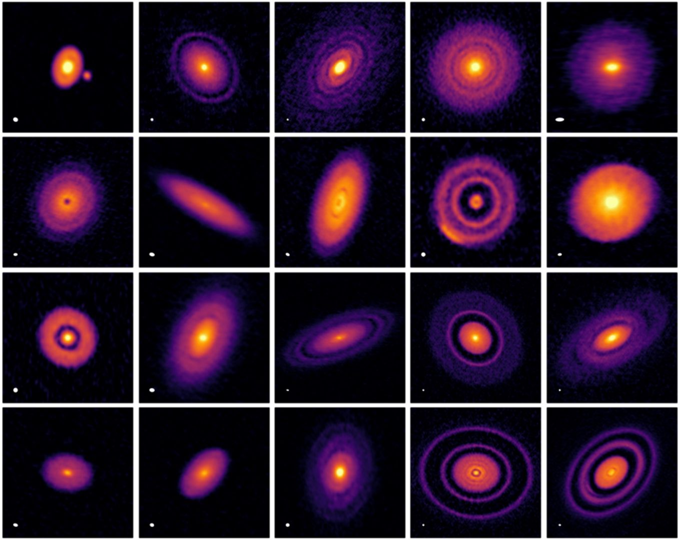
Viscous heating (Nakamoto+1994; Oka+ 2011) also included

Evolution of $\Sigma_{\rm gas}$

- $\alpha_{r\phi} = 8 \times 10^{-5}$
  Dead Zone Level
  - w/o or w/ DW
  - w/o or w/
    Wind Torque
- Dispersal Time:
  $\tau = \Sigma/(\rho v_z)_{\rm w}$
  $\propto r^{-3/2}$

t=9.99E+06

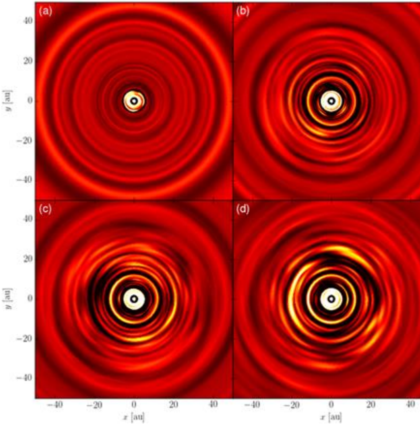
Substructures of Disks



DSHARP/ALMA

Mechanisms for Substructures

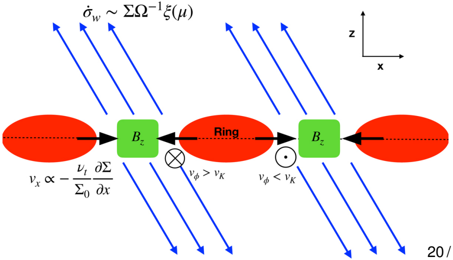
- Instability of MHD accreting flows Suriano+ 2017;2018;2019
- Embedded planets
  Kley & Nelson 2012; Baruteau+ 2014; Dong+ 2015
  but need very rapid formation of planets for younger disks, e.g. HL Tau (< 1 Myr)
- Secular gravitation instability of dust + gas
  Takahashi & Inutsuka 2014
- Edges of dead zones Flock+ 2015
- Snowlines of different species Okuzumi+ 2016



Suriano+ 2019

Previous works for Stability of Wind-torque driven MHD Accreting Flows

- "Discovery" of Instability with a simple scaling model Lubow+1994... but criticized Königl & Wardle 1996
- Linear perturbation analysis Cao & Spruit 2002; Campbell 2009 too complicated to understand the physics (at least too me)
- More intuitive L. P. A. Riols & Lesur 2019... but
  - Unrealistic $\boldsymbol{B}$-field configuration
  - Cartesian coordinates



Basic Equations & Setup

Tokuno+ in prep.

- $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0$
- $\frac{\partial}{\partial t}(\rho \vec{u}) + \vec{\nabla} \cdot (\rho \vec{u} \otimes \vec{u}) = -\vec{\nabla} p - \vec{\nabla} \frac{B^2}{8\pi} + \frac{1}{4\pi} \vec{\nabla} \cdot (\vec{B} \otimes \vec{B})$
- $\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{u} \times \vec{B}) - \eta \vec{\nabla} \times (\vec{\nabla} \times \vec{B})$

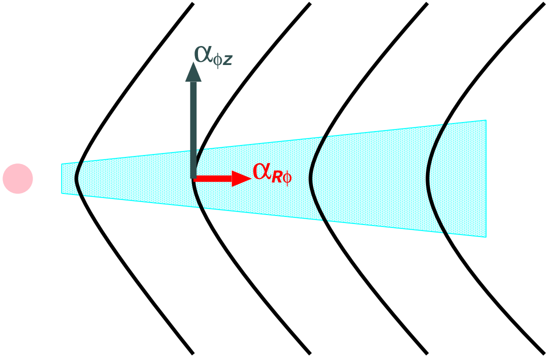
Setup

- $T(\propto c_s^2)$ profile: $c_s^2 = c_{s,0}^2 \left(\frac{R}{R_0}\right)^{-1/2}$
- Cylindrical coordinates, (R, ϕ, z)
- $\vec{v}$ in Keplerian corotating frames: $\vec{v} = \vec{u} - R\Omega_K \hat{\phi}$

  where $\Omega_K = \sqrt{GM_\star / R^3}$
- Surface density $\Sigma = \int_{z_-}^{z_+} \rho dz = \sqrt{2\pi} \rho_{mid} H$,

  where $H = c_s / \Omega_K$ is a scale height.
- $\alpha_{\mu\nu} \equiv \left[\rho v_\mu v_\nu - B_\mu B_\nu / 4\pi\right] / (\rho c_s^2) \propto \beta^{-q}$

Accretion by $\alpha_{R\phi}$ & $\alpha_{\phi z}$



- $\alpha_{R\phi}$ (= α of Shakura & Sunyaev (1973)): Turbulent viscosity
  Outward transport of A.M.
  ⇒ Inner Accretion + Outer Expansion
- $\alpha_{\phi z}$: Removal of A.M. by Wind Torque
  ⇒ Accretion without Expansion

Suzuki+ 2016

Linear Perturbation Analyses

Axisymmetric perturbation $\delta \propto \exp(i\omega t - ikR)$ with

$\alpha_{R\phi} = 0$, $\alpha_{\phi z} \neq 0$, $\eta \neq 0 \Rightarrow$

$$(\omega^2 - i\omega k^2 \eta)(\omega^2 - \Omega_K^2 - k^2 c_s^2)$$
$$+ v_{R,0}\Omega_K^2[(1+q)\omega k - i(1-q)k^3\eta - 2qk^2 v_{R,o}] \approx 0$$

Focus on

- $\omega^2 \ll \Omega_K^2 \Rightarrow \omega^2 - \Omega_K^2 - k^2 c_s^2 \approx -\Omega_K^2 - k^2 c_s^2$

$$\omega^2 - \left[ik^2\eta + \frac{(1+q)k}{1+k^2H^2} v_{R,0}\right]\omega$$
$$+ \frac{k^2}{1+k^2H^2} v_{R,0}[i(1-q)k\eta + 2qv_{R,0}] = 0$$

(note: $H = c_s/\Omega_K$)

Growth Rate

Growth rate: $\gamma = \frac{1}{2}k^2\eta\left[\sqrt{1+\frac{8(1+3k^2H^2)}{9(1+k^2H^2)^2}\frac{v_{R,0}^2}{k^2\eta^2}}-1\right]$

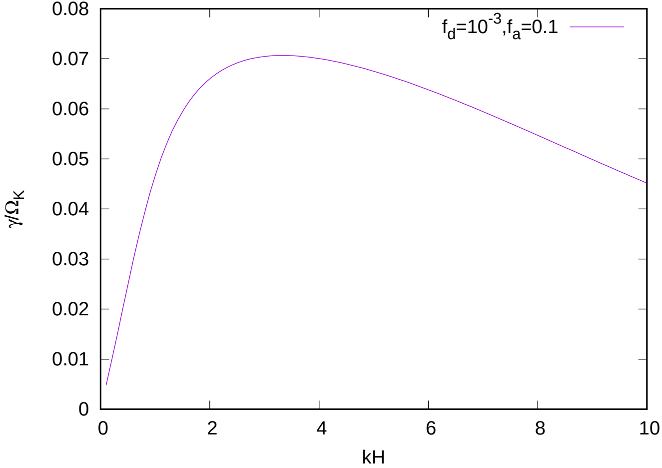
Normalization for accretion & diffusion

- accretion velocity $v_{R,0} = f_\mathrm{a} c_\mathrm{s}$
- magnetic diffusivity $\eta = f_\mathrm{d} H c_\mathrm{s} = f_\mathrm{d} H^2 \Omega_\mathrm{K}$
  f_d^{-1}: Reynolds(-like) number

Dimensionless Growth Rate:

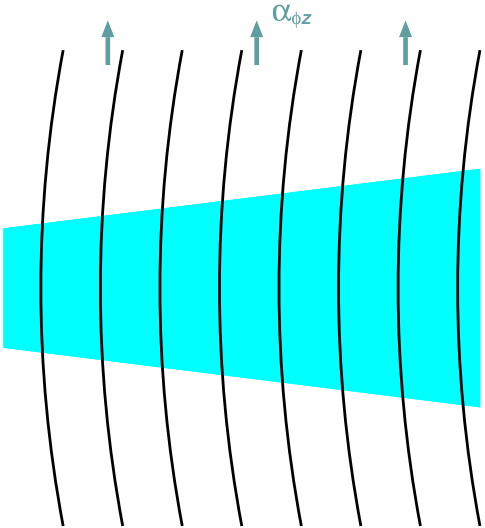
$$\frac{\gamma}{\Omega_\mathrm{K}} = \frac{1}{2}f_\mathrm{d}k^2H^2\left[\sqrt{1+\frac{8(1+3k^2H^2)}{9(1+k^2H^2)^2}\frac{f_\mathrm{a}^2}{f_\mathrm{d}^2}\frac{1}{k^2H^2}}-1\right]$$

Dispersion Relation



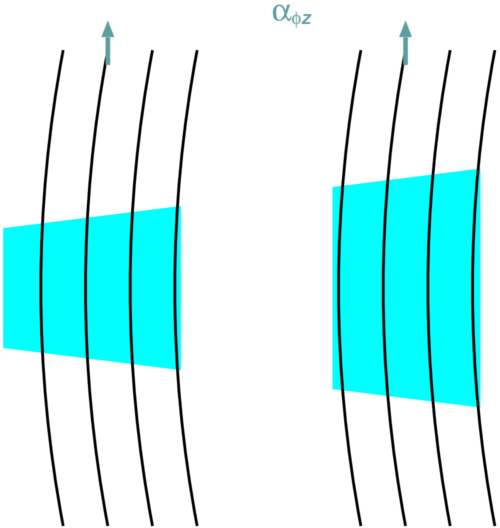
$$\frac{\gamma}{\Omega_{\mathrm{K}}} = \frac{f_{\mathrm{d}} k^2 H^2}{2}\left[\sqrt{1 + \frac{8(1+3k^2H^2)}{9(1+k^2H^2)^2}\frac{f_{\mathrm{a}}^2}{f_{\mathrm{d}}^2}\frac{1}{k^2H^2}} - 1\right]$$ with $f_{\mathrm{d}} = 10^{-3}$ & $f_{\mathrm{a}} = 0.1$

Physical Interpretation 1/4



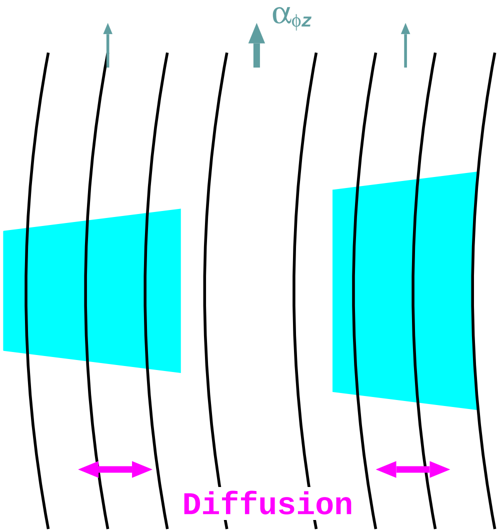
Initial Condition:
Smooth Profile

Physical Interpretation 2/4



Perturbation:
$\delta\Sigma/\Sigma_0 \propto \delta B_z/B_{z,0}$

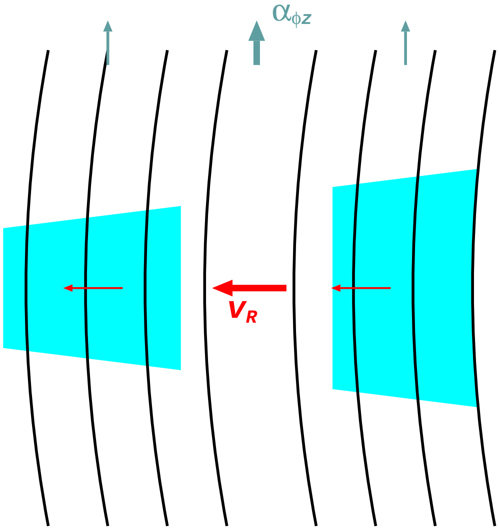
Physical Interpretation 3/4



B_z diffuses to regions with $\delta\Sigma < 0$

$\Rightarrow$ More efficient removal of angular momentum because $\alpha_{\phi z} \propto (B_z^2/\Sigma)^q$

Physical Interpretation 4/4



- Larger mass loss
- Faster accretion

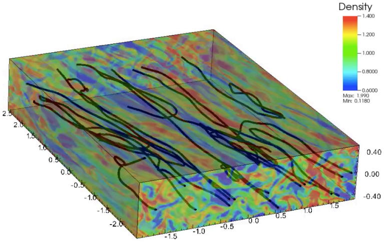
in regions with
$\delta\Sigma < 0$

$\Rightarrow$ Reinforce the initial perturbation (Unstable)

Cartesian Shearing Box

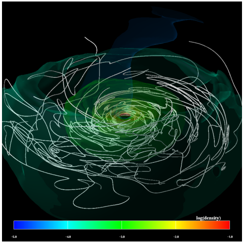
Some Disadvantages

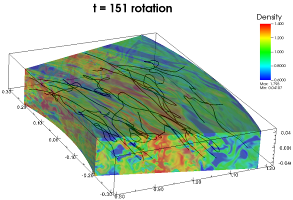
- Neglect the Curvature
- $\pm x$ symmetry
  The central star located on either left or right
- No Net Gas Accretion
- The direction of angular momentum NOT defined
- Removal of Angular Momentum by Disk Winds NOT well-defined

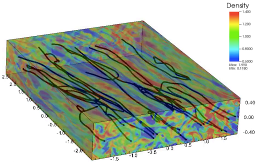


Zoom-in & Zoom-out

Global ⇐ ⇒ Local







Spherical	Cylindrical	Cartesian
(r, θ, ϕ)	(r, ϕ, z)	(x, y, z)

A New Approach: "Cylindrical Shearing Box"

- Break the Symmetry
- Introduce the Curvature

⇒ can handle the net accretion ?

Previous Attempts: Brandenburg+ 1996; Klahr & Bodenheimer 2003; Obergaulinger+ 2009

Previous Attempts

- Nonlocal Shearing Box
  Add curvature terms (Brandenburg+ 1996)
- Radiation HD simulations in “Shearing Disks” in spherical coordinated (Klahr & Bodenheimer 2003)
- Semi-global MHD Simulations for supernovae
  extension of KB03 in Cylindrical Coordinates
  (Obergaulinger+ 2009)

Unphysical oscillations excited
$\Rightarrow$ Damping zone treatment

Cylindrical Shearing Box

Key : Boundary Condition at $R_{\pm}$

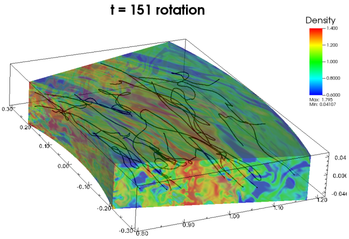
- Shear: $A(R_{\pm}, \phi, z) = A(R_{\mp}, \phi \pm \Delta\Omega_{\mathrm{eq}} t, z)$

  where $\Delta\Omega_{\mathrm{eq}} = \Omega_{\mathrm{eq},-} - \Omega_{\mathrm{eq},+}$
- Radial Boundary Condition

  $\Leftarrow$ Conservation Laws

  of Mass+Momentum+(Energy)+B

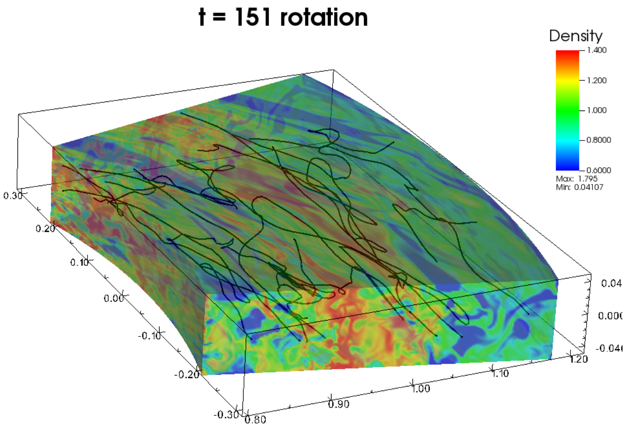
Conserved quantities, A, at R_{-} & R_{+}



$$A = \begin{cases} \rho v_R R \\ \rho v_R^2 R \\ (\rho v_R v_\phi + B_\phi B_R / 4\pi)/\Omega_{\mathrm{eq}} \\ \rho v_R v_z R \\ v_R B_\phi - v_\phi B_R \\ (v_z B_R - v_R B_z) R \\ \mathbf{Energy} \end{cases}$$

Cylindrical Shearing Box (CySB)

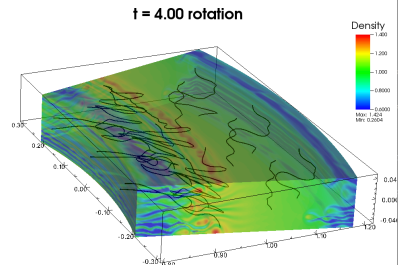
Suzuki+ (2019)

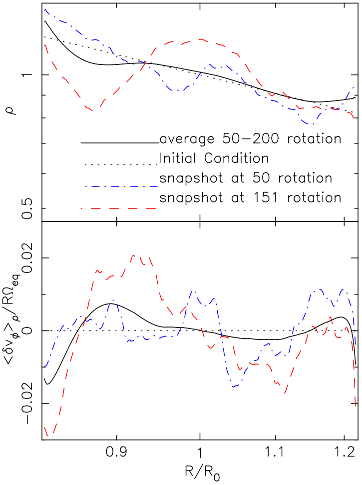


Discussion

$\kappa_+ \neq \kappa_-$ (epicycle frequency at $R_\pm$)

- Shearing periodic condition: Not consistent
- Wave reflection at $R_\pm$
- A zonal flows via boundary effects ?
- Unphysical Oscillation ?





Need further elaboration

Summary

- MHD winds
  - Global $\boldsymbol{B}$ –Magnetocentrifugal Winds
  - Turbulent $\boldsymbol{B}$ –Fluctuation driven upflows & Failed winds
    $\Rightarrow$ Accretion
- Long-time evolution of disks with winds
  Affect the radial profile of surfacce density
- Local substructure – Wind-induced instability:
  on-going research
- Cylindrical shearing box: on-going research

Axisymmetric ($\partial_\phi = 0$) Equations

- Mass: $\frac{\partial \Sigma}{\partial t} + \frac{1}{R}\frac{\partial}{\partial R}(\Sigma v_R R) + [\rho v_z]_{z_-}^{z_+} = 0$
- R momentum:
$$\frac{\partial}{\partial t}(\Sigma v_R) + \frac{1}{R}\frac{\partial}{\partial R}(R\Sigma c_s^2)\alpha_{RR} + \left[\rho c_s^2 \alpha_{Rz}\right]_{z_-}^{z_+}$$
$$= 2\Omega_K \Sigma v_\phi - \frac{\partial}{\partial R}(\Sigma c_s^2) + \Sigma\frac{v_\phi^2}{R} - \frac{\partial}{\partial R}\int B^2 dz - \frac{1}{R}\int \frac{B_\phi^2}{4\pi} dz$$
- ϕ momentum:
$$\frac{\partial}{\partial t}(\Sigma v_\phi R) + \frac{1}{R}\frac{\partial}{\partial R}(R^2 \Sigma c_s^2 \alpha_{R\phi}) + \left[\rho c_s^2 \alpha_{\phi z} R\right]_{z_-}^{z_+}$$
$$+ \frac{1}{2}\Omega_K \Sigma v_R R = 0$$
- B_z evolution:
$$\frac{\partial B_z}{\partial t} = \frac{1}{R}\frac{\partial}{\partial R}\left[R(v_z B_R - v_R B_z)\right] - \eta \frac{1}{R}\frac{\partial}{\partial R}\left[R\left(\frac{\partial B_R}{\partial z} - \frac{\partial B_z}{\partial R}\right)\right]$$

Unperturbed State ($\alpha_{R\phi} = 0$, $\alpha_{\phi z} \neq 0$, $\eta \neq 0$)

Steady-state equations

- $\Sigma v_R R$ = const.
- $2\Omega_{\mathrm{K}}\Sigma v_\phi - \frac{\partial}{\partial R}(\Sigma c_{\mathrm{s}}^2) = 0.$
- $\left[\rho c_{\mathrm{s}}^2 \alpha_{\phi z} R\right]_{z_-}^{z_+} + \frac{1}{2}\Omega_{\mathrm{K}}\Sigma v_R R = 0$

  where $\left[\rho c_{\mathrm{s}}^2 \alpha_{\phi z} R\right]_{z_-}^{z_+} = 2\rho_{\mathrm{mid}} c_{\mathrm{s}}^2 \alpha_{\phi z} R = \frac{2}{\sqrt{2\pi}}\Sigma c_{\mathrm{s}}\Omega_{\mathrm{K}}\alpha_{\phi z} R$
- $R(v_z B_R - v_R B_z) - \eta R\left(\frac{\partial B_R}{\partial z} - \frac{\partial B_z}{\partial R}\right)$ = const.

give a set of unperturbed-state solutions:

- $\Sigma = \Sigma_0\left(\frac{R}{R_0}\right)^{-1}$
- $v_{R,0} = -\frac{4}{\sqrt{2\pi}}\alpha_{\phi z} c_{\mathrm{s}}$
- $v_{\phi,0} = -\frac{3}{4}\frac{c_{\mathrm{s},0}^2}{\Omega_{\mathrm{K},0} R_0}$

  Sub-Keplerian Rotation
- $B_z = B_{z,0}\left(\frac{R}{R_0}\right)^{-1}$

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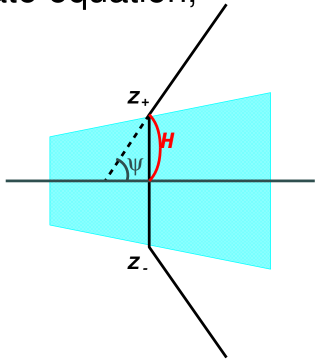
Advection & Diffusion of B_z

Assuming $v_z = 0$, we get a steady-state equation,

$$-v_R B_z - \eta \frac{\partial B_R}{\partial z} + \eta \frac{\partial B_z}{\partial R} = \frac{c_1}{R} \qquad (1)$$



- An hour-glass shape B field
  $\Rightarrow B_R = \pm B_z \cot\psi$
  $\Rightarrow \frac{\partial B_R}{\partial z} \approx \frac{B_R^+ - B_R^-}{2H} = \frac{\Omega_K}{c_s} B_z \cot\psi$
- $\left|\frac{\partial B_z}{\partial R}\right| \ll \left|\frac{\partial B_R}{\partial z}\right| \approx \left|\frac{B_z}{H}\right|$

Eq.(1) $\Rightarrow \left(v_R + \eta \frac{\Omega_K}{c_s} \cot\psi\right) B_z = -\frac{c_1}{R}$

$$\Rightarrow B_z \propto R^{-1}$$

Linear Perturbation Analyses

Axisymmetric perturubation $\delta \propto \exp(i\omega t - ikR) \Rightarrow$

$$\omega^4 - \left[k\left\{v_{R,0} + \eta\left(\frac{\Omega_\mathrm{K}}{\cot\Psi + \frac{1}{R}}\right)\right\} + ik^2\eta\right]\omega^3$$

$$-\left[\Omega_\mathrm{K}^2 + k^2 c_\mathrm{s}^2\left(-\frac{3i}{2R} + k\right)\right]\omega^2$$

$$+\left[\Omega_\mathrm{K}^2 k\left\{(2+q)v_{R,0} + ik\eta + \eta\left(\frac{\Omega_\mathrm{K}}{c_\mathrm{s}}\cot\Psi + \frac{1}{R}\right)\right\}\right.$$
$$\left. + c_\mathrm{s}^2 k^2\left(-\frac{3i}{2R} + k\right)\left\{v_{R,0} + ik\eta + \eta\left(\frac{\Omega_\mathrm{K}}{c_\mathrm{s}}\cot\Psi + \frac{1}{R}\right)\right\}\right]\omega$$

$$-v_{R,0}\Omega_\mathrm{K}^2 k^2\left[(1+q)v_{R,0} + (1-q)\eta\left(-\frac{3i}{2R} + k\right)i(1-q)k\eta\right]$$

$$= 0$$