Three-generation models in SO(32) heterotic string theory

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Introduction

The Standard Model of particle physics

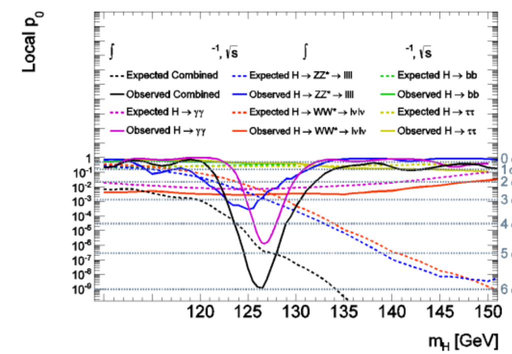
Gauge group : SU(3) × SU(2) × U(1)

Matter content :

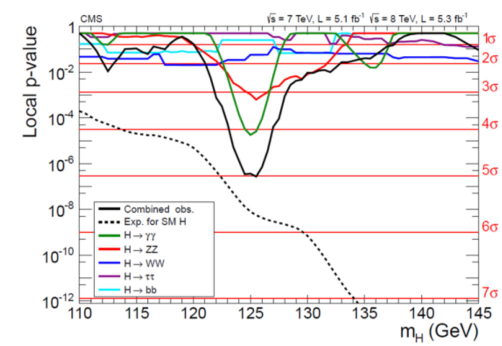
	spin1/2	$SU(3)_c,\ SU(2)_L,\ U(1)_Y$
quarks (×3 families)	$Q^i=(u_L,\ d_L)^i$	(3, 2, 1/6)
	u^i_R	$(\bar{3}$, 2, -2/3)
	d^i_R	$(\bar{3}$, 1, 1/3)
leptons (×3 families)	$L^i=(\nu,\ e_L)^i$	(1, 2, -1/2)
	e^i_R	(1, 1, 1)
	spin0	
Higgs	$H=(H^+,\ H^0)$	(1, 2, -1/2)

	spin1	$SU(3)_c,SU(2)_L,U(1)_Y$
gluon	g	(8, 1, 0)
W bosons	$W^\pm\ \ W^0$	(1, 3, 0)
B boson	B^0	(1, 1, 0)

ATLAS



CMS



Introduction

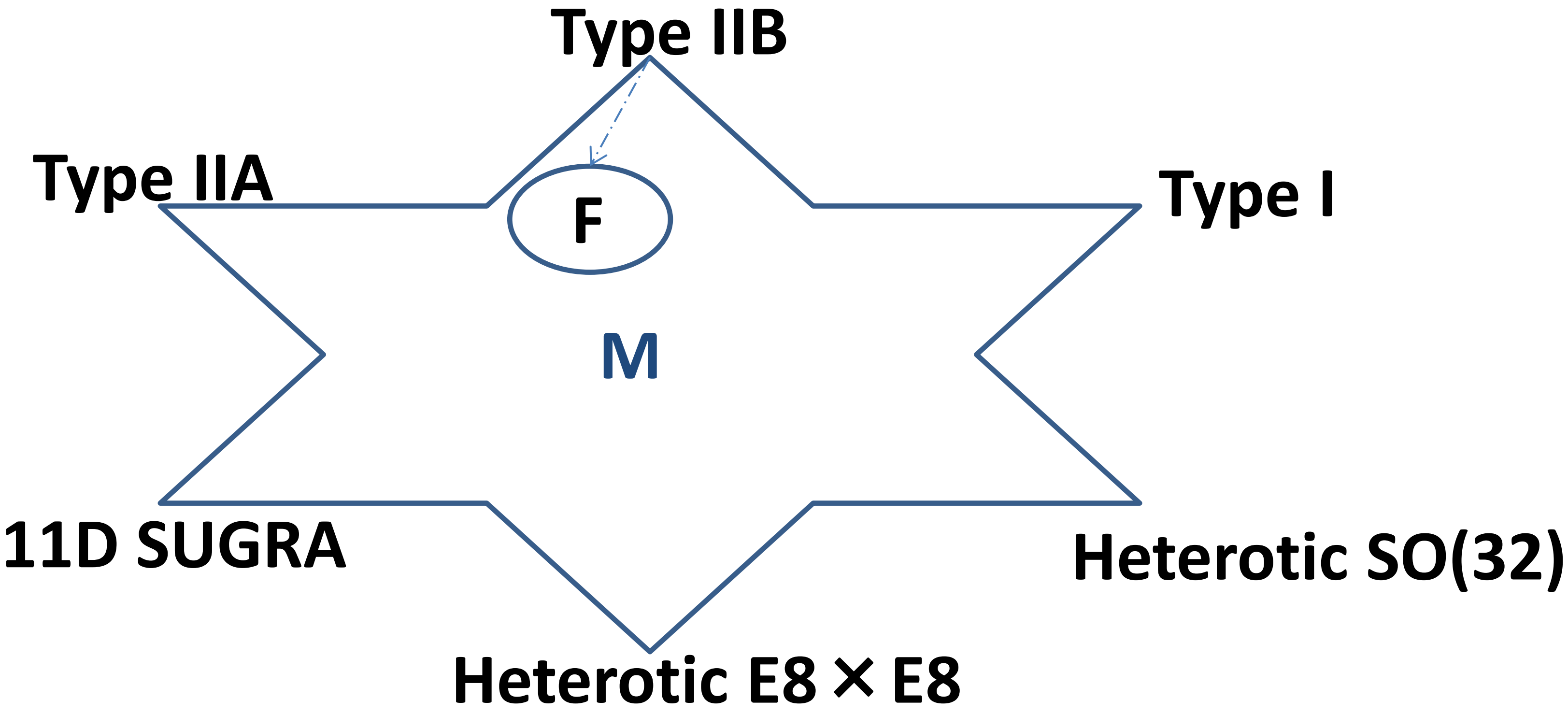
Problem:

No gravitational interaction in the standard model

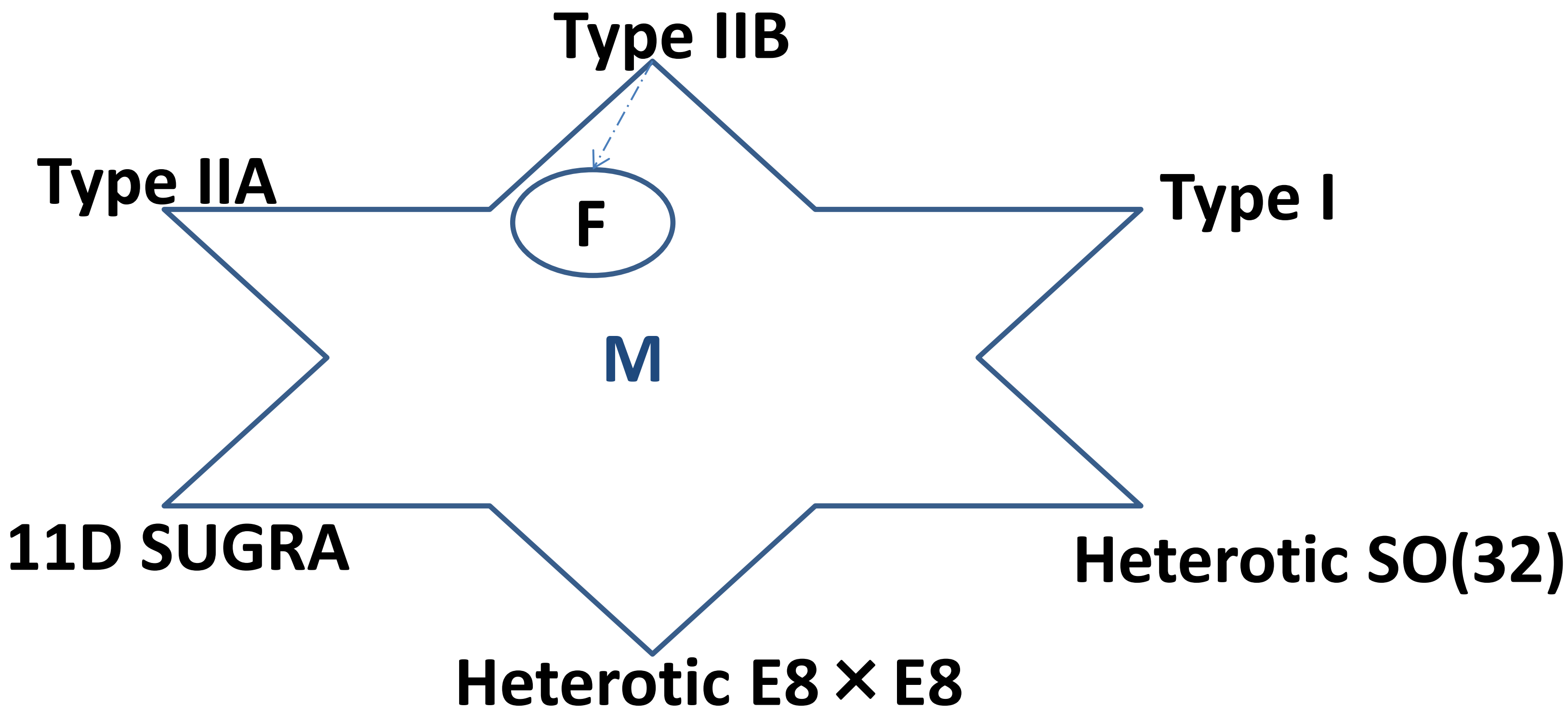
String theory

A good candidate for the unified theory of the gauge and gravitational interactions

Superstring theory / M theory



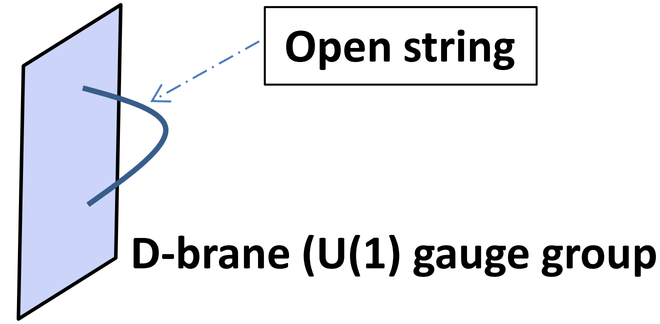
Superstring theory / M theory

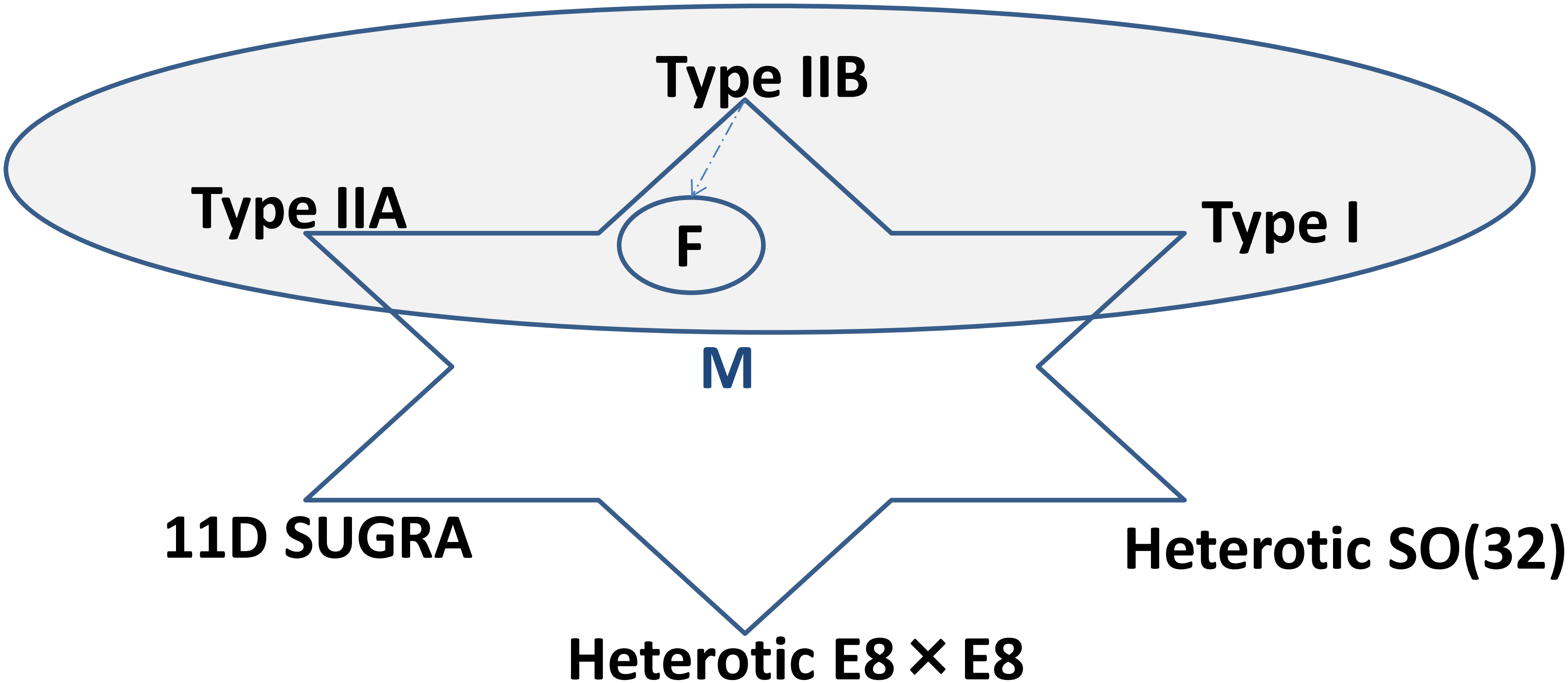


Where is the standard model ?

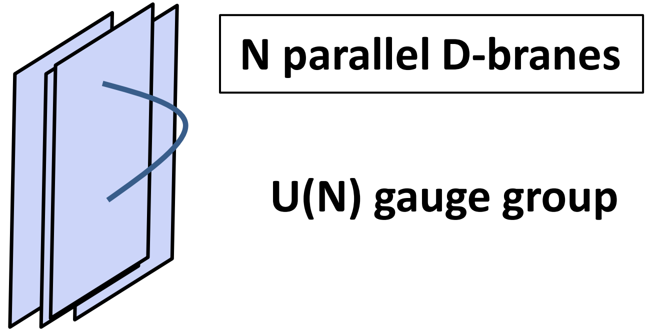
Why are there three generations ?

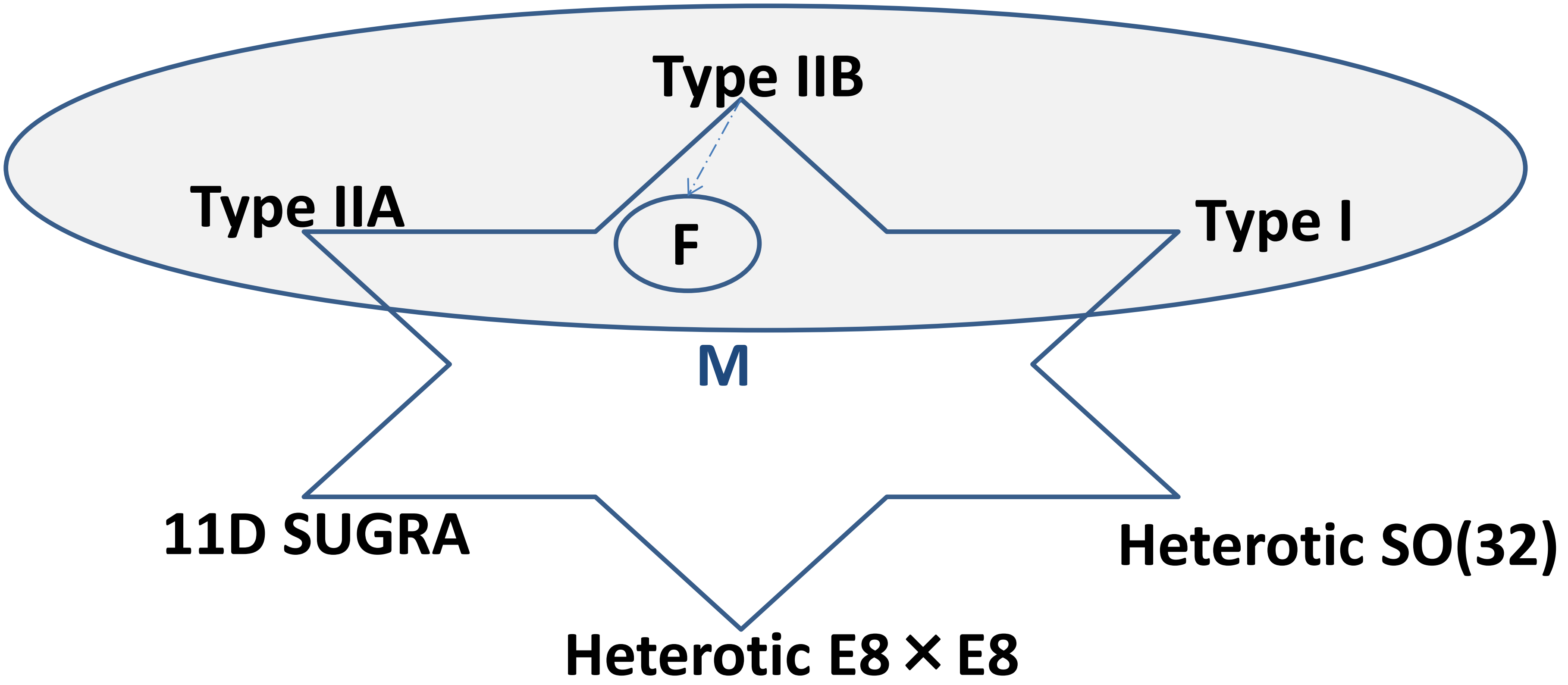
Superstring theory / M theory





Superstring theory / M theory





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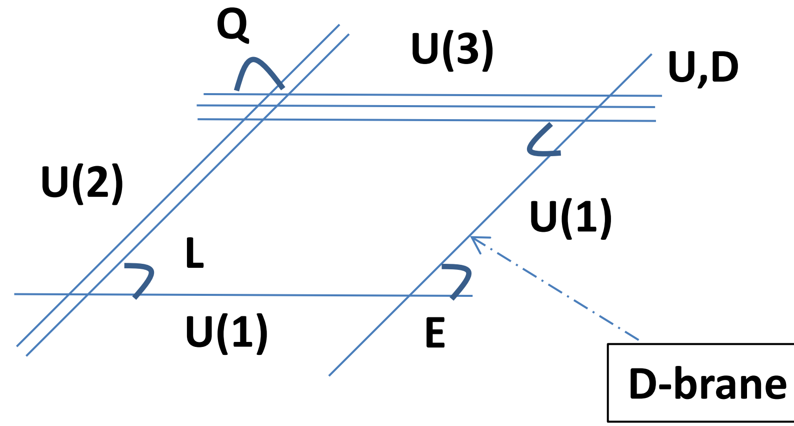
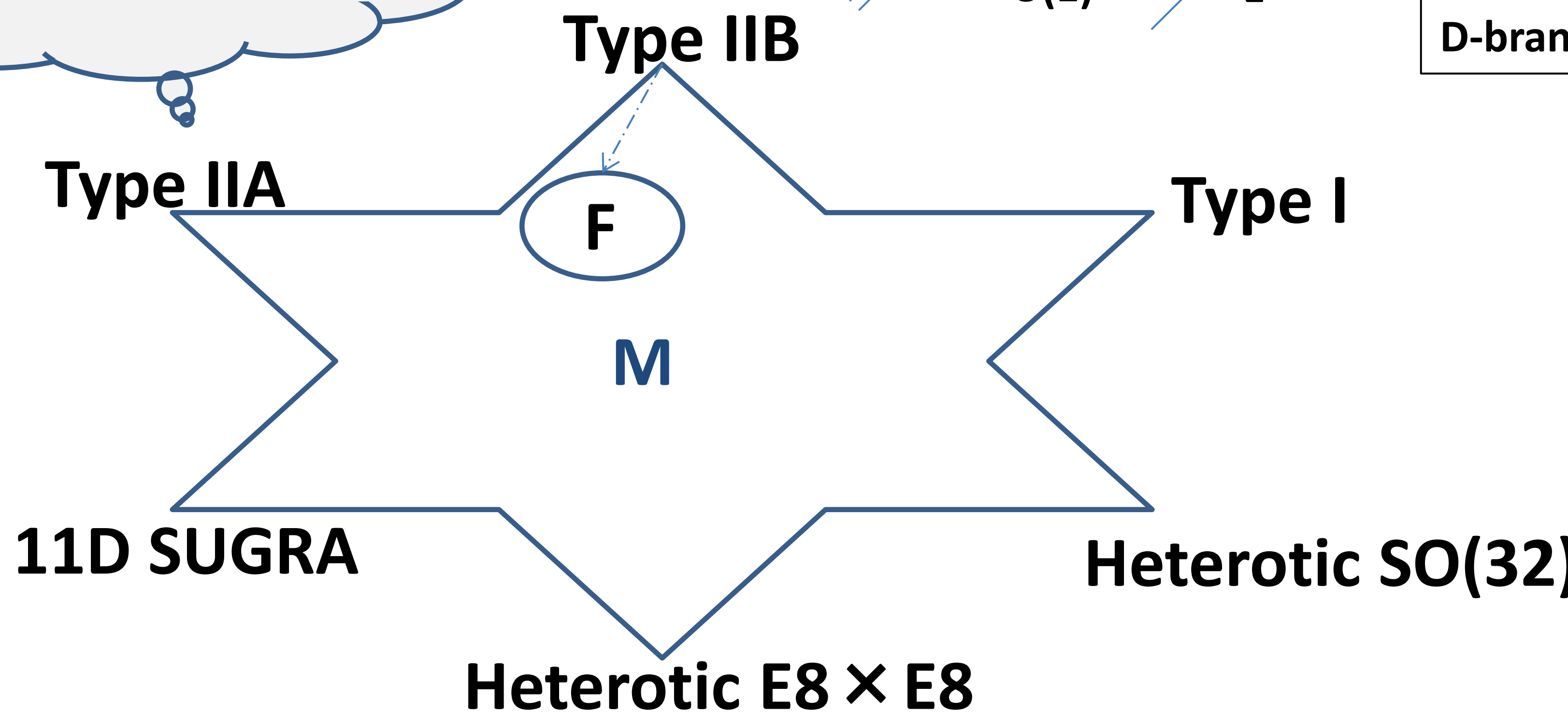
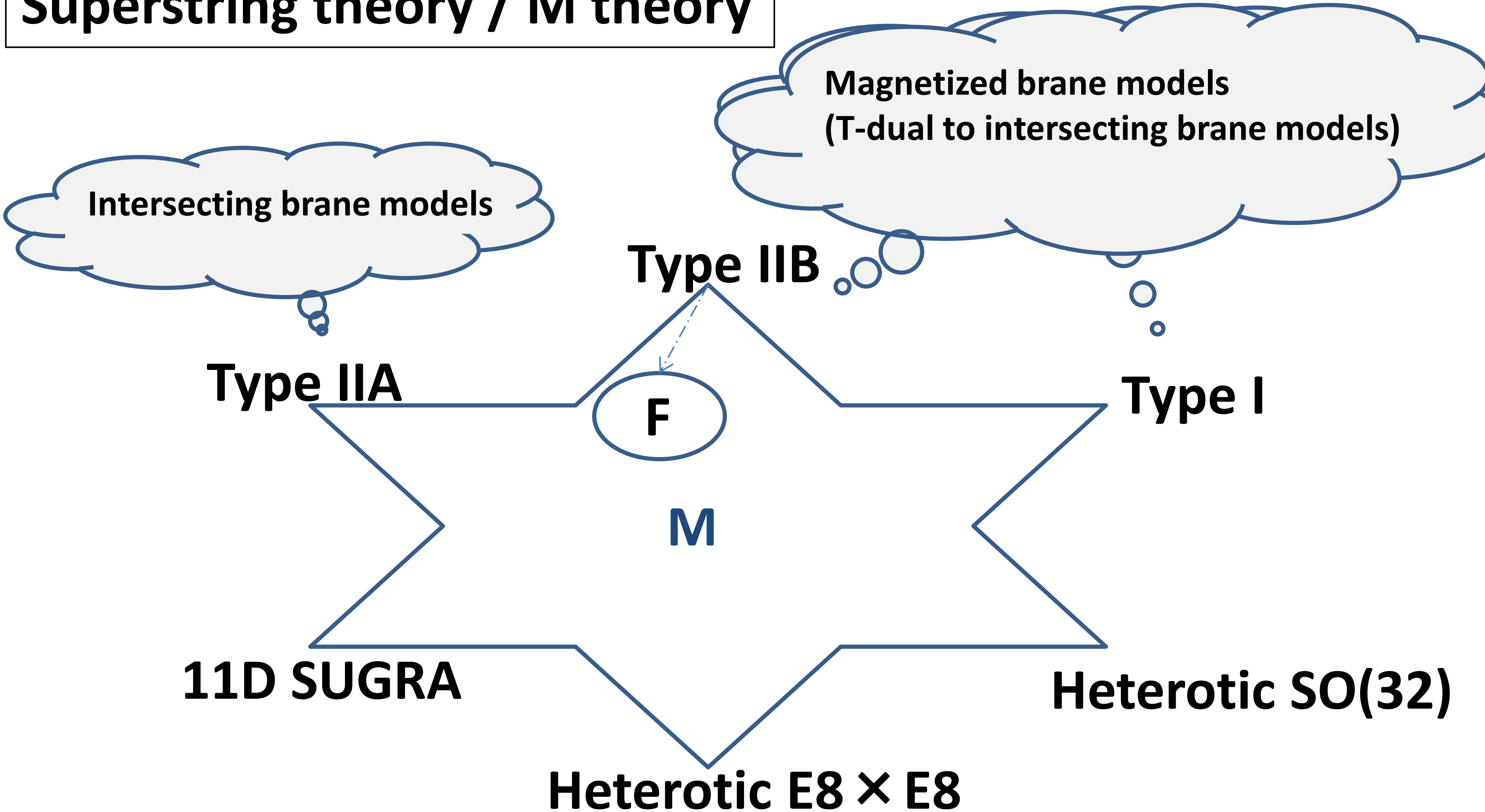


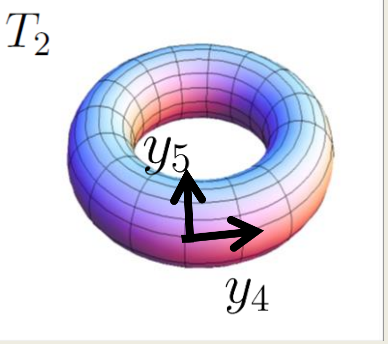


Superstring theory / M theory



D5-brane wrapping on magnetized torus

D. Cremades, L. E. Ibanez & F. Marchesano '04



Effective action: U(N) Super Yang-Mills theory

$$\mathcal{L}_{\rm SYM} = -\frac{1}{4g^2}{\rm Tr}(F^{MN}F_{MN}) + \frac{i}{2g^2}{\rm Tr}(\bar{\lambda}\Gamma^M D_M \lambda)$$

$$M, N = 0,1,2,3,4,5$$

Constant U(1) flux

$$N = N_a + N_b$$

$$F_{45} = 2\pi \begin{pmatrix} M_a \mathbf{1}_{N_a \times N_a} & 0 \\ 0 & M_b \mathbf{1}_{N_b \times N_b} \end{pmatrix}.$$

$$M_a, M_b \in \boldsymbol{Z}$$

Gauge symmetry breaking

$$U(N) \to U(N_a) \times U(N_b)$$

$$M_a \neq M_b$$

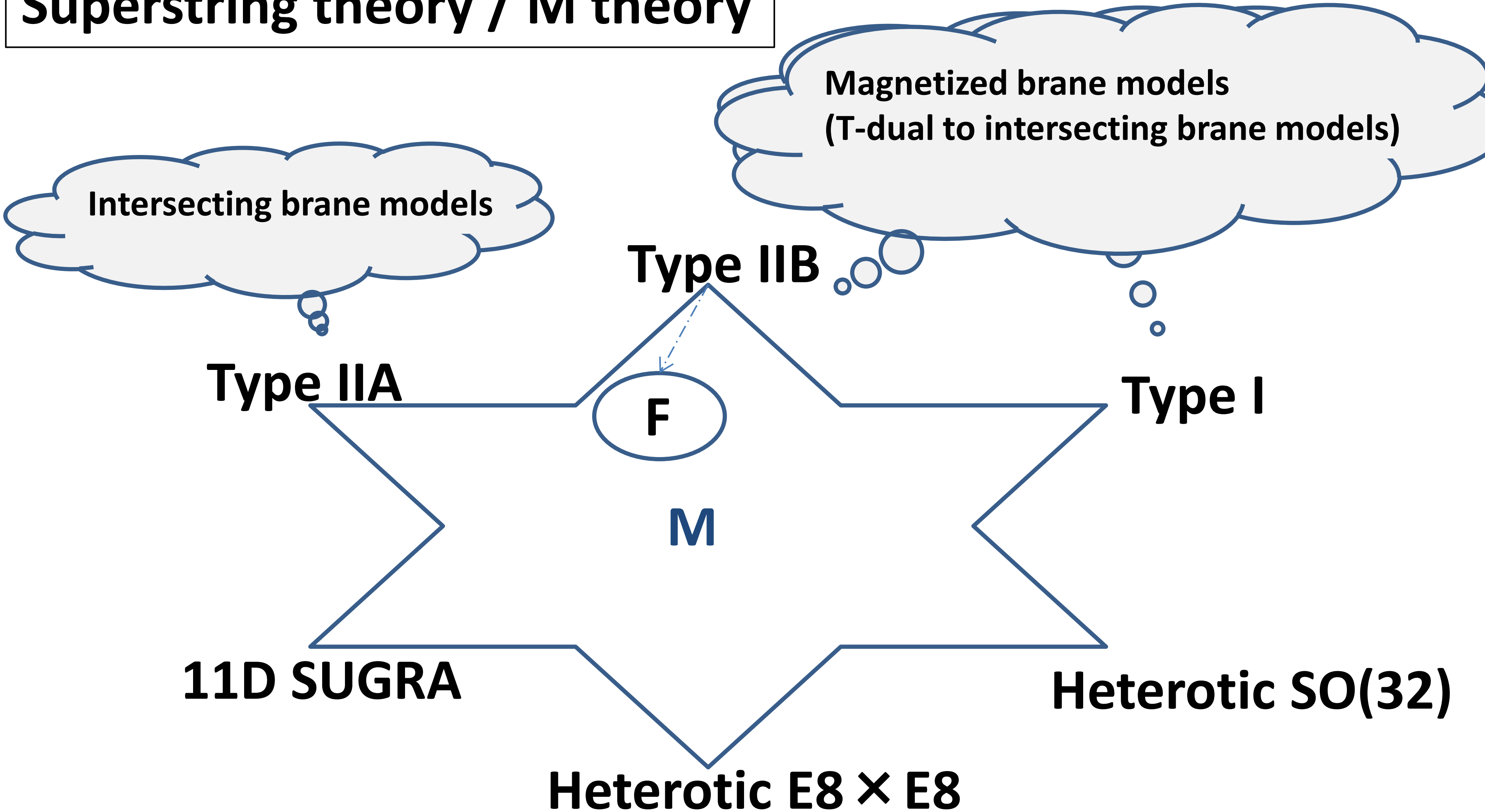
Gaugino field

$$\lambda(x,y) = \begin{pmatrix} \lambda^{aa}(x,y) & \lambda^{ab}(x,y) \\ \lambda^{ba}(x,y) & \lambda^{bb}(x,y) \end{pmatrix}.$$

Bi-fundamental field (quarks, leptons)

$$(N_a, \bar{N}_b), (\bar{N}_a, N_b).$$

Superstring theory / M theory



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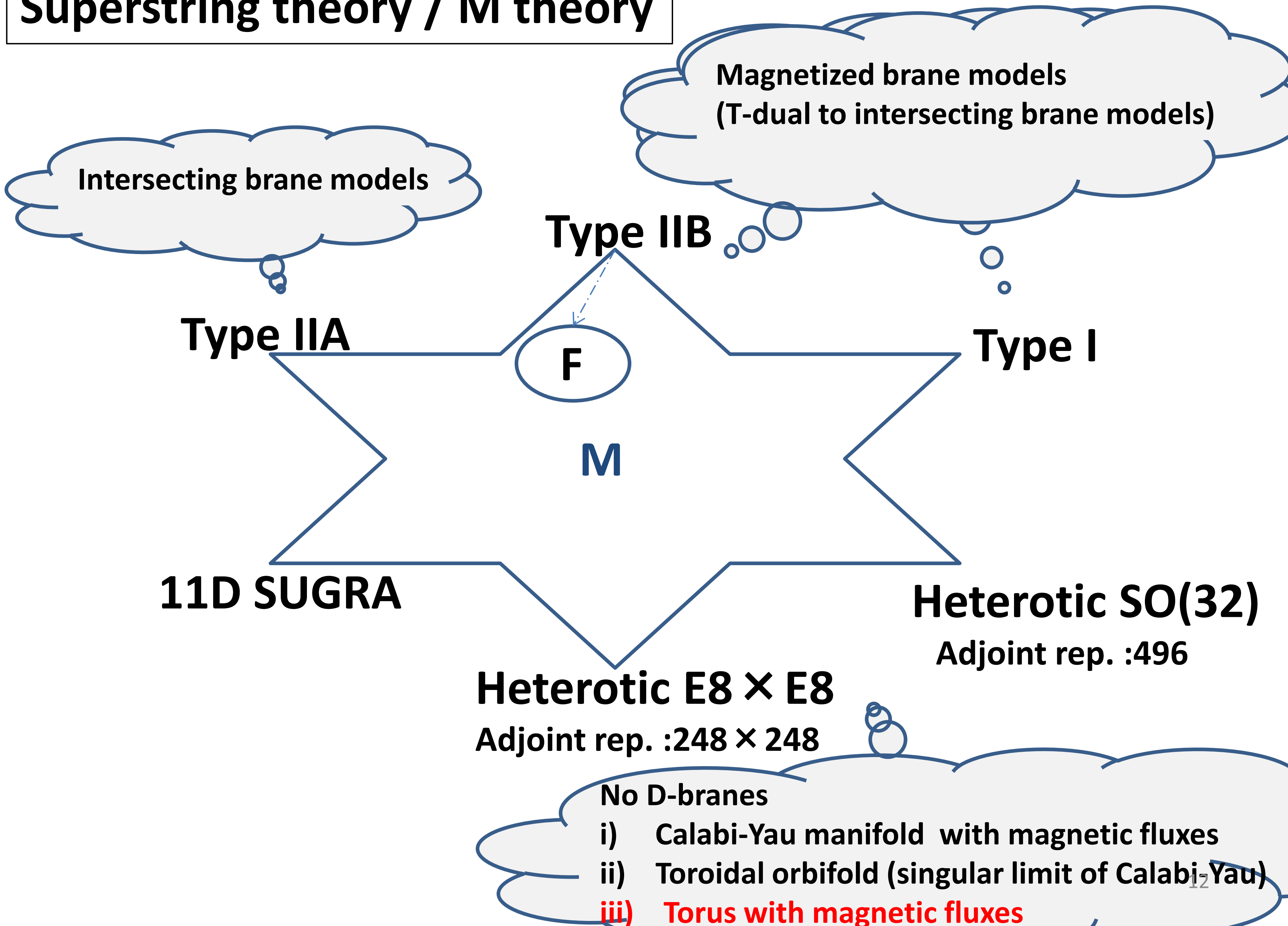


Outline

Introduction

Heterotic string

i) Decomposition of the gauge groups
ii) Chiral matters and degenerate zero-modes
iii) Three-generation models
iv) Summary
v) Gauge coupling unification in SO(32) heterotic string

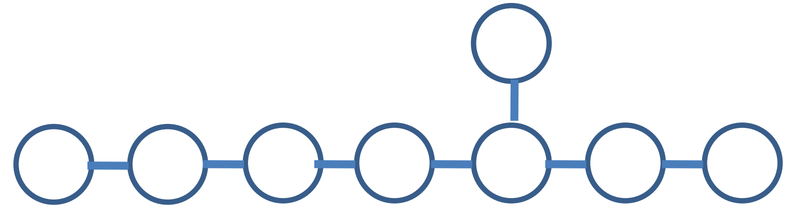
Conclusion

Heterotic E8 × E8 vs **Heterotic SO(32)**

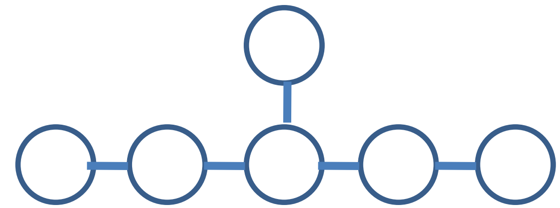
Adjoint rep. :248 × 248 vs **Adjoint rep. :496**

Adjoint rep. of the E8 includes the rep. of E6,SO(10),SU(5)GUT

248 **E8**

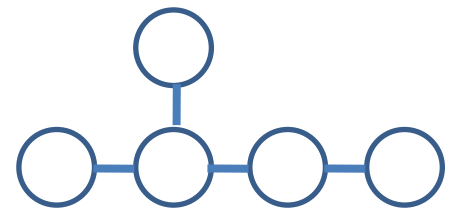


78 **E6**



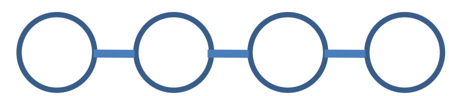
$78 \rightarrow 45 + \mathbf{16} + \overline{\mathbf{16}} + 1$

16 **SO(10)**



$16 \rightarrow \mathbf{10} + \overline{\mathbf{5}} + \mathbf{1}$

10, $\overline{5}$,1 **SU(5)**

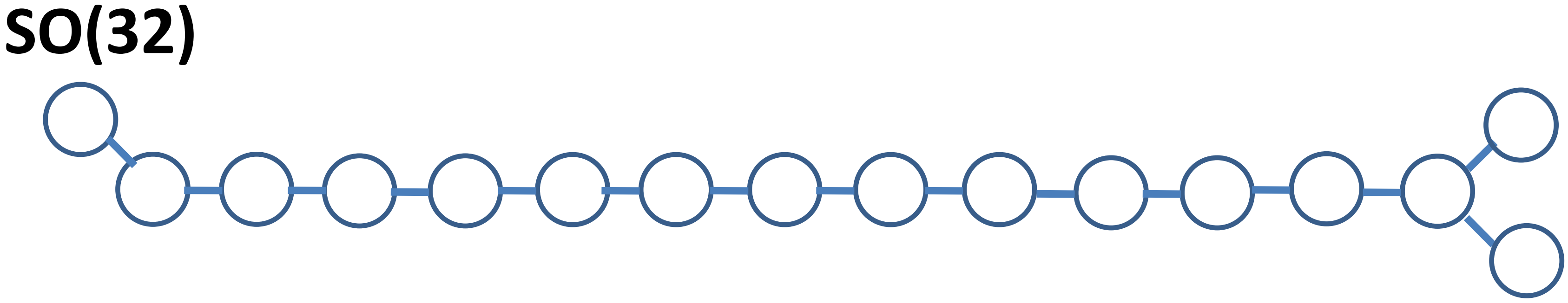


SU(3) × SU(2) × U(1)

$\overline{\mathbf{5}} \rightarrow (\overline{\mathbf{3}}, \mathbf{1})_{\mathbf{1/3}} + (\mathbf{1}, \mathbf{2})_{-\mathbf{1/2}}$

$10 \rightarrow (\mathbf{3}, \mathbf{2})_{\mathbf{1/6}} + (\overline{\mathbf{3}}, \mathbf{1})_{-\mathbf{2/3}} + (\mathbf{1}, \mathbf{1})_{\mathbf{1}}$

However, the adj. rep. of SO(32) does not include the spinor rep. of SO(10).



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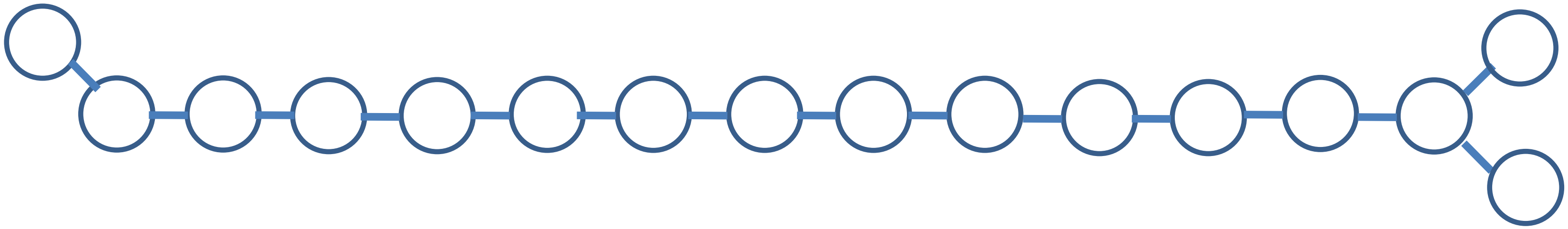
SO(32)



Thus, SU(5) and SO(10) GUT cannot be realized.

arXiv:1503.06770
H.Abe, T. Kobayashi, H.O., and Y. Takano

SO(32)→SO(12)×SO(20)

496→(66,1)+(12,20)+(1,190)

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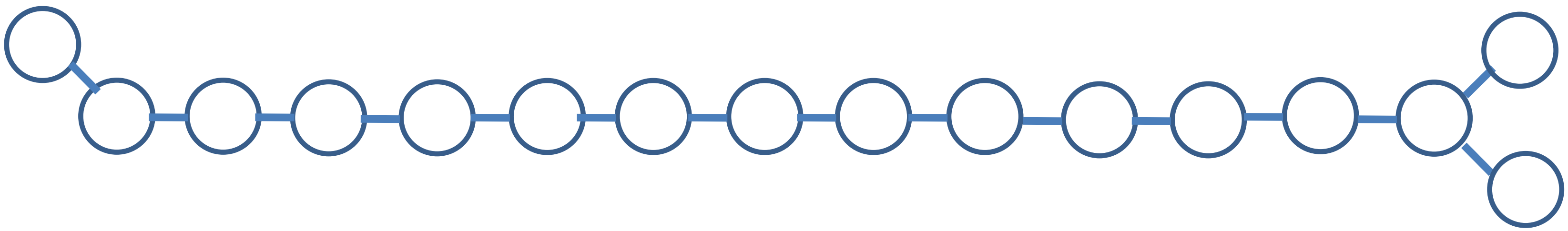

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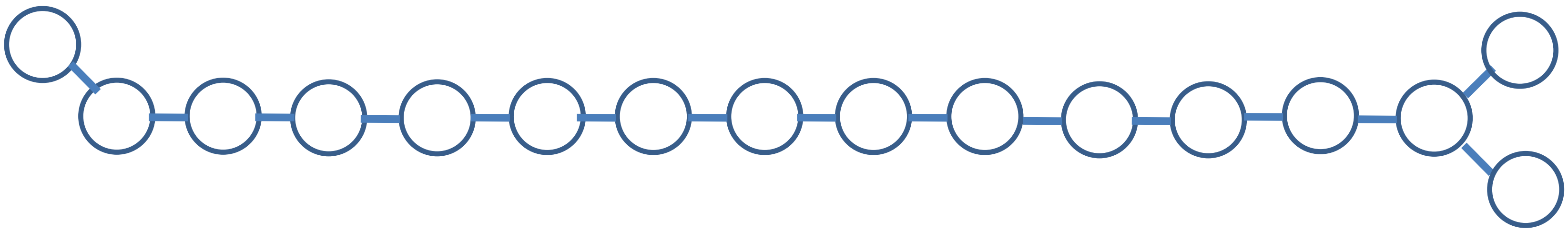
SO(32)



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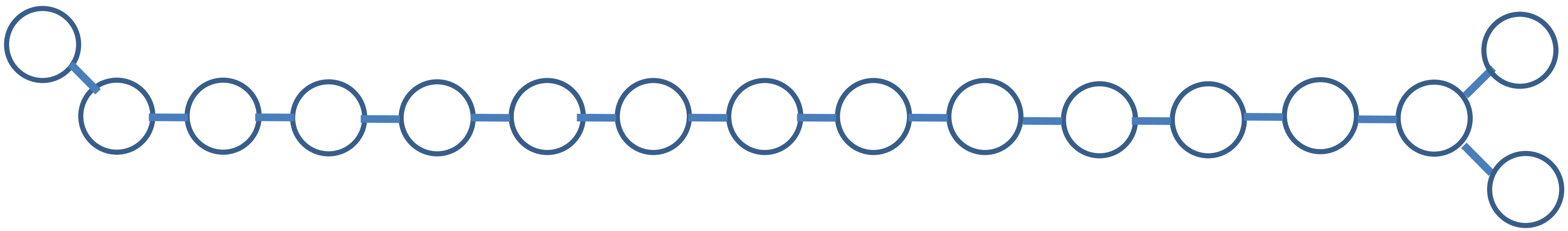
SO(32)



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$$496 \rightarrow (\mathbf{66},1)+(12,20)+(1,190)$$

$$\mathrm{SO}(12) \rightarrow \mathrm{SO}(8) \times \mathrm{SO}(4)$$

$$\rightarrow \mathrm{SO}(8) \times \mathrm{SU}(2) \times \mathrm{U}(1)_1$$

$$\rightarrow \mathrm{SU}(3) \times \mathrm{U}(1)_3 \times \mathrm{U}(1)_2 \times \mathrm{SU}(2) \times \mathrm{U}(1)_1$$

SO(32)→SO(12) × SO(20)

496→(66,1)+(12,20)+(1,190)

SO(12)→SO(8) × SO(4)

→SO(8) × SU(2) × $U(1)_1$

→SU(3) × SU(2)

× $U(1)_1$ × $U(1)_2$ × $U(1)_3$

$$U(1)_Y = U(1)_3/6$$

$$Q : \begin{cases} Q_1 = (3,2)_{1,1,1} \\ Q_2 = (3,2)_{-1,1,1} \end{cases}$$

$$d_R^c : \begin{cases} d^c_{R_1} = (\bar{3},1)_{0,2,2} \\ d^c_{R_2} = (\bar{3},1)_{0,-2,2} \end{cases}$$

$$66 \begin{cases} \cdots \begin{cases} \begin{cases} (8,1)_{0,0,0} \\ (3,1)_{0,0,4} \\ (\bar{3},1)_{0,0,-4} \\ (1,1)_{0,0,0} \end{cases} \\ \begin{cases} (3,1)_{0,2,-2} \\ (\bar{3},1)_{0,2,2} \end{cases} \\ \begin{cases} (3,1)_{0,-2,-2} \\ (\bar{3},1)_{0,-2,2} \end{cases} \\ \begin{cases} (3,2)_{1,1,1} \\ (1,2)_{1,1,-3} \end{cases} \\ \begin{cases} (\bar{3},2)_{1,-1,-1} \\ (1,2)_{1,-1,3} \end{cases} \\ \begin{cases} (3,2)_{-1,1,1} \\ (1,2)_{-1,1,-3} \end{cases} \\ \begin{cases} (\bar{3},2)_{-1,-1,-1} \\ (1,2)_{-1,-1,3} \end{cases} \end{cases} \\ (1,3)_{0,0,0} \\ (1,1)_{2,0,0} \\ (1,1)_{-2,0,0} \\ (1,1)_{0,0,0} \end{cases}$$

Only right-handed leptons do not appear from the adj. rep. of SO(12).

SO(32)→SO(12)×SO(20)

496→(66,1)+(12,20)+(1,190)

SO(12)→SO(8)×SO(4)

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$$L : \begin{cases} L_1 = (1,2)_{1,1,-3} \\ L_2 = (1,2)_{-1,1,-3} \end{cases}$$

$$u_R^c : \{ \; u_{R_1}^c = (\bar{3},1)_{0,0,-4}$$

$$66 \left\{ \begin{array}{l} \left\{ \begin{array}{l} (8,1)_{0,0,0} \\ (3,1)_{0,0,4} \\ (\bar{3},1)_{0,0,-4} \\ (1,1)_{0,0,0} \end{array} \right. \\ \left\{ \begin{array}{l} (3,1)_{0,2,-2} \\ (\bar{3},1)_{0,2,2} \end{array} \right. \\ \left\{ \begin{array}{l} (3,1)_{0,-2,-2} \\ (\bar{3},1)_{0,-2,2} \end{array} \right. \\ \quad \cdots \\ \left\{ \begin{array}{l} (3,2)_{1,1,1} \\ (1,2)_{1,1,-3} \end{array} \right. \\ \left\{ \begin{array}{l} (3,2)_{1,-1,-1} \\ (1,2)_{1,-1,3} \end{array} \right. \\ \left\{ \begin{array}{l} (3,2)_{-1,1,1} \\ (1,2)_{-1,1,-3} \end{array} \right. \\ \left\{ \begin{array}{l} (\bar{3},2)_{-1,-1,-1} \\ (1,2)_{-1,-1,3} \end{array} \right. \\ (1,3)_{0,0,0} \\ (1,1)_{2,0,0} \\ (1,1)_{-2,0,0} \\ (1,1)_{0,0,0} \end{array} \right.$$

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→SU(3) × SU(2)

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$$d_R^c : \begin{cases} d_{R_1}^c = (\bar{3},1)_{0,2,2} \\ d_{R_2}^c = (\bar{3},1)_{0,-2,2} \end{cases} \qquad n_1 = (1,1)_{2,0,0}.$$

Then, we decompose SO(20) into multiple U(1)s.

SO(32)→SO(12)×SO(20)→SU(3)×SU(2)×U(1) ×U(1) ×･･･×U(1)

496→(66,1)+(12,20)+(1,190)

$$U(1)_Y = \frac{1}{6}\left(U(1)_3 + 3\sum_{c=4}^{13} U(1)_c \right)$$

Right-handed leptons

There are all the matter contents in the standard model.

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ii) **Chiral matters and degenerate zero-modes**

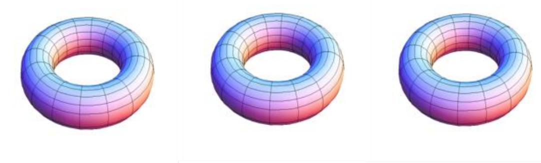
iii) **Three-generation models**

iv) **Summary**

v) **Gauge coupling unification in SO(32) heterotic string**

Conclusion

Heterotic string on three 2-tori

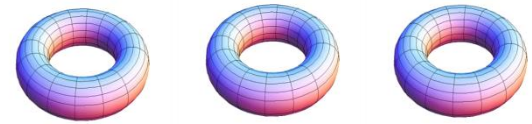


Set-up:

Effective action : 10D N=1 SUGRA + SO(32) Super Yang-Mills

How do we realize the standard model gauge groups and chiral matters from SO(32) heterotic string?

Heterotic string on three 2-tori

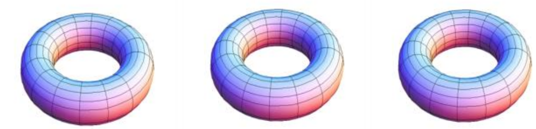


$$SO(32) \to SU(3)_C \otimes SU(2)_L \otimes_{a=1}^{13} U(1)_a$$

$$U(1)_Y = \frac{1}{6}\left(U(1)_3 + 3\sum_{c=4}^{13} U(1)_c\right)$$

$$\begin{aligned}
U(1)_1 &: \ (0,0,0,0,1,1;0,0,\cdots,0),\\
U(1)_2 &: \ (1,1,1,1,0,0;0,0,\cdots,0),\\
U(1)_3 &: \ (1,1,1,-3,0,0;0,0,\cdots,0),\\
U(1)_4 &: \ (0,0,0,0,0,0;1,0,\cdots,0),\\
U(1)_5 &: \ (0,0,0,0,0,0;0,1,\cdots,0),\\
&\ \vdots\\
U(1)_{13} &: \ (0,0,0,0,0,0;0,0,\cdots,1),
\end{aligned}$$

Heterotic string on three 2-tori



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$$U(1)_Y = \frac{1}{6}\left(U(1)_3 + 3\sum_{c=4}^{13} U(1)_c\right)$$

$$\begin{aligned}
U(1)_1 &: \ (0,0,0,0,1,1;0,0,\cdots,0),\\
U(1)_2 &: \ (1,1,1,1,0,0;0,0,\cdots,0),\\
U(1)_3 &: \ (1,1,1,-3,0,0;0,0,\cdots,0),\\
U(1)_4 &: \ (0,0,0,0,0,0;1,0,\cdots,0),\\
U(1)_5 &: \ (0,0,0,0,0,0;0,1,\cdots,0),\\
&\ \ \vdots\\
U(1)_{13} &: \ (0,0,0,0,0,0;0,0,\cdots,1),
\end{aligned}$$

We consider the Abelian flux background.

$$\bar{f}_a^{(i)} = dA_a^{(i)}(z^i) \propto 2\pi m_a^{(i)}$$

z^i **:coordinate of torus** $(T^2)_i$

The vanishing fluxes lead to the gauge enhancements.

Chiral matters

First, we define the 10D Majorana-Weyl (MW) spinor,

$$\Gamma\lambda = \lambda$$

Γ is the 10D chirality matrix

10D MW spinor →four 4D Weyl spinors

$$\lambda_0 = \lambda_{+++}, \quad \lambda_1 = \lambda_{+--}, \quad \lambda_2 = \lambda_{-+-}, \quad \lambda_3 = \lambda_{--+},$$

where the subscript indexes denote the eigenvalues of Γ^i with $i = 1, 2, 3$.

Γ_i 2D chirality operators

$$\Gamma^i \lambda_0 = \lambda_0, \quad \Gamma^i \lambda_j = \begin{cases} +\lambda_j & (i = j) \\ -\lambda_j & (i \neq j) \end{cases}$$

Chiral matters

Four 4D Weyl spinors

$$\lambda_0 = \lambda_{+++}, \ \ \lambda_1 = \lambda_{+--}, \ \ \lambda_2 = \lambda_{-+-}, \ \ \lambda_3 = \lambda_{--+},$$

where the subscript indexes denote the eigenvalues of Γ^i with $i = 1, 2, 3$.

When we insert the magnetic fluxes on three 2-tori, one of the four 4D Weyl spinors would be chosen → Chiral fermion

Zero-mode equation for the fermion

D. Cremades, L. E. Ibanez & F. Marchesano '04

KK decomposition for the gaugino field

$$\lambda(x^\mu, z^i) = \sum_n \chi_n(x^\mu) \otimes \psi_n^{(1)}(z^1) \otimes \psi_n^{(2)}(z^2) \otimes \psi_n^{(3)}(z^3)$$

$z^i = y^{2+2i} + \tau^i y^{3+2i}$:coordinate of torus

Dirac equations:

$$\not{D}_i \psi^{(i)}(z^i) = (\Gamma^{z^i}\nabla_{z^i} + \Gamma^{\bar{z}^i}\nabla_{\bar{z}^i})\psi^{(i)}(z^i) = 0$$

$$\psi^{(i)}(z^i) = \begin{pmatrix} \psi_+^{(i)}(z^i) \\ \psi_-^{(i)}(z^i) \end{pmatrix}$$

$$\Gamma^{z^i} = \frac{1}{2\pi R_i}\begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}, \quad \Gamma^{\bar{z}^i} = \frac{1}{2\pi R_i}\begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix}$$

$$\nabla_{z^i} = \partial_{z^i} - iq_a(A_a^{(i)})_{z^i}$$

$$\nabla_{\bar{z}^i} = \partial_{\bar{z}^i} - iq_a(A_a^{(i)})_{\bar{z}^i}$$

Zero-mode equation for the fermion

D. Cremades, L. E. Ibanez & F. Marchesano '04

The zero-mode equations:

$$\left(\bar{\partial}_{\bar{z}^i} + \frac{\pi q^a m_a^i}{2\mathrm{Im}\,\tau_i} z^i\right) \psi_+^{(i)}(z^i, \bar{z}^i) = 0,$$
$$\left(\partial_{z^i} - \frac{\pi q^a m_a^i}{2\mathrm{Im}\,\tau_i} \bar{z}^i\right) \psi_-^{(i)}(z^i, \bar{z}^i) = 0.$$

$$\psi_0^{(i)}(z^i) = \begin{pmatrix} \psi_+^{(i)}(z^i) \\ \psi_-^{(i)}(z^i) \end{pmatrix}$$

$$M^i = q_a m_a^i$$

$\psi_+^{(i)}(z^i, \bar{z}^i)$ has zero-modes only if $M^i > 0$

$\psi_-^{(i)}(z^i, \bar{z}^i)$ has zero-modes only if $M^i < 0$

○By introducing a non-trivial flux, we select one of the two chiralities of the two-dimensional spinor.

Zero-mode equation for the fermion

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○ **By introducing a non-trivial flux, we select one of the two chiralities of the two-dimensional spinor.**

$$\lambda_0 = \lambda_{+++}, \quad \lambda_1 = \lambda_{+--}, \quad \lambda_2 = \lambda_{-+-}, \quad \lambda_3 = \lambda_{--+},$$

○ $M = |M^1||M^2||M^3|$ **independent solutions of the Dirac equations. (The number of generation)**

Outline

Introduction

Heterotic string

i) Decomposition of the gauge groups
ii) Chiral matters and degenerate zero-modes
iii) Three-generation models
iv) Summary
v) Gauge coupling unification in SO(32) heterotic string

Conclusion

The fluxes for each matter:
(The number of generation)

$SO(32) \to SU(3)_C \otimes SU(2)_L \otimes_{a=1}^{13} U(1)_a$

$$m_{Q_1} = \prod_{i=1}^{3} m_{Q_1}^i = \prod_{i=1}^{3}(m_1^i + m_2^i + m_3^i), \quad m_{Q_2} = \prod_{i=1}^{3} m_{Q_2}^i = \prod_{i=1}^{3}(-m_1^i + m_2^i + m_3^i),$$
$$m_{L_1} = \prod_{i=1}^{3} m_{L_1}^i = \prod_{i=1}^{3}(m_1^i + m_2^i - 3m_3^i), \quad m_{L_2} = \prod_{i=1}^{3} m_{L_2}^i = \prod_{i=1}^{3}(-m_1^i + m_2^i - 3m_3^i),$$
$$m_{u_{R_1}^c} = \prod_{i=1}^{3} m_{u_{R_1}^c}^i = \prod_{i=1}^{3}(-4m_3^i), \quad m_{n_1} = \prod_{i=1}^{3} m_{n_1}^i = \prod_{i=1}^{3}(2m_1^i),$$
$$m_{d_{R_1}^c} = \prod_{i=1}^{3} m_{d_{R_1}^c}^i = \prod_{i=1}^{3}(2m_2^i + 2m_3^i), \quad m_{d_{R_2}^c} = \prod_{i=1}^{3} m_{d_{R_2}^c}^i = \prod_{i=1}^{3}(-2m_2^i + 2m_3^i),$$

$$m_{L_3^a} = \prod_{i=1}^{3} m_{L_3^a}^i = \prod_{i=1}^{3}(m_1^i - m_a^i), \quad m_{L_4^a} = \prod_{i=1}^{3} m_{L_4^a}^i = \prod_{i=1}^{3}(-m_1^i - m_a^i),$$
$$m_{u_{R_2}^{c\,a}} = \prod_{i=1}^{3} m_{u_{R_2}^{c\,a}}^i = \prod_{i=1}^{3}(-m_2^i - m_3^i - m_a^i), \quad m_{d_{R_3}^{c\,a}} = \prod_{i=1}^{3} m_{d_{R_3}^{c\,a}}^i = \prod_{i=1}^{3}(-m_2^i - m_3^i + m_a^i),$$
$$m_{e_{R_1}^{c\,a}} = \prod_{i=1}^{3} m_{e_{R_1}^{c\,a}}^i = \prod_{i=1}^{3}(-m_2^i + 3m_3^i + m_a^i), \quad m_{n_2^a} = \prod_{i=1}^{3} m_{n_2^a}^i = \prod_{i=1}^{3}(-m_2^i + 3m_3^i - m_a^i),$$

Yang-Mills fluxes satisfy the following consistency conditions.
①The massless condition for $U(1)_Y$ gauge boson
②Tadpole condition
③D-term (SUSY) condition
④K-theory condition

○Result

MSSM + extra Higgs + vector-like matters

$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	$U(1)_{10}$
(m_1^1, m_1^2, m_1^3)	(m_2^1, m_2^2, m_2^3)	(m_3^1, m_3^2, m_3^3)	(m_4^1, m_4^2, m_4^3)	$(m_{10}^1, m_{10}^2, m_{10}^3)$
$(1, 0, \frac{1}{2})$	$(2, 1, \frac{1}{2})$	$(0, 0, 0)$	$(-1, -2, \frac{1}{2})$	$(0, 1, -\frac{1}{2})$

$$m_4^i = m_5^i = m_6^i = -m_7^i = -m_8^i = -m_9^i,$$
$$m_{10}^i = m_{11}^i = -m_{12}^i = -m_{13}^i,$$

The number of generation:

$(Q_1, Q_2, L_1, L_2, u_{R_1}^c, d_{R_1}^c, d_{R_2}^c, n_1)$	$(3, 0, 3, 0, 0, 8, -8, 0)$
$(L_3^4, L_4^4, u_{R_2}^{c\,4}, d_{R_3}^{c\,4}, e_{R_1}^{c\,4}, n_2^4)$	$(0, 0, 1, 0, 0, 1)$
$(L_3^5, L_4^5, u_{R_2}^{c\,5}, d_{R_3}^{c\,5}, e_{R_1}^{c\,5}, n_2^5)$	$(0, 0, 1, 0, 0, 1)$
$(L_3^6, L_4^6, u_{R_2}^{c\,6}, d_{R_3}^{c\,6}, e_{R_1}^{c\,6}, n_2^6)$	$(0, 0, 1, 0, 0, 1)$
$(L_3^7, L_4^7, u_{R_2}^{c\,7}, d_{R_3}^{c\,7}, e_{R_1}^{c\,7}, n_2^7)$	$(0, 0, 0, 1, 1, 0)$
$(L_3^8, L_4^8, u_{R_2}^{c\,8}, d_{R_3}^{c\,8}, e_{R_1}^{c\,8}, n_2^8)$	$(0, 0, 0, 1, 1, 0)$
$(L_3^9, L_4^9, u_{R_2}^{c\,9}, d_{R_3}^{c\,9}, e_{R_1}^{c\,9}, n_2^9)$	$(0, 0, 0, 1, 1, 0)$
$(L_3^{10}, L_4^{10}, u_{R_2}^{c\,10}, d_{R_3}^{c\,10}, e_{R_1}^{c\,10}, n_2^{10})$	$(-1, 0, 0, 0, 0, 0)$
$(L_3^{11}, L_4^{11}, u_{R_2}^{c\,11}, d_{R_3}^{c\,11}, e_{R_1}^{c\,11}, n_2^{11})$	$(-1, 0, 0, 0, 0, 0)$
$(L_3^{12}, L_4^{12}, u_{R_2}^{c\,12}, d_{R_3}^{c\,12}, e_{R_1}^{c\,12}, n_2^{12})$	$(0, 1, 0, 0, 0, 0)$
$(L_3^{13}, L_4^{13}, u_{R_2}^{c\,13}, d_{R_3}^{c\,13}, e_{R_1}^{c\,13}, n_2^{13})$	$(0, 1, 0, 0, 0, 0)$

○Result

MSSM + extra Higgs + vector-like matters

$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	$U(1)_{10}$
(m_1^1, m_1^2, m_1^3)	(m_2^1, m_2^2, m_2^3)	(m_3^1, m_3^2, m_3^3)	(m_4^1, m_4^2, m_4^3)	$(m_{10}^1, m_{10}^2, m_{10}^3)$
$(1, 0, \frac{1}{2})$	$(2, 1, \frac{1}{2})$	$(0, 0, 0)$	$(-1, -2, \frac{1}{2})$	$(0, 1, -\frac{1}{2})$

$$m_4^i = m_5^i = m_6^i = -m_7^i = -m_8^i = -m_9^i,$$
$$m_{10}^i = m_{11}^i = -m_{12}^i = -m_{13}^i,$$

The Yukawa couplings of quarks and leptons are allowed in terms of the renormalizable operators.

We require the Wilson lines into the internal component of $U(1)_3$.
(SU(4)→SU(3) × $U(1)_3$)

Summary

○ **We can realize the three-generation standard-like model from the SO(32) heterotic string theory.**

U(1) magnetic fluxes and Wilson lines
→ Three-generation of quarks and leptons
Gauge symmetry breaking

$$SO(32) \to SU(3)_C \otimes SU(2)_L \otimes_{a=1}^{13} U(1)_a$$

○ **Such fluxes are constrained by**
①The massless condition for $\mathrm{U}(1)_Y$ gauge boson
②Tadpole condition
③D-term (SUSY) condition
④K-theory condition

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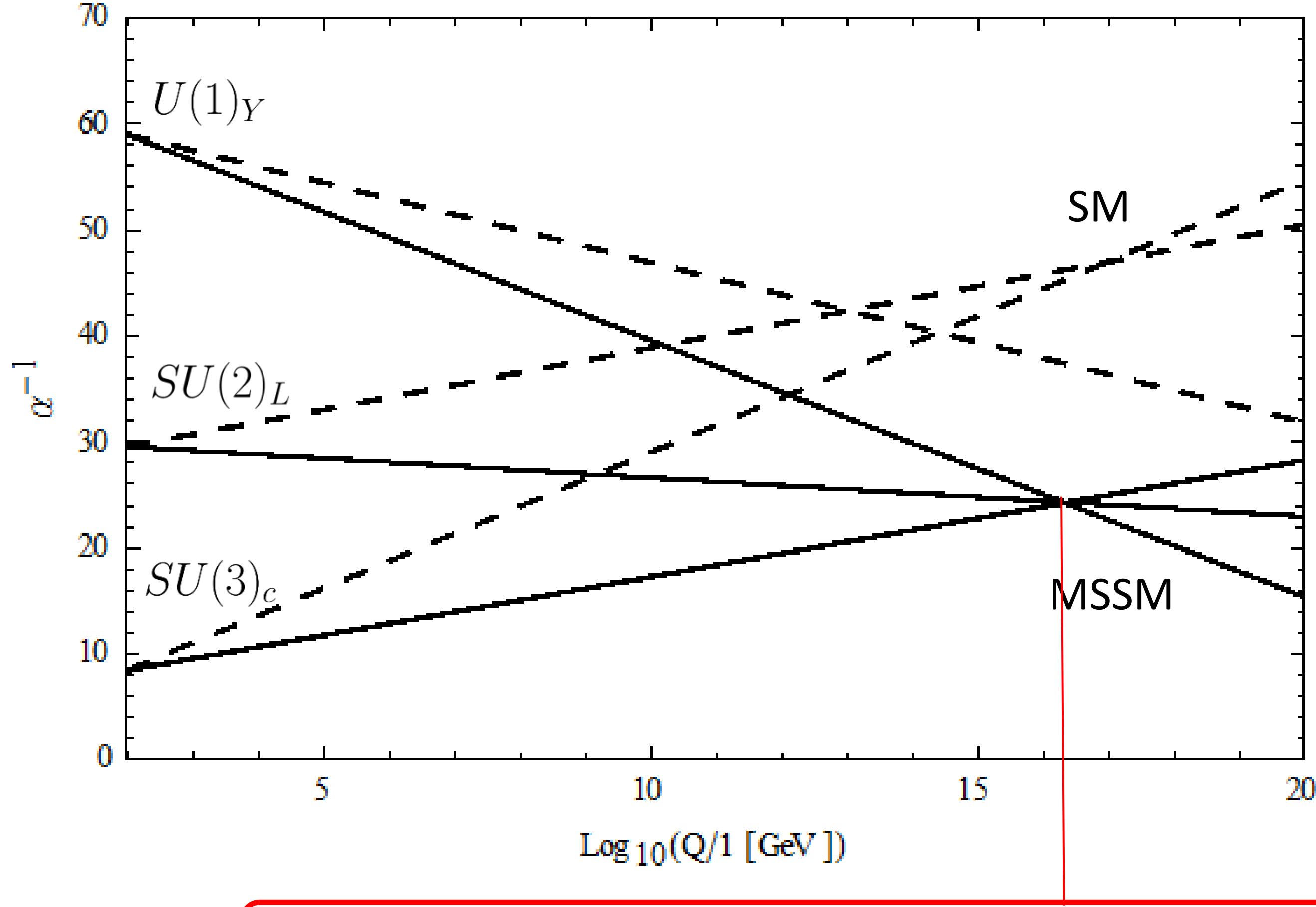
Gauge coupling unification



$$g(m_X) \simeq 0.7 \text{ at a scale } m_X \simeq 2 \times 10^{16} \text{ GeV}$$

MSSM supersymmetric Standard Model with just two Higgs doublets

Gauge coupling unification

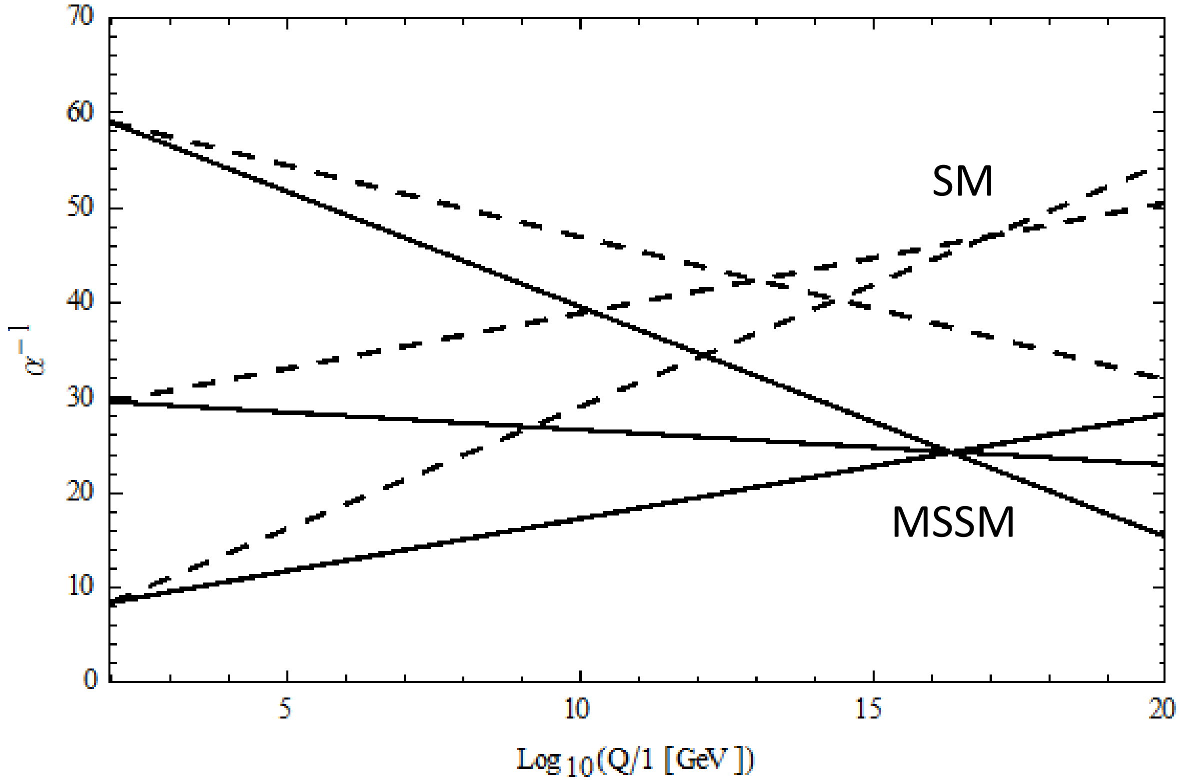
in the context of heterotic string theory

$$M_s^2 = \frac{M_{\rm Pl}^2}{4\pi\alpha_4^{-1}}$$

$$M_s \simeq 1 \times 10^{17}\,\mathrm{GeV}$$

String scale is larger than the "observed" unification scale.

$$m_X \simeq 2 \times 10^{16}\,\mathrm{GeV}$$



MSSM

$$\frac{g^2_{SU(3)}(M_s)}{g^2_{U(1)_Y}(M_s)} \simeq 0.881$$

$$\frac{g^2_{SU(2)}(M_s)}{g^2_{U(1)_Y}(M_s)} \simeq 0.994$$

SM

$$\frac{g^2_{SU(3)}(M_s)}{g^2_{U(1)_Y}(M_s)} \simeq 0.763$$

$$\frac{g^2_{SU(2)}(M_s)}{g^2_{U(1)_Y}(M_s)} \simeq 0.775$$

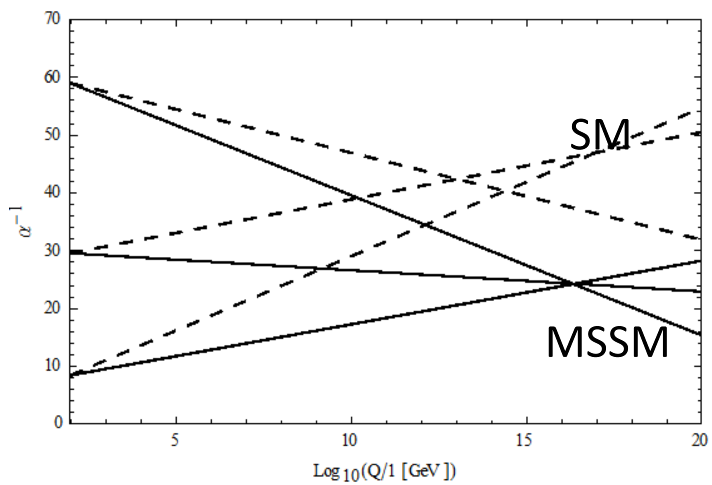
Gauge coupling constants in heterotic string

$$\frac{1}{g_a^2(\mu)} = \frac{1}{g_{\text{tree}}^2(M_s)} + \frac{b_a}{16\pi^2} \ln \frac{M_s^2}{\mu^2} + \frac{1}{16\pi^2} \Delta_a$$

Moduli-dependent threshold corrections

$T \cdots$ Kähler moduli

In the case of toroidal orbifold,
the gauge coupling unification can only be achieved
by the large values of Kähler moduli, $T \sim 26$



L. E. Ibanez and D. Lust

[hep-th/9202046]

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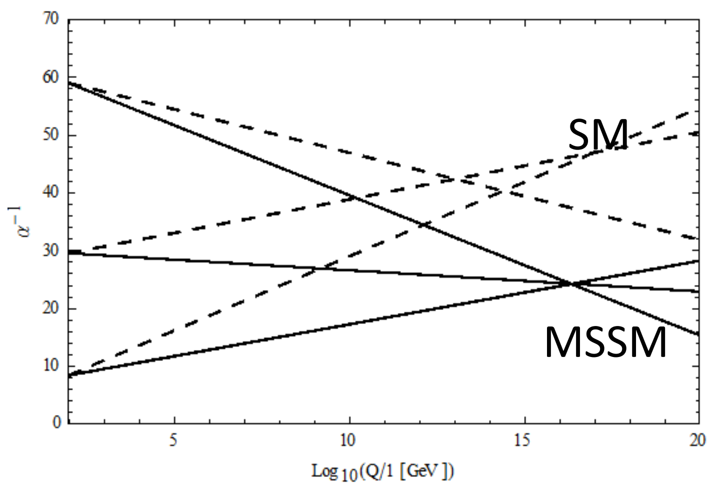
L. E. Ibanez and D. Lust [hep-th/9202046]



$$\left(\frac{M_{\rm Pl}}{M_s}\right)^2 \frac{1}{4\pi} = \frac{{\rm Vol}(T)}{g_s^2} \simeq 23 \qquad {\rm Vol}(T) \simeq T_1 T_2 T_3$$

The string coupling g_s may be strong.
Can we explain the experimental values of gauge couplings by $T \sim 1$?

Green-Schwarz term

L. E. Ibanez and H. P. Nilles ('86)

$$S_{\rm GS} = \frac{1}{24(2\pi)^5\alpha'}\int B^{(2)} \wedge X_8$$

$$X_8 = \frac{1}{24}{\rm Tr}F^4 - \frac{1}{7200}({\rm Tr}F^2)^2 - \frac{1}{240}({\rm Tr}F^2)({\rm tr}R^2) + \frac{1}{8}{\rm tr}R^4 + \frac{1}{32}({\rm tr}R^2)^2$$

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Dimensional reduction

$$\beta_3^k = \frac{(d_2)^2}{8\pi} d_{ijk} m_2^i m_2^j, \quad \beta_2^k = \frac{(d_1)^2}{4\pi} d_{ijk} m_1^i m_1^j,$$

with $d_1 = \sqrt{2}$ and $d_2 = 2$

$$\langle \mathrm{Re}\, S \rangle = \frac{1}{g_{4D}^2}$$

$$\langle \mathrm{Re}\, S + \beta_3^k \mathrm{Re}\, T^k \rangle = \frac{1}{g_{SU(3)}^2}$$

$$\langle \mathrm{Re}\, S + \beta_2^k \mathrm{Re}\, T^k \rangle = \frac{1}{g_{SU(2)}^2}$$

One-loop corrections

Gauge couplings

$$\langle \mathrm{Re}\, S \rangle = \frac{1}{g_{4D}^2} \Rightarrow$$

$$\langle \mathrm{Re}\, S + \beta_3^k \mathrm{Re}\, T^k \rangle = \frac{1}{g_{SU(3)}^2}$$

$$\langle \mathrm{Re}\, S + \beta_2^k \mathrm{Re}\, T^k \rangle = \frac{1}{g_{SU(2)}^2}$$

$$\left(\frac{1}{3} + \frac{N-3}{4}\right) \langle \mathrm{Re}\, S \rangle = \frac{1}{g_{U(1)_Y}^2}$$

$$\beta_3^k = \frac{(d_2)^2}{8\pi} d_{ijk} m_2^i m_2^j, \quad \beta_2^k = \frac{(d_1)^2}{4\pi} d_{ijk} m_1^i m_1^j, \qquad \text{with } d_1 = \sqrt{2} \text{ and } d_2 = 2$$

Because of such non-universality,
we have found that certain models can lead to gauge couplings consistent with experimental values.

MSSM

$$\frac{g_{SU(3)}^2(M_s)}{g_{U(1)_Y}^2(M_s)} \simeq 0.881$$

$$\frac{g_{SU(2)}^2(M_s)}{g_{U(1)_Y}^2(M_s)} \simeq 0.994$$

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$$\frac{g_{SU(2)}^2(M_s)}{g_{U(1)_Y}^2(M_s)} \simeq 0.775$$

Conclusion

○ We have constructed the standard-like models from SO(32) heterotic string theory, which are realistic from both viewpoints of massless spectrum and the gauge couplings.

Ongoing work

○ Phenomenological aspects of SO(32) heterotic string
  - The phenomenology of the non-univeresal gaugino mass
  - Yukawa hierarchies between the generation
  - Cosmological point of view (e.g., the inflation mechanism)

○ The formulation of the Yang-Mills flux on 2D world-sheet.