ICRR
Institute for Cosmic Ray Research
University of Tokyo

Revisiting Metastable Cosmic String Breaking

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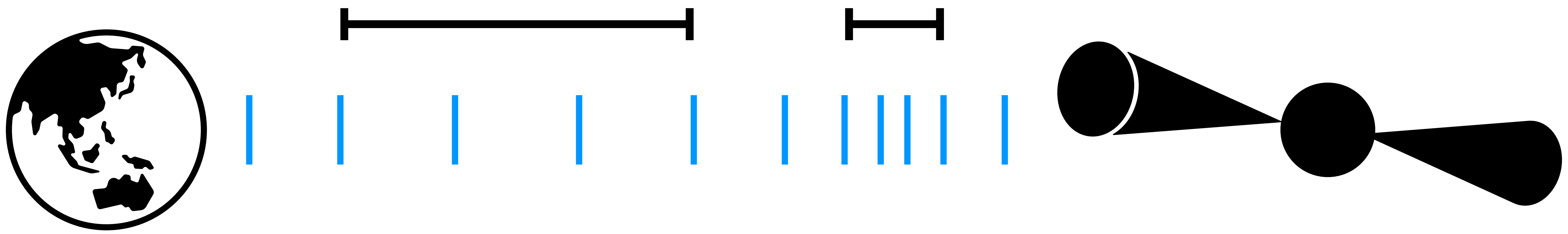
Akifumi Chitose (ICRR, U. Tokyo)

Based on:
JHEP 04 (2024) 068 [arXiv:2312.15662]
Akifumi Chitose, Masahiro Ibe, Yuhei Nakayama, Satoshi Shirai and Keiichi Watanabe

Stochastic Gravitational Wave Background

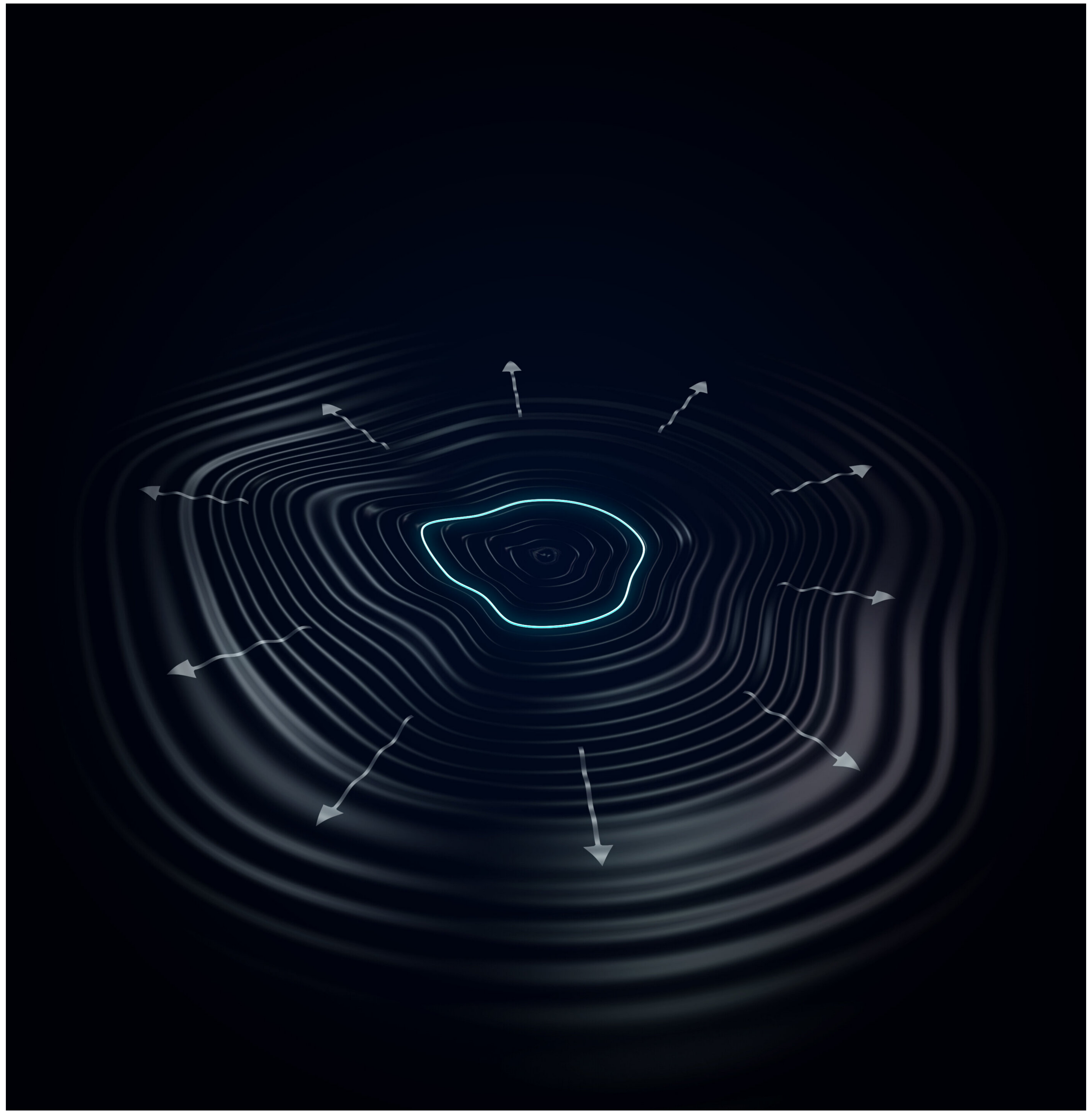
- Evidenced by PTA observations (NANOGrav, InPTA, EPTA, PPTA, CPTA)
  - Observed at nHz range
- Many possible origins
  - Black holes?
  - Phase transition?
  - Domain walls?
  - ….



Cosmic Strings

Probing BSM with GW

- Linear solitons in QFT
- Created in the Universe by
  e.g. spontaneous U(1) breaking
- Predicted by many BSM physics
  - GUT
  - Dark photon

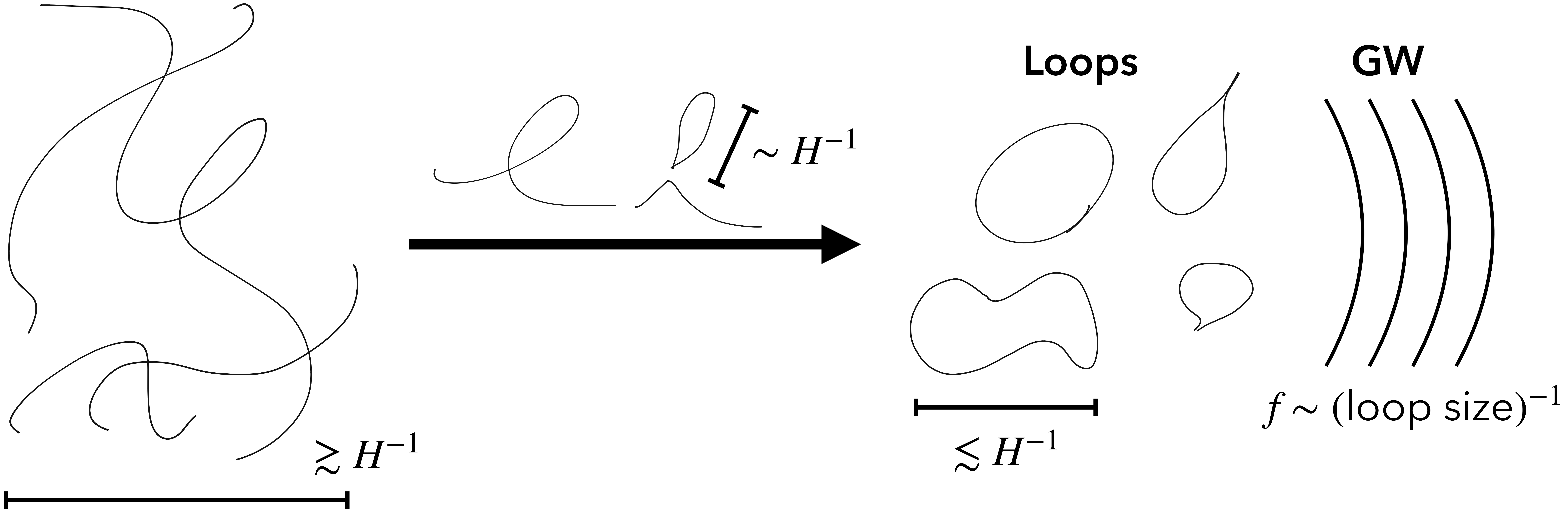


Credit: Daniel Dominguez from CERN's Education, Communications & Outreach (ECO) Department.

Cosmic Strings

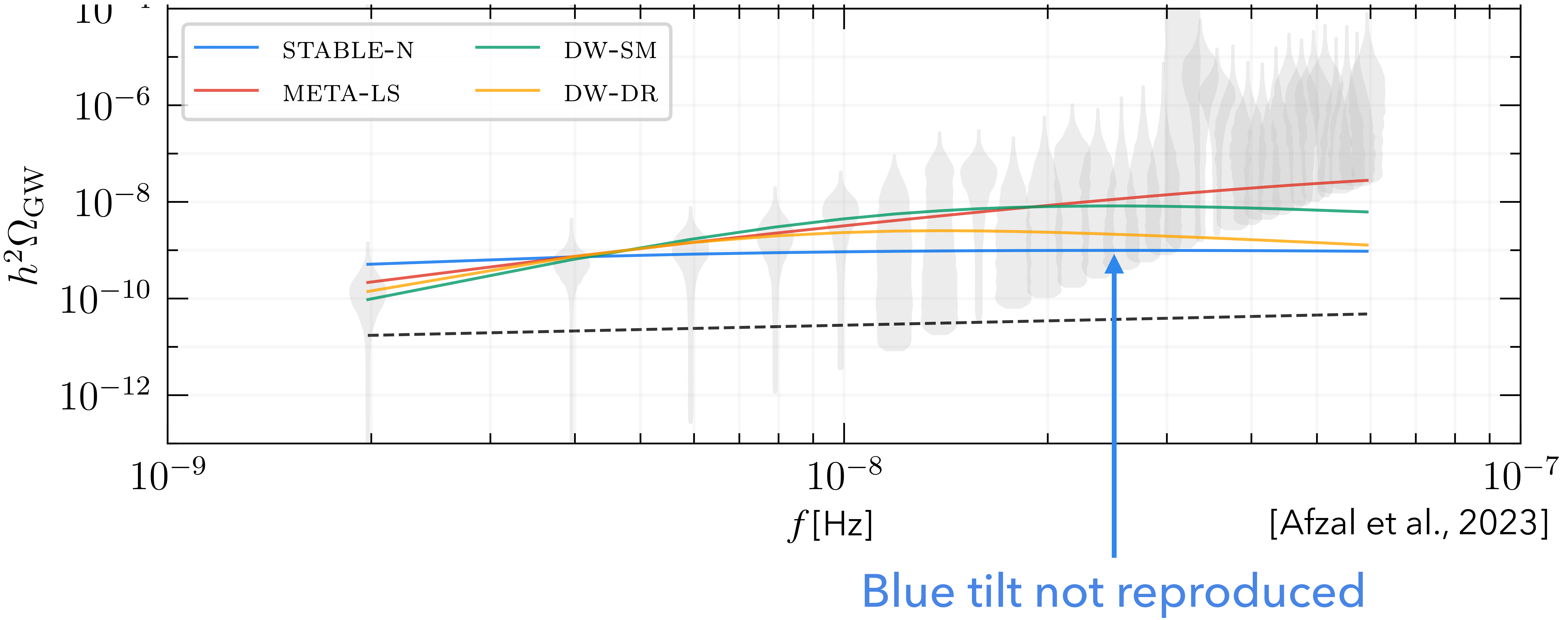
Gravitational waves from loops

Network of long strings



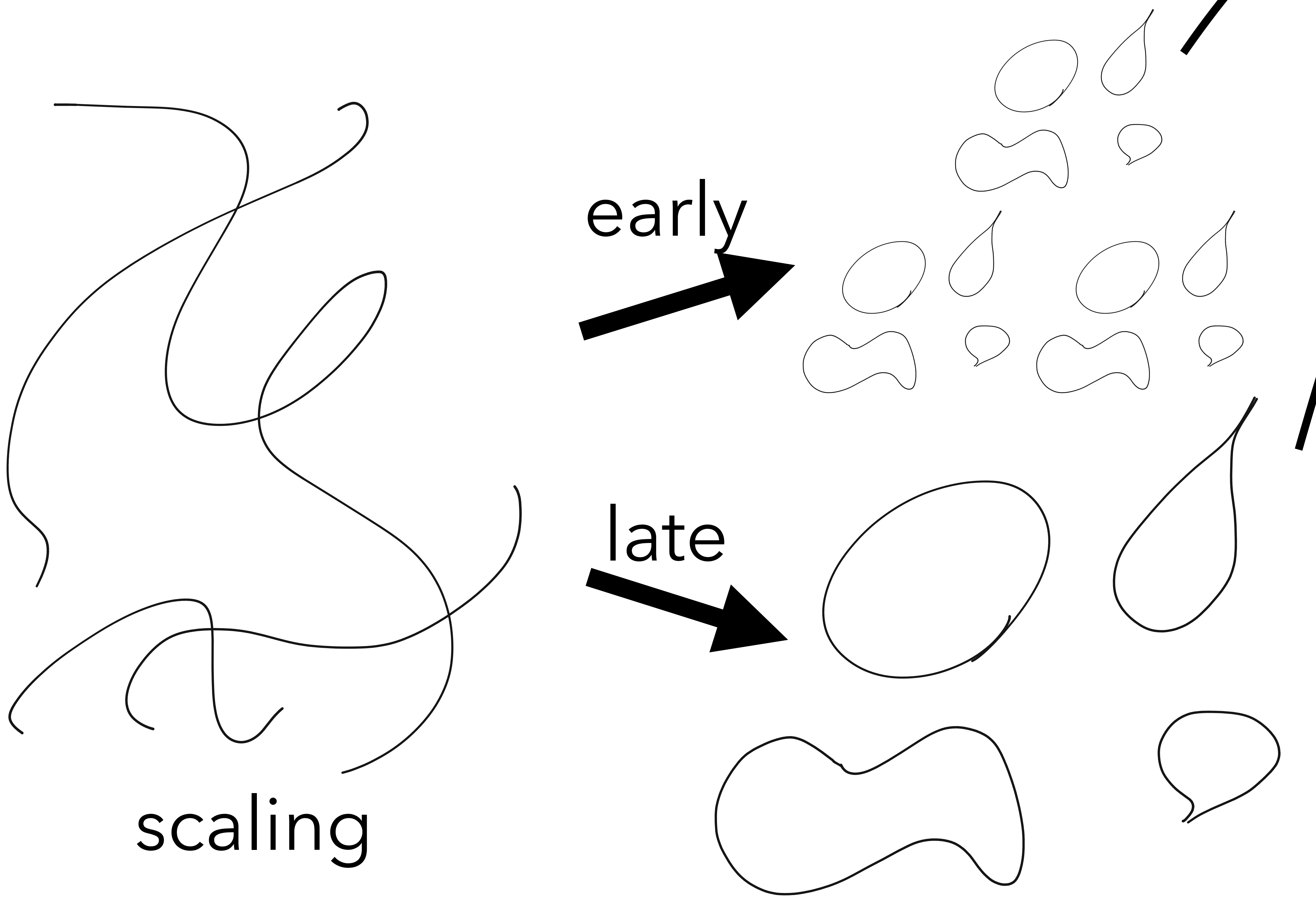
Cosmic Strings for PTA

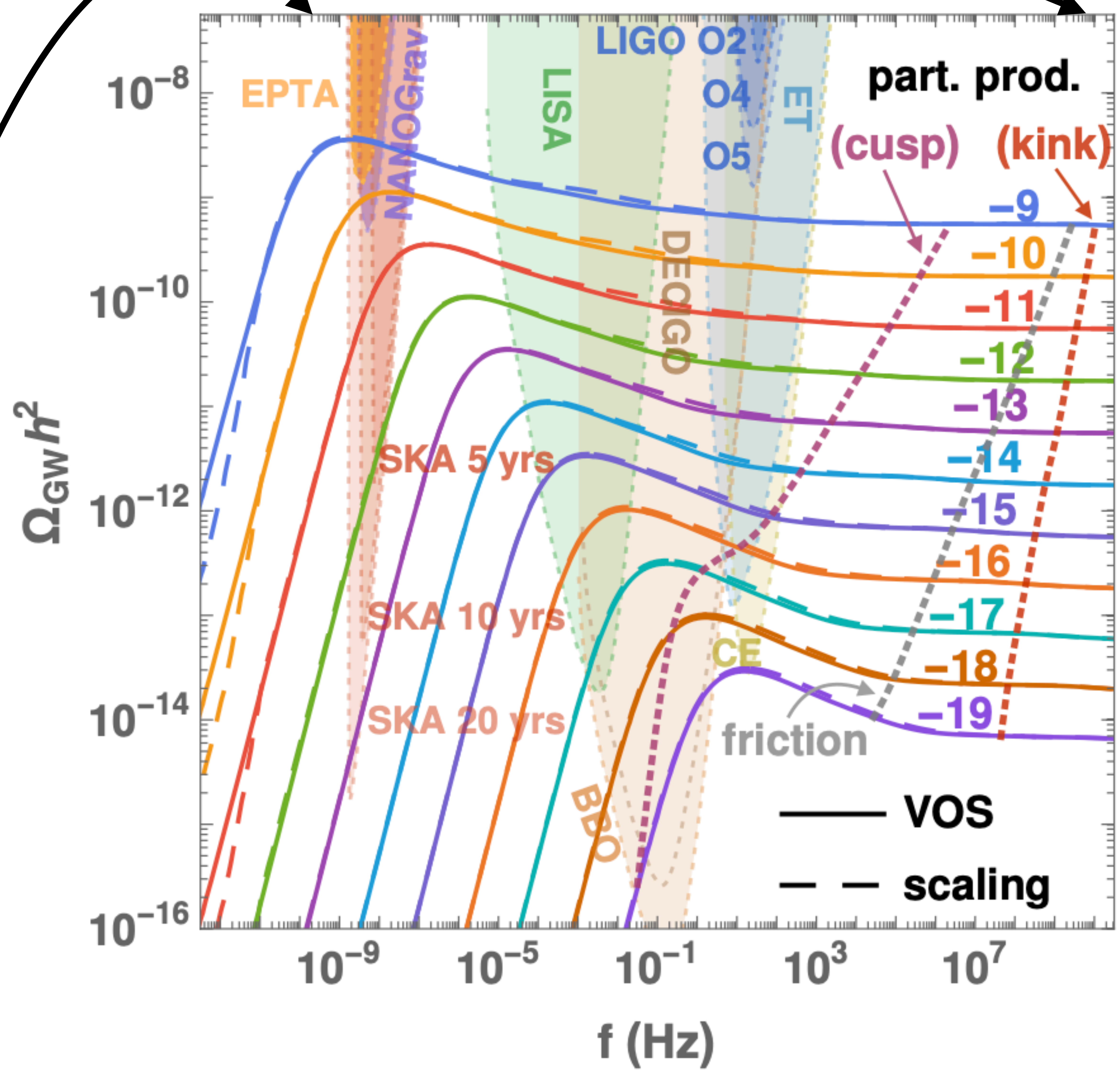
Failure of stable cosmic strings



Cosmic Strings for PTA

Stable strings vs. PTA





Only parameter: tension

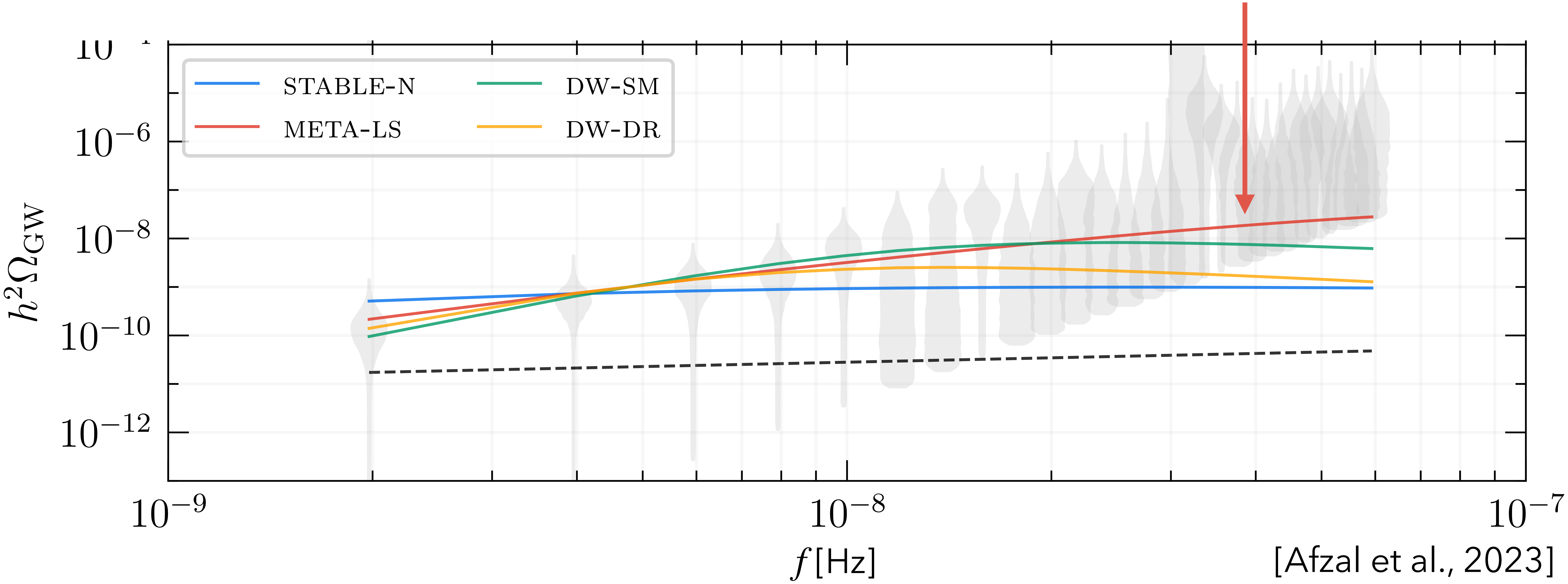
→ cannot fit height & tilt

[Gouttenoire et al., 2019]

Cosmic Strings for PTA

Failure of stable cosmic strings

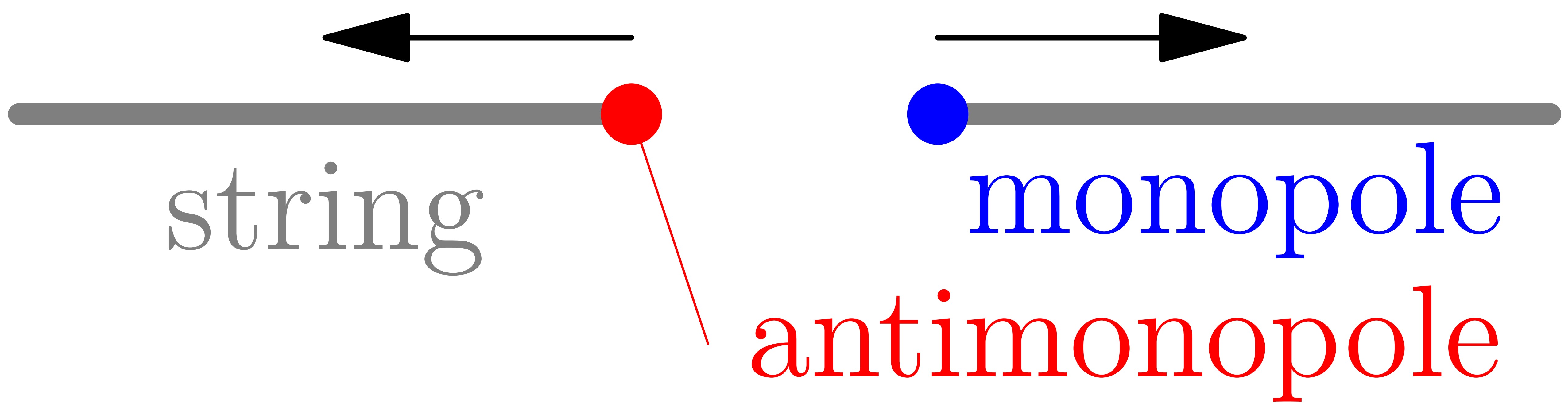
Metastable cosmic strings work nicely



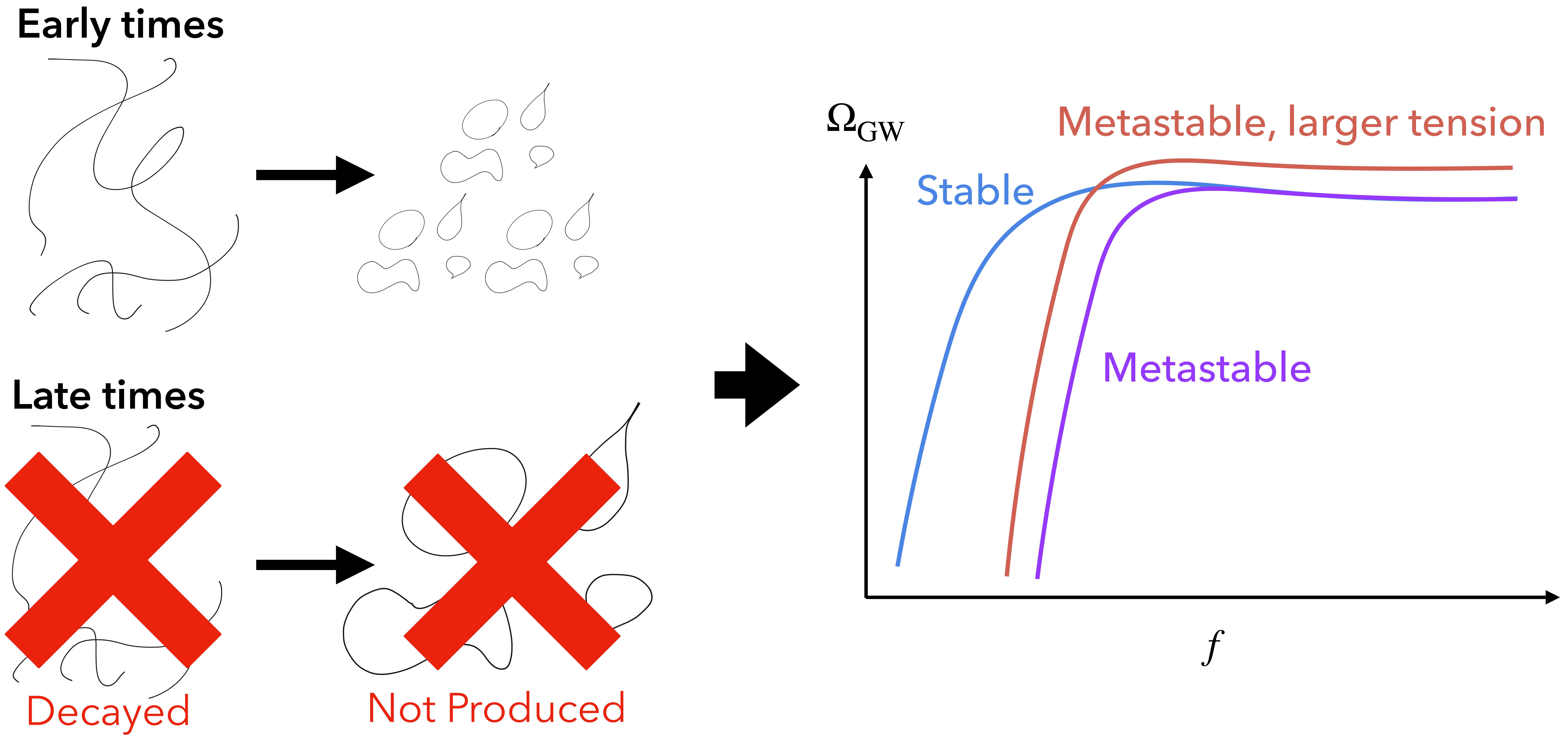
[Afzal et al., 2023]

Metastable Cosmic Strings

- Spontaneously cut by monopole-antimonopole pair creation
- Arise from e.g. $G \to \mathrm{U}(1) \to 1$ w/ G: simply connected



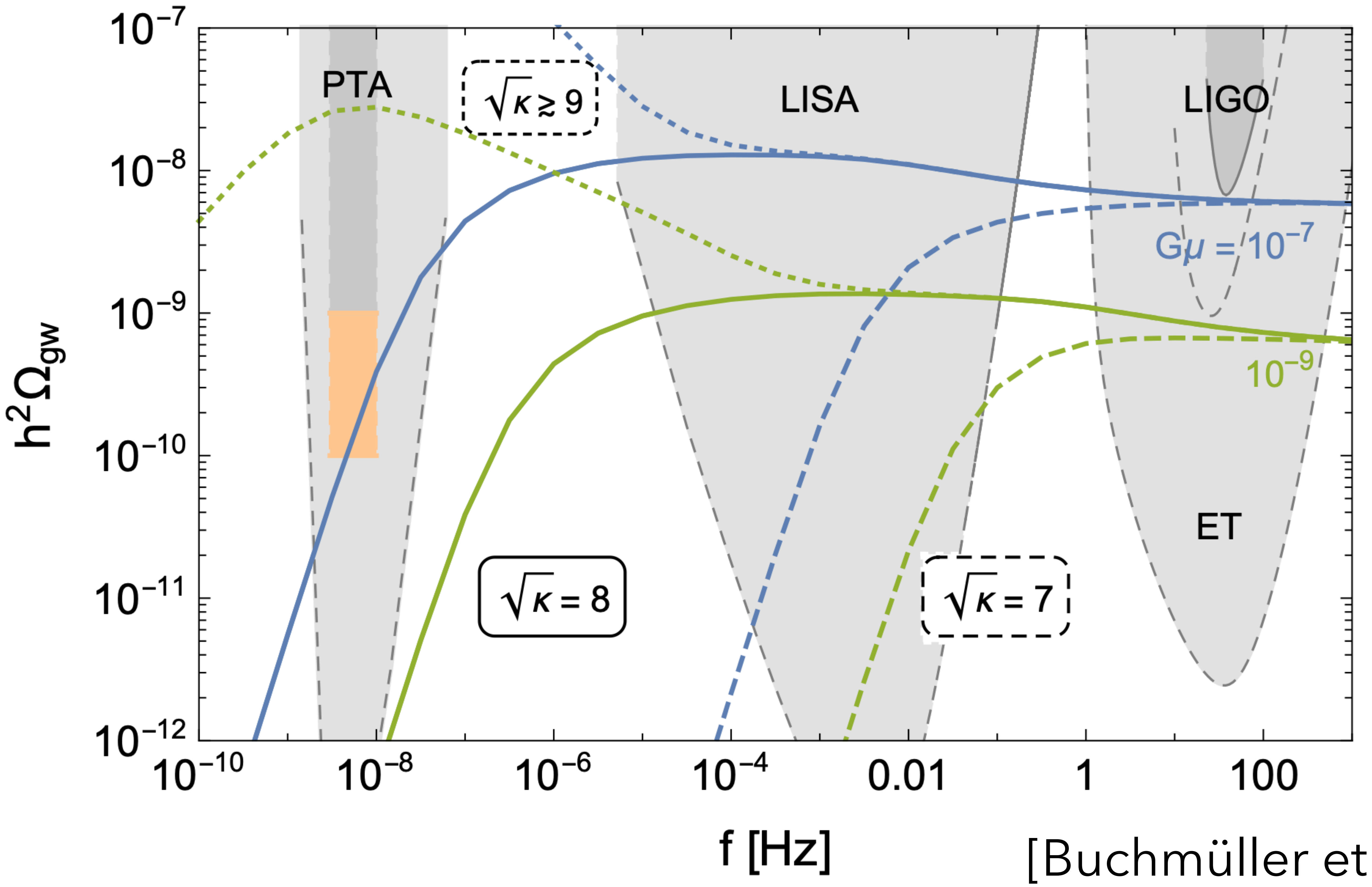
Metastable Cosmic Strings



Metastable Cosmic Strings

GW spectrum depends on the decay rate

- Precise estimate of the decay rate is crucial
  - For decay rate $\propto \exp[-\pi\kappa]$, $\sqrt{\kappa} \sim 8$

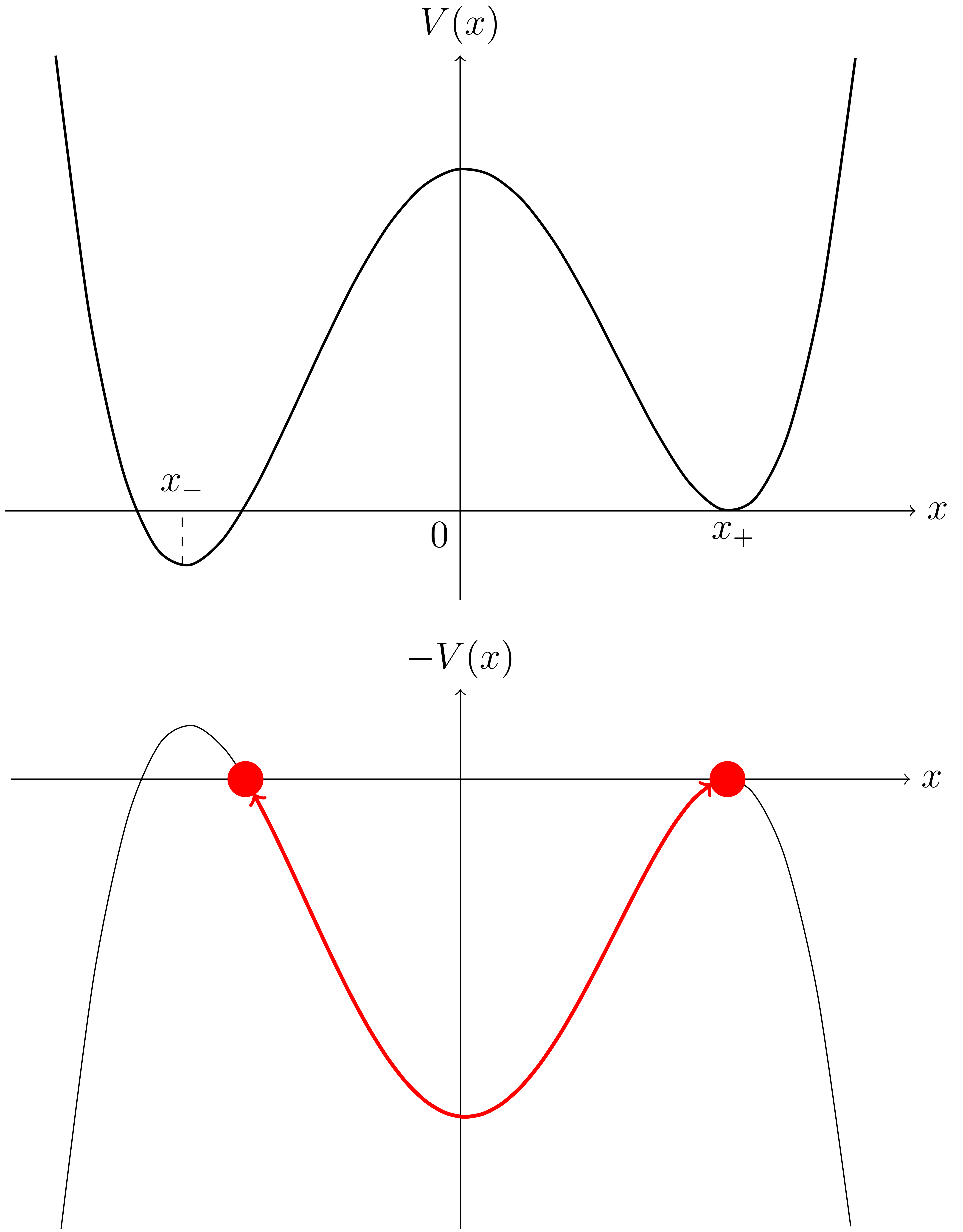


[Buchmüller et al., 2023]

String breaking rate

Tunneling and bounce see e.g. [Coleman, 1985]

- Procedure:
  - Go to imaginary time
    - $\approx$ invert the potential
  - Find the bounce solution
    - Action: S_B
  - Decay rate: $\Gamma \sim \exp[-S_B]$



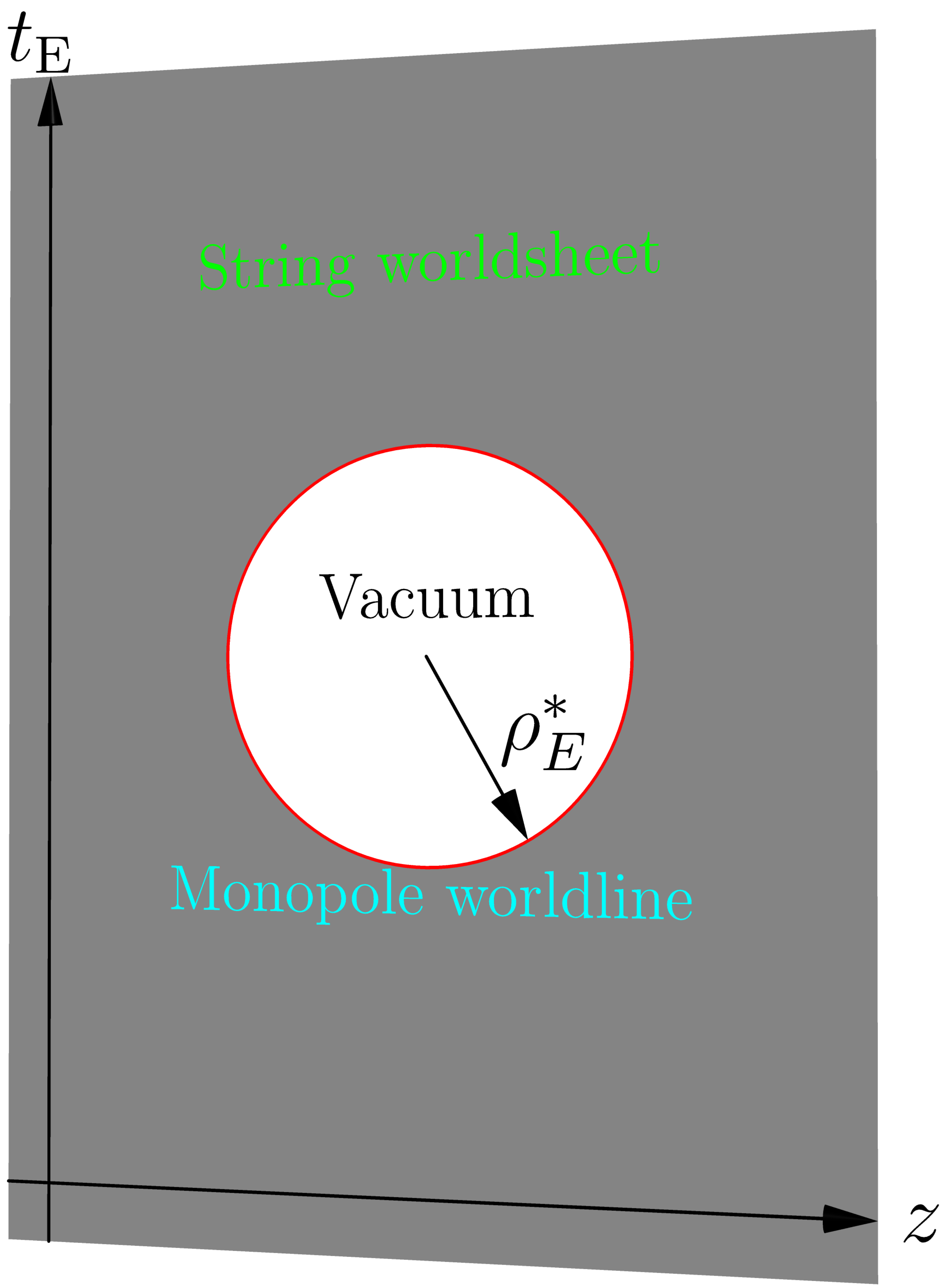
String breaking rate

Preskill-Vilenkin approximation [Preskill & Vilenkin, 1992]

- Neglect monopole size and string width
- $S_E = 2\pi\rho_E^* M_M - \pi\rho_E^{*2} T_{\rm str}$
  - $\rightarrow \rho_E^* = M_M / T_{\rm str}$, $S_B = \pi M_M^2 / T_{\rm str} = \pi\kappa$
    - M_M: monopole mass, $T_{\rm str}$: string tension
- String width $T_{\rm str}^{-1/2} \ll \rho_E^*$ required

$\rightarrow \sqrt{\kappa} \gg 1 \cdots$ **Is this OK for PTA ($\sqrt{\kappa} \sim 8$)?**

$\rightarrow$ Alternative evaluation desired



Re-evaluation of Bounce Action

Setup

SU(2) gauge theory w/ adjoint Higgs & fundamental Higgs

- $\mathscr{L} = -\frac{1}{4g^2}F^2 - |Dh|^2 - \left(D\overrightarrow{\phi}\right)^2 - V_{\text{Higgs}}(h, \phi)$
  - h: SU(2) fundamental, ϕ: SU(2) adjoint
- $V_{\text{Higgs}}(h, \phi) = \lambda\left(|h|^2 - v^2\right)^2 + \tilde{\lambda}\left(\overrightarrow{\phi}^2 - V^2\right)^2 + \gamma\left|\left(\phi^a\frac{\tau^a}{2} - \frac{V}{2}\right)h\right|^2$
  - Assumptions: λ, $\tilde{\lambda}$, $\gamma > 0$, $V > v$

Setup

Symmetry breaking pattern

- $V_{\text{Higgs}}(h,\phi) = \lambda\left(|h|^2 - v^2\right)^2 + \tilde{\lambda}\left(\vec{\phi}^2 - V^2\right)^2 + \gamma\left|\left(\phi^a\frac{\tau^a}{2} - \frac{V}{2}\right)h\right|^2$

$$\phi^a = V\delta_3^a$$

$$\gamma V^2 |h_2|^2$$

- SU(2) → U(1) by $\phi^a = V\delta_3^a$
  - U(1) generator: $\tau^3/2$
- U(1) → 1 by $h_i = v\delta_i^1$

Setup

Cosmic Strings and Monopoles

- 1st SSB: SU(2) → U(1) by $\phi = V\delta_3^a$
  - Monopoles formed by ϕ
- 2nd SSB: U(1) → 1 by $h_1 = ve^{i\times 0}$
  - Cosmic strings formed by h_1
  - But SU(2) is simply connected → only metastable

$\sqrt{\kappa_{PV}} \propto V/v$

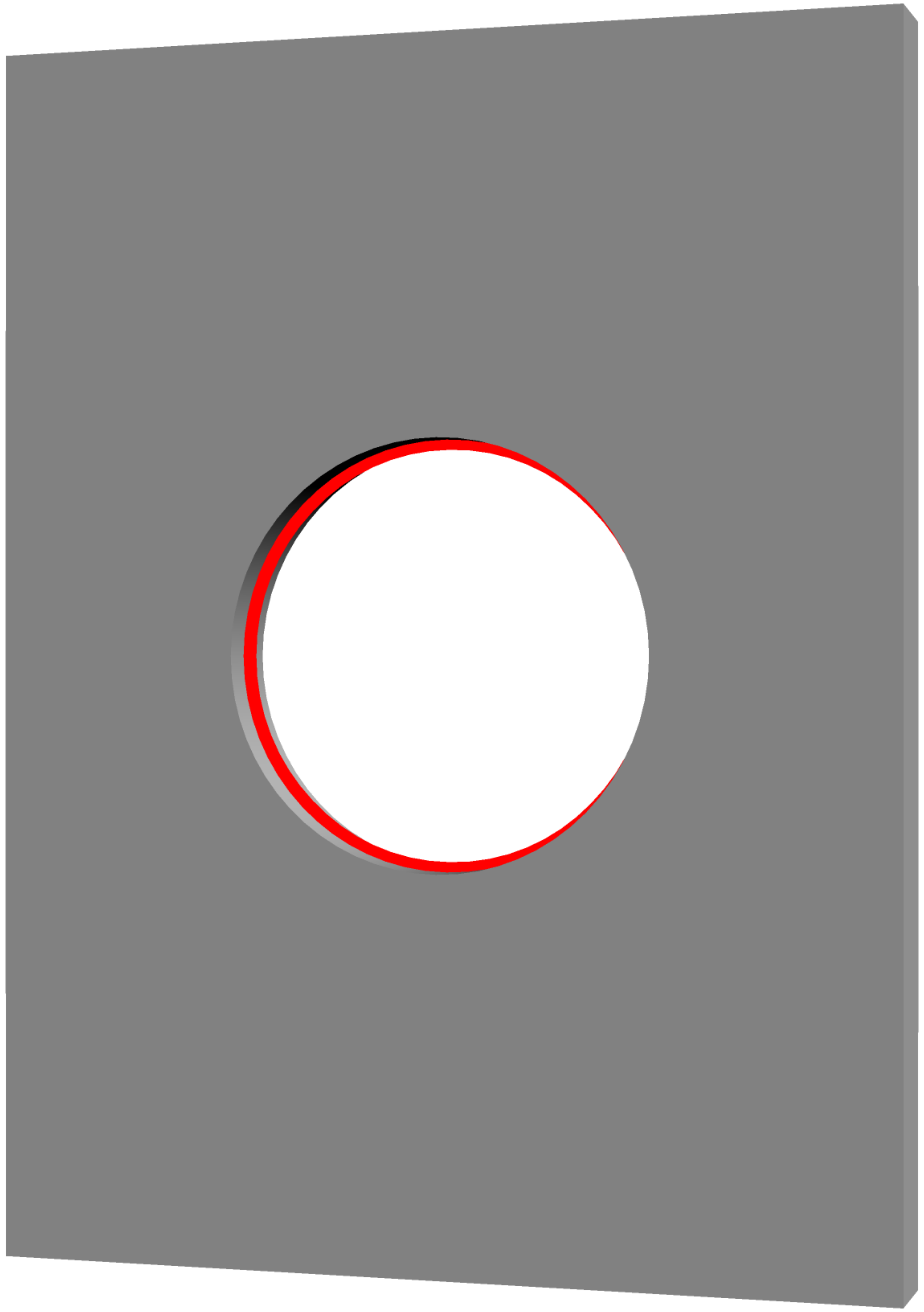
→ interested in $V/v = \mathcal{O}(1)$

Strategy

How to evaluate the bounce action?

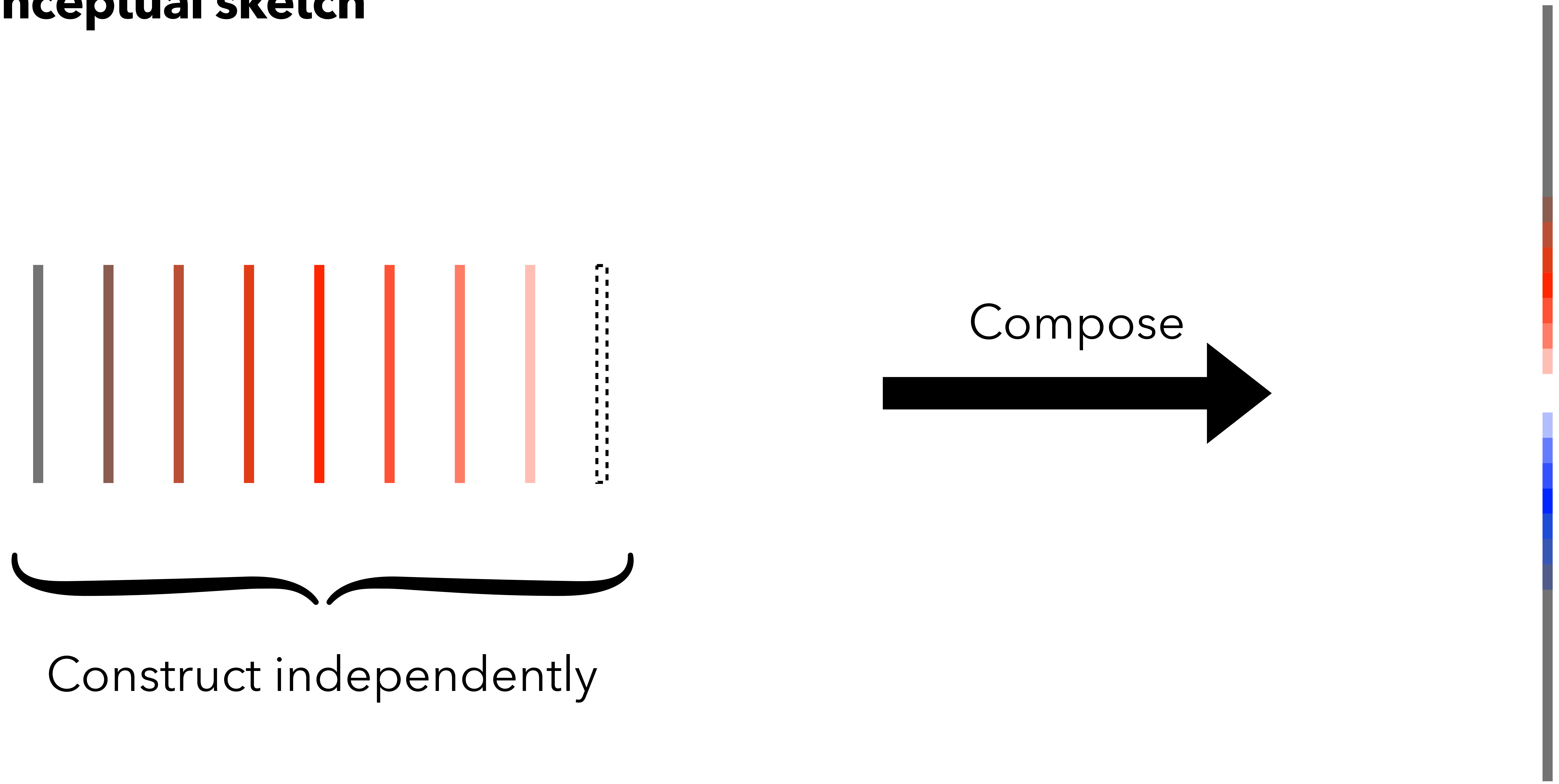
- Solve 4D Euclidean field equation?
  - Bounce: saddle point of S_E
    → nontrivial algorithm needed
  - Less symmetric than vacuum decay

→ Alternative strategy



Strategy

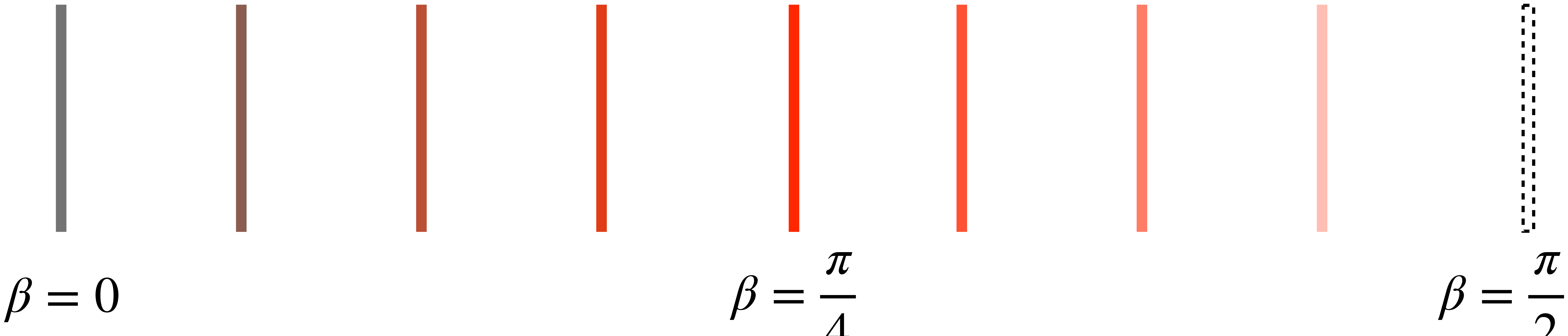
Conceptual sketch



Strategy

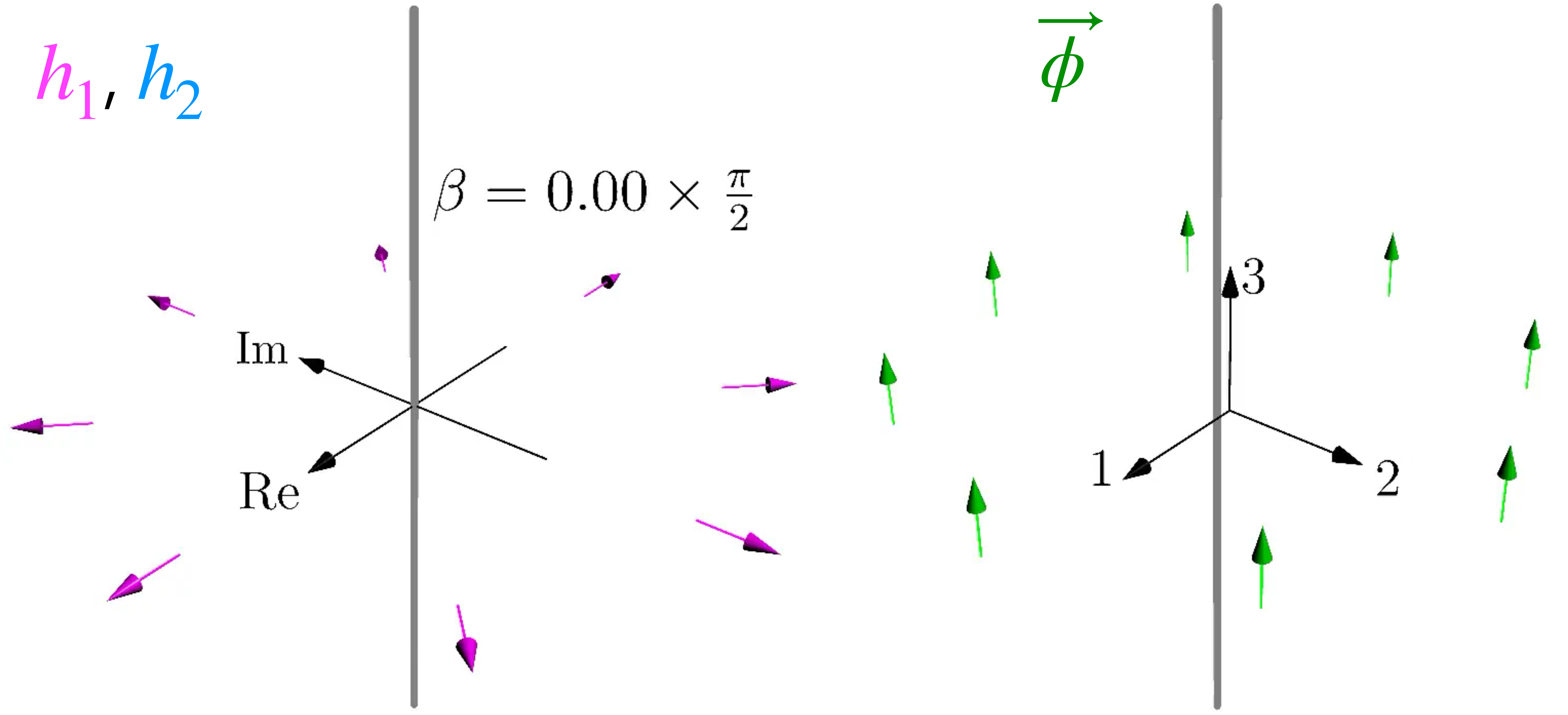
Step 1: Build "excited strings" with an Ansatz

- Make β-dependent static string configuration
  - β: unwinding parameter (ordinary string at $\beta = 0$, vacuum at $\beta = \pi/2$)
  - 4D field configuration $\leftarrow$ β-dependent Ansatz [Shifman & Yung, 2002]



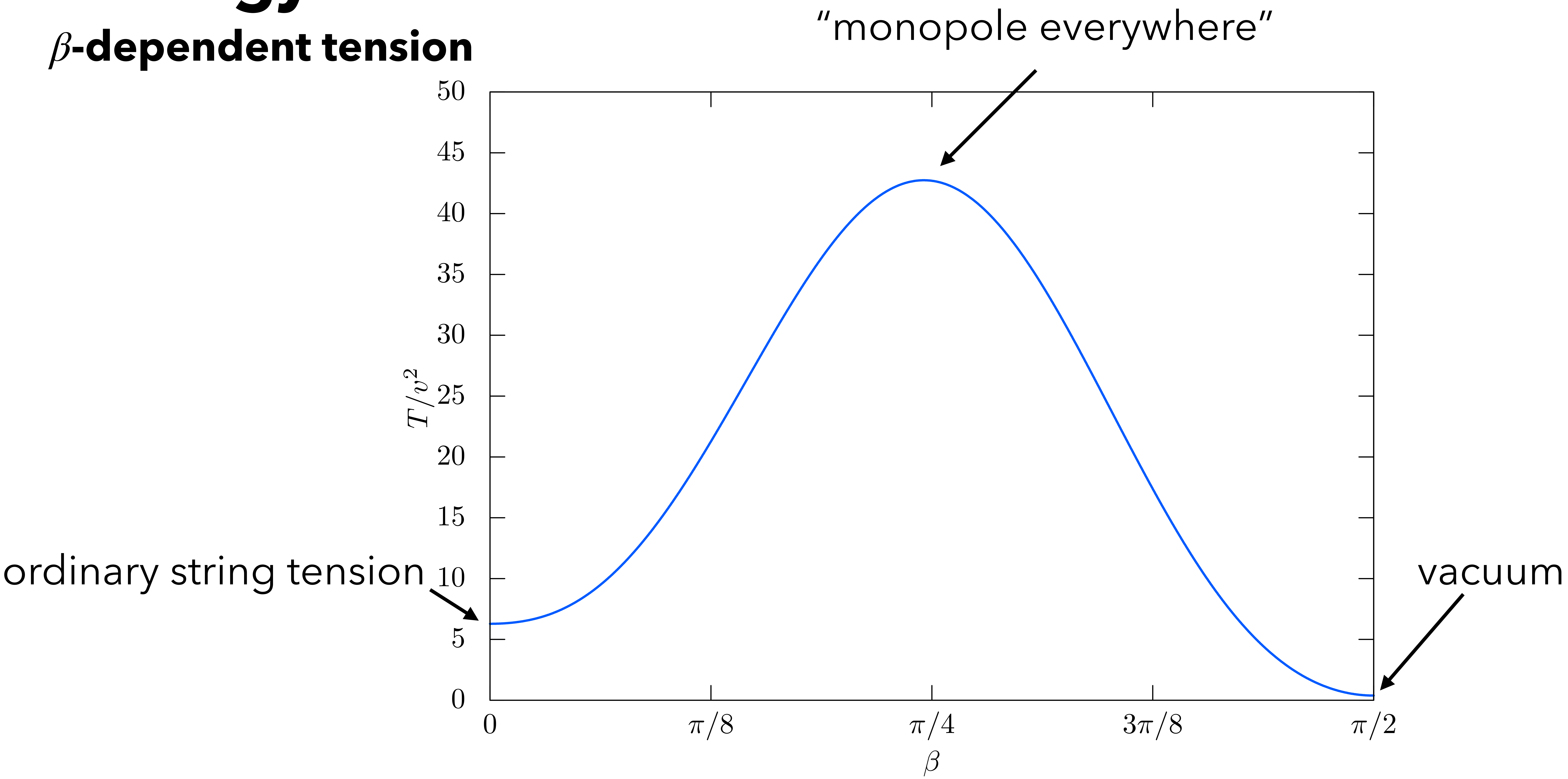
Strategy

Unwinding the string (ctd.)



Strategy

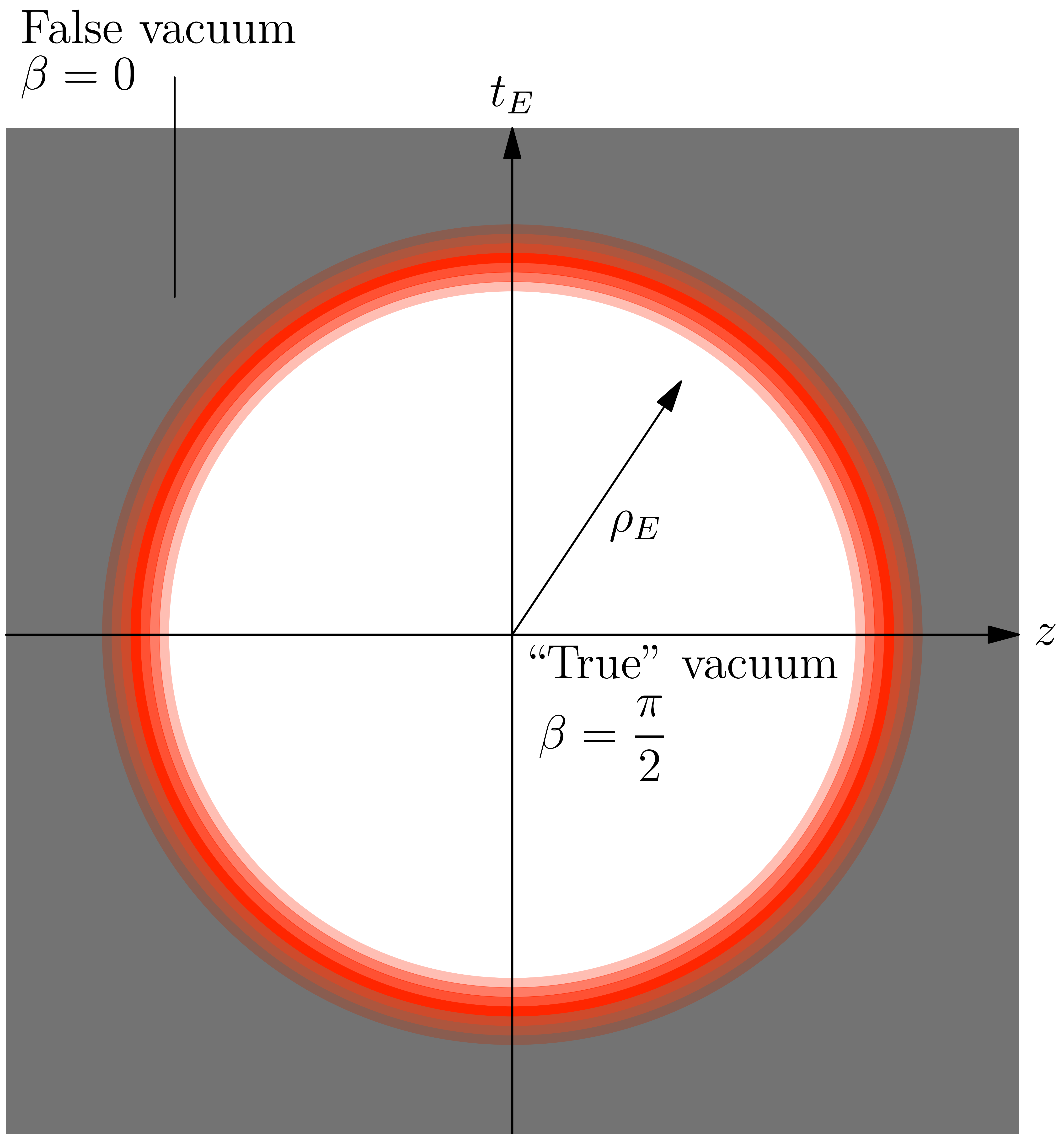
β-dependent tension



Strategy

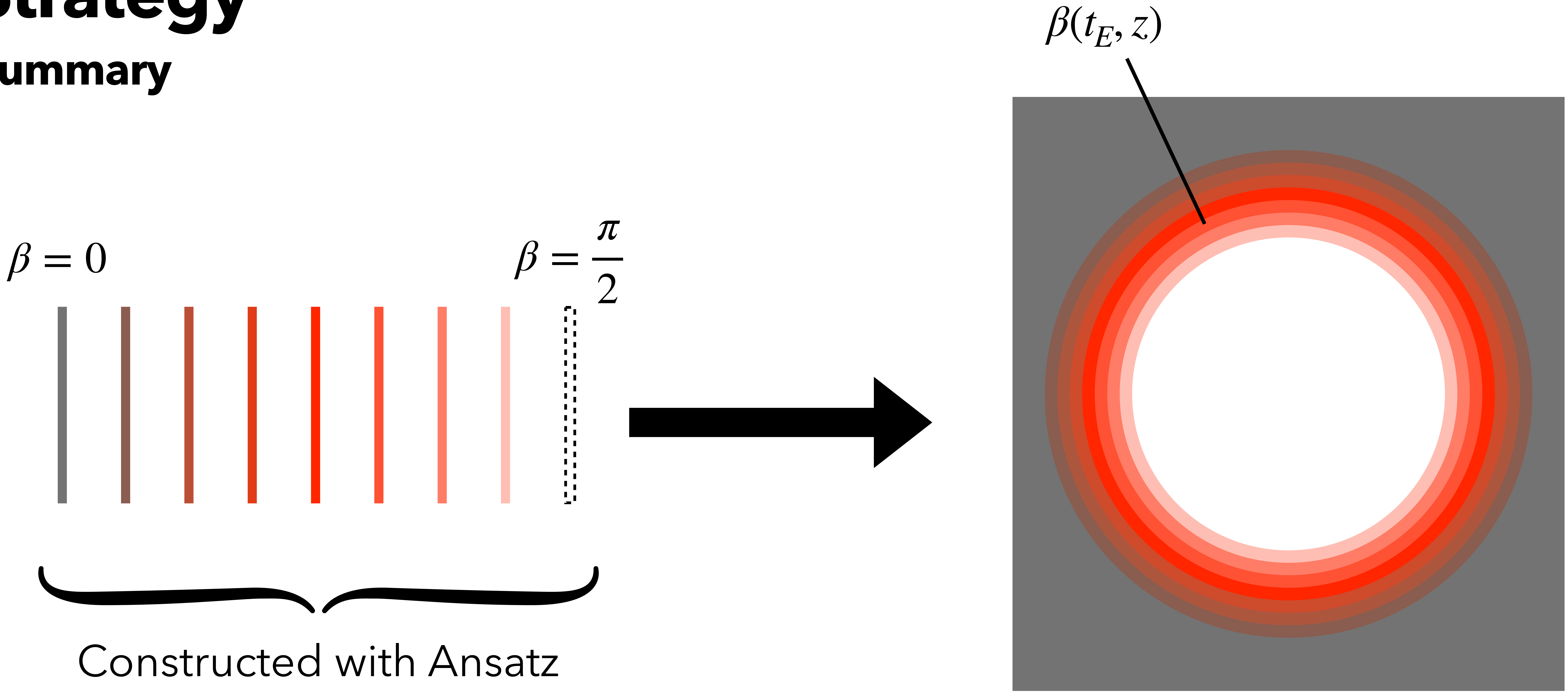
Step 2: Promote β to a field on the string

- Construct effective 2D theory about $\beta(t_E, z)$
- The bubble is circular
  - Reduces to 1D theory:
    $$S_E = 2\pi \int_0^\infty \rho_E \, \mathrm{d}\rho_E \left[\frac{1}{2} \mathscr{K}_{\mathrm{eff}}(\beta) \beta'^2 + T(\beta) \right]$$
  - EoM solvable → bounce action



Strategy

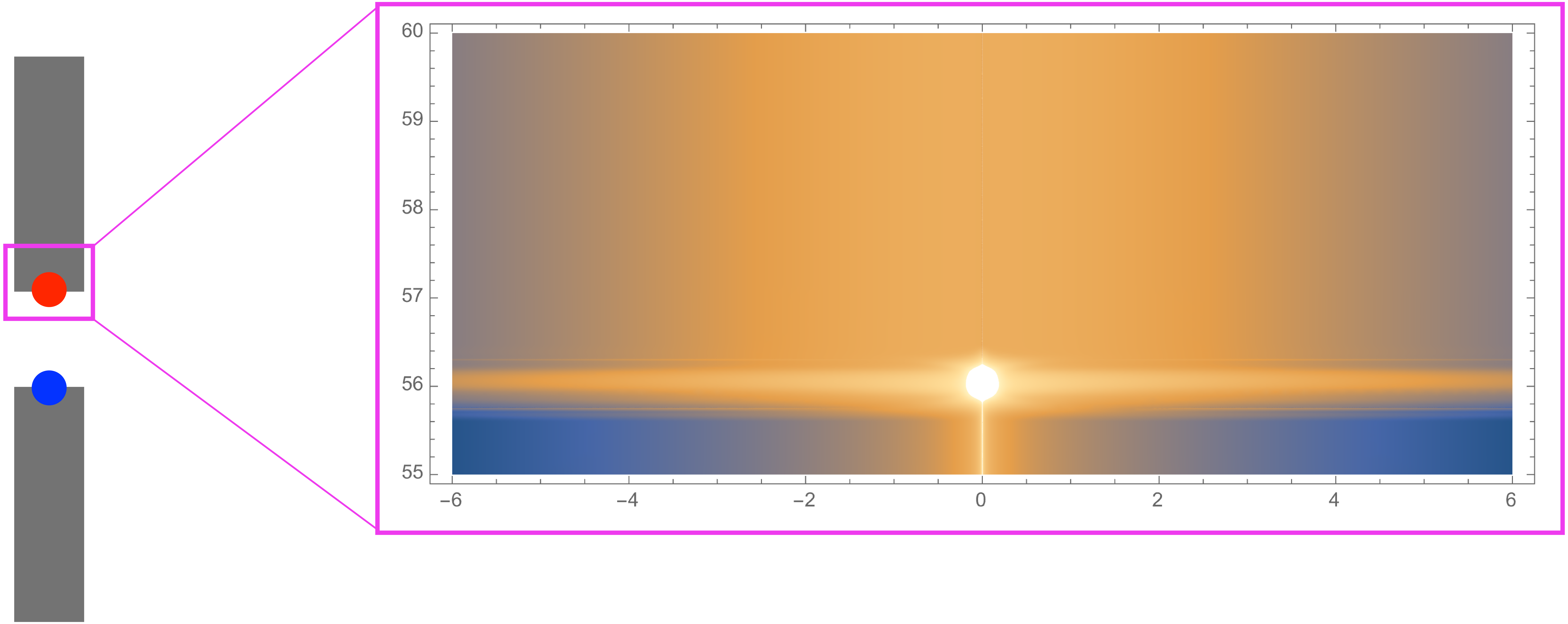
Summary



→ Upper bound on the actual bounce action

Results

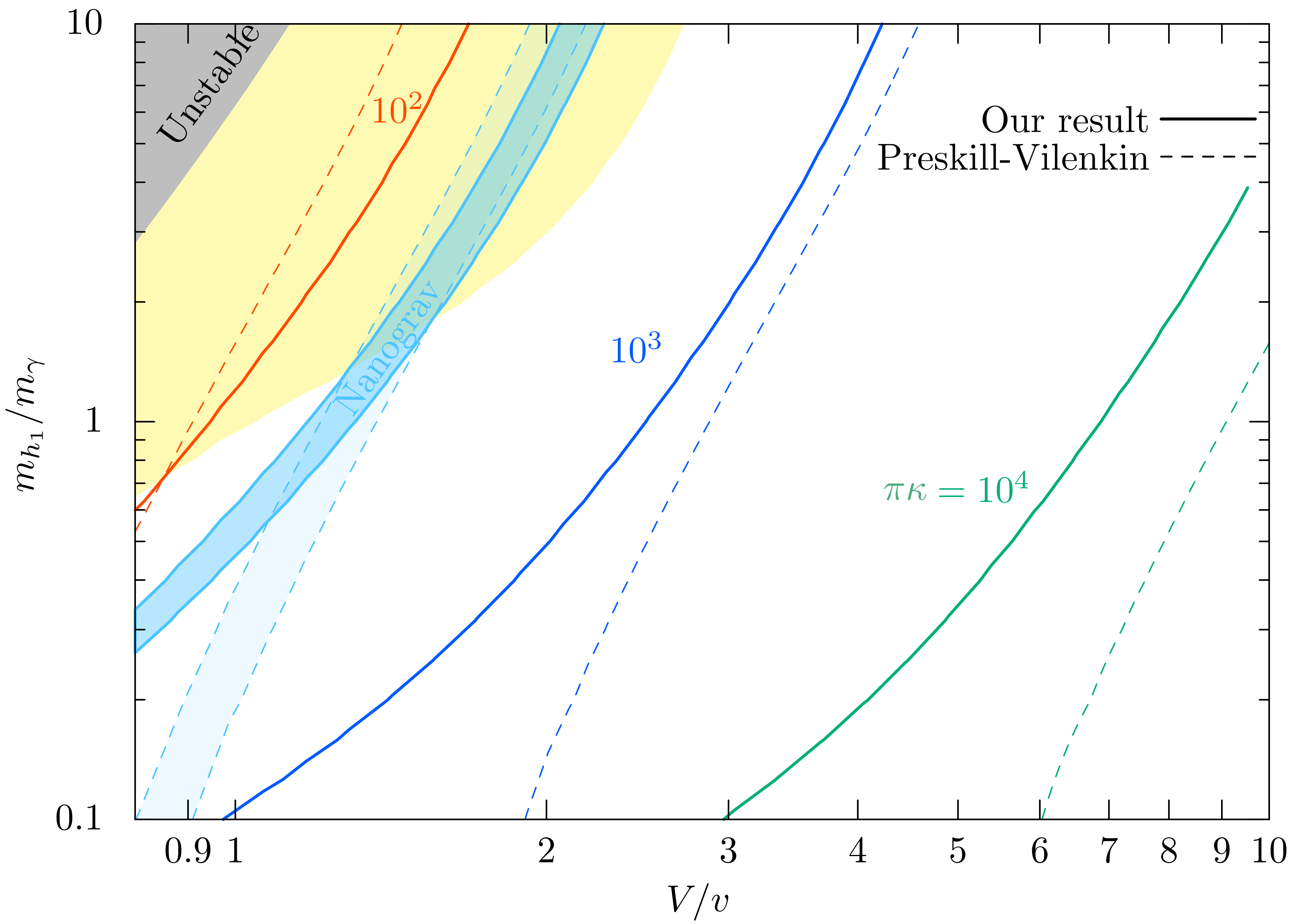
Sample field configuration



Results

Interpretation of NANOGrav results

- Yellow: our S_B < Preskill-Vilenkin
  - Overlaps with NANOGrav region
  - Modifies the interpretation



Conclusions & Outlook

- A robust upper bound on the bounce action for string breaking was calculated
  - free of the conventional assumption
- The Preskill-Vilenkin approximation can be inappropriate to interpret the PTA data
- Next steps:
  - Optimal bounce action?
  - More realistic setup?
  - Inflation model?

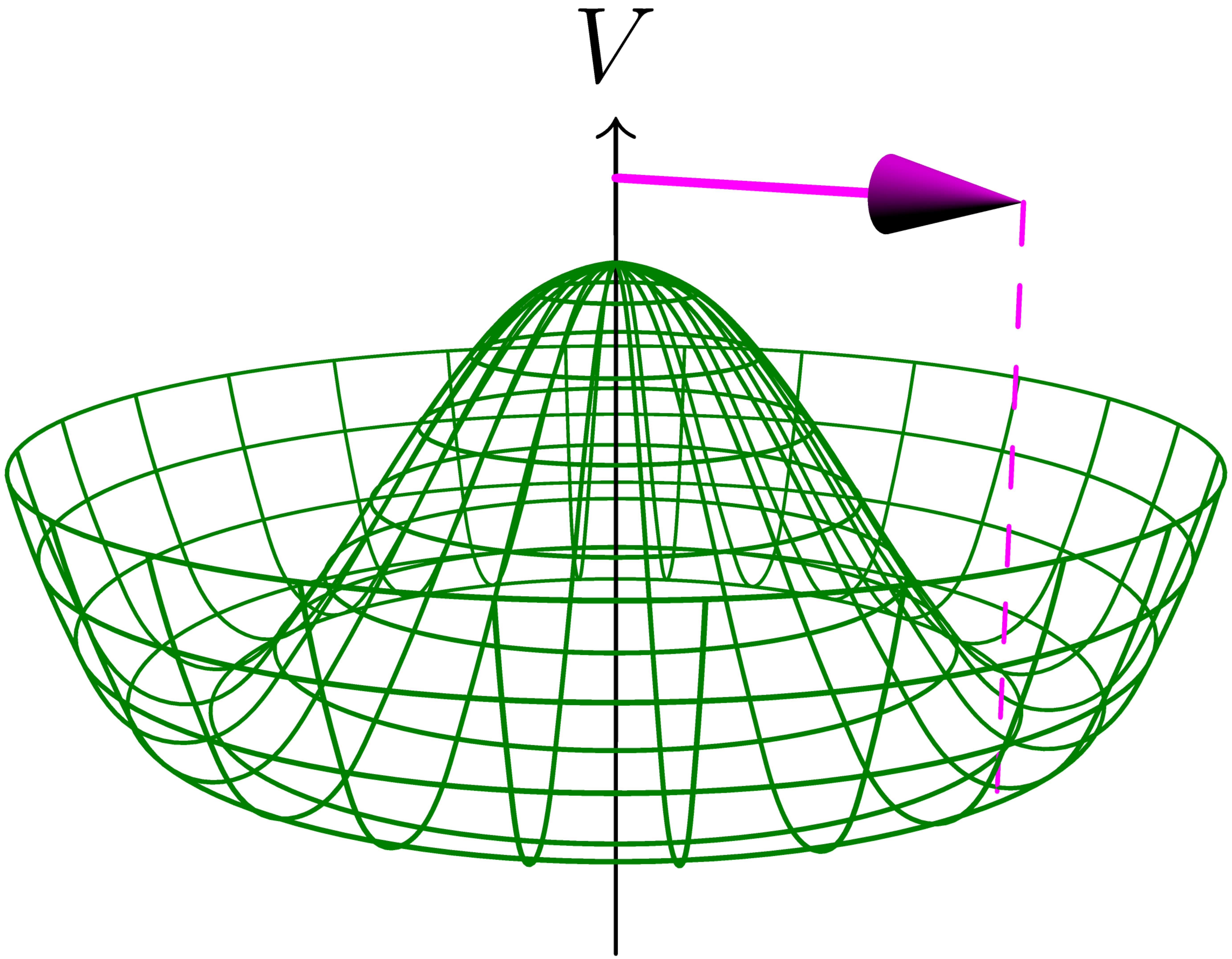
Thank you!

Backup

Cosmic Strings

from U(1) breaking

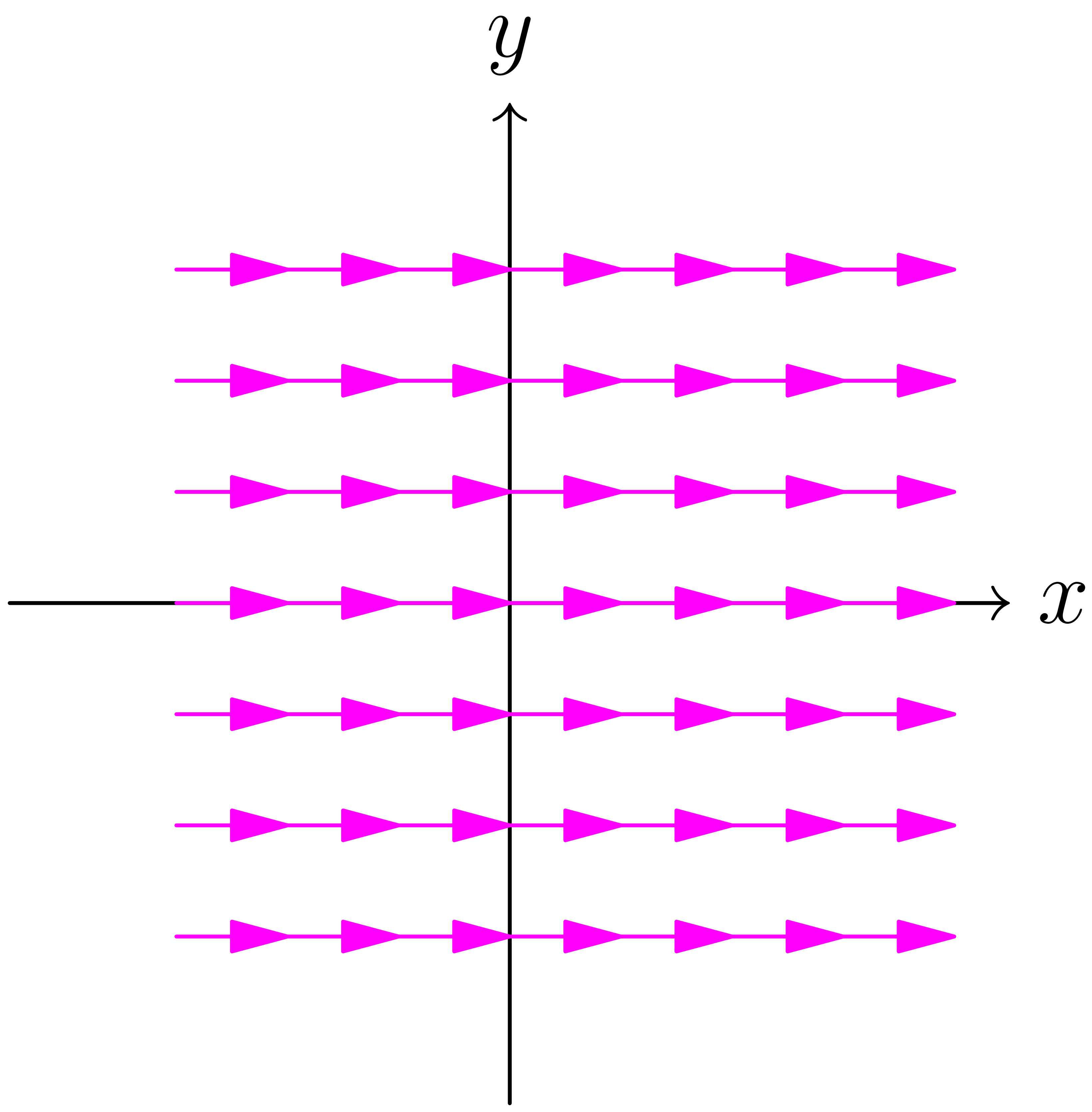
- Simplest setup: abelian Higgs
- $V(\phi) = \lambda \left(\phi^\dagger \phi - v^2\right)^2$
- U(1): $\phi \to e^{i\alpha}\phi$
  - broken by $\langle \phi \rangle = v$



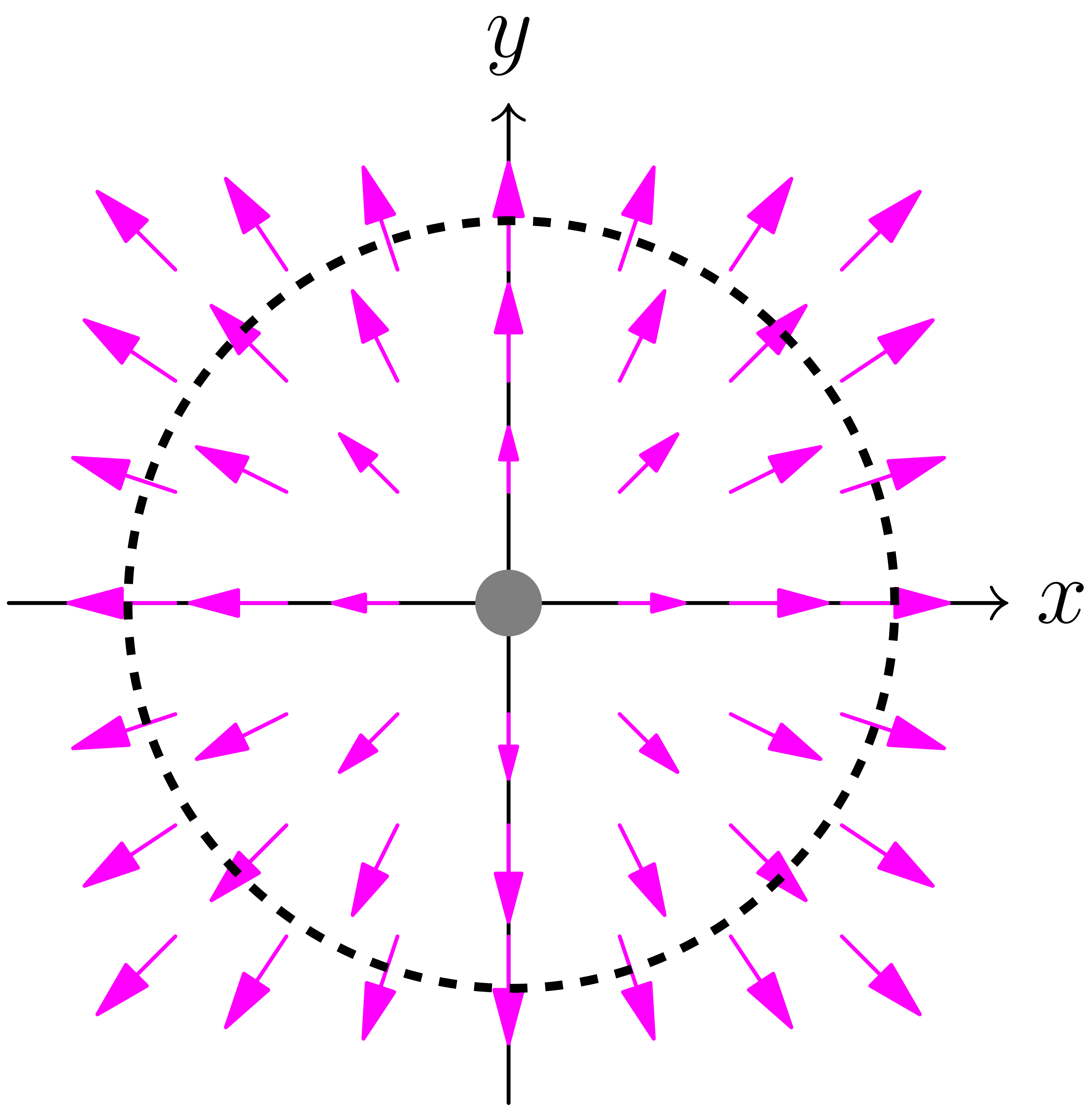
Cosmic Strings

from U(1) breaking (ctd.)

Vacuum

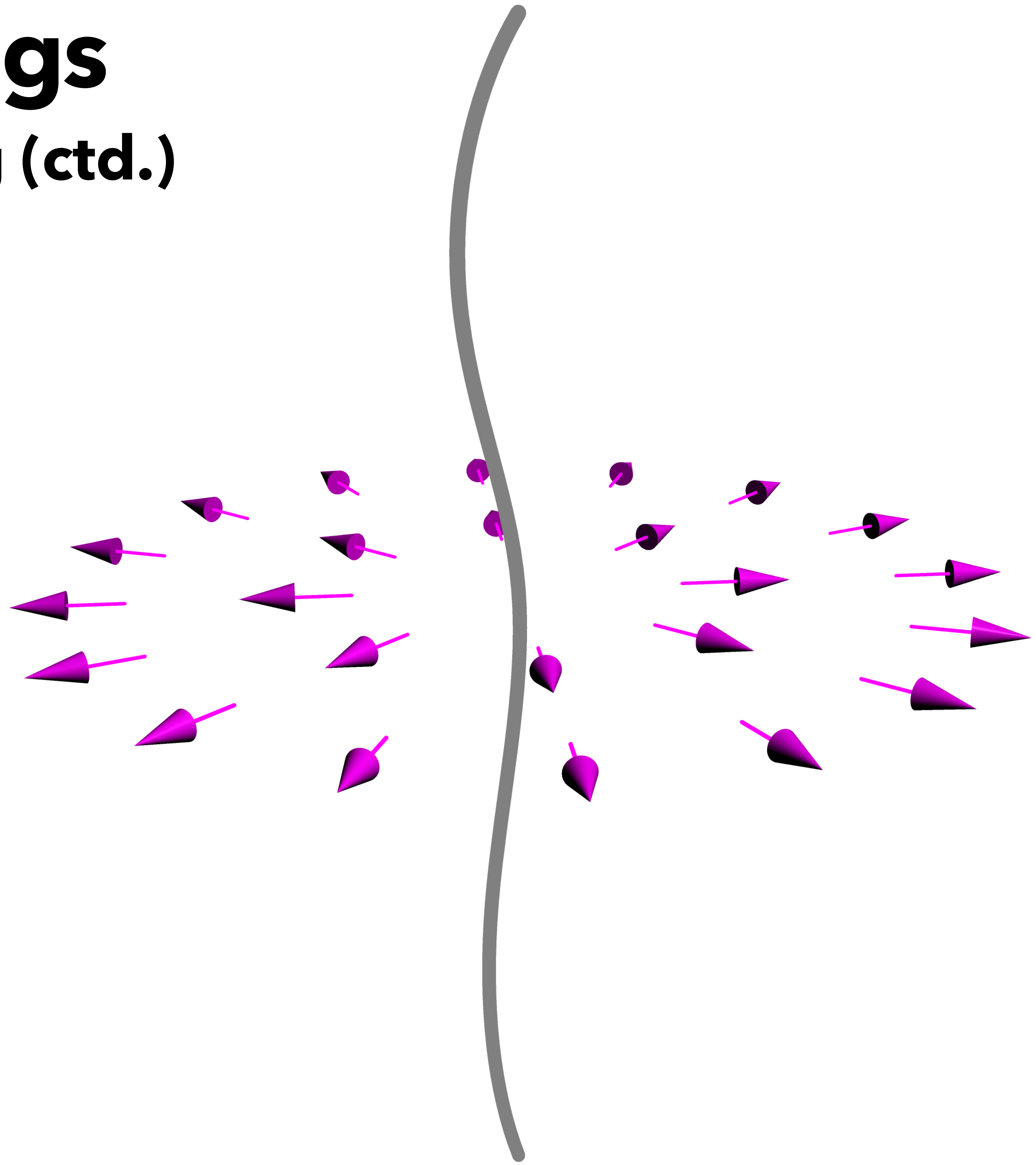


Wound about z axis



Cosmic Strings

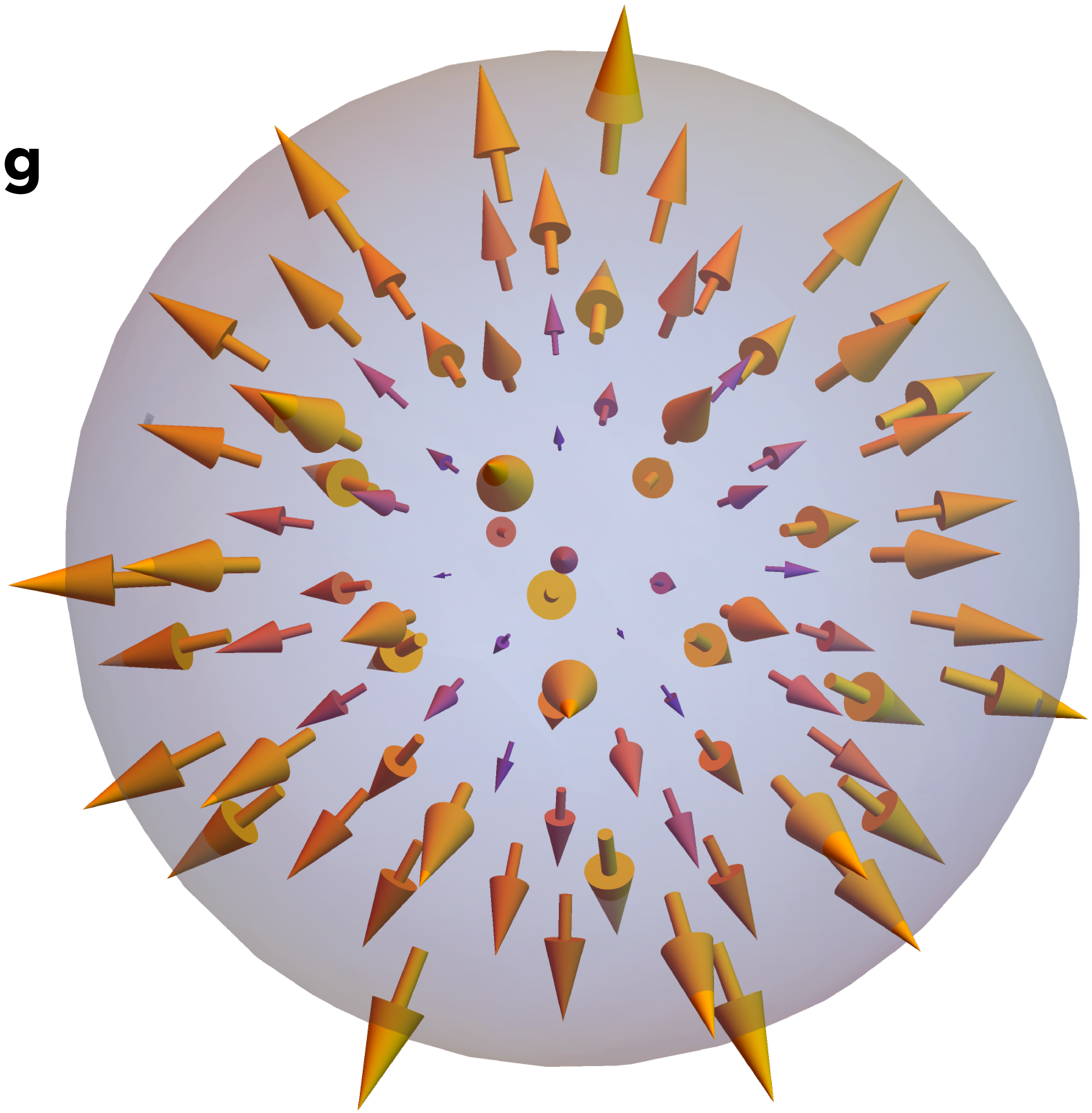
from U(1) breaking (ctd.)



Monopoles

0 dimensional cousin of cosmic string

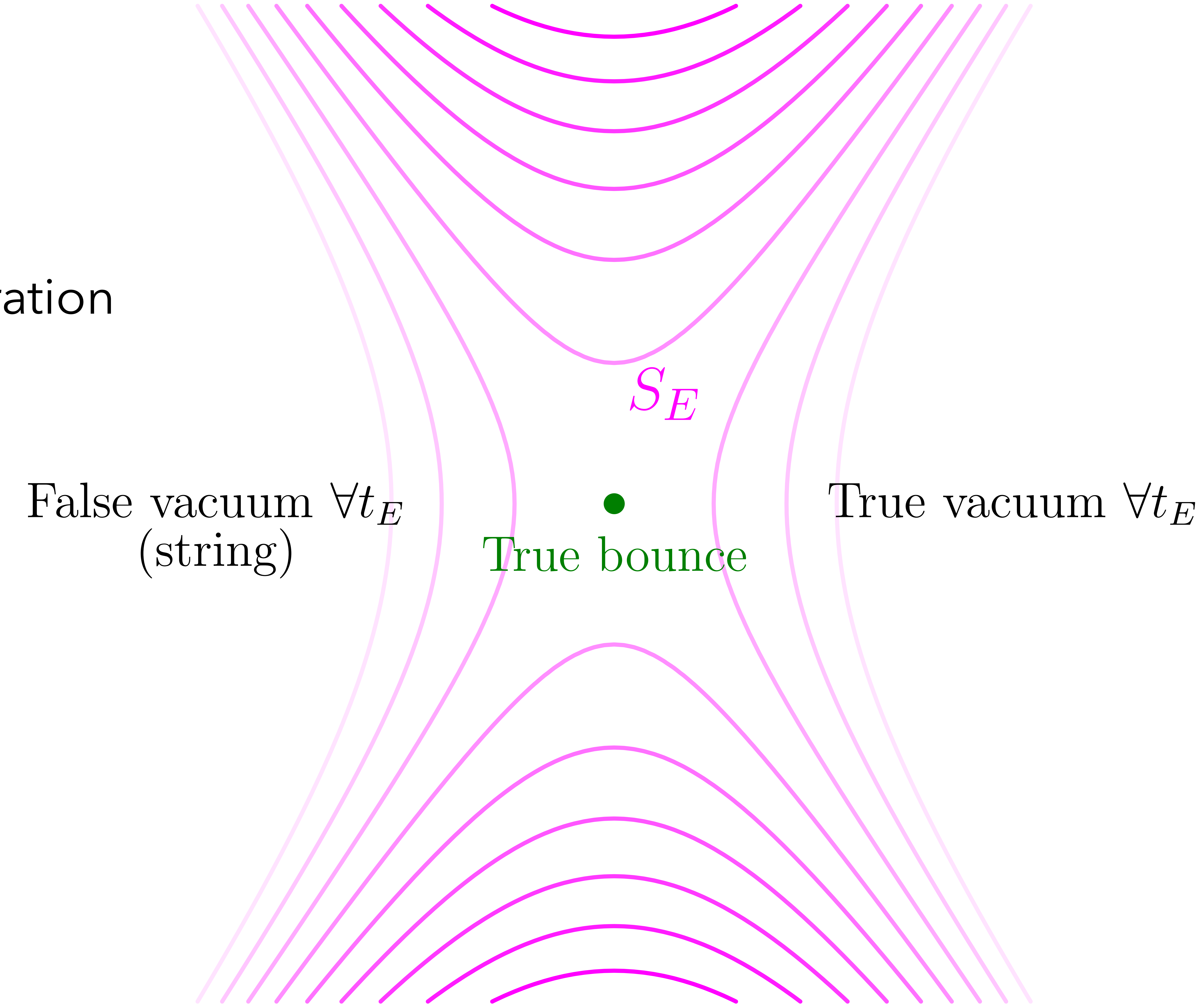
- Arise from winding on 2D sphere
- Behave like point-like particles*



Strategy

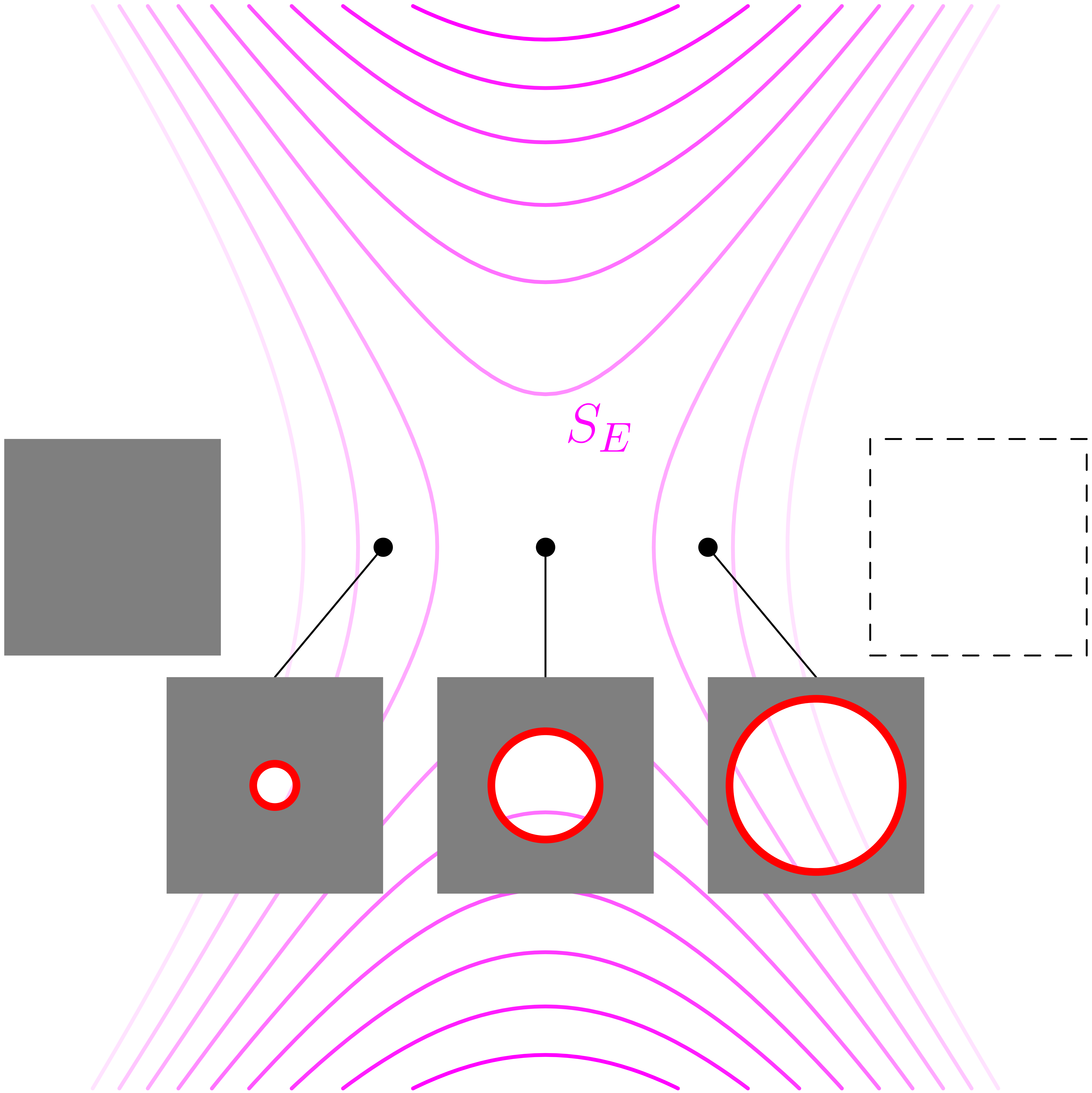
Why it is an upper bound

Each point: 4D field configuration



Strategy

Why it is an upper bound

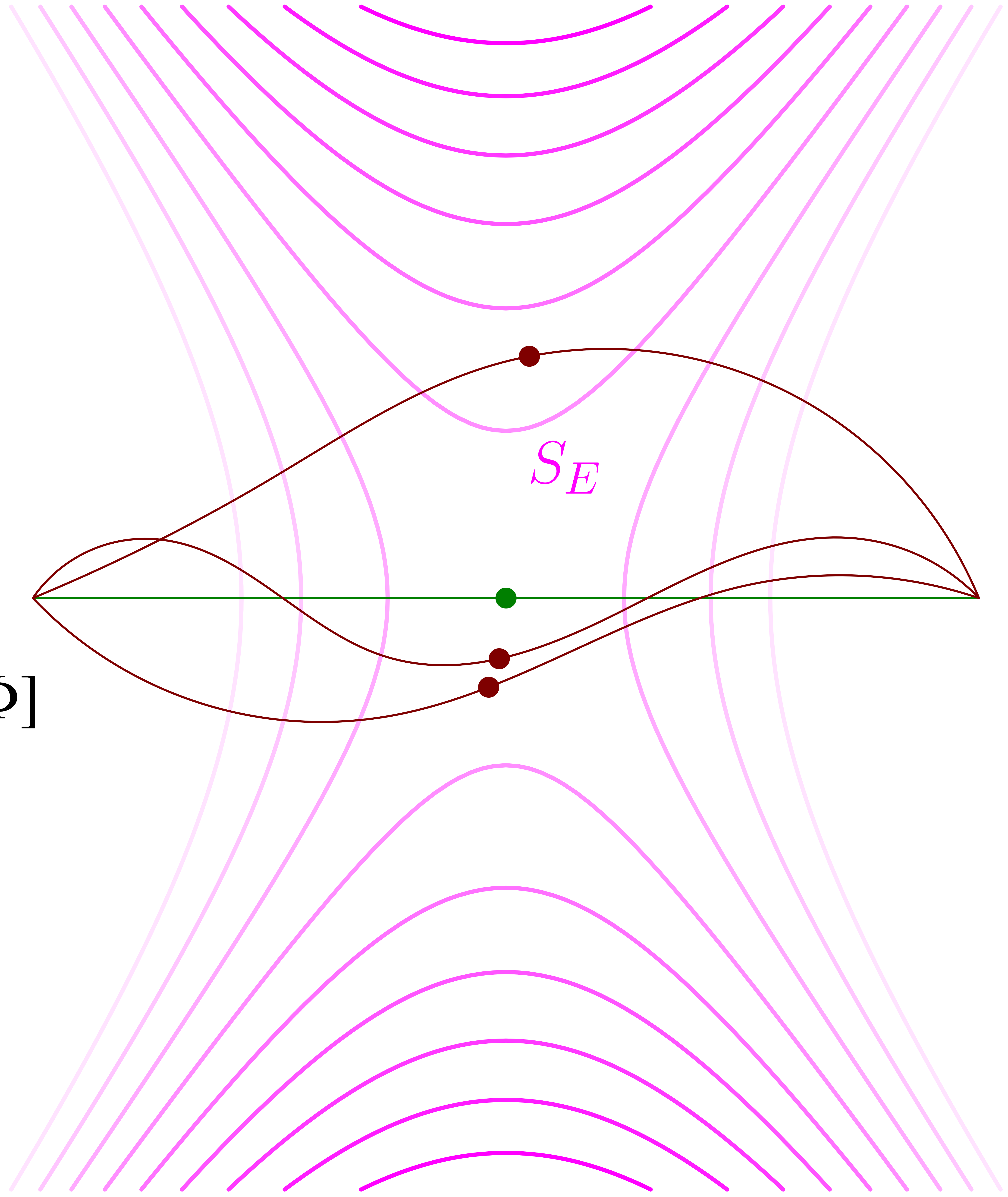


Strategy

Why it is an upper bound

True (optimal) bounce action:

$$S_E[\bullet] = \min_{\text{path joining the two sides}} \max_{\Phi \in \text{path}} S_E[\Phi]$$



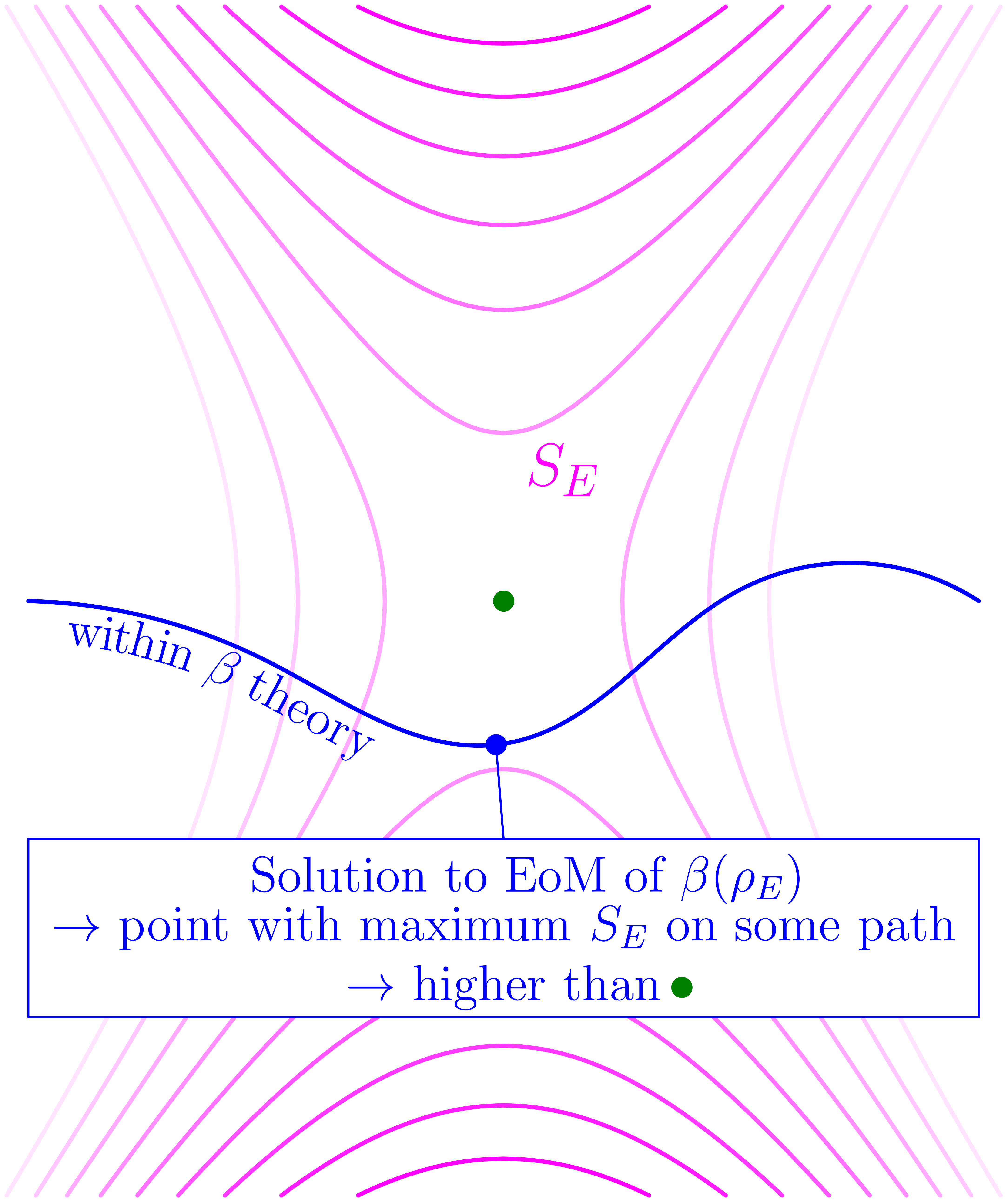
Strategy

Why it is an upper bound

∃path that

- joins the two vacua
- stays within the effective β theory
- has maximum S_E at ● (blue)

→ S_E[● (blue)] ≥ S_E[● (green)]



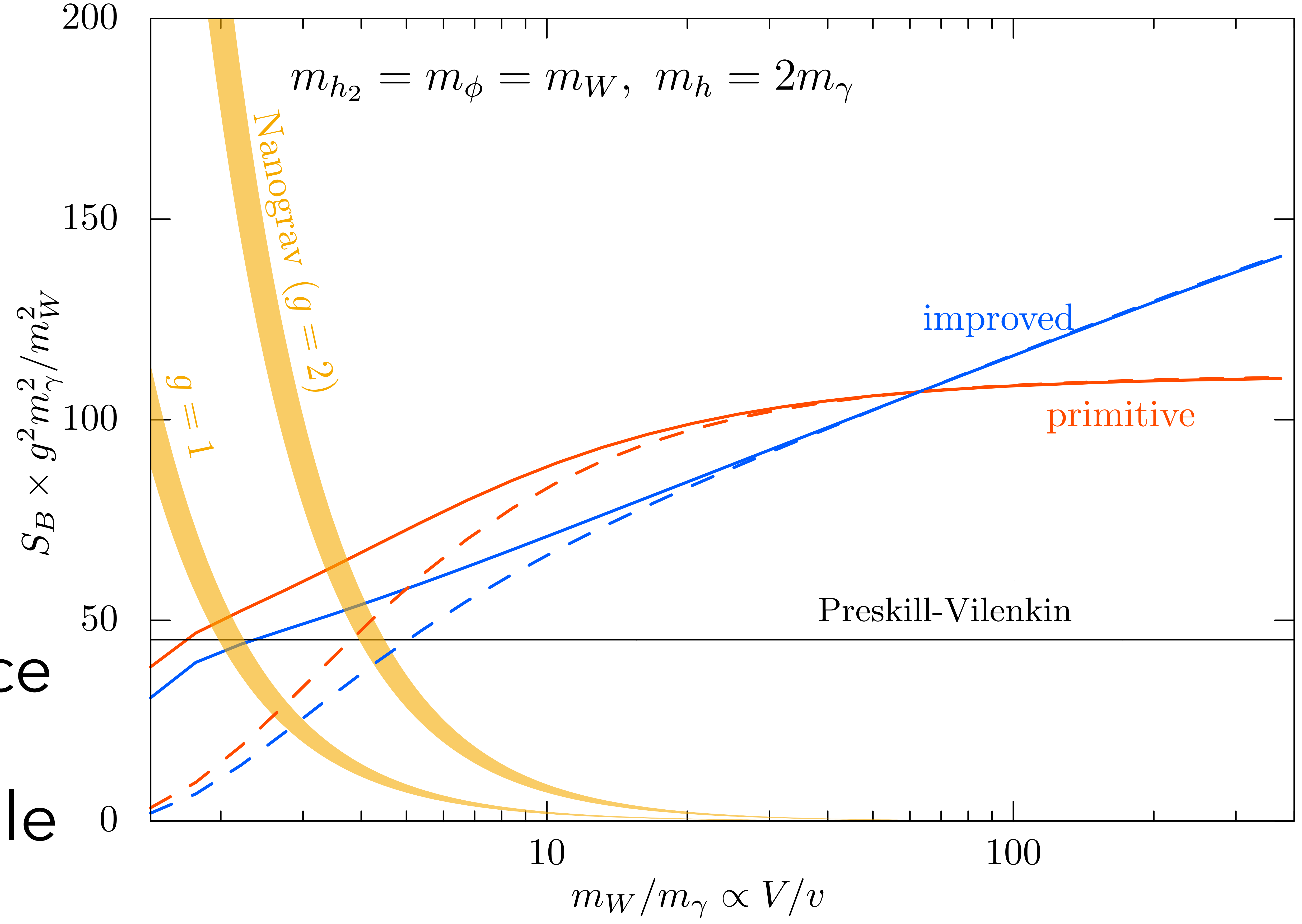
When does thin-wall break down?

- Introduce "β-thin-wall approximation"
- Thin-wall approximation to the 1D effective theory of $\beta(\rho_E)$
  - Valid only for $V \gg v$
- Preskill-Vilienkin approximation: similar but different
  - β-thin-wall: Ansatz → effective 1D theory → thin-wall
  - Preskill-Vilenkin: assume thin-wall in the 4D theory

Results

Hint for stronger results?

- solid: bounce, dashed: β-thin-wall
- For large hierarchy:
  - Primitive: Preskill-Vilenkin $\times\ \mathcal{O}(1)$
- For small hierarchy:
  - β-thin-wall deviates from the bounce
    - Preskill-Vilenkin: also questionable



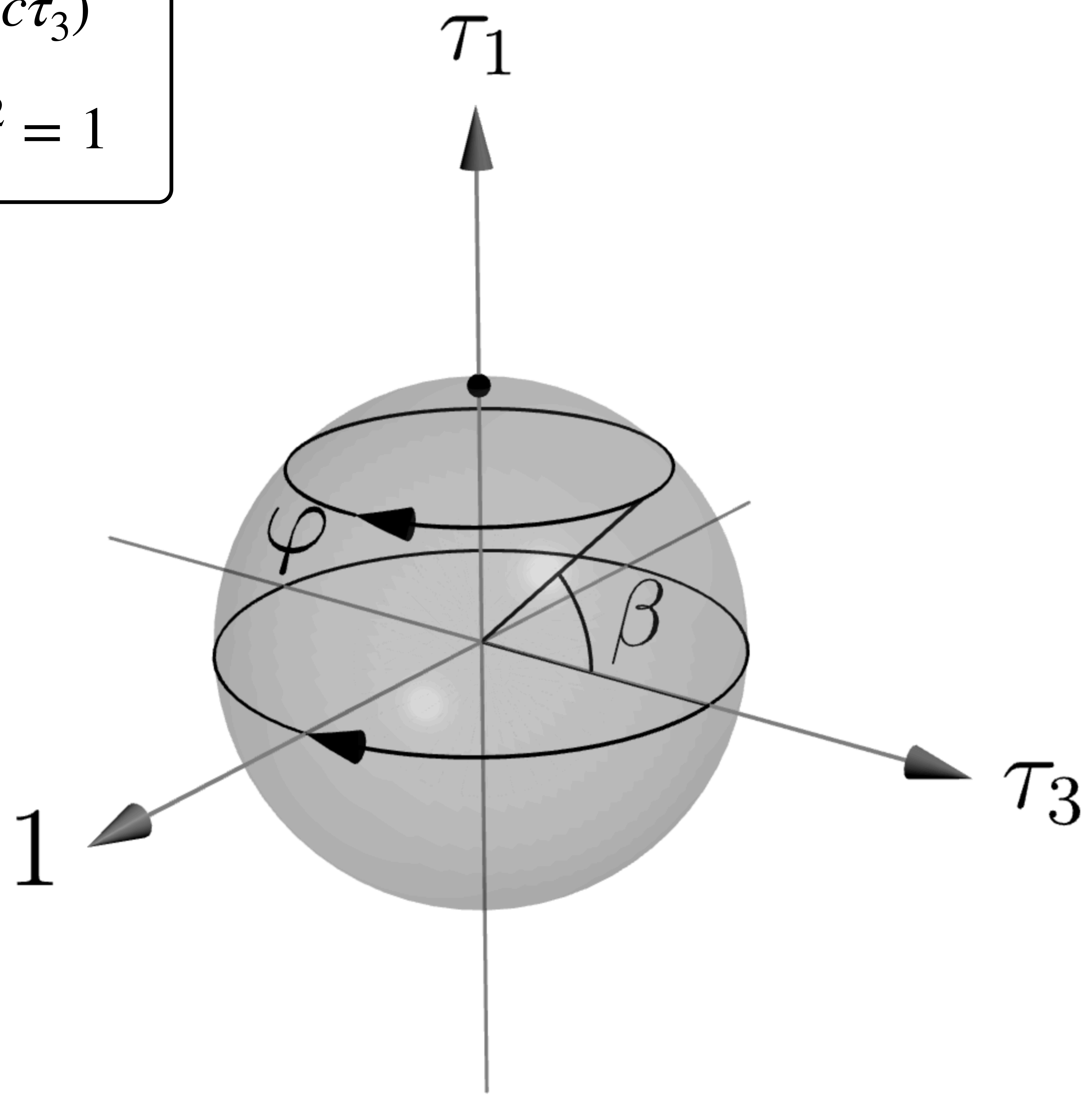
Strategy

Unwinding the string

$a + i(b\tau_1 + c\tau_3)$

$a^2 + b^2 + c^2 = 1$

- $U = e^{-i\tau_3\varphi}\cos\beta + i\tau_1 \sin\beta \in S^2 \subset \mathrm{SU}(2)$
  - $h = U(v\ 0)^\top,\ \phi = U(\tau_3/2)U^\dagger$
  - controls the U(1) winding
    - $h_1 = e^{-i\varphi}v$ for $\beta = 0$
  - $U = i\tau_1 = \mathrm{const.}$ for $\beta = \pi/2$
    - completely unwound

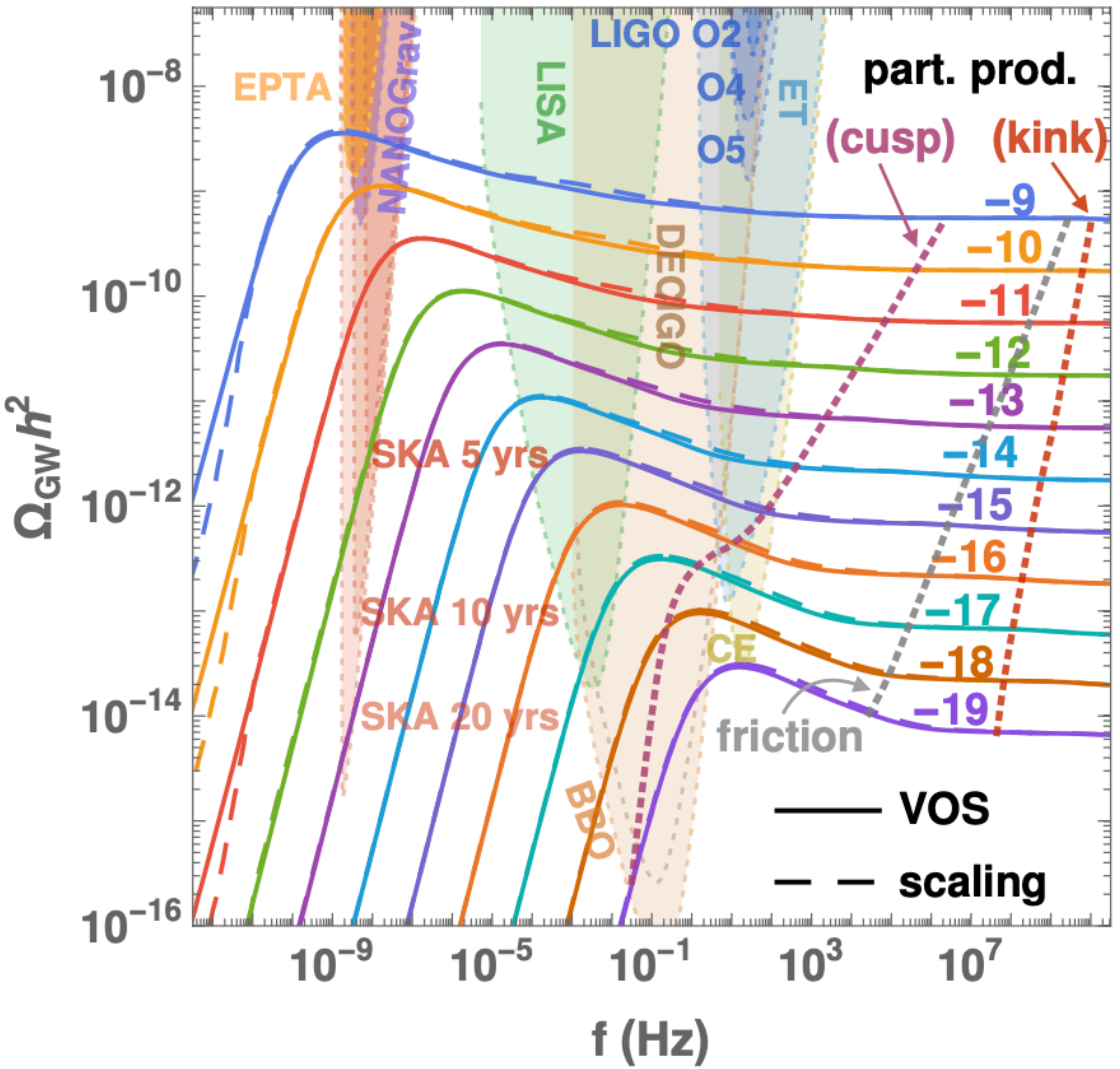


On metastability

Stable strings vs. PTA

- Nanograv's spectrum: blue tilted
- GW spectrum from stable cosmic strings →
  - The amplitude and the low-frequency cutoff correlate
  - → Mismatch with Nanograv

late emission ← early emission

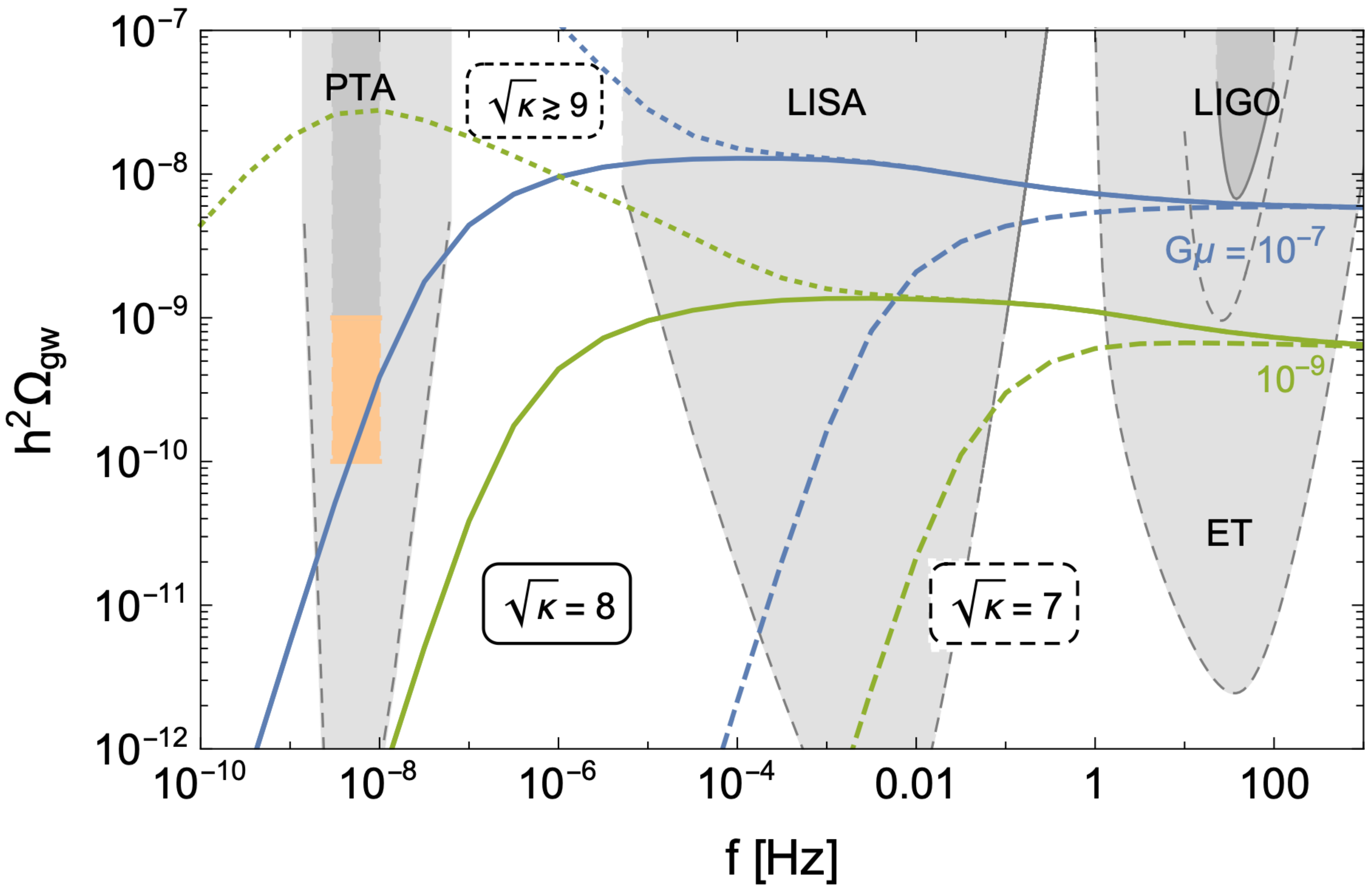


[Gouttenoire et al., 2019]

On metastability

Metastable strings vs. PTA

- Finite lifetime moves the cutoff to the right
  - → better fit with the PTA data

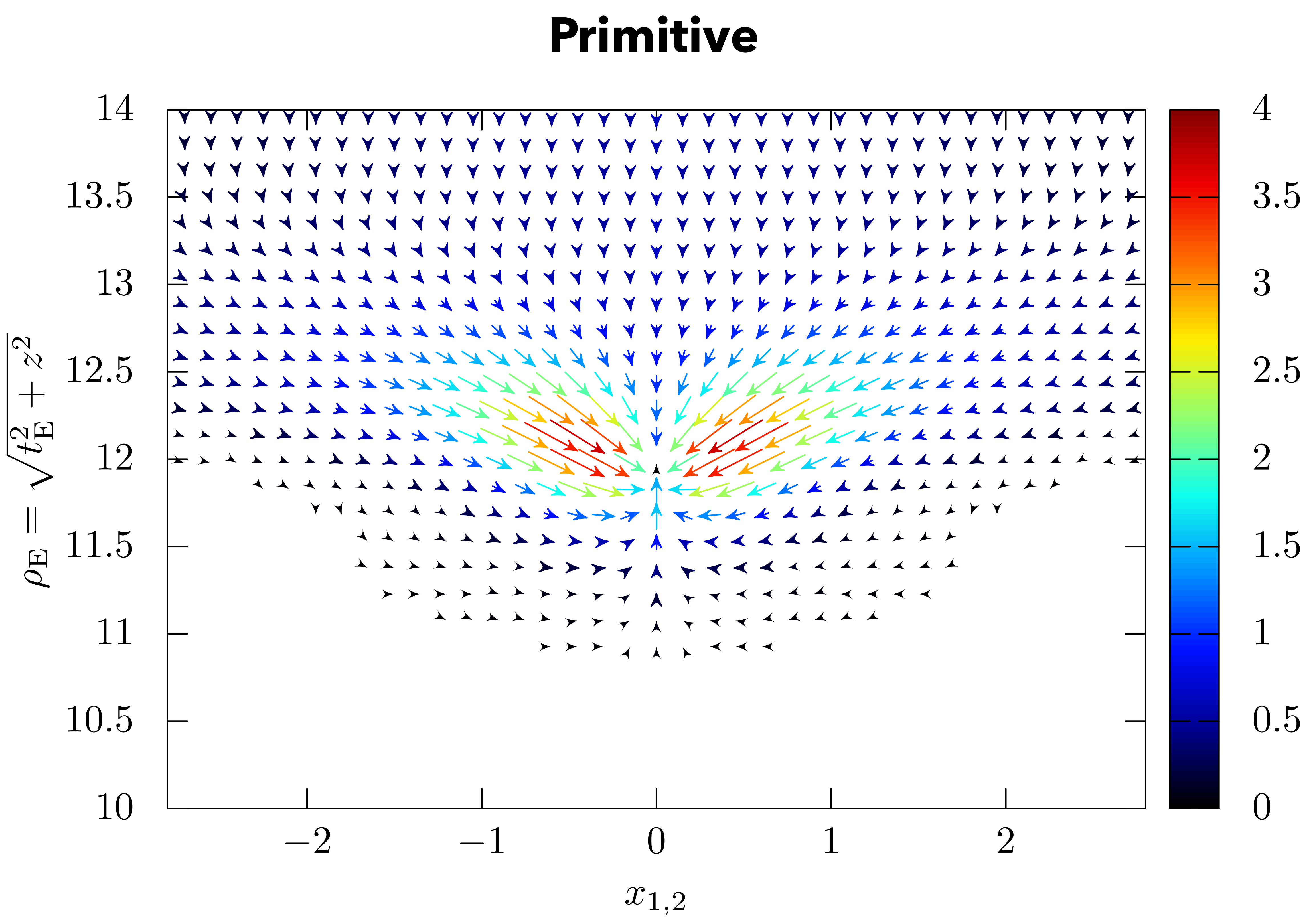


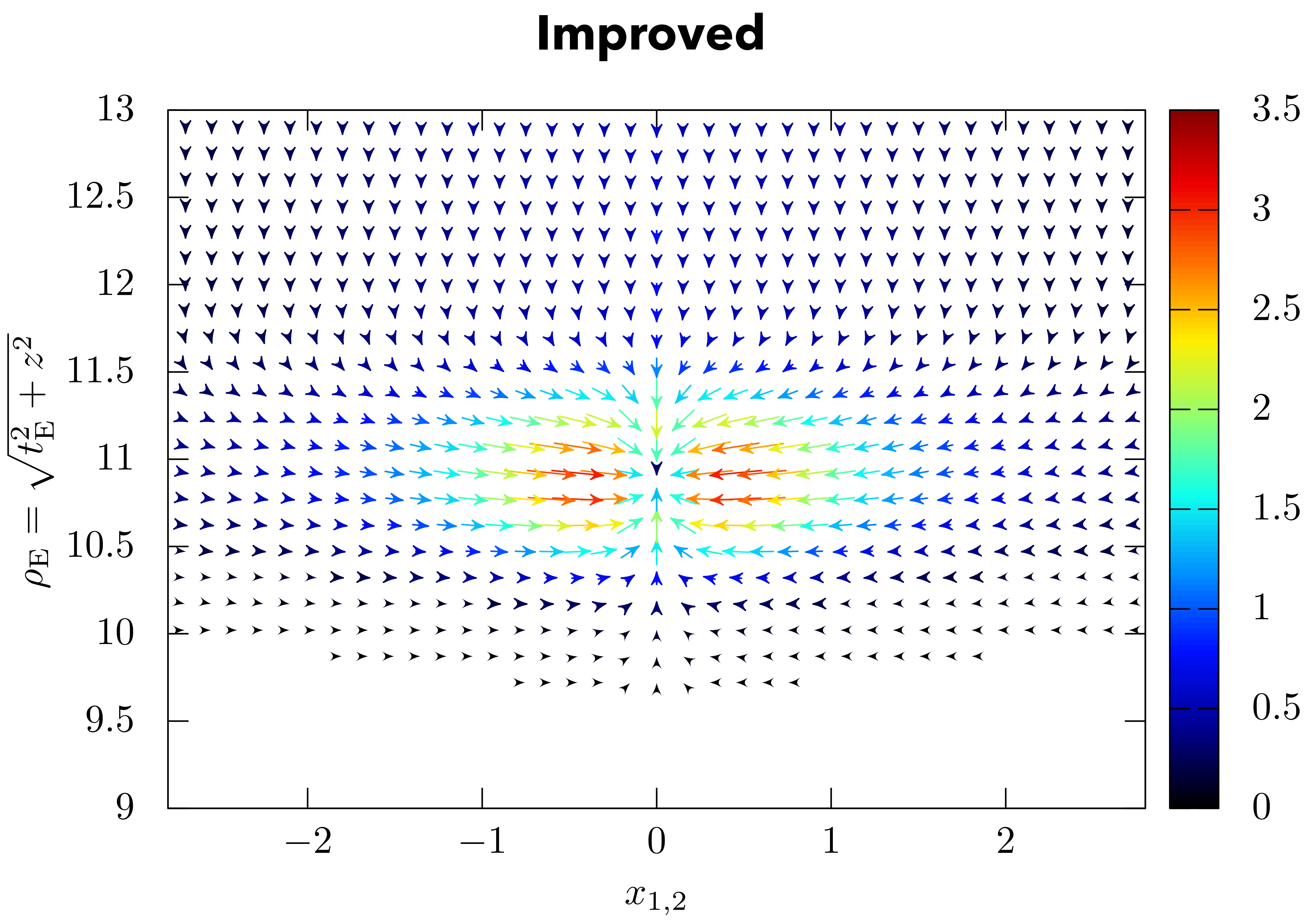
[Buchmüller et al., 2023]

Magnetic fields

Cross section of the breaking string

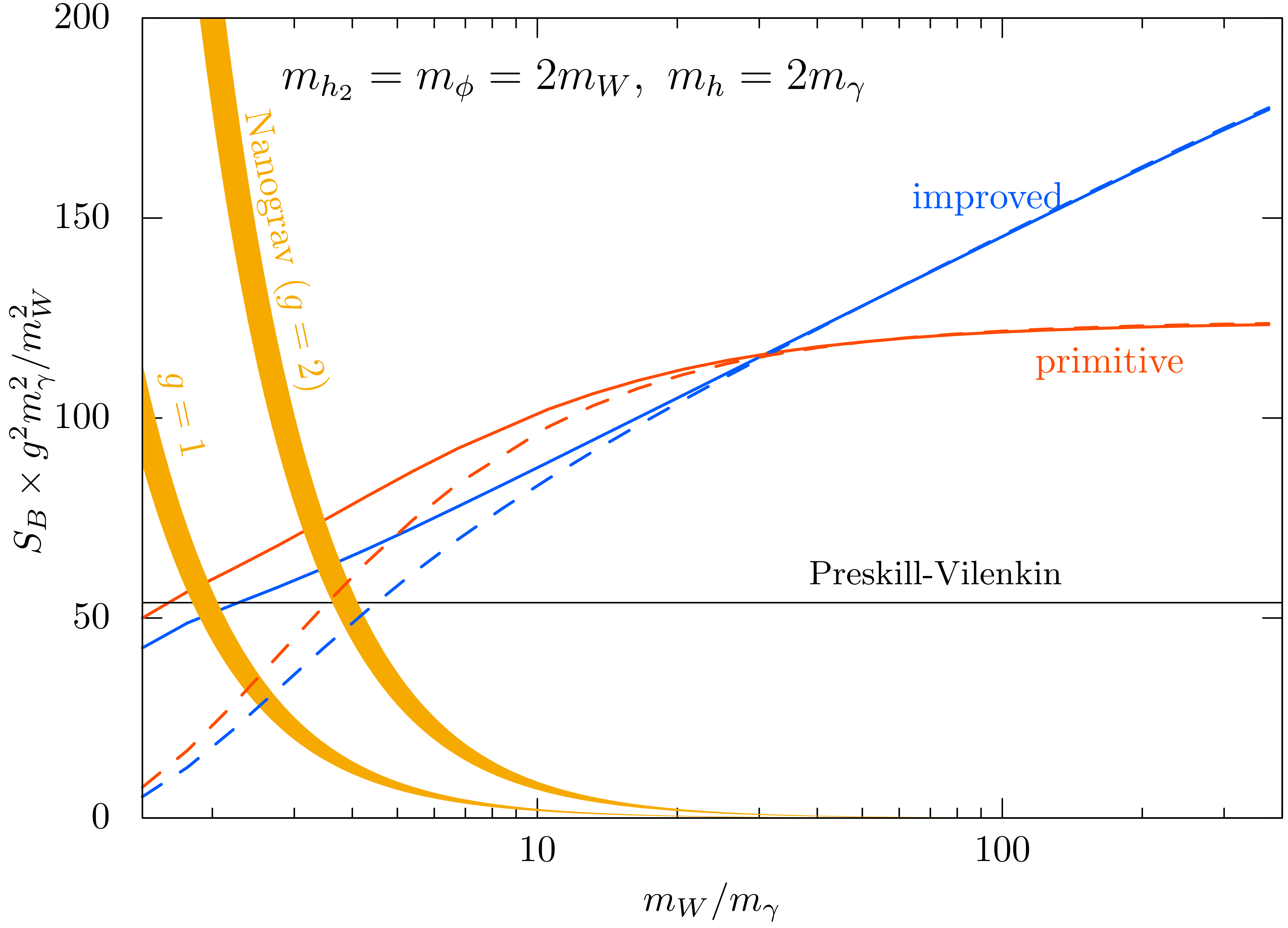
$$B_i = \frac{1}{2}\epsilon^{ijk}\frac{\phi^a}{V}F^a_{jk}$$





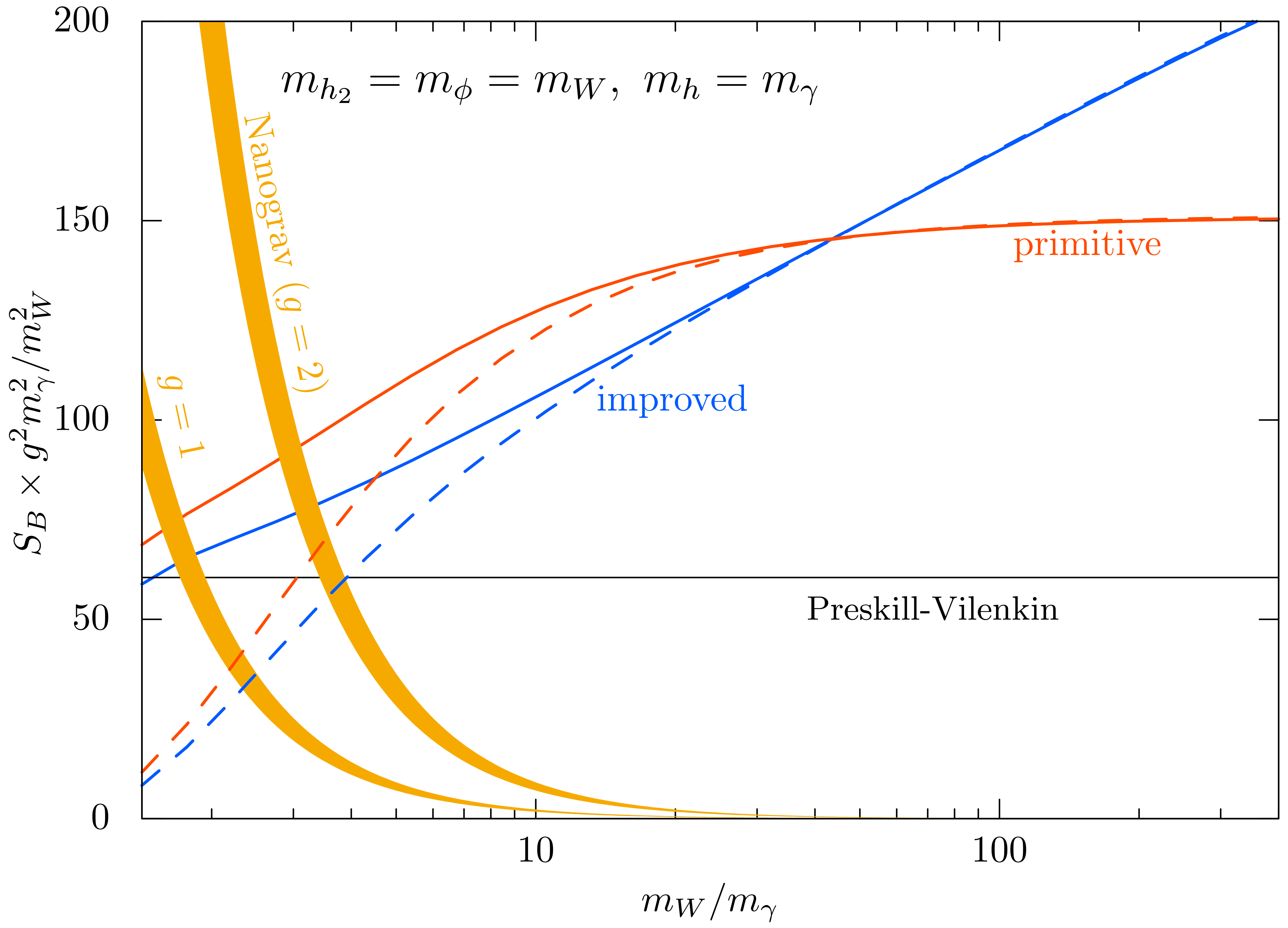
Other parameters

Light W



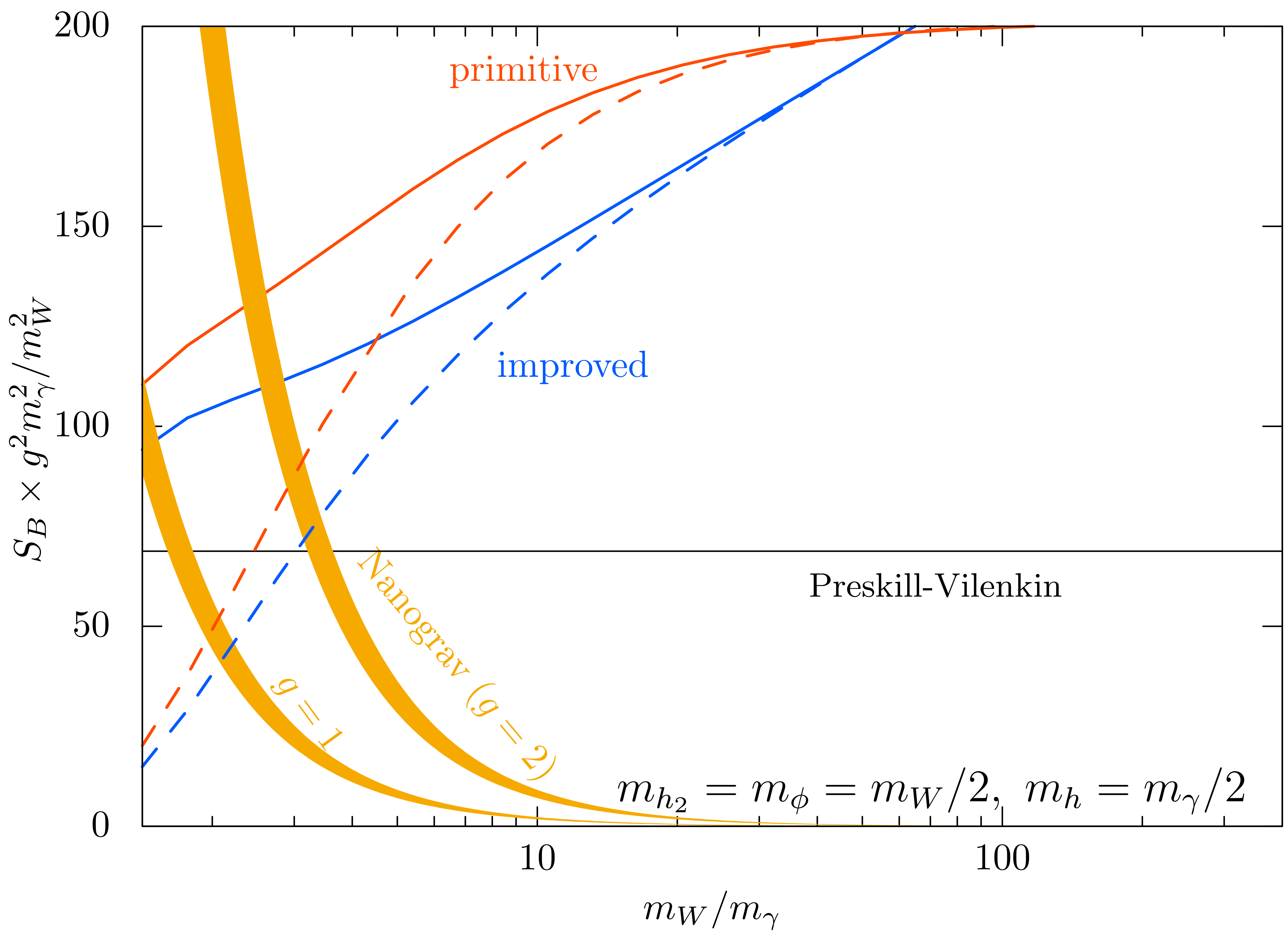
Other parameters

SUSY-like



Other parameters

Heavy W

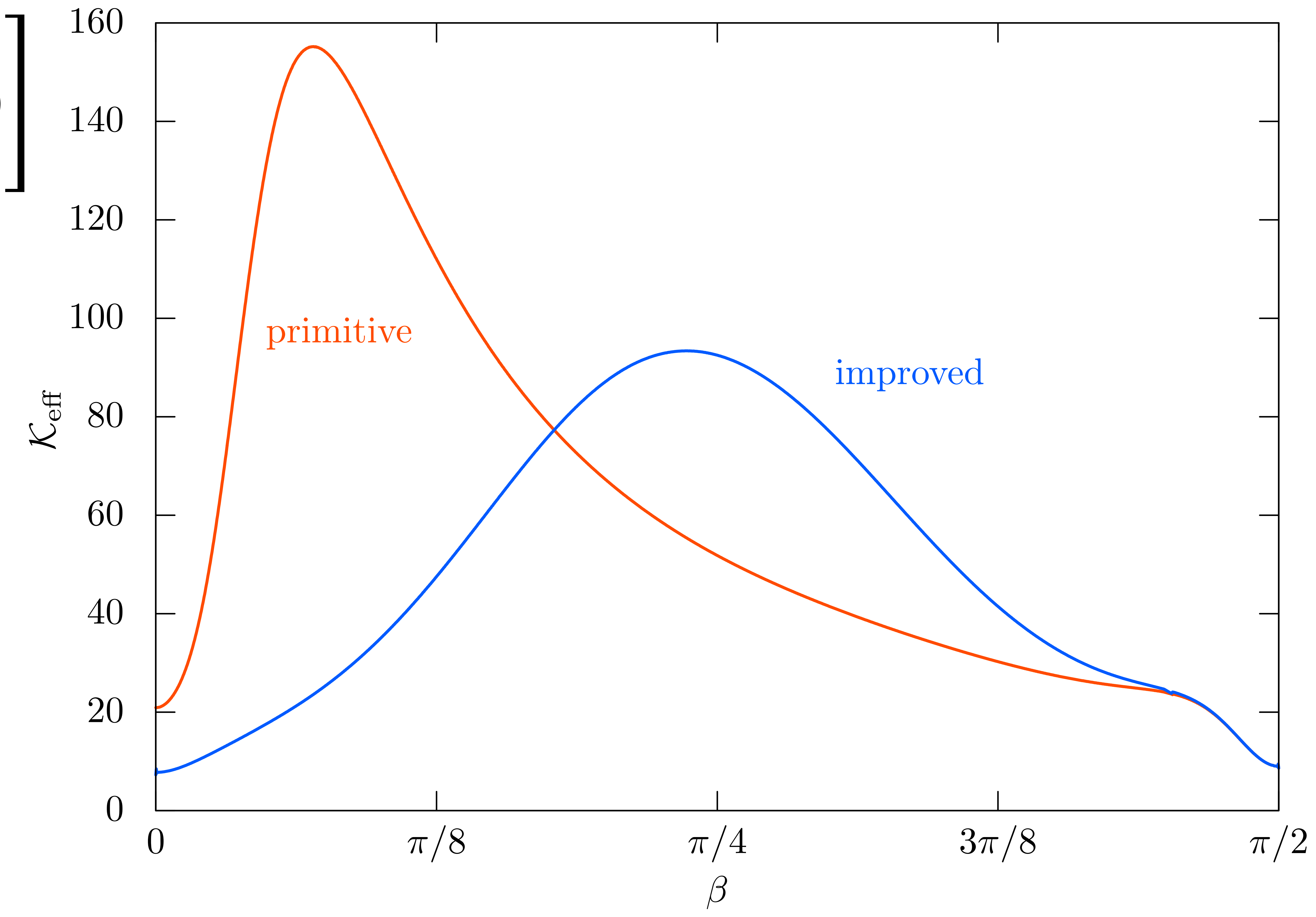


β-thin-wall approximation

- $$S_B = 2\pi \int_0^\infty \rho_E \mathrm{d}\rho_E \left[\frac{1}{2}\mathscr{K}_{\text{eff}}(\beta)\beta'^2 + T(\beta) - T(0)\right]$$
$$\approx -\pi\rho_E^{*2}\left[T(0) - T\left(\frac{\pi}{2}\right)\right] + 2\pi\rho_E^* \int_{\text{wall}} \mathrm{d}\rho_E \left[\frac{1}{2}\mathscr{K}_{\text{eff}}(\beta)\beta'^2 + T(\beta) - T(0)\right]$$
$$= -\pi\rho_E^{*2}\left[T(0) - T\left(\frac{\pi}{2}\right)\right] + 2\pi\rho_E^* m_{\text{eff}}$$
  - $$m_{\text{eff}} := \int_0^{\frac{\pi}{2}} \mathrm{d}\beta \sqrt{2\mathscr{K}_{\text{eff}}(\beta)(T(\beta) - T(0))}$$
- Maximum: $S_B = \pi \dfrac{m_{\text{eff}}^2}{T(0) - T(\pi/2)}$

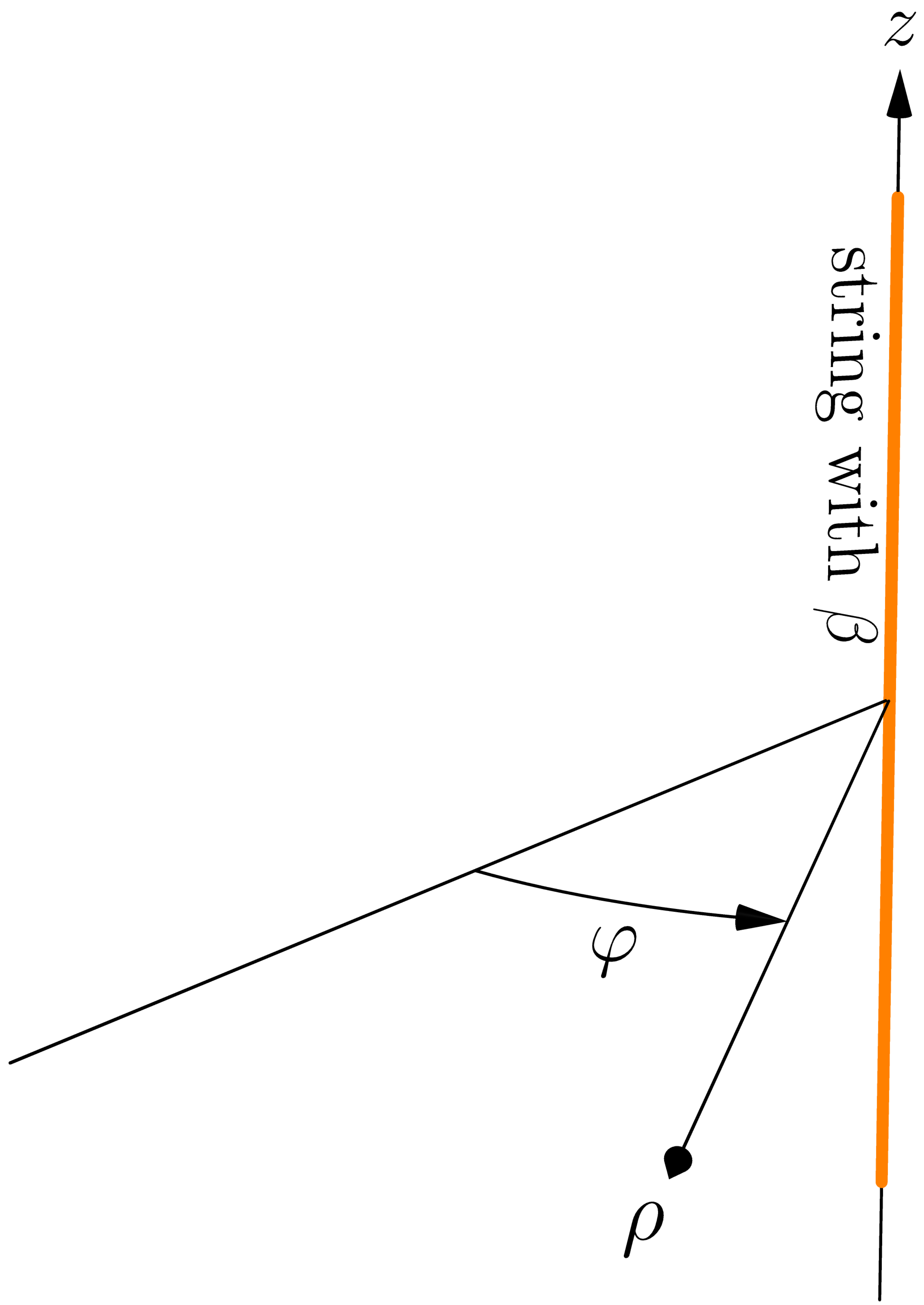
Kinetic term

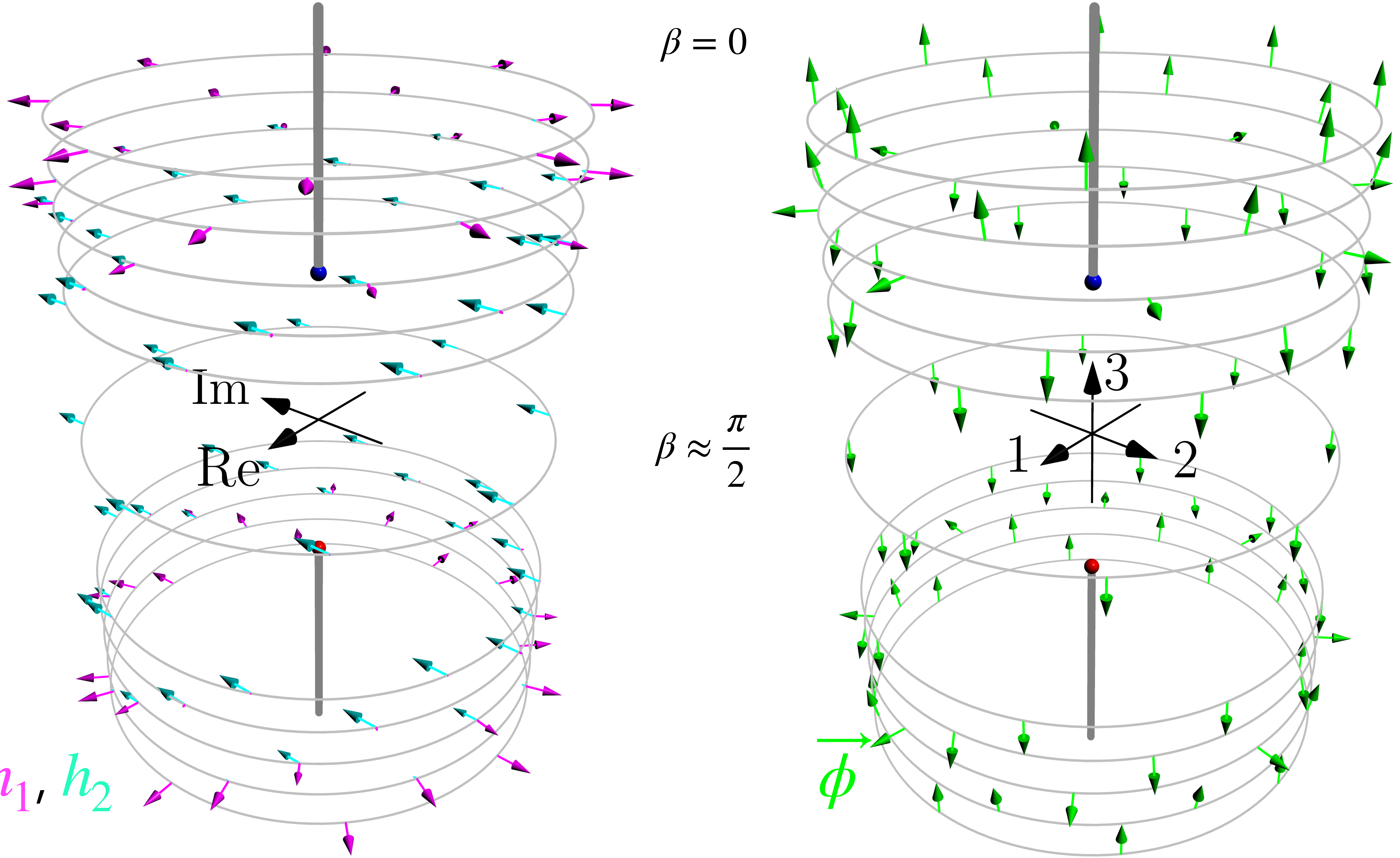
- $S_E = 2\pi \int_0^\infty \rho_E \mathrm{d}\rho_E \left[\frac{1}{2} \mathscr{K}_{\text{eff}}(\beta)\beta'^2 + T(\beta) \right]$



Primitive Ansatz [Shifman & Yung, 2002]

- $h(x) = U \begin{pmatrix} \xi_\beta(\rho) \\ 0 \end{pmatrix}$
- $A_\theta(x) = iU\partial_\varphi U^{-1}[1 - f_\beta(\rho)]$, other components: 0
- $\phi(x) = VU\frac{\tau_3}{2}U^{-1} + \varphi_\beta(\rho)\left[\frac{\tau_1}{2}\sin\beta - \frac{\tau_2}{2}\cos\beta\right]$
- $U = e^{-i\tau_3\varphi}\cos\beta + i\tau_1\sin\beta$
- $\xi_\beta(0) = 0, \xi_\beta(\infty) = v, f_\beta(0) = 1, f_\beta(\infty) = 0, \varphi_\beta(0) = V\sin 2\beta, \varphi_\beta(\infty) = 0$





$\beta = 0$
$\beta \approx \frac{\pi}{2}$
Im
Re
3
1
2
h_1, h_2
$\vec{\phi}$

Setup

Couplings vs. Masses

- Scale hierarchy: $\sqrt{\kappa_{PV}} = M_M/\sqrt{T_{\rm str}} \sim V/v \propto m_W/m_\gamma$
  - Gauge field : $m_W = gV$, $m_\gamma = \frac{1}{\sqrt{2}} gv$
  - (Scalars : $m_\phi = \sqrt{8\tilde{\lambda}}V$, $m_{h_1} = 2\sqrt{\lambda}v$, $m_{h_2} = \sqrt{\gamma}V$)
- Euclidean action in terms of the masses: $S_E = \frac{1}{g^2}[g \text{ independent}]$

Couplings vs. Masses (detailed)

- Gauge field : $m_W = gV$, $m_\gamma = \frac{1}{\sqrt{2}} g v$
  - Scale hierarchy: $V/v \propto m_W/m_\gamma$
- Scalar triplet : $m_\phi = \sqrt{8\tilde{\lambda}}V$
- Scalar doublet: $m_{h_1} = 2\sqrt{\lambda}v$, $m_{h_2} = \sqrt{\gamma}V$
- Euclidean action:

$$g^2\mathscr{H} = \frac{1}{4}F^2 + \left|D\hat{h}\right|^2 + \frac{1}{2}\left(D\hat{\phi}\right)^2 + \frac{m_\phi^2}{8m_W^2}\left(\hat{\phi}^2 - m_W^2\right)^2 + \frac{m_{h_1}^2}{4m_\gamma^2}\left(|\hat{h}|^2 - 2m_\gamma^2\right)^2 + \frac{m_{h_2}^2}{m_W^2}\left|\left(\hat{\phi} - \frac{m_W}{2}\right)\hat{h}\right|^2$$