CKM行列要素決定のための格子QCD計算の現状

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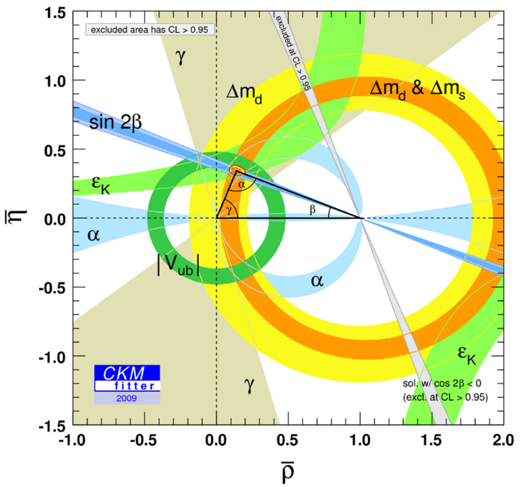
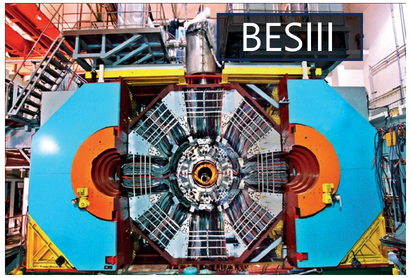
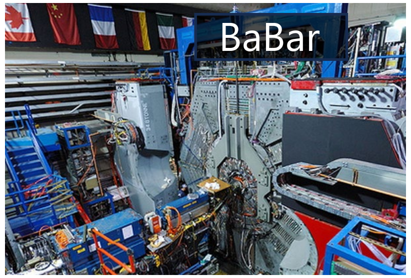
introduction

Cabibbo-Kobayashi-Maskawa (CKM) matrix elements

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$







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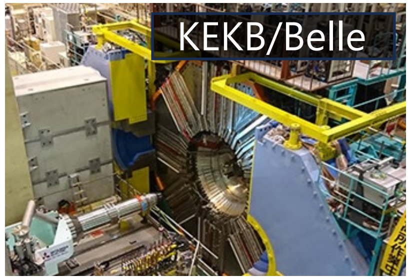
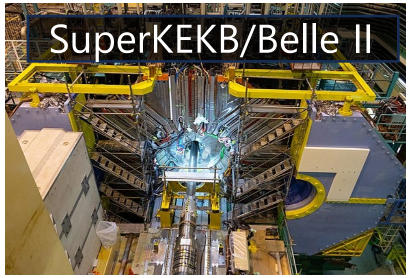




…

theory vs experiment

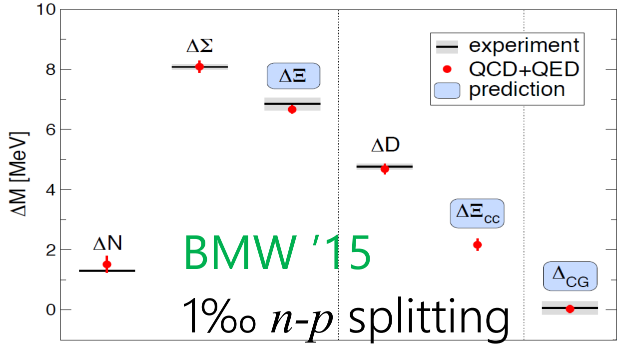
steadily improved accuracy

lattice QCD ⇒ for hadronic inputs

1st simulations: 0.8 fm box, 2 GeV cut-off, quenched, 1 MFLOPS computer

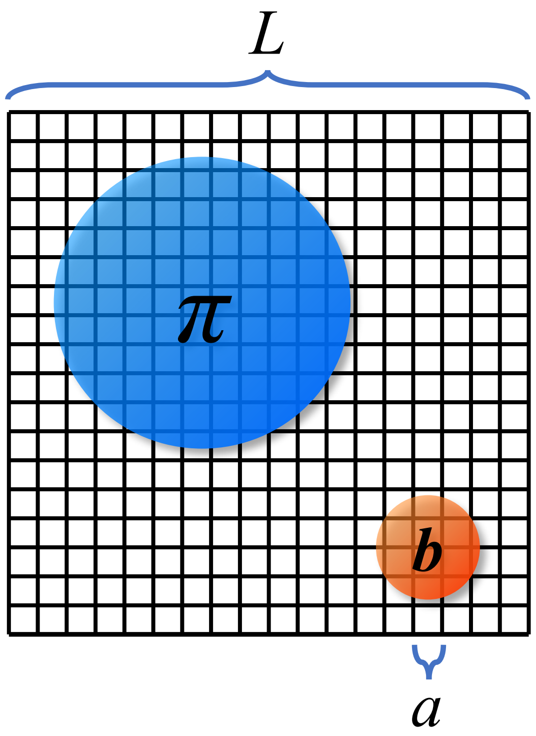
↓ dominant uncertainty ≲ 2015

Fugaku 0.5 EFLOPS, N_f = 2+1(+1), physical $m_{ud,s,c}$, 10 fm and/or 4.5 GeV



this talk : status of lattice QCD in comparison with experiment

simulation of flavor physics



multi-scale problem

$$L^{-1} < M_\pi(m_{ud}) < M_K(m_s) < M_D(m_c) < M_B(m_b) < a^{-1}$$

+ hierarchical quark masses (m_u=m_d, top=perturbation)
+ lattice size $L > M_\pi^{-1}$ ⇔ finite volume effects
+ cutoff $a^{-1} > m_b$ ⇔ discretization effects

B physics

– need high resolution $L/a > O(100)$ and cost ∝ $(L/a)^7 L^2$

⇒ O(10) EFLOPS for fully realistic simulation of b hadrons ⇒ Fugaku NEXT?

⇒ current simulations are limited to $a^{-1} \lesssim m_b$

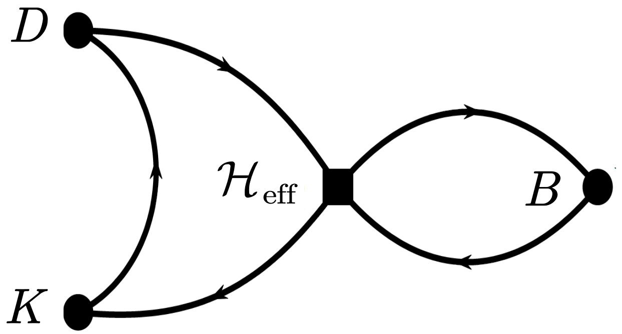
+ QCD action for b quarks w/ unphsically small m_b ⇔ sizable $O((am_b)^n)$ errors
+ effective-theory-based action w/ physical m_b ⇔ perturbative matching to QCD

kaon, charm physics : cost suppressed by $(M_{K,D}/M_B)^7$ ⇒ realistic studies w/ controlled systematics

improving accuracy by developing algorithms & new powerful computers

multi-hadron or unstable hadron state

e.g. $B \to DK$ for UT angle



$$\langle \mathcal{O}_K(t',\mathbf{p}')\mathcal{O}_D(t',\mathbf{p})\mathcal{H}_{\rm eff}(t)\mathcal{O}_B^\dagger(0,\mathbf{q})\rangle \xrightarrow[t,t'\to\infty]{} \langle D(\mathbf{0})K(\mathbf{0})|\mathcal{H}_{\rm eff}|B\rangle + \cdots$$

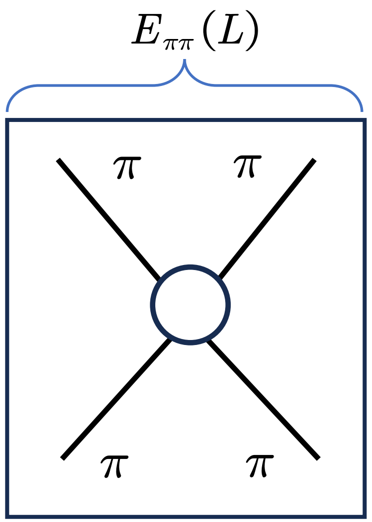
correlation functions on a finite volume (V) Euclidean lattice

⇒ "ground-state" + "signal of interest" × $e^{-\Delta E(t'-t)}$ × finite V corrections…

① "gold-plated" : ≤ 1 "hadron stable in QCD" for initial/final state

- $B \to \ell\nu$, $B \to \pi\ell\nu$, $B \to K\nu\nu$, …
- lattice correlation functions ⇒ MEs of interest

Lüscher '86, '91



② non "gold-plated" : w/ muti-hadron states, unstable particles

- $K \to \pi\pi$, $B \to \rho\ell\nu$, $B \to K^*\ell\ell$, $B \to X_{\{c,u\}}\ell\nu$, …
- need a framework to extract MEs of interest from lattice corr. functions

wider application w/ newly developed frameworks

Outline

status and prospects of lattice studies in comparison w/ experiment

NOT comprehensive : SUBJECTIVELY selected studies

- Introduction: challenges in lattice QCD
- kaon decays
- $B_{(s)}$ exclusive decays
- B and τ inclusive decays
- $D_{(s)}$ decays

FLAG & B2TIP

two references for lattice and Belle II precision

Flavor Lattice Averaging Group (FLAG)

review of recent lattice studies on
flavor physics and world average

- ~ 40 lattice experts
- ≥ 400 pages
- regularly published reviews ‘10, ‘13 , ‘16, ‘19, ‘21 editions
- 2111.09849 ⊕ ‘23 web update

Belle II Theory Interface Platform (B2TIP)

Belle II and theory expected precision
& impact to new physics search

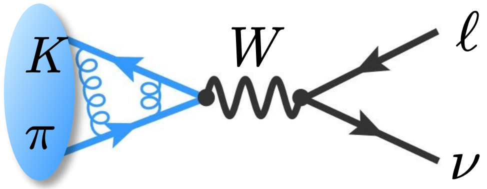
- led by E. Kou & P. Urquijo
- ≥ 500 theorists & experimentalists
- ~ 650 pages
- white paper ’19 :

kaon decays

$|V_{us}|/|V_{ud}|$ from leptonic decays ($K_{\ell 2}$, $\pi_{\ell 2}$)

Marciano '04

$$\frac{\Gamma(K\to\ell\nu[\gamma])}{\Gamma(\pi\to\ell\nu[\gamma])}=\left(\frac{|V_{us}|}{|V_{ud}|}\right)^2\left(\frac{f_{K^\pm}}{f_{\pi^\pm}}\right)^2\frac{M_K\left(1-m_\ell^2/M_K^2\right)^2}{M_\pi\left(1-m_\ell^2/M_\pi^2\right)^2}\left(1+\delta_{\mathrm{EM}}\right)^2$$



$$\langle 0|A_\mu|P\rangle = i p_\mu f_P$$ w/ strong isospin correction

exactly cancel in the ratio f_K/f_π

- renormalization factor of A_μ
- lattice scale by $(aM_H)_{\mathrm{lat}}\ a^{-1}=M_{H,\mathrm{exp}}$

} often limit lattice precision of MEs

may partially cancel

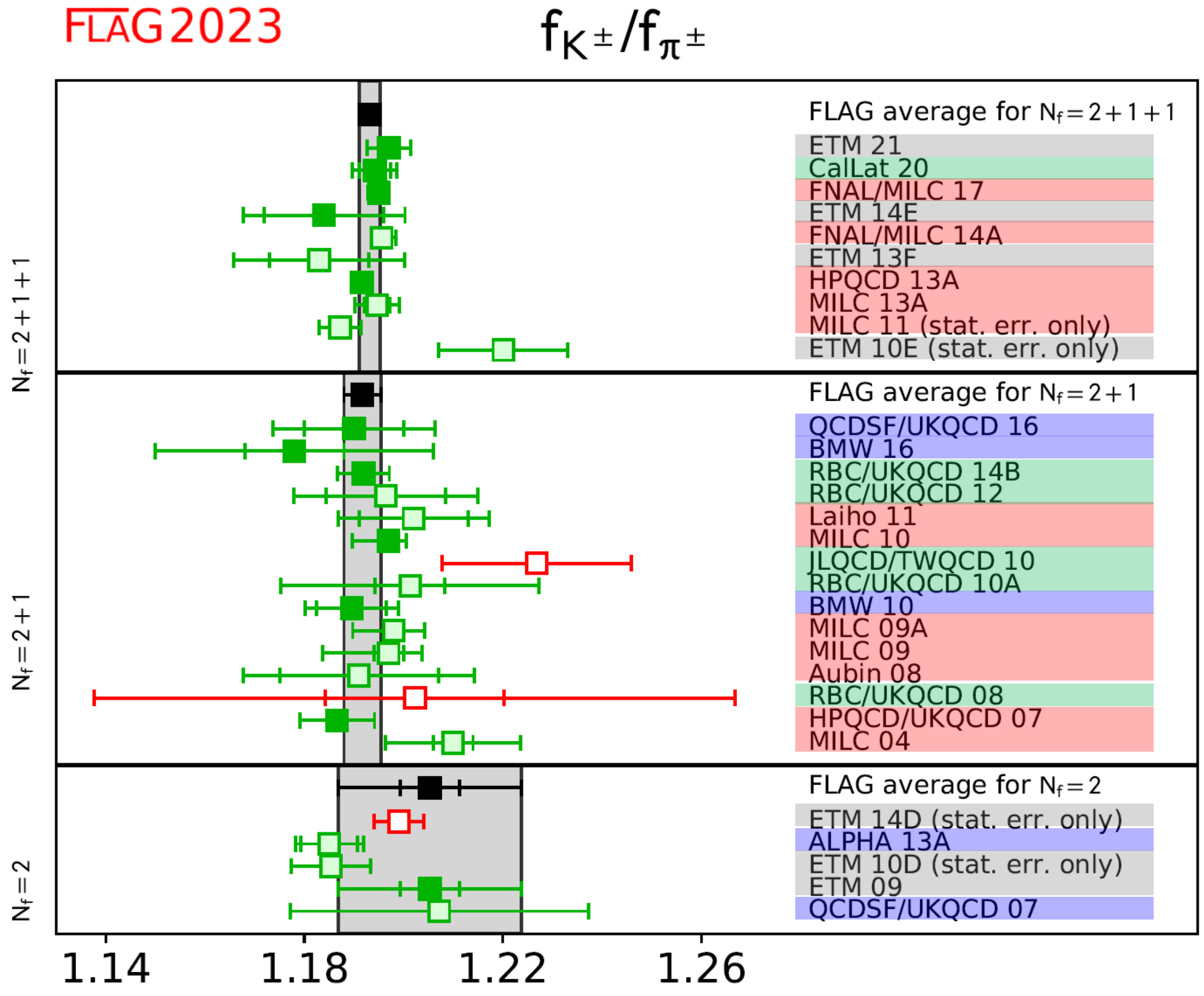
- M_π, M_K dependences
- finite volume corrections (FVCs)
- isospin correction
- $a\neq 0$ errors

can be corrected by ChPT

- NNLO SU(3) ChPT [Ananthanarayan+ '17]
- NNLO [Bijnens-Rössler '14]
- NLO [Cirigliano-Neufeld '11]

$f_{K^\pm}/f_{\pi^\pm}$

FLAG '23 web update



independent calculations w/ different setups

Nielsen-Ninomiya '82 ⇒ doublers or ~~chiral sym~~

- Wilson-type : break chiral symmety
- staggered-type : keep doubler d.o.f.
- chiral fermions : chiral symmetric
- twisted mass : $O(a^2)$ parity violation

⇒ consistency among precision calculations

FLAG average

$N_f=4$ 1.1934(19) [0.16%]

$N_f=3$ 1.1917(37) [0.31%]

⇔ HALQCD 2406.16665 1.1875(35)

⇒ average = 1.1899(26)

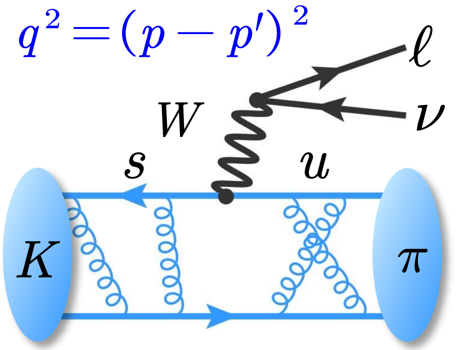
⇔ $|V_{us}|\, f_{K^\pm} / |V_{ud}|\, f_{\pi^\pm}$ = 0.27599(37) [0.13%] PDG'22

≤ 0.2% accuracy comparable to experimental input

$K \to \pi\ell\nu$ ($K_{\ell 3}$) semileptonic form factors (FFs)

$$\Gamma(K\to\pi\ell\nu) = \frac{G_F^2}{192\pi^3}|V_{us}|^2 C_K^2 S_{\rm EW} f_+^{K^0\pi^-}(0)^2 M_K^5 I_{K\ell}\left(1+\delta_{\rm EM}^{K\ell}+\delta_{SU(2)}^{K\pi}\right)^2$$

$$\langle\pi(p')|V_\mu|K(p)\rangle = \left(p+p'+\frac{M_K^2-M_\pi^2}{q^2}q\right)_\mu f_+(q^2) + \frac{M_K^2-M_\pi^2}{q^2}q_\mu f_0(q^2)$$



basic strategy for hadronic uncertainty

- chiral expansion of $f_+(0)$: NNLO in SU(3) ChPT Post-Schilcher '01, Bijnens-Talavera '03

$$f_+(0) = 1 + f_2 + \Delta f \quad (f_2 \sim O(p^2))$$

- NLO $f_2 \sim O((m_s - m_{ud})^2)$ Ademollo-Gatto '64 ⇒ no $O(p^4)$ couplings L_i ⇒ fixed in ξ – expansion ($F \to F_\pi$)

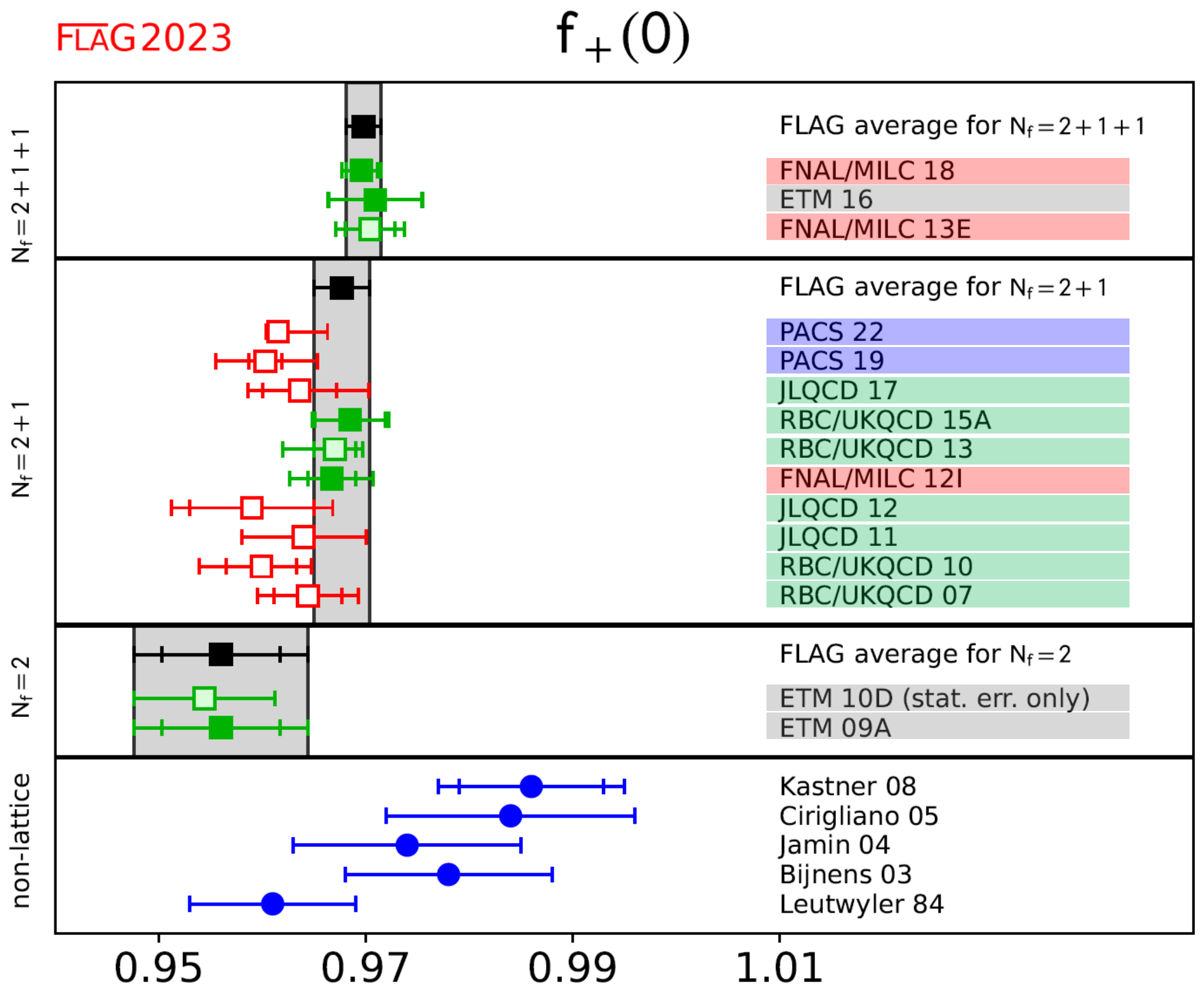
$$f_2 = H_{K^0\pi} + \frac{1}{2}H_{K^+\pi} + \frac{1}{2}H_{K^+\eta} + \sqrt{3}\,\varepsilon(H_{K\pi} - H_{K\eta}) = -0.023 \qquad H_{PQ} = -\frac{1}{128\pi^2 F_\pi^2}\left\{M_P^2 + M_Q^2 + \frac{2M_P^2 M_Q^2}{M_P^2 - M_Q^2}\ln\left[\frac{M_Q^2}{M_P^2}\right]\right\}$$

- task of lattice QCD : estimate small higher-order correction Δf

  e.g. 10% lattice error for small correction Δf ⇒ sub-% determination of $f_+(0)$

$f_+(0)$

FLAG '23 web update



$N_f = 4$ 0.9698(17) [0.18%]

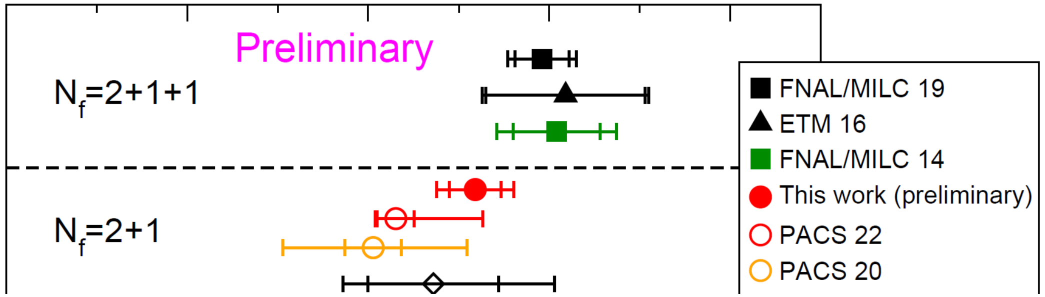
$\Leftrightarrow |V_{us}| f_+(0) = 0.21654(41)$ [0.19%] CKM16

- dominated by Fermilab/MILC '18
- independent calculations are very welcome !

$N_f = 3$ 0.9677(27) [0.28%]

PACS PRD '19, '22 : only two a's

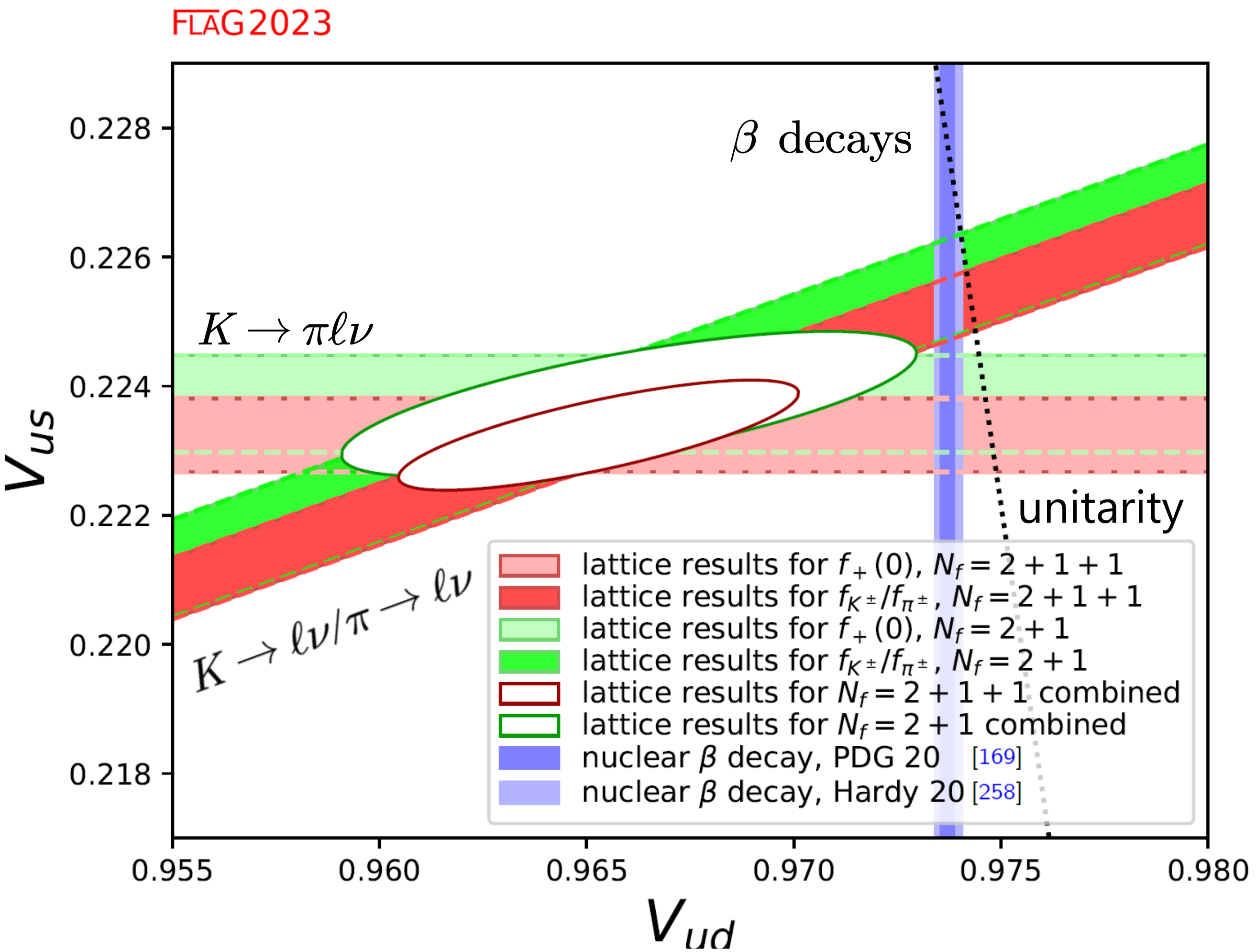
PACS Lattice '24 w/ a^{-1} = 2.3, 3.1, 4.9 GeV



$\leq$ 0.2% accuracy comparable to experimental input / welcome more studies

"Cabibbo angle anomaly"

FLAG '23 web update



$K_{\ell 2}$, $\pi_{\ell 2}$ ⇒ $|V_{us}|/|V_{ud}|$ = 0.23131(50) [0.22%]

$K_{\ell 3}$ ⇒ $|V_{us}|$ = 0.22328(56) [0.22%]

$0^+ \to 0^+$ nuclear decay ⇒ $|V_{ud}|$ = 0.97373(31) [0.03%]

$|V_{ub}| \approx 0.004$ from $B \to \pi \ell \nu$, $X_u \ell \nu$: too small

$$\Delta_{\mathrm{CKM}} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1$$

$= -\ 0.0180(51)_{K\ell 2}(41)_{K\ell 3}$ [2.8σ] [$K_{\ell 2}+K_{\ell 3}$]

$= -\ 0.00198\ (25)_{K\ell 3}(60)_{0+}$ [3.0σ] [$K_{\ell 3}$ + $0^+ \to 0^+$]

$= -\ 0.00112(22)_{K\ell 2}(64)_{0+}$ [1.7σ] [$K_{\ell 2}$ + $0^+ \to 0^+$]

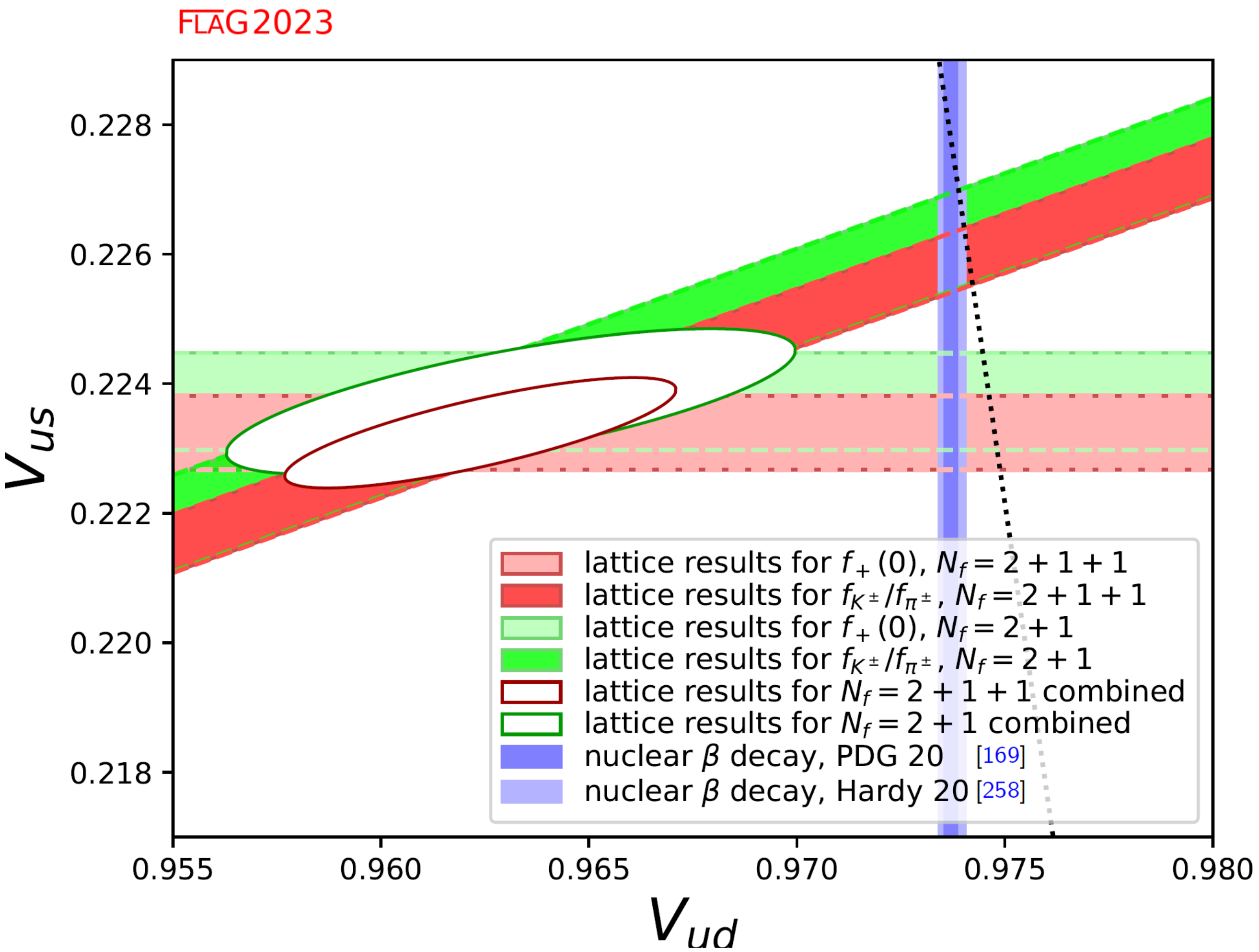
PDG'24: inconsistency in $|V_{us}|$ b/w the latter two

⇒ scale factor $\sqrt{\chi^2}$=2.5 ⇒ 2.3σ tension

3σ anomaly w/ $K_{\ell 3}$; independent calculations are very welcome

isospin correction for $K_{\ell 2}/\pi_{\ell 2}$

FLAG '23 web update + EM



$O(e^2p^2)$ ChPT [Cirigliano-Nerfeld '11]

$\delta_{\mathrm{EM+SU(2)}}$ = -0.56(0.11)% [LEC, higher orders]

Rome-SOTON '19 for $K \to \ell\nu + \ell\nu\gamma$

$$\Gamma_{\ell\nu+\ell\nu\gamma} = \left[\Gamma^{pt}_{\ell\nu}(\mu_\gamma) + \Gamma^{pt}_{\ell\nu\gamma}(\mu_\gamma)\right]_{\mathrm{PT}} + \left[\Gamma_{\ell\nu}(L) - \Gamma^{pt}_{\ell\nu}(L)\right]_{\mathrm{lat}}$$

– divide into IR safe pieces

– $\delta_{\mathrm{EM+SU(2)}}$ = -0.63(7)%

– improvable w/ realistic simulations

– indep. calc. by RBC/UKQCD '22 -0.43(21)%

for $K_{\ell 3}$

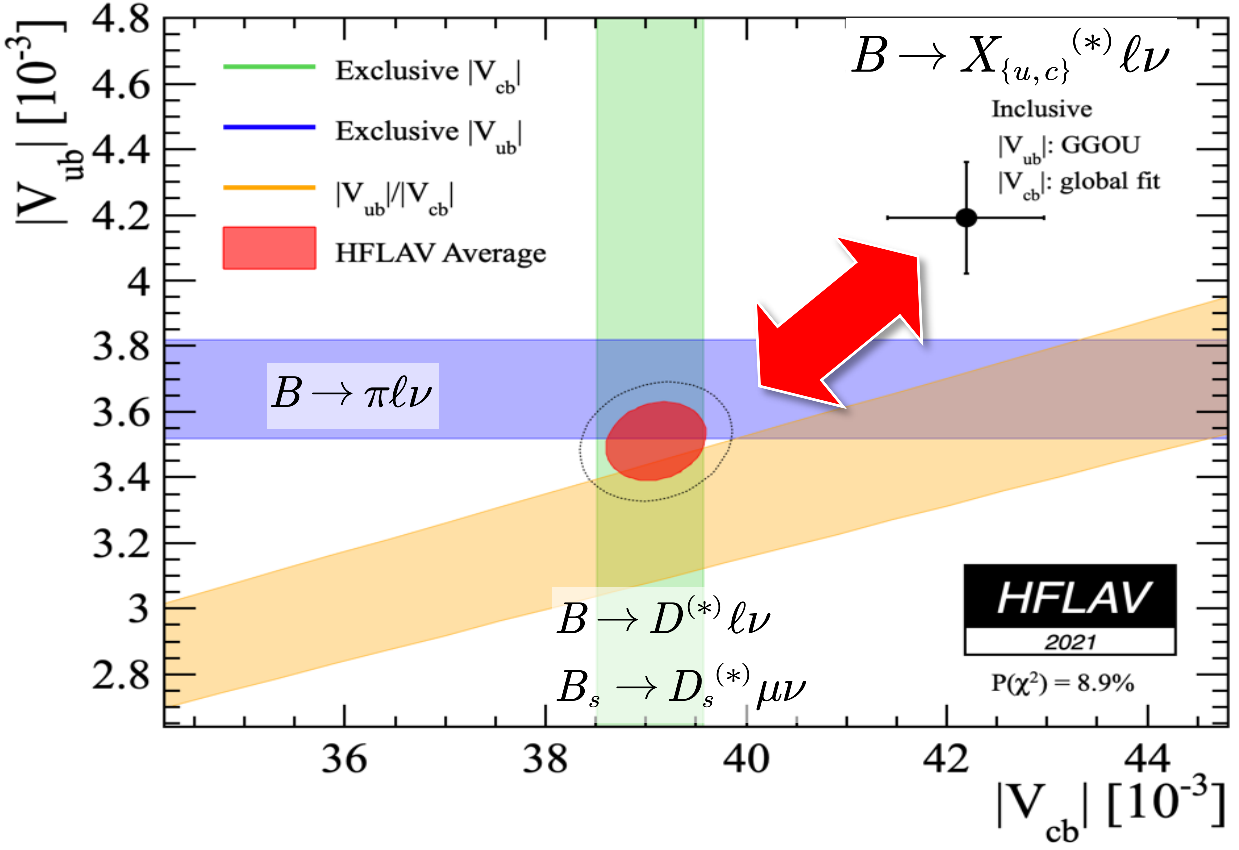
– $\Delta\delta_{\mathrm{EM+SU(2)}}$ = 0.10 – 0.14% ⇔ $\Delta f_+(0)$ = 0.18%

– Christ+ '23: proposal of a framework

$B_{(s)}$ exclusive decays

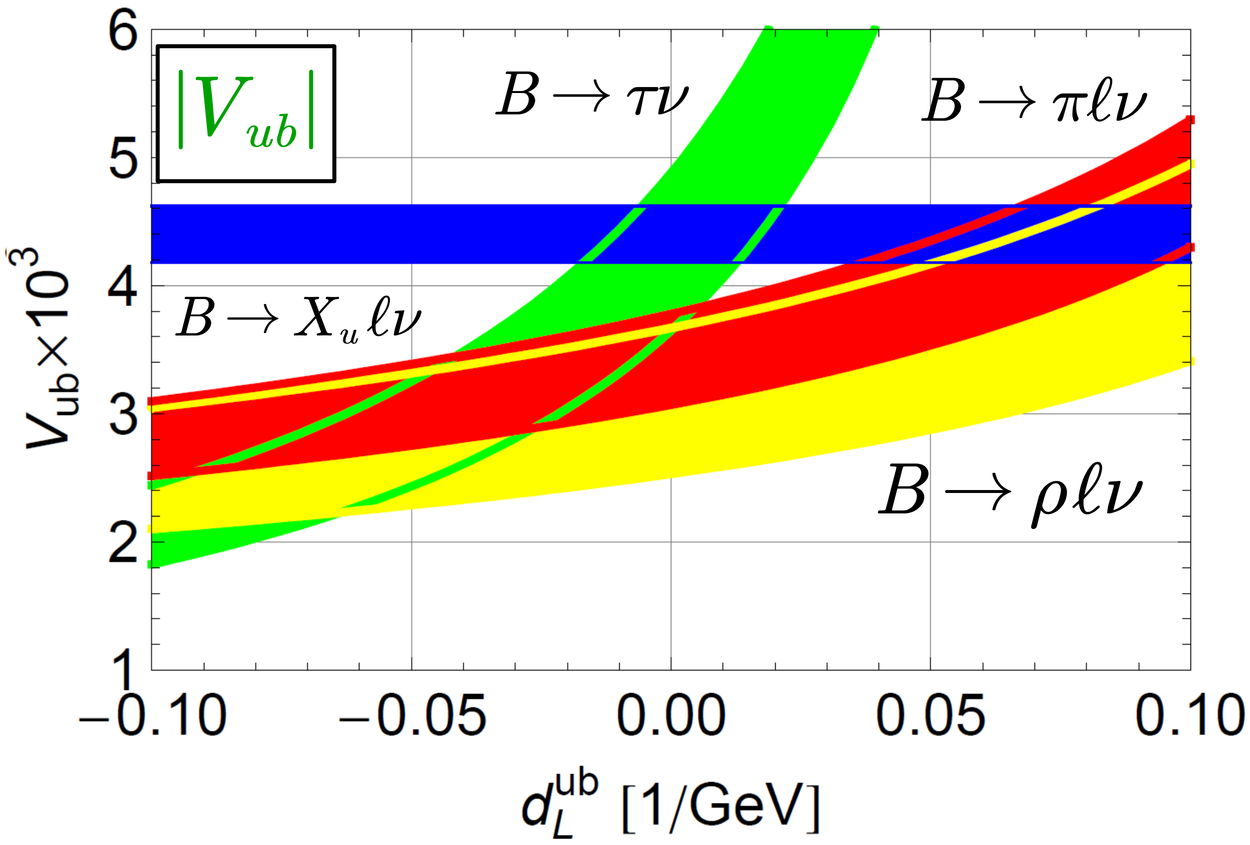
$|V_{cb}|$ and $|V_{ub}|$ tensions

tension b/w exclusive and inclusive decays



$\geq$ 8%, 3σ tension for more than 10 years ...

Crivellin-Pokorski '18



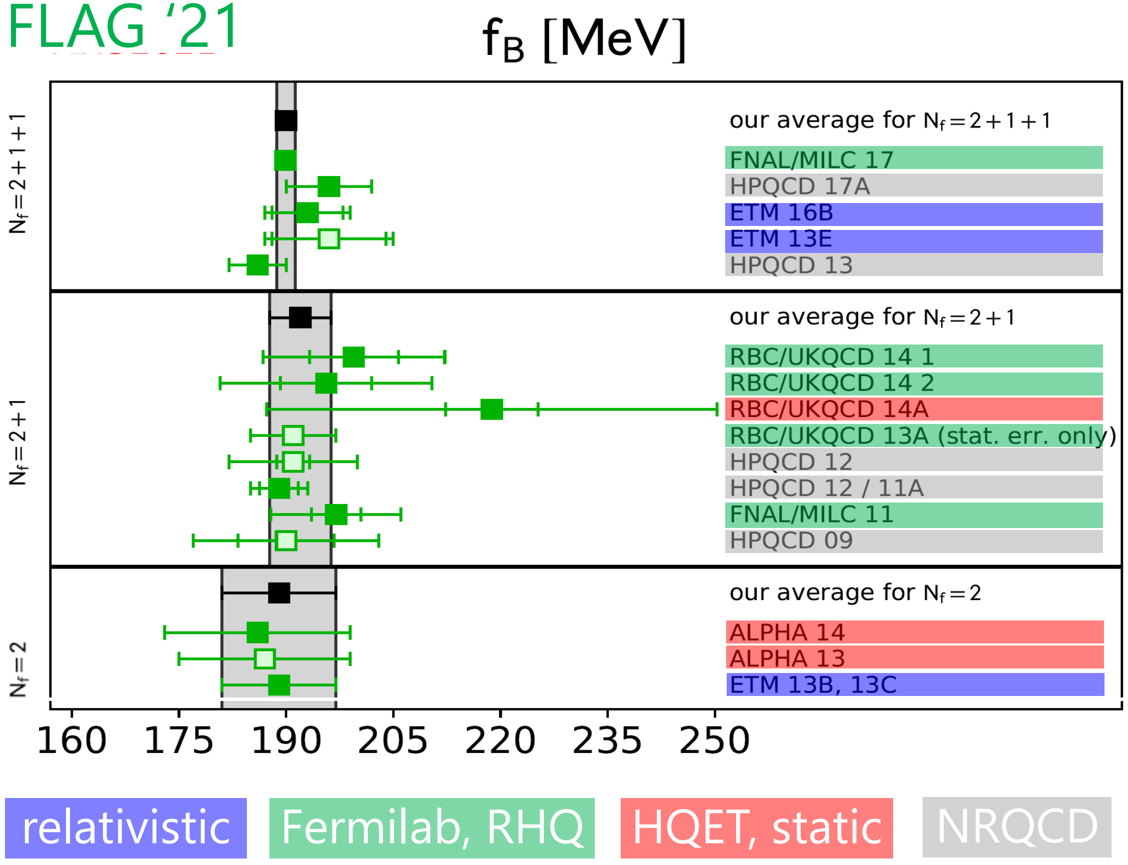
BSM tensor interaction can explain ....

$\Rightarrow$ new tension w/ $B \to \tau\nu$; too large $\Gamma(Z \to bb)$

th. and/or exp. uncertainties have NOT yet been fully understood $\Leftrightarrow$ hadonic inputs

B meson decay constant

FLAG '21



– independent studies⇒ f_B = 190.0(1.3) MeV [0.7%]

– $|V_{ub}|$ = 4.05(3)$_{\rm lat}$(64)$_{\rm exp}$ × 10^{-3} ⇔ helicity suppression

⇒ B2TiP '19 $\Delta|V_{ub}|_{\rm exp}$ ~ 3% @ Belle II 50 ab^{-1}

⇒ competitive to conventional $B \to \pi\ell\nu$

decay constant of other mesons

Parisi '83, Lepage '89

– $ud \to s$, c quarks ⇒ less statistical error

$$\frac{\Delta C_{\rm 2pt}^{\bar{Q}q}(t)}{C_{\rm 2pt}^{\bar{Q}q}(t)} \sim \exp[\alpha t], \quad \alpha_{\bar{Q}q} = M_{\bar{Q}q} - \frac{M_{\bar{Q}Q} + M_{\bar{q}q}}{2}$$

$$\alpha_B = 0.51, \ \alpha_{Bs} = 0.31, \ \alpha_{Bc} = 0.08$$

B_s

– needed for new physics search on $B_s \to \mu\mu$

– Δf_{Bs} ~ 0.6% ⇒ $\Delta\mathcal{B}$ ~ 4%

⇔ 11% CMS@ICHEP'22 ⇒ 4% [HL-LHC]

B_c

– HPQCD '15 f_{Bc} = 434(15) MeV [3.5%]

⇔ FCC-ee tera-Z $\Delta\mathcal{B}$ ~ 2% Amhis+ '21

toward radiative leptonic decay $B \to \ell \nu \gamma$ ($\ell = e, \mu$)

- lift helicity suppression Belle: $\mathcal{B} < 4(e) - 3(\mu) \times 10^{-6}$ $\Leftrightarrow$ SM: $\ell\nu < 10^{-11} - 10^{-6}$
- $|V_{ub}|$ from $\ell = e, \mu$ channel $\Leftrightarrow$ Belle II (B2TIP '19) $(\Delta\mathcal{B})_{\rm stat} \sim 4\%$ (!)
- hard $\gamma \Rightarrow$ structure of B meson (LCDA, …)

$$T_{\mu\nu} = -i \int d^4x\, e^{ip_\gamma x} \langle 0 | T(J^{\rm EM}_\mu(x) J^{\rm weak}_\nu(0)) | B(p) \rangle = \varepsilon_{\mu\nu\rho\sigma} p_\gamma^\rho v_B^\sigma F_V + i(-g_{\mu\nu} p_\gamma v_B + v_{B,\mu} p_{\gamma,\nu}) F_A + \cdots$$

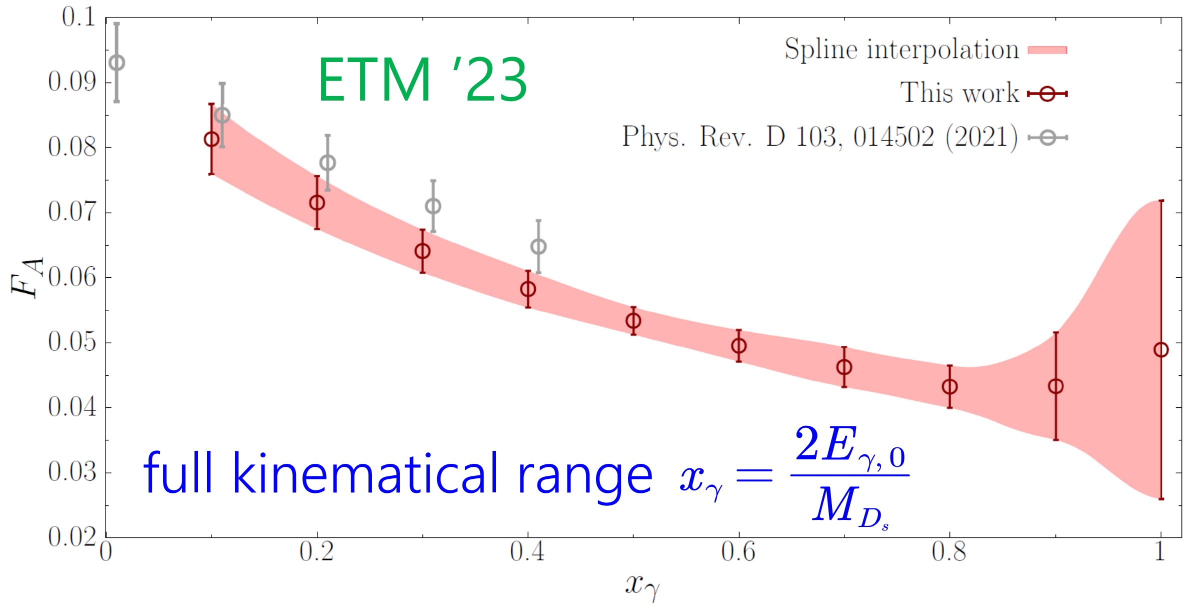
first lattice studies of $D_s \to \ell \nu \gamma$

Giusti+ 2302.01298 [a^{-1} = 1.8GeV, $M_\pi \sim$ 340 MeV]

ETM 2306.05904 [@ m_{ud}, m_s, $m_{c,\rm phys}$, $a \to 0$]

- exponentially suppressed signal
- 5-10%accuracy for FFs (50% at $E_{\gamma,0,\rm max}$ by $a \to 0$)
- ETM $\mathcal{B}$ [SM] = 4.4(3) × 10^{-6} : 7% accuracy

  + consistent w/ BESIII upper limit 1.3 × 10^{-4}



extension to $B \to \ell\nu\gamma$ $\Rightarrow$ much largr $E_{\gamma,0,\rm max} = M_{Ds}/2 \to M_B/2$ (somewhat straightforward)

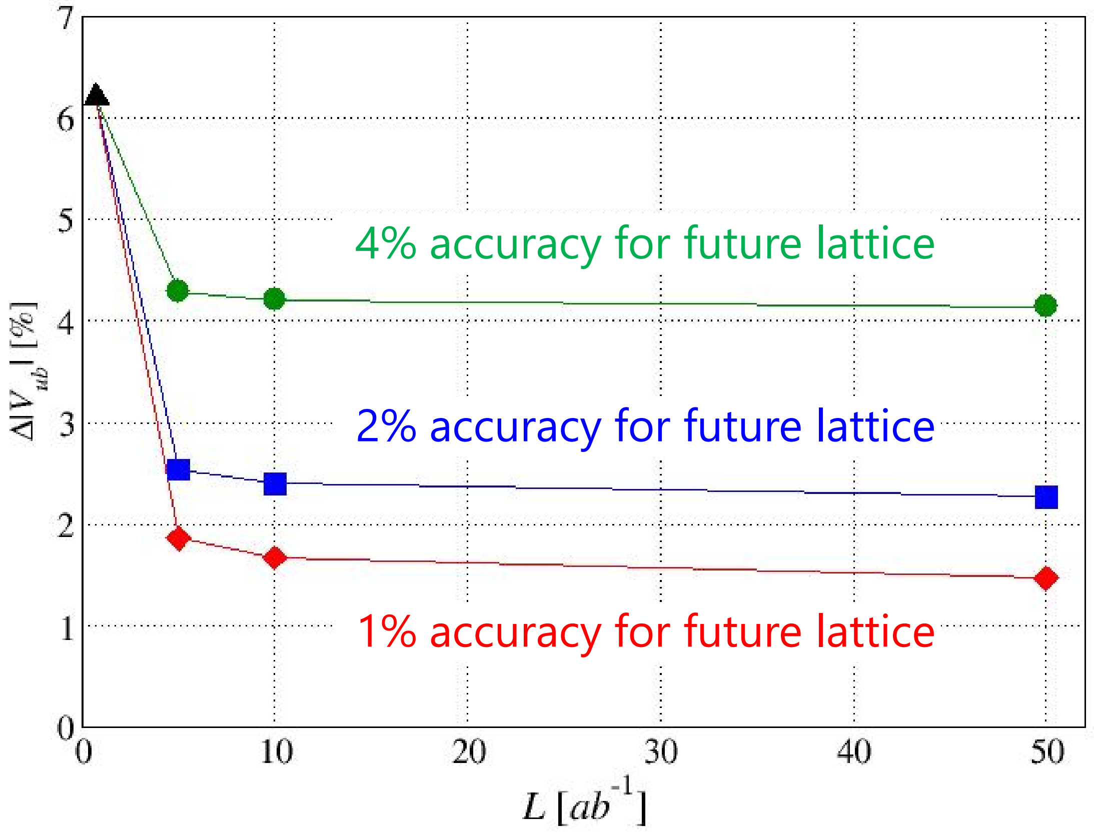
$B \to \pi \ell \nu$ for $|V_{ub}|$

- $B \to \pi e \nu, \pi \mu \nu \Rightarrow$ conventional determination of $|V_{ub}|$
  - ⇔ 2.2σ (12%) tension w/ inclusive decay
- to be improved by Belle II
  - + CKM suppressed: $\mathcal{B} \sim 1.5 \times 10^{-4}$ % ⇔ 2-5% $D^{(*)}\ell\nu$
  - + non-small statistical error reduced by ×50 data
  - + 1 – 2 % accuracy @ 10 ab^{-1} (~2028?)
- $B \to \pi \tau \nu$: new physics in LFUV?
  - + not yet measured : $\mathcal{B} < 2.5 \times 10^{-4}$ %
  - + 14% LFUV ratio by Belle II

$$R(\pi) \sim \Gamma(B \to \pi \tau \nu)/\Gamma(B \to \pi \{e, \mu\} \nu)$$

⇔ 3.4 tension w/ SM in $R(D^{(*)})$

B2TIP '19 : $|V_{ub}|$ expected accuracy (tagged)



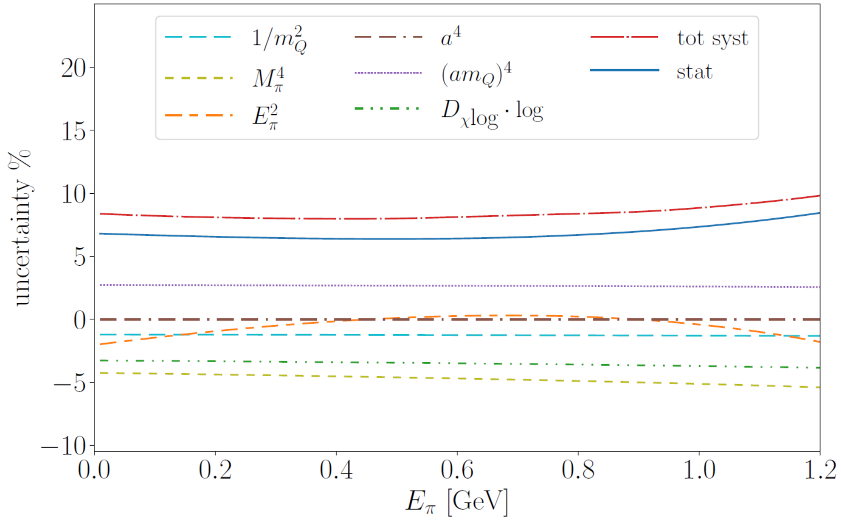
$B \to \pi \ell \nu$ for $|V_{ub}|$

-2020

- 3 studies using NRQCD, HQET b & unphysically large M_π
- best accuracy $\Delta|V_{ub}|$ ~ 5% : Fermilab/MILC @ $M_\pi \geq$ 165MeV

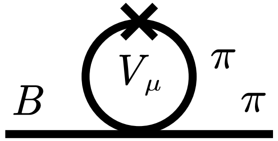
JLQCD 2203.04938 : chiral fermions @ $M_\pi \geq$ 230MeV

- 10% accuracy: largest from stat. and chiral extrap to $M_{\pi,\text{phys}}$



HMChPT (Bijnens-Jemos '11)

$$\delta f = -\frac{3}{4}(1+3g^2_{B^*B\pi})M_\pi^2 \ln\left[\frac{M_\pi^2}{\Lambda^2}\right]$$



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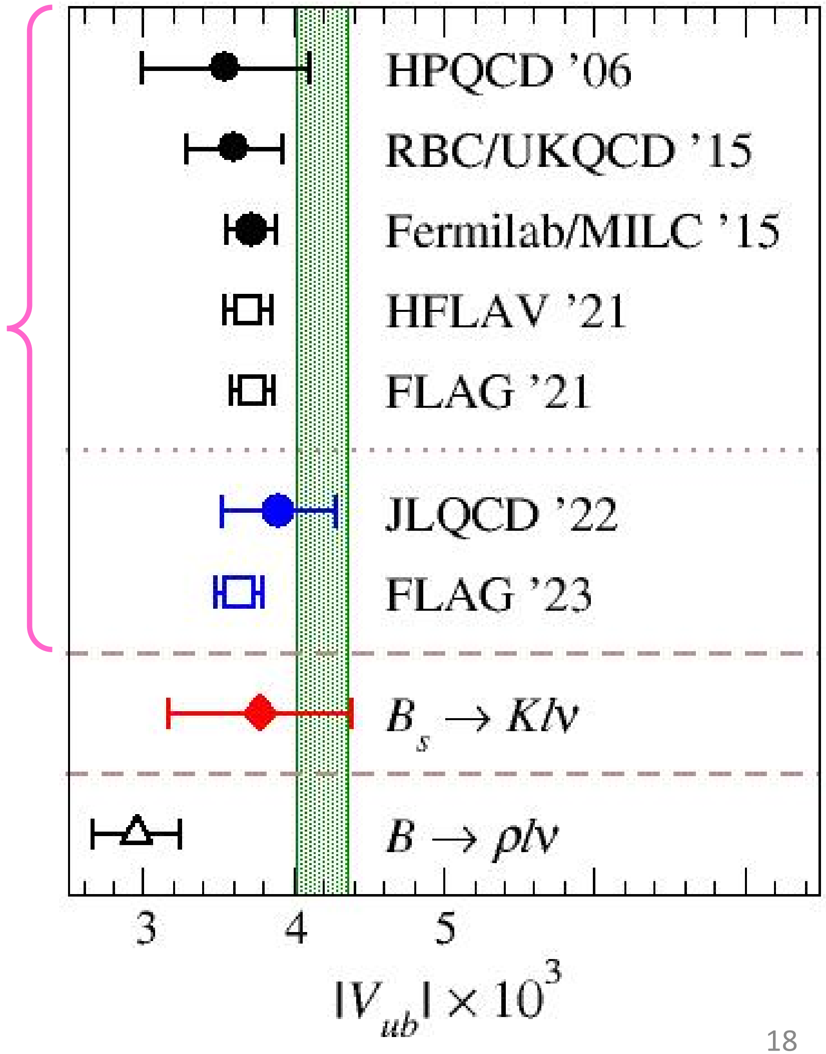
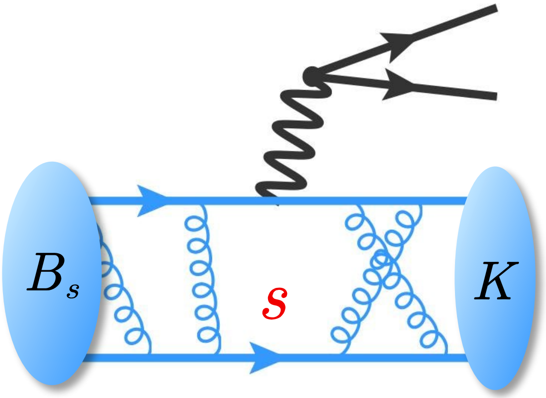


- STILL, world average dominated by "OLD" Fermilab/MILC ...

⇒ Fermilab/MILC, RBC/UKQCD, JLQCD: high statistics @ $M_{\pi,\text{phys}}$

a target : a few % accuracy by different groups in 5 years

$B_s \to K\ell\nu$: an alternative for $|V_{ub}|$

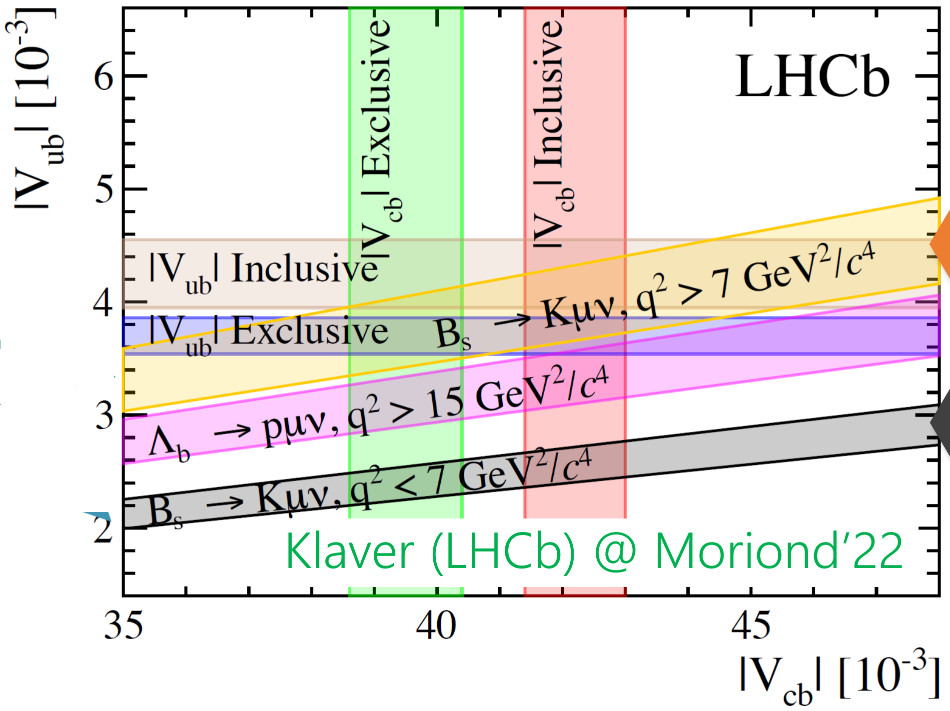


"$B \to \pi\ell\nu$"

w/ s spectator

"advantages" in lattice QCD

- suppressed chiral log. ⇒ better control of chiral extrap
- Parisi '83, Lepage '89 ⇒ less statistical error

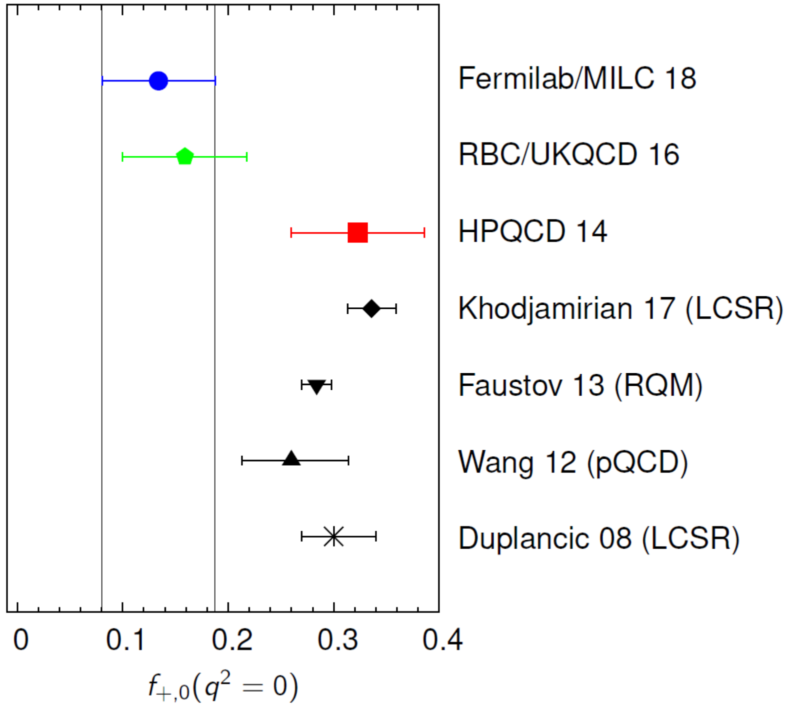


$$\frac{B_s \to K\mu\nu}{B_s \to D_s\mu\nu} \Rightarrow \frac{|V_{ub}|}{|V_{cb}|}$$

q^2 > 7GeV^2
w/ lattice FFs
(Fermilab/MILC'19)l

q^2 < 7GeV^2
w/ LCSR FFs
(Khodjamirian+'17)

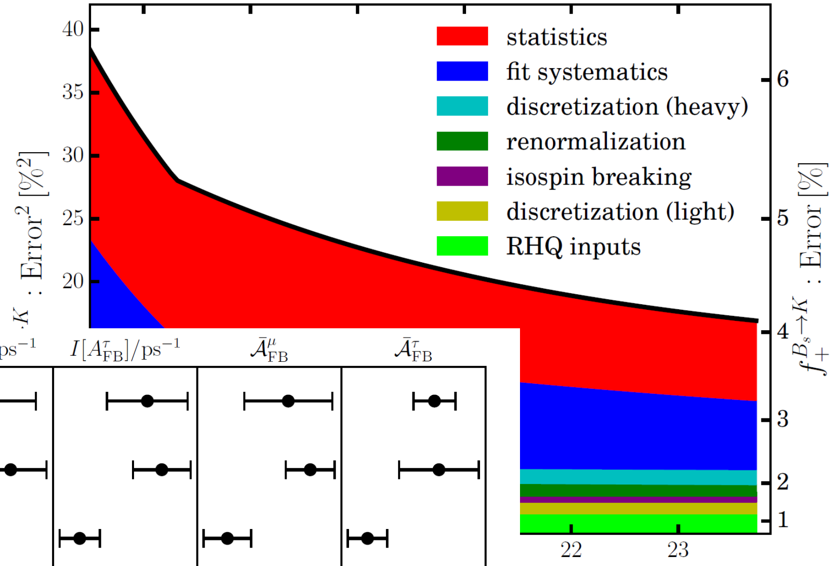
Fermilab/MILC '19

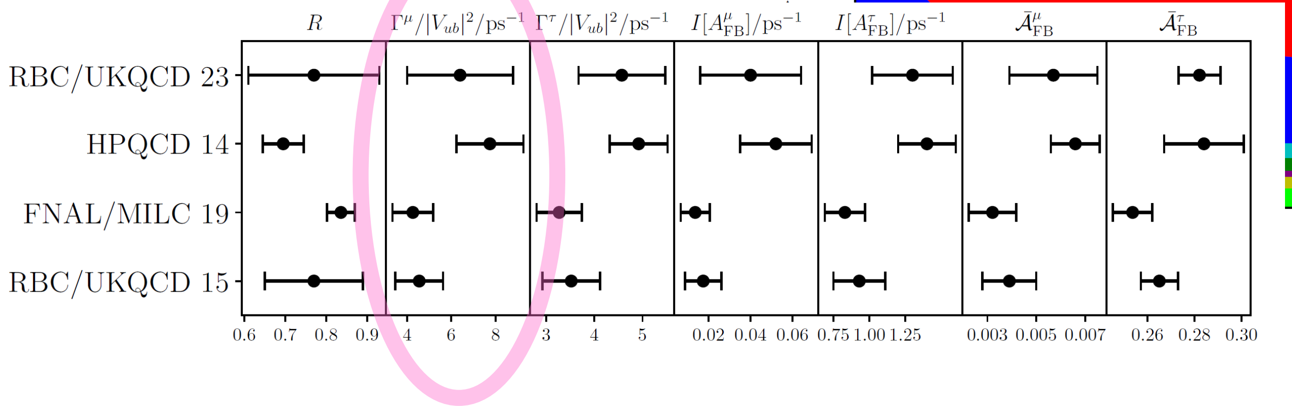


$B_s \to K\ell\nu$: an update

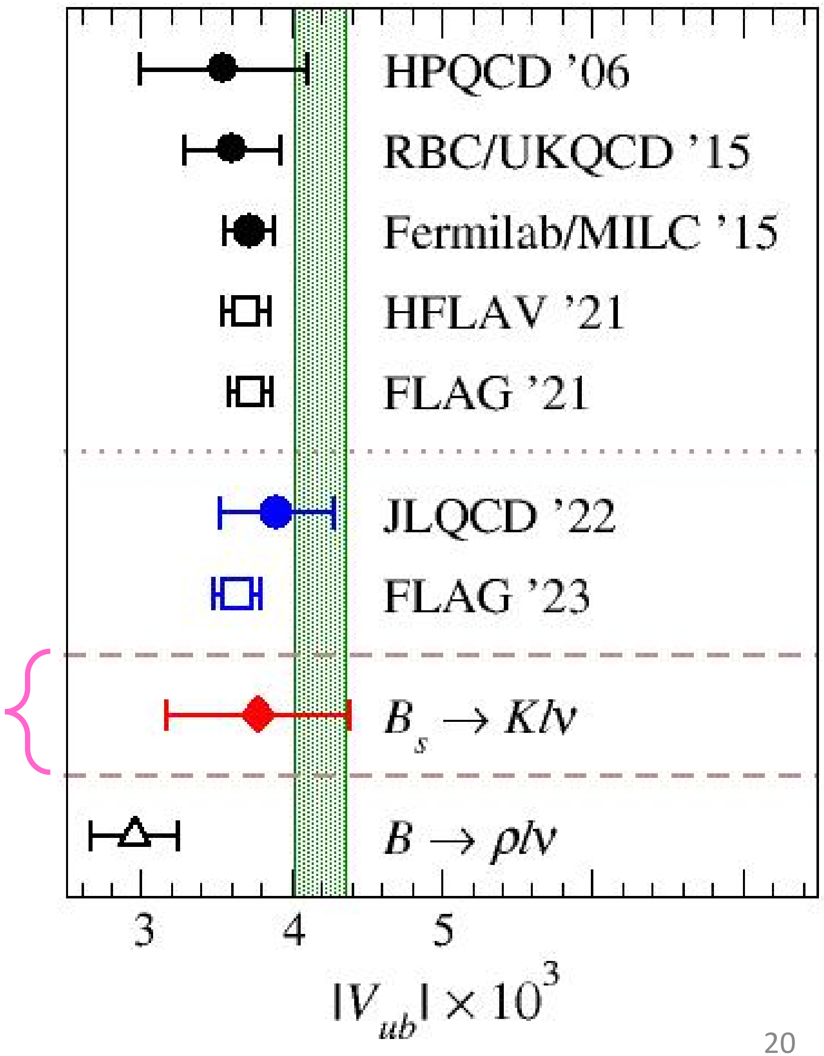
RBC/UKQCD 2303.11280 w/ larger a^{-1} ~ 2.8GeV (M_π ~ 260 MeV)

- 5% error at simulated q^2
- ≥ 10% towards $q^2 = 0$
- largest from stat., extrap.
- consistent w/ previous studies





inclusive



- w/ LHCb $\mathcal{B}(B_s \to K\ell\nu) / \mathcal{B}(B_s \to D_s\ell\nu) \otimes \mathcal{B}(B_s \to D_s\ell\nu) \otimes \tau_{Bs}$

⇒ $|V_{ub}| = 3.78(61) \times 10^{-3}$ w/ 16% error ⇒ 10% th, 13% ex

⇒ on-going JLQCD (indep), HPQCD, Fermilab/MILC, RBC/UKQCD (smaller a) : better than $B \to \pi\ell\nu$ in 5y

Lat '24

$B \to \rho \ell \nu$: non-gold-plated for $|V_{ub}|$

Bernlochner+ 2104.05739

– w/ LCSR estimate for FFs

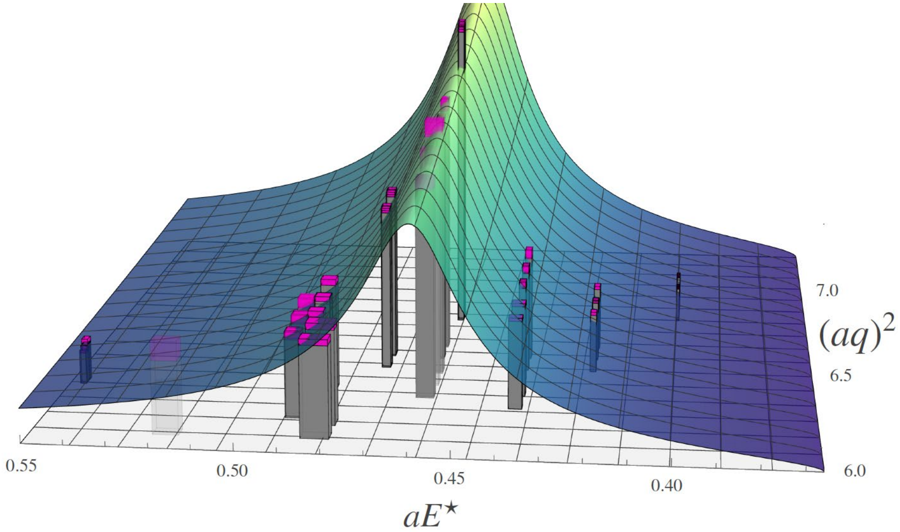
– $|V_{ub}|$ = 2.96(29) × 10^{-3} [10% error, -2.1σ below from $B \to \pi \ell \nu$]

Leskovec+ @ Lattice '22-23, CKM'23

– unstable ρ (non-gold-plated), but decays almost only to $\rho \to \pi\pi$

– framework to extract MEs from lattice corr. functions (Briceno+ '21)

$$\langle \pi\pi | V_\mu | B \rangle = i\epsilon_{\mu\nu\rho\sigma} \varepsilon'^{\nu} v'^{\rho} v^{\sigma} F_V(q^2, E^*)$$



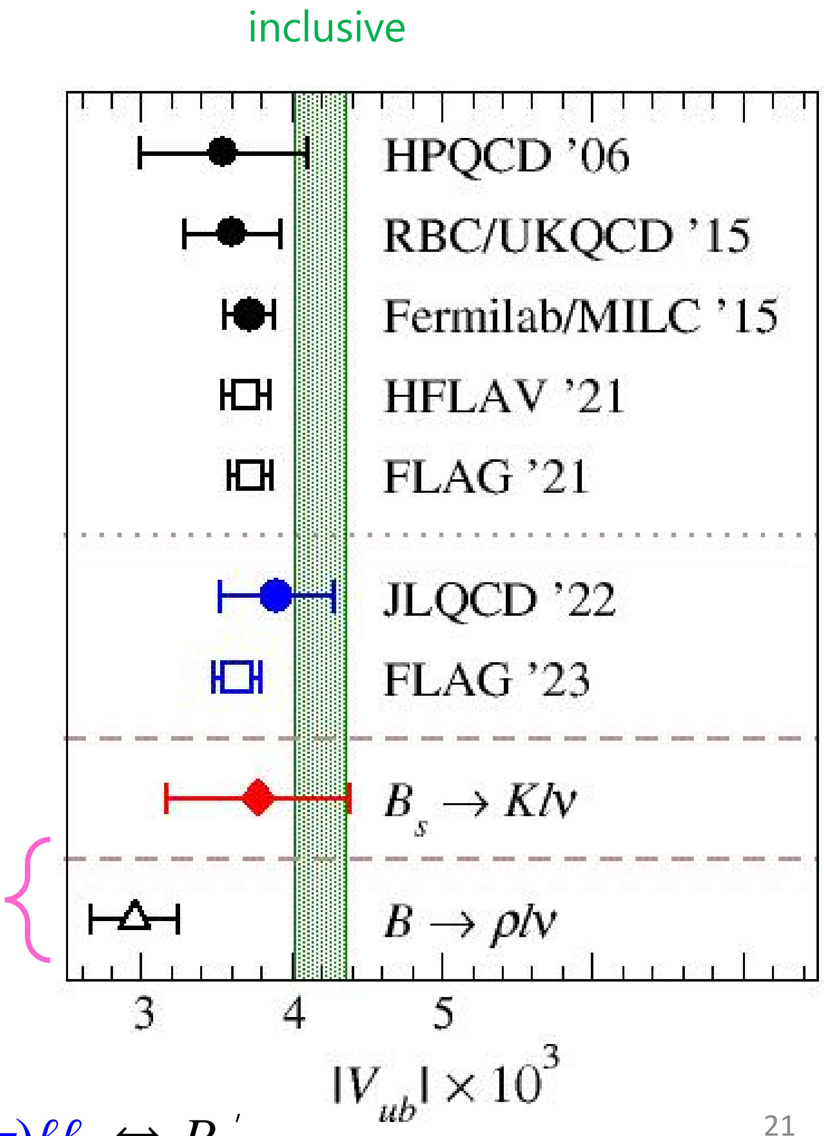
1st lattice attempt

– no $a \to 0$ / chiral extrap.s

– no axial FFs

amplitudes as a func. of 2 Lorentz invariants

may extend to $B \to K^*(\to K\pi)\ell\ell \Leftrightarrow P_5'$



$B \to D^* \ell \nu$ FFs

$$\langle D^*(p',\epsilon)|V^\mu|\bar{B}(p)\rangle = ig\epsilon^{\mu\alpha\beta\gamma}\epsilon^*_\alpha p'_\beta p_\gamma,$$

$$\langle D^*(p',\epsilon)|A^\mu|\bar{B}(p)\rangle = f\epsilon^{*\mu} + (\epsilon^*\cdot p)\left[a_+(p+p')^\mu + a_-(p-p')^\mu\right]$$

$$\mathcal{F}_1 = \frac{1}{M_D^*}\left\{2k^2q^2a_+ - \frac{1}{2}\left(q^2 - M_B^2 + M_{D^*}^2\right)f\right\} \qquad \mathcal{F}_2 = \frac{1}{M_D^*}\left\{f + \left(M_B^2 - M_{D^*}^2\right)a_+ + q^2a_-\right\}$$

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{192\pi^3 M_B^3}|V_{cb}|^2\frac{k}{q^5}\left(q^2 - m_\ell^2\right)\left\{\left(2q^2 + m_\ell^2\right)\left(2q^2f^2 + \mathcal{F}_1^2 + 2k^2q^4g^2\right) + 3k^2q^2m_\ell^2\mathcal{F}_2^2\right\}$$

-2020 : only f was calculated @ w=1 on the lattice [Fermilab/MILC '14, HPQCD '17]

– other FFs ($f(w\neq1)$, g, $\mathcal{F}_1$, $\mathcal{F}_2$) fixed to reproduce experiment ⇒ "SM value of $R(D^*)$" w/ exp input

2021- : 3 lattice calculations of all FFs also @ $w\neq1$

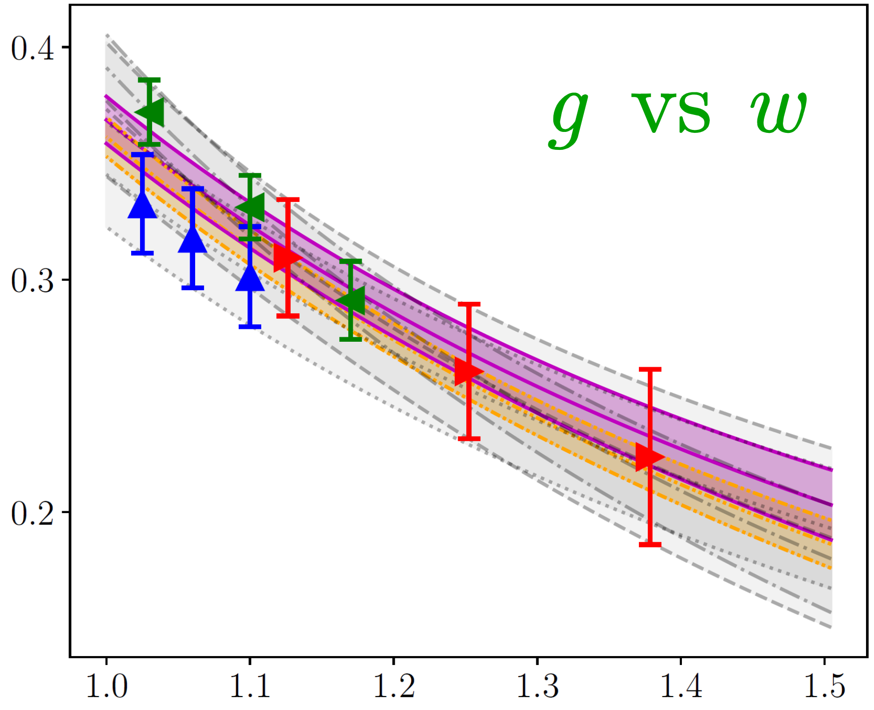
– Fermilab/MILC '21: 1st calc w/ HQET-based b quarks @ physical $m_{b,\mathrm{phys}}$

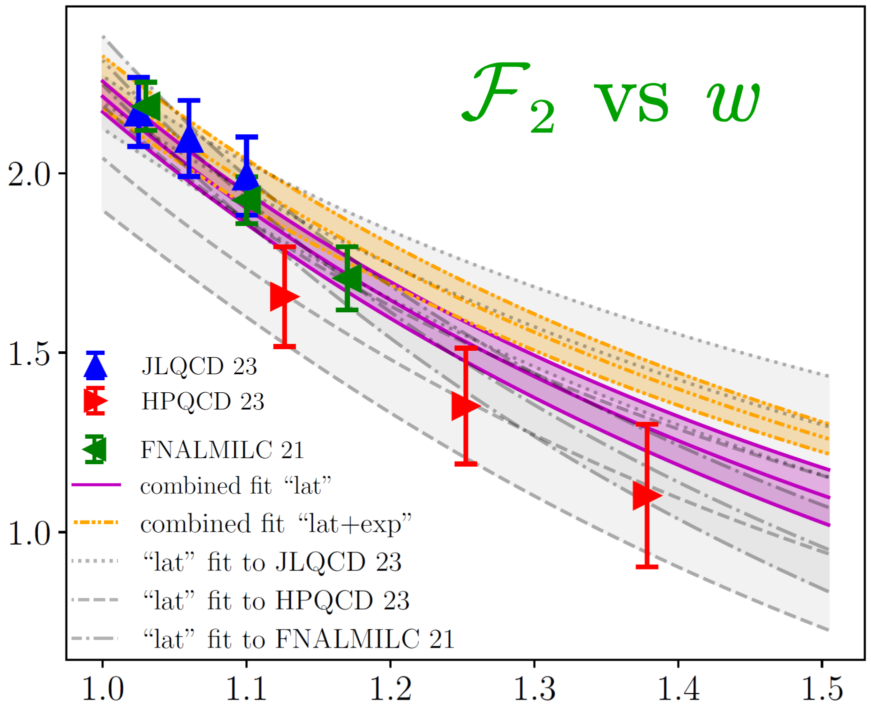
– HPQCD '23: inexpensive relativistic (~~chiral sym.~~, ~~locality~~) b quarks @ unphysically small m_b

– JLQCD '23: relativistic b quarks w/ chiral symmetry @ unphysically small m_b

tension on FFs ?

comparison of FFs : Bordone+ 2406.10074





w/ HPQCD's update Jan '24

⇔ conservative renorm. error

- const, slope of g
  Fermilab/MILC vs JLQCD
- const of $\mathcal{F}_2$
  HPQCD vs others
- slope of $\mathcal{F}_2$
  Fermilab/MILC vs JLQCD

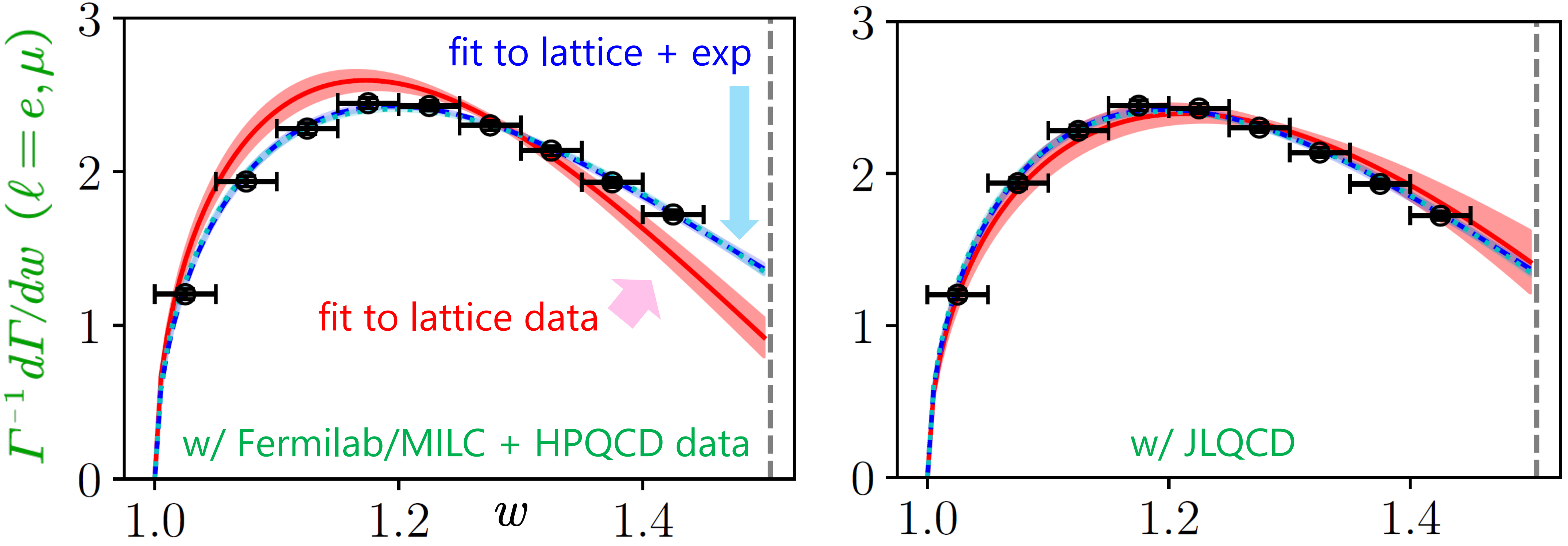
– reasonably consistent @ $w\sim1$ ⇒ individual fits develop difference

– fit of all lattice data w/, w/o exp data ⇒ χ_2/dof ≤ 1, difference ~ w_{max}

⇔ less constrained @ large recoils even w/ model-indep parametrization

tension on FFs ?

comparison of FFs : Bordone+ 2406.10074



does NOT necessarily means JLQCD is correct

– reasonably consistent @ w~1 ⇒ individual fits develop difference

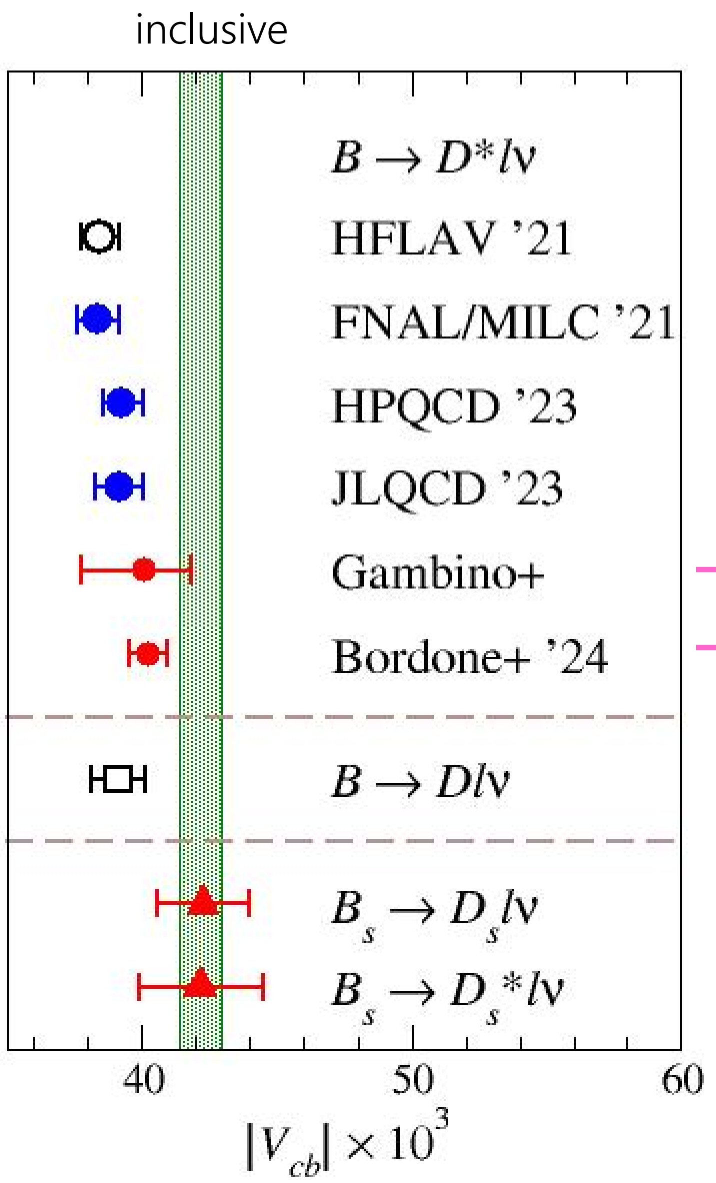
– fit of all lattice data w/, w/o exp data ⇒ χ_2/dof ≤ 0.1, difference ~ w_{max}

⇔ less constrained @ large recoils even w/ model-indep parametrization

⇒ need "safe" simulations @ large recoils for theoretical predictions ⇔ (maybe) OK for $|V_{cb}|$

$|V_{cb}|$

$|V_{cb}|$ from B decays



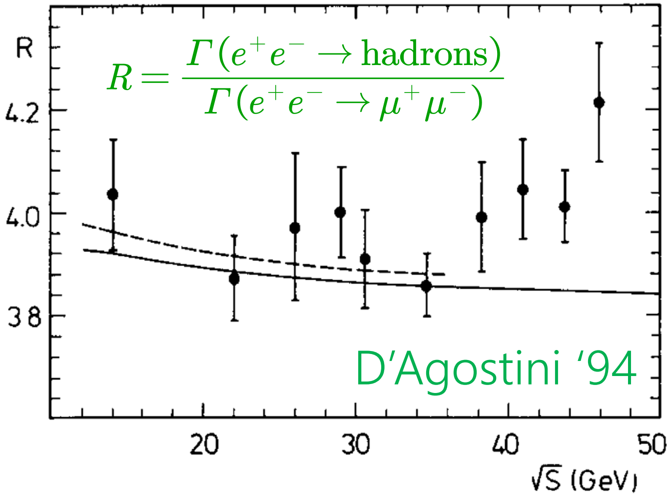
– HFLAV –'21 : $f(w=1)$ from lattice QCD (Fermilab/MILC '14, HPQCD '17)

recent 3 lattice calculation of all FFs ⇒ consistent w/ previous

– Fermilab/MILC: bin analysis ⇒ bin-dependent $|V_{cb}|$

– HPQCD: w/ $B_{(s)} \to D_{(s)}^{(*)} \ell\nu$ ⇔ total $\Gamma(B \to D^* \ell\nu)$ ⇒ $|V_{cb}|$ = 44.6(1.6) ×10^{-3}

$|V_{cb}|$ tension remains unsolved ? ⇔ "Belle II Physics Week '23" @ KEK



D'Agostini effects [Gambino]

– due to stong correlation of ex data

– JLQCD $|V_{cb}| \times 10^3$ = 39.2(9) ⇒ 40.8(+1.8/-2.3)

parametrization of FFs [Ligeti, Gambino, ...]

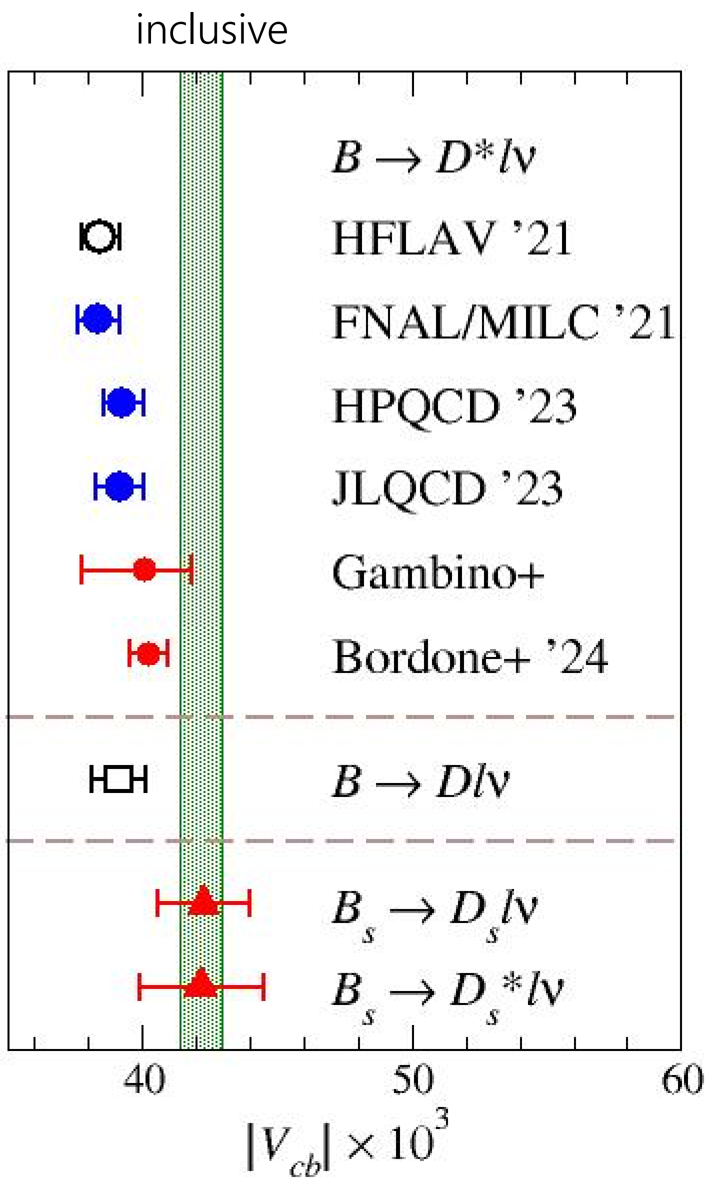
$$f_X(w) \propto \sum_n^{N_X} a_n^X z^n, \quad z = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}$$

Bordone+ '24: Akaike information criteria + recent Belle [II] data '23

need more discussions among theorists & experimentalists

alternatives : $B_s \to D_s^{(*)}\ell\nu$, $B_c \to J/\psi\ell\nu$

|V$_{cb}$| from B_s decays



$B_s \to D_s^{(*)}\ell\nu$ for |V$_{cb}$|

- advantageous on the lattice

  statistical accuracy, chiral extrap (log), gold-plated (*e.g.* $D_s^* \to D_s\gamma$)

HPQCD 2105.11433

- relativistic b, $m_b \leq 3.6$ GeV

  $|V_{cb}| \times 10^{-3} = 42.3(1.2)_{\text{lat}}(1.2)_{\text{exp}}$ $[B \to D_s]$

  $43.0(2.1)_{\text{lat}}(1.7)_{\text{exp}}(0.4)_{\text{EM}}$ $[B \to D_s^*]$

- consistent w/ both B exclusive & inclusive w/ 4-6 % error

HPQCD, RBC/UKQCD @ Lattice '24: reduce largest err from stat, $a \neq 0$

$B_c \to J/\psi\ell\nu$ for LFUV: HPQCD 2007.06957(PRD), 2007.06956

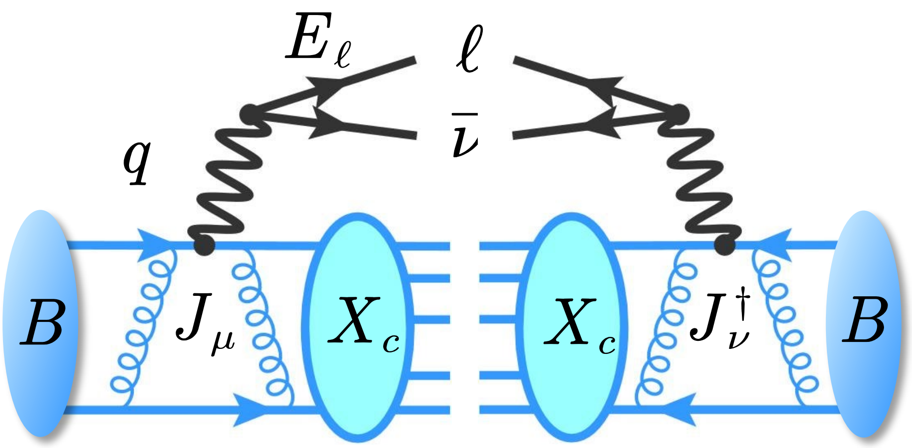
$R(J/\psi) = 0.258(4) \Leftrightarrow 0.71(25)$ LHCb '18

- LHCb 1808.08865: $\Delta(R(J/\psi)) = 0.07$ (Run-3), 0.02 (HL-LHC)
- HPQCD @ Lattice '24: $a^{-1} = 4.5 \to 6.6$ GeV

inclusive decays

B meson inclusive semileptonic decays

$B \to X_c \ell \nu$ inclusive rate



$$\frac{d\Gamma(B \to X_c \ell \nu)}{d\mathbf{q}^2 dq_0 dE_\ell} = \frac{G_F^2}{8\pi^3} |V_{cb}|^2 L^{\mu\nu} W_{\mu\nu}$$

$$W_{\mu\nu} \sim \sum_{X_c} \langle B | J_\mu^\dagger | X_c \rangle \langle X_c | J_\nu | B \rangle = \mathrm{im} \langle B | J_\mu^\dagger \otimes J_\nu | B \rangle$$

hadronic tensor optical theorem

conventional analysis : OPE

$$W_{\mu\nu} = \sum_{\mathcal{O}} \frac{c_{\mathcal{O}}(\alpha_s)}{m_b^{n_{\mathcal{O}}}} \langle B | \mathcal{O} | B \rangle$$

double expansion in α_s, $1/m_Q$: convergence?

how to provide non-perturbative MEs?

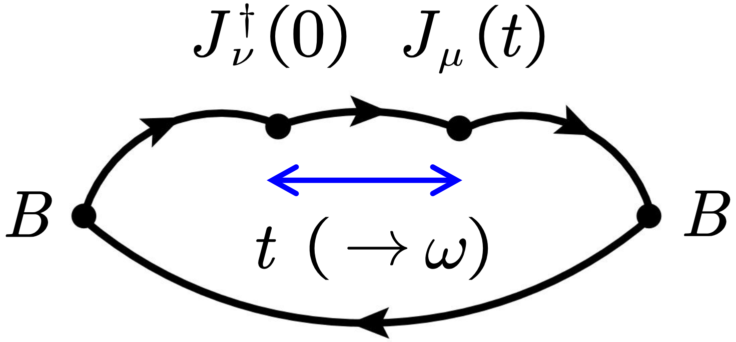
⇔ lattice QCD : *e.g.* JLQCD '02

inclusive ≠ NOT simple sum of exclusive ⇒ non-trivial crosscheck of CKM MEs

lattice QCD ⇔ direct determination of $W_{\mu\nu}$ from 1st principles

new idea

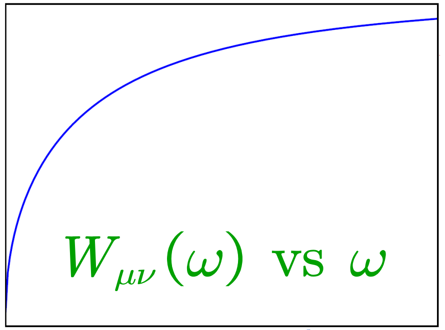
$$C_{\mu\nu}(t,\boldsymbol{q}) = \langle B | J_\mu^\dagger(-\boldsymbol{q}) e^{-Ht} J_\nu^\dagger(\boldsymbol{q}) | B \rangle = \int_0^\infty d\omega W_{\mu\nu}(\omega,\boldsymbol{q}) e^{-\omega t}$$



$C_{\mu\nu}$(t) $\Rightarrow$ $W_{\mu\nu}$: ill-posed inverse problem

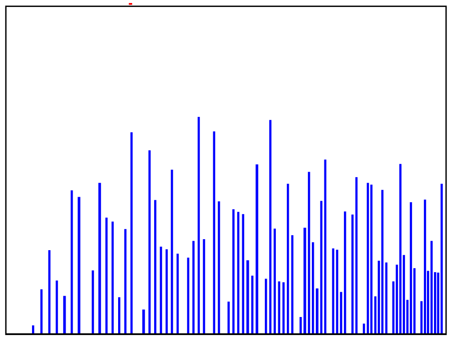
- finite $V \Rightarrow$ severely deformed $W_{\mu\nu}(\omega)$
- limited info of $C_{\mu\nu}(t)$ @ discrete t w/ stat. error

$$\int d\omega K(\omega)$$



$L \rightarrow \infty$

$$\sim \int d\omega K_{L,\sigma}(\omega)$$



finite V

Hashimoto '17, Hansen+ '17, Gambino-Hashimoto '20

may evaluate the energy integral of $W_{\mu\nu}(\omega)$ w/ good precision (various technical issues)

$$\frac{d\Gamma}{d\boldsymbol{q}^2} = \frac{G_F^2}{24\pi^3} |V_{cb}|^2 \sqrt{\boldsymbol{q}^2}\, \bar{X}(\boldsymbol{q}^2) \qquad \bar{X}(\boldsymbol{q}^2) = \int_0^\infty d\omega K_{\mu\nu,\sigma}(\omega,\boldsymbol{q}^2) W_{\mu\nu,L}(\omega,\boldsymbol{q}^2)$$

feasibility study

Gambino+ 2203.11762

– computationally inexpensive setup

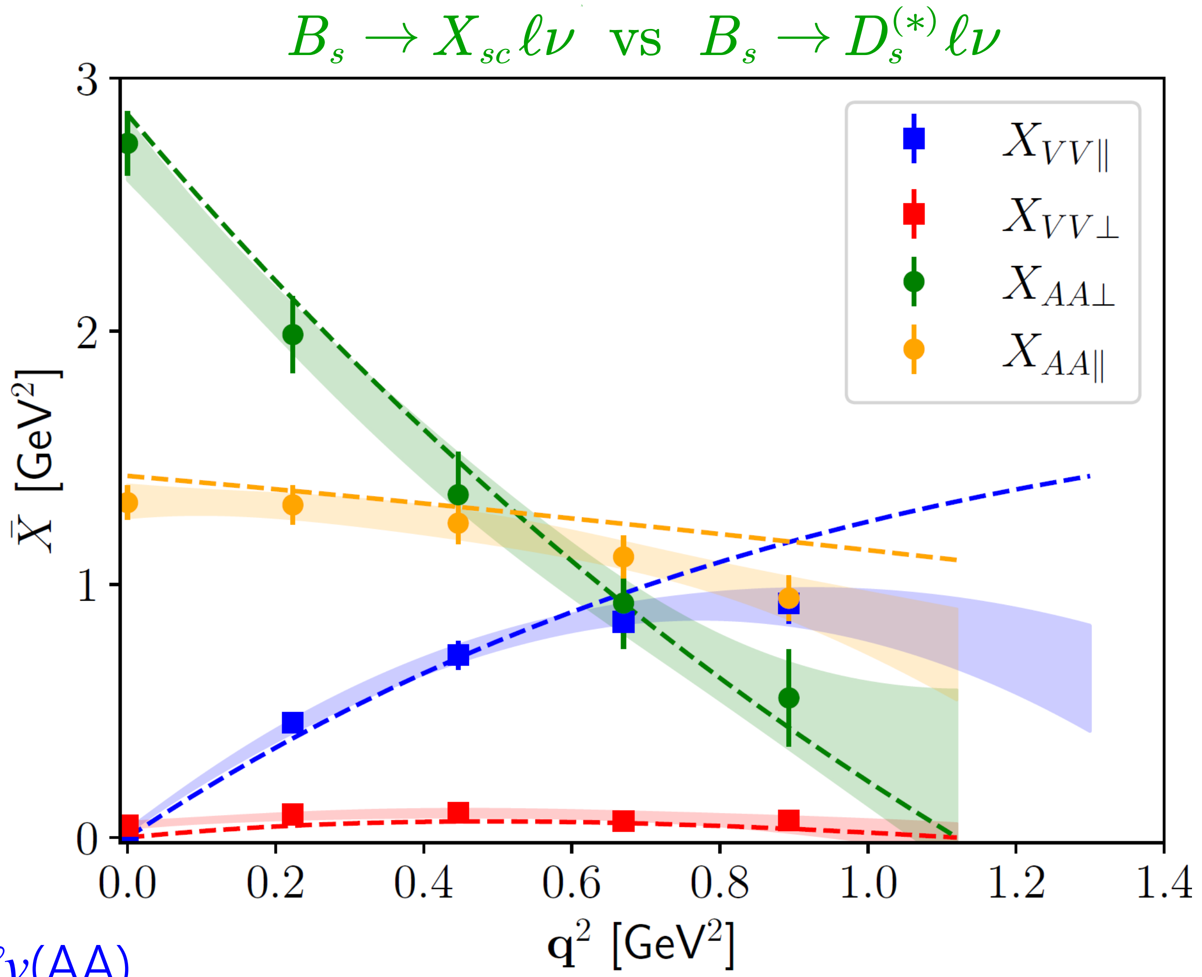
+ $B_s \to X_{sc}\ell\nu$

+ a = 0.1 fm, $m_b = 2.4\, m_c$

– inclusive rates for $J_{\mu'}$ J_ν comb. & polarization

$$\bar{X}(\boldsymbol{q}^2) = \int_0^\infty d\omega\, K_{\mu\nu,\sigma}(\omega,\boldsymbol{q}^2)\, W_{\mu\nu,L}(\omega,\boldsymbol{q}^2)$$

$$\frac{d\Gamma}{d\boldsymbol{q}^2} = \frac{G_F^2}{24\pi^3}|V_{cb}|^2\sqrt{\boldsymbol{q}^2}\,\bar{X}(\boldsymbol{q}^2)$$



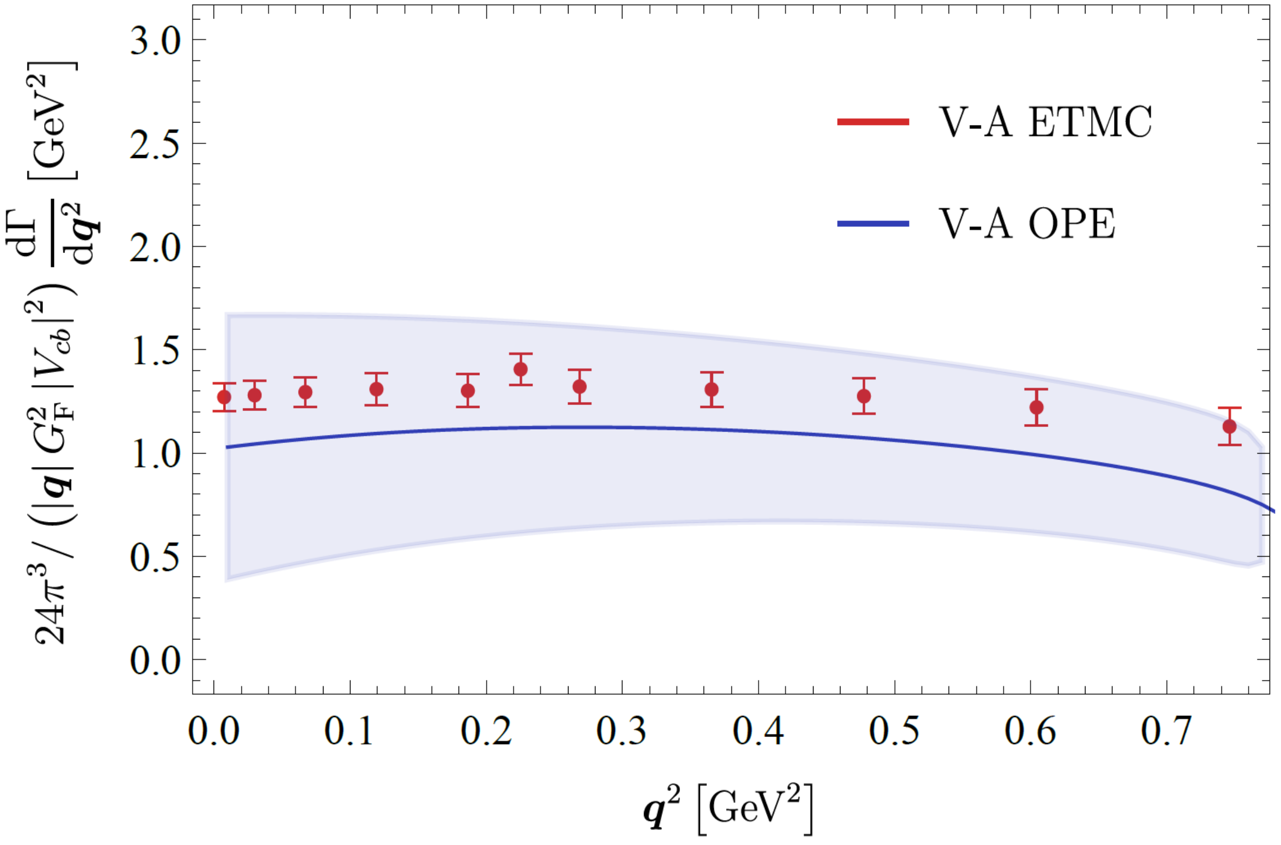
– close to dominant exclusive rate $B_s \to D_s\ell\nu$(VV), $D_s^*\ell\nu$(AA)

⇒ validity of the strategy

– too close? ⇒ limited phase space by $m_b \sim m_{c,\mathrm{phys}}$

vs OPE

even at unphysical m_q's, rigorous comparison w/ conventional OPE is possible



Gambino+ 2203.11762 (cont'd)

– $B_s \to X_{sc} \ell \nu$
– unphysical m_b both for lattice & OPE

– OPE
  + NP MEs from fit to exp
  + $O(1/m_b{}^3)$ correction
  + $O(\alpha_s)$ radiative correction

– lattice inclusive : statistical error only

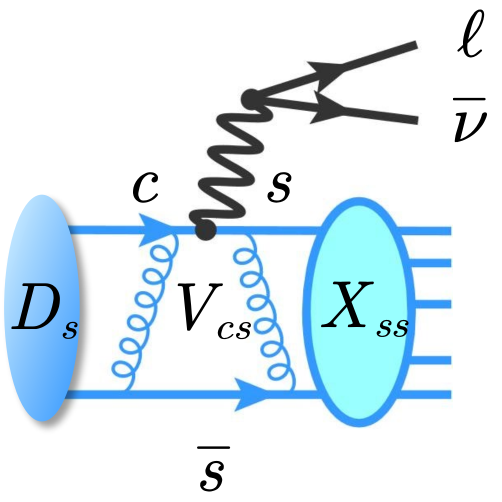
confirmed good consistency b/w lattice and OPE calculations

Gambino-Hashimoto '20 : better consistency w/ higher order for OPE

toward quantitative calculation

$D_s \to X_{ss}\ell\nu$ inclusive decay

– all conventional errors controlled ⇒ good testing ground of systematics
  all valence m_q's to physical / $a^{-1} >> m_c$ / statistics
– most non-trivial systematics ~ finite V effects (FVEs)

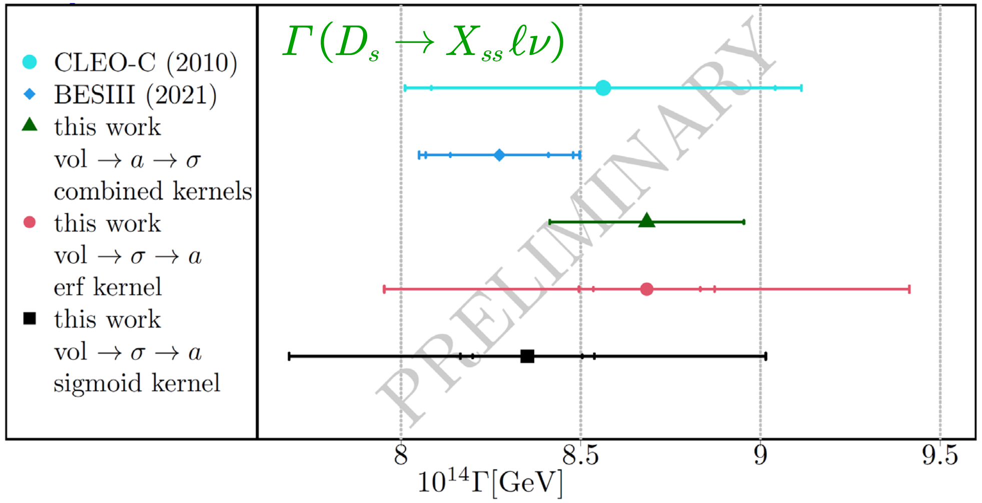


Kellermann+ (KEK-CERN) @ Lattice '23, '24

– develop a model to evaluate non-trivial FVEs via $D_s \to \Phi(\to KK)\ell\nu$
– for a chosen set of parameters, 5% calculation of $d\Gamma/d\boldsymbol{q}^2$ is possible

ETM @ Lattice '24

– study FVEs by directly simulating different V's
– 3-7% calculation of Γ is possible

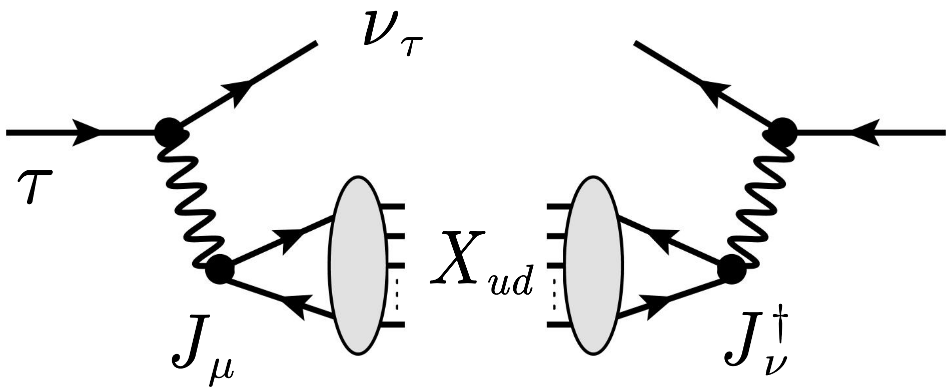


next target : B decays w/ systematics controlled

inclusive τ decays

$\tau \to X_{ud} \nu_\tau$ inclusive rate

$$\Gamma(\tau \to X_{ud}\nu_\tau) = \frac{G_F^2}{4m_\tau}|V_{ud}|^2 \int \frac{d^3q}{(2\pi)^3 2E_\nu} L^{\mu\nu}(p_\tau, p_\nu)\rho_{\mu\nu}(q)$$



$$\rho_{\mu\nu} \sim \sum_{X_c} \langle 0|J_\mu^\dagger|X_{ud}\rangle \langle X_{ud}|J_\nu|0\rangle = \mathrm{im}\langle 0|J_\mu^\dagger \otimes J_\nu|0\rangle$$

no initial/final state hadron ⇒ lattice 2-pt funcions

normalized rate to determine $|V_{ud}|$

$$R_{ud} = \frac{\Gamma(\tau \to X_{ud}\nu_\tau)}{\Gamma(\tau \to e\bar{\nu}_e\nu_\tau)} = 6\pi S_{\mathrm{EW}}|V_{ud}|^2 \int_0^1 \frac{dE}{2\pi} K_{\mu\nu}\rho_{\mu\nu}(s) \qquad \Gamma(\tau \to e\bar{\nu}_e\nu_\tau) = \frac{G_F^2 m_\tau^5}{192\pi^2}$$

$$C_{\mu\nu}(t, \boldsymbol{q}) = \int_0^\infty \frac{dE}{2\pi} e^{-Et} \rho_{\mu\nu}(E, \boldsymbol{q})$$

– $\rho_{\mu\nu}$ largely distorted in a finite volume ⇒ $C_{\mu\nu} \to \rho_{\mu\nu}$: ill-posed problem again

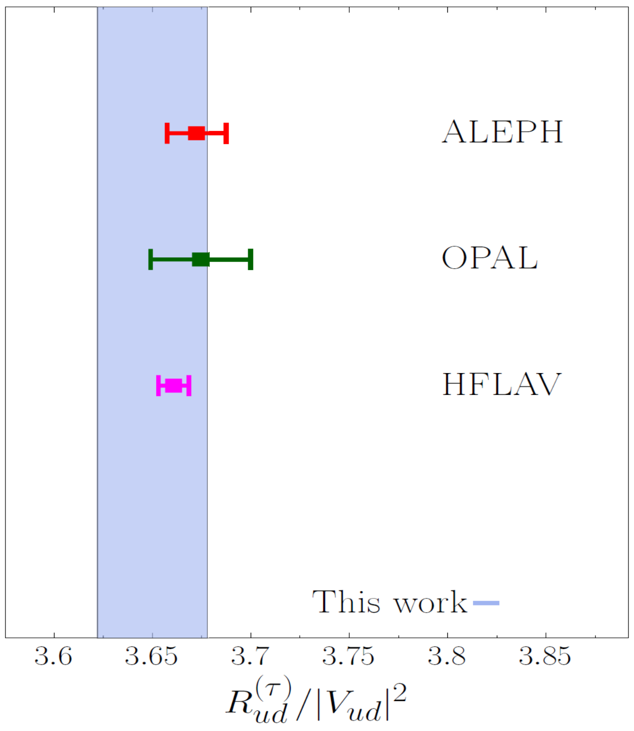
– may evaluate its energy integral (=inclusive rate) a la Hashimoto-Gambino, Hansen+

– 2pt func ⇒ much better control of systematics than 4-pt for B decays !!

first lattice study of "fully" inclusive τ decays

ETM 2308.03125 calculation of $R_{ud}/|V_{ud}|^2$

- physical m_{ud} → no m_q extrapolation
- 3 a's → controlled a=0 limit
- 2 V's → study of finite V effects



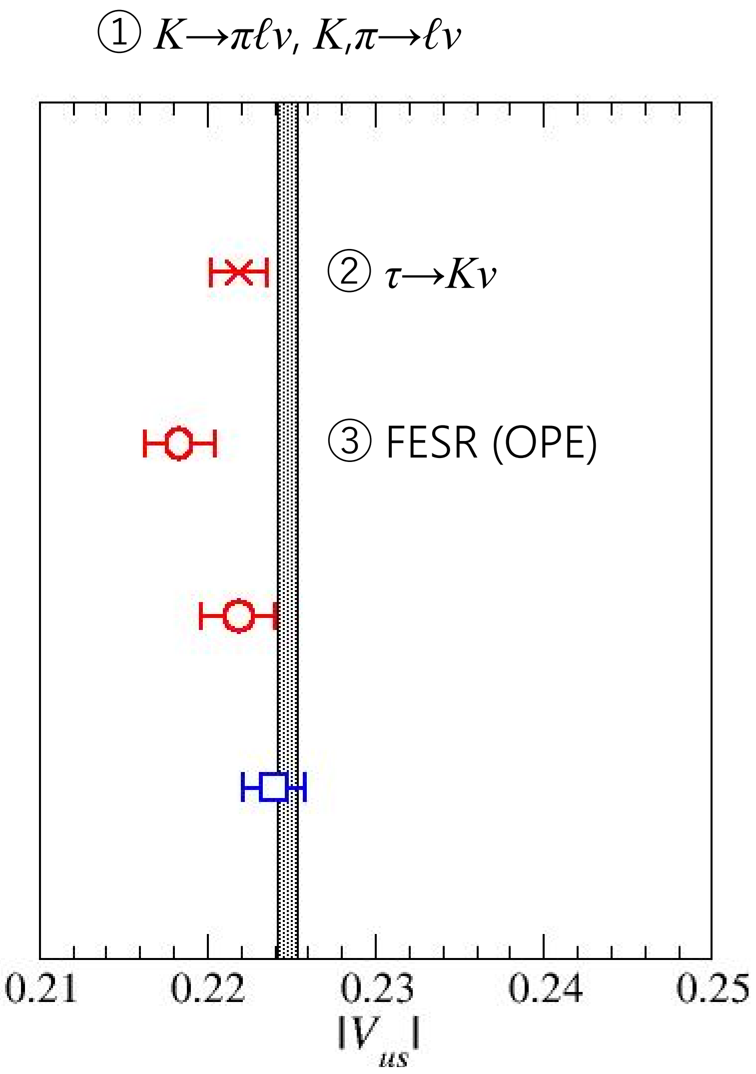
w/ HFLAV R_{ud} ⇒ $|V_{ud}|$ = 0.9752(37)$_{th}$(10)$_{ex}$

- independent determination from inclusive τ decay
- 0.4% determination - error dominated by theory

  statistics, isospin $(m_u\text{-}m_d)/\Lambda_{QCD}$ ~ 0.5%, finite V effects
- consistent w/ conventional 0.03% determination from nuclear β decay : Hardy-Towner '20 $|V_{ud}|$ = 0.97373(31)
- (would-be) competitive to $|V_{ud}|$ determinations from $n\to pe\nu$ [0.1 - 0.2%] and $\pi^+\to\pi^0 e^+\nu$ [0.3%]

extension to $|V_{us}|$

long standing tension b/w τ and K decays



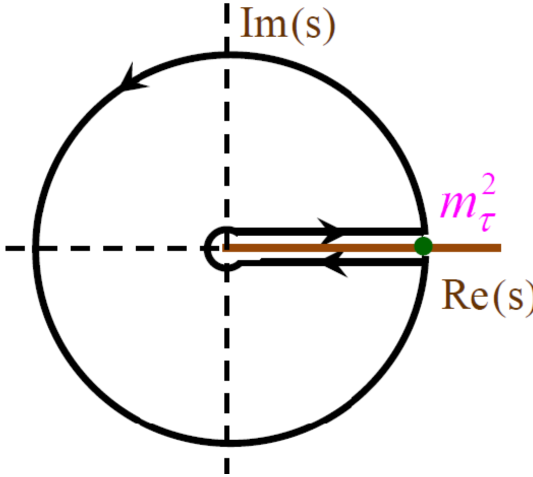
① $K\to\pi\ell\nu$, $K,\pi\to\ell\nu$ ⇒ 0.3% determination of $|V_{us}|$

② $\tau\to K\nu$ ⇒ 0.8% determination consistent w/ kaon decays

③ finite energy sum rule (FESR) + OPE: Gamiz+ '06

$$R_{us} = \int_0^{s_0} ds\; w_{\mu\nu}(s)\,\rho_{\mu\nu}(s) = -\frac{1}{2\pi i}\oint_{s_0} ds\; w_{\mu\nu}(s)\,\Pi_{\mu\nu}(s)$$

$$\Pi_{\mu\nu}(s) = \sum_{\mathcal{O}} \frac{c_{\mathcal{O}}(\alpha_s)}{s^{n_{\mathcal{O}}}}\langle 0|\mathcal{O}|0\rangle$$

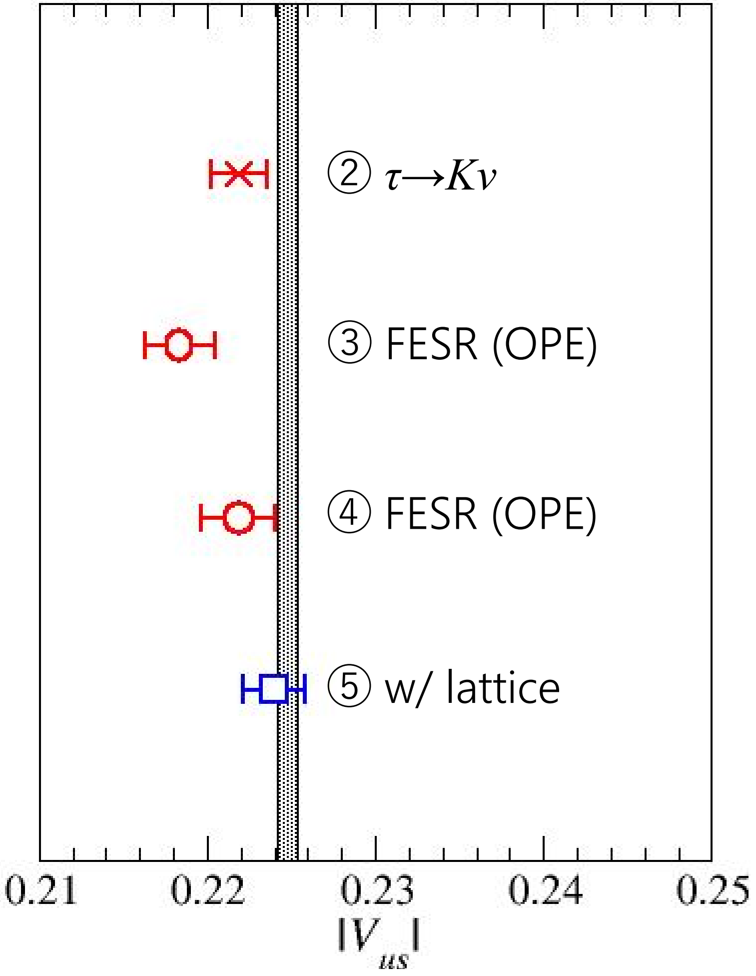


- $s_0 = m_\tau$
- $w(s)$: phase factor ⊗ leptonic
- vacuum saturation for $D\geq 6$
- $\Delta R = R_{ud}/|V_{ud}|^2 - R_{us}/|V_{us}|^2$ (=0 in SU(3))

$|V_{us}|$ = 0.2184(21) 1% determination, 3σ tension w/ kaon decays

extension to $|V_{us}|$

long standing tension b/w τ and K decays



$$R_{us} = \int_0^{s_0} ds\ w_{\mu\nu}(s)\rho_{\mu\nu}(s) = -\frac{1}{2\pi i}\oint_{s_0} ds\ w_{\mu\nu}(s)\Pi_{\mu\nu}(s)$$

④ finite energy sum rule (FESR) + OPE: Maltman+ '15-

– $w(s)$: modified to suppress higher order OPE corrections

– non-perturbative MEs for $D \geq 6$: fixed from fit to lattice data

– slightly different exp data for $K\pi\nu$ (Babar preliminary)

$|V_{us}|$ = 0.2228(23) 1% determination, consistent w/ kaon decays

⑤ w/ 2pt func from lattice: RBC/UKQCD '18

– avoid OPE

– $w(s)$: enhanced weight for $K\nu$, $K\pi\nu$ = "partially" inclusive

$|V_{us}|$ = 0.2240(18) 0.8% determination, consistent w/ kaon decays

$|V_{us}|$ tension resolved ?

first lattice study of "fully" inclusive τ decays

ETM 2403.05404 calculation of $R_{us}/|V_{us}|^2$

- physical $m_{ud,s}$ → no m_q extrapolation
- 4 a's → controlled a=0 limit
- 2 V's → study of finite V effects

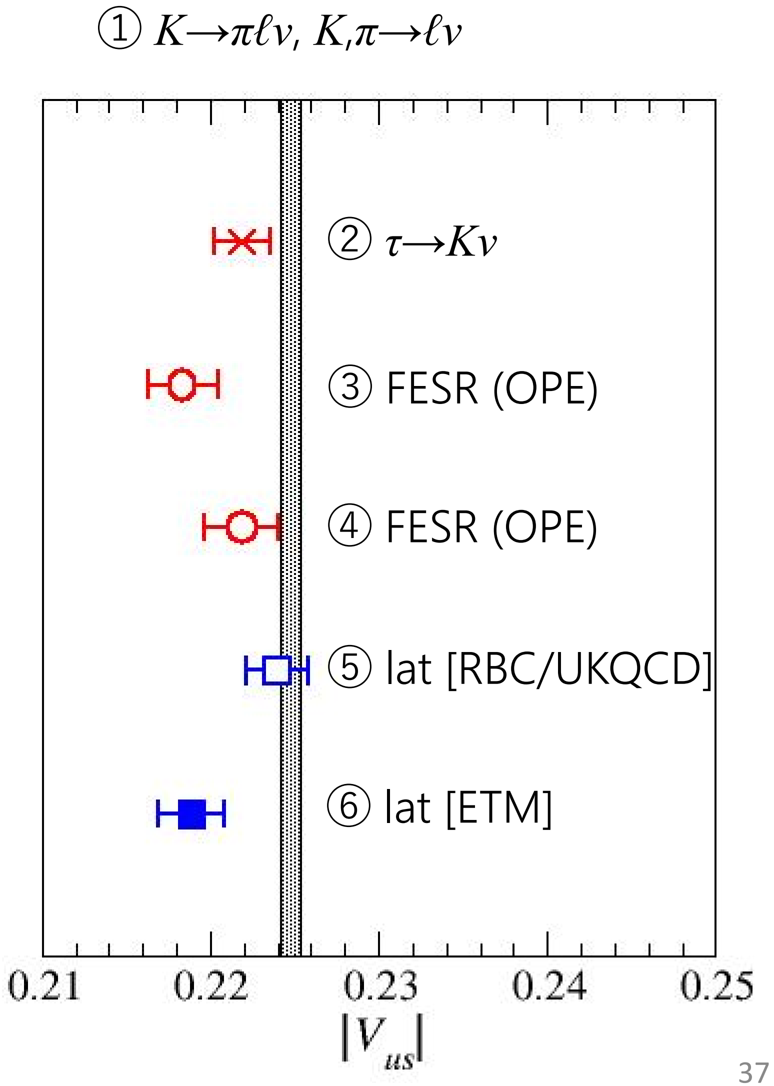
$R_{us}/|V_{us}|^2$ = 3.407(22)

$|V_{us}|$ = 0.2189(7)$_{\text{th}}$(18)$_{\text{ex}}$

- no OPE → use R_{us} rather than ΔR
- 0.6% error of $R_{us}/|V_{us}|^2$ dominated by stat, finite V

  better statistical accuracy w/ $m_{q,\text{val},1} \sim m_{q,\text{val},2}$
- 0.9% determination of $|V_{us}|$ w/ error dominated by exp
- consistent w/ conventional FESR/OPE result !!

$|V_{us}|$ tension revived



$D_{(s)}$ decays

CKM unitarity in the 2nd row

CKM elements

$|V_{cd(cs)}|$ from $D_{(s)}\to\ell\nu$: limited by exp (HFLAV'22)

$D_s\to\ell\nu \Rightarrow |V_{cs}| = 0.9820(96)_{\rm exp}(20)_{\rm lat}$ [1.0%]

$D\to\ell\nu \Rightarrow |V_{cd}| = 0.2181(49)_{\rm exp}(7)_{\rm lat}$ [2.3%]

$|V_{cd(cs)}|$ from $D\to\pi(K)\ell\nu$: limited by lat → theory

⇒ [a]HPQCD'21 [b]Fermilab/MILC '22

$D\to K\ell\nu\ |V_{cs}| = 0.9663(39)_{\rm exp}(53)_{\rm lat}(44)_{\rm EW}$ [0.8%][a]

$0.9589(23)_{\rm exp}(40)_{\rm lat}(96)_{\rm EW}$ [1.1%][b]

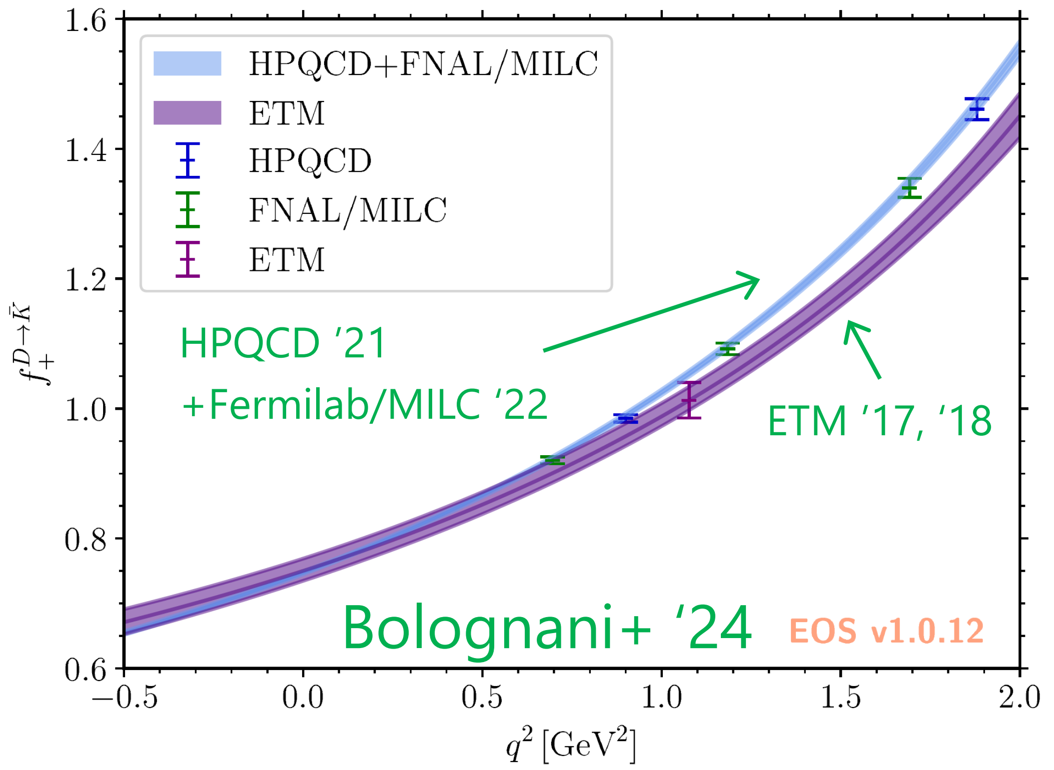
$D\to\pi\ell\nu\ |V_{cd}| = 0.2238(11)_{\rm exp}(15)_{\rm lat}(22)_{\rm EW}$ [1.3%][b]

unitarity

$$|V_{cd}|^2+|V_{cd}|^2+|V_{cb}|^2=0.984(1)_{cd}(15)_{cs}(0)_{cb}$$

a concern

– tensions in f_+ and f_T near $q^2_{\rm max}$ (= 1.88GeV^2)



ETM@Lattice '17 – ~~large rotational sym~~

consistency b/w HPQCD+Fermilab/MILC and ETM?, $|V_{cd}|$ for unitarity in 2nd row

summary

recent progress in lattice QCD to determine CKM MEs

gold-plated

– becoming accurate, more calculations, systematics to be studied carefully

Tobi Tsang @Lattice '24: tensions ⇔ interpolation to $m_{q,\mathrm{ref}}$, $q^2{}_{\mathrm{ref}}$, …

– tensions to be understood : $B_s \to K\ell\nu$, $B \to D^*\ell\nu$, $D \to K\ell\nu$

– systematic error as |"fit A"-"fit B"| ⇔ "1σ" ?

non gold-plated

– new applications: inclusive analysis for B, τ decays, finite volume framework $B \to \rho\ell\nu$, …

– a realistic calculations for inclusive τ; and B in the "near" future

研究員公募 KEK素核研 研究員24-6

国際先導研究「スーパーBファクトリー研究による素粒子物理学フロンティアの開拓と若手研究者の育成」

1. 公募職種・人員
   研究員・１名

2. 研究（職務）内容
   素粒子現象論、あるいは、格子 QCD の研究を行う。KEK 素粒子原子核研究所理論センターに在籍し、金児隆志や遠藤基と協力して共同研究を推進する。在任期間のうち3ヶ月以上は海外の研究機関に滞在して広い意味でのフレーバー物理に関する研究を行うことを推奨する(旅費･滞在費を支給する)。

4. 着任時期
   2025 年 1 月以降（応相談）

5. 任期
   単年度契約で、着任から 3 年間。評価により、最長で 2029 年 3 月 31 日まで更新可能

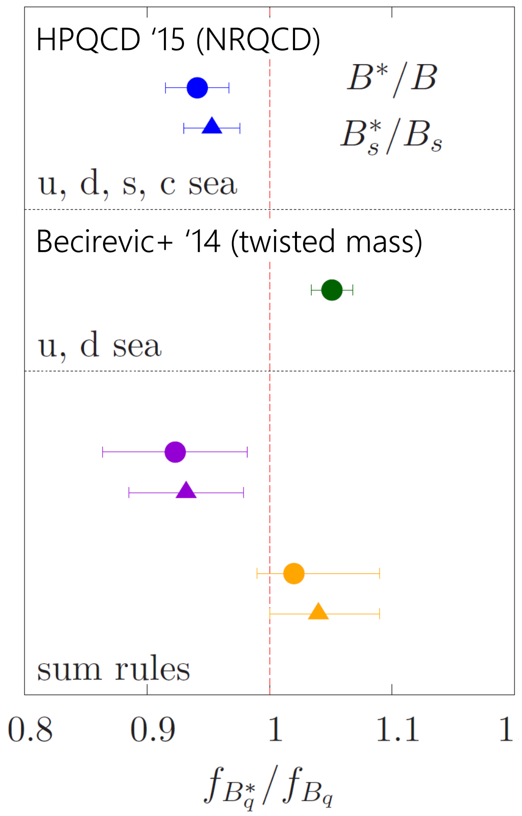
7. 待遇等
   原則として専門業務型裁量労働制を適用する。（みなし勤務時間：1 日 7 時間 45 分）
   給与　月額 45 万円程度

公募締切 2024年9月30日(月): https://www.kek.jp/ja/career/researcher24-6

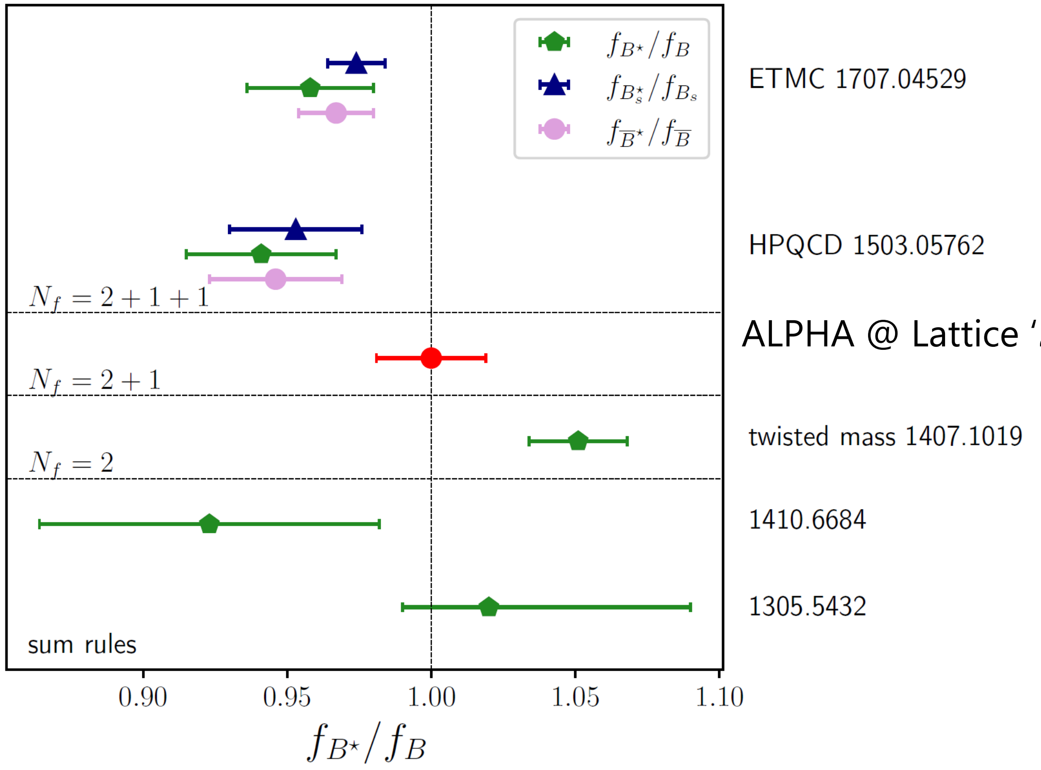
Back Up

leptonic decays of other mesons

$B_{(s)}^*$ ⇒ most simplest matrix elements of V_k ⇒ ratio method w/ more involved MEs



↔ HPQCD @ Lattice '24



B_c ⇒ easier than f_B *cf.* HPQCD '15 f_{Bc} = 434(15) MeV, f_{Bc*}/f_{Bc} = 0.988(27)

good phenomenological applications?

$B^*\pi$ contamination

review by Hashimoto @ Lattice '18

- $H\pi$ state contamination towards $M_{\pi,\mathrm{phys}}$

Bär, Mon, 15:00-, Broll, Mon, 15:20-

$B\pi$ state contamination within HMChPT

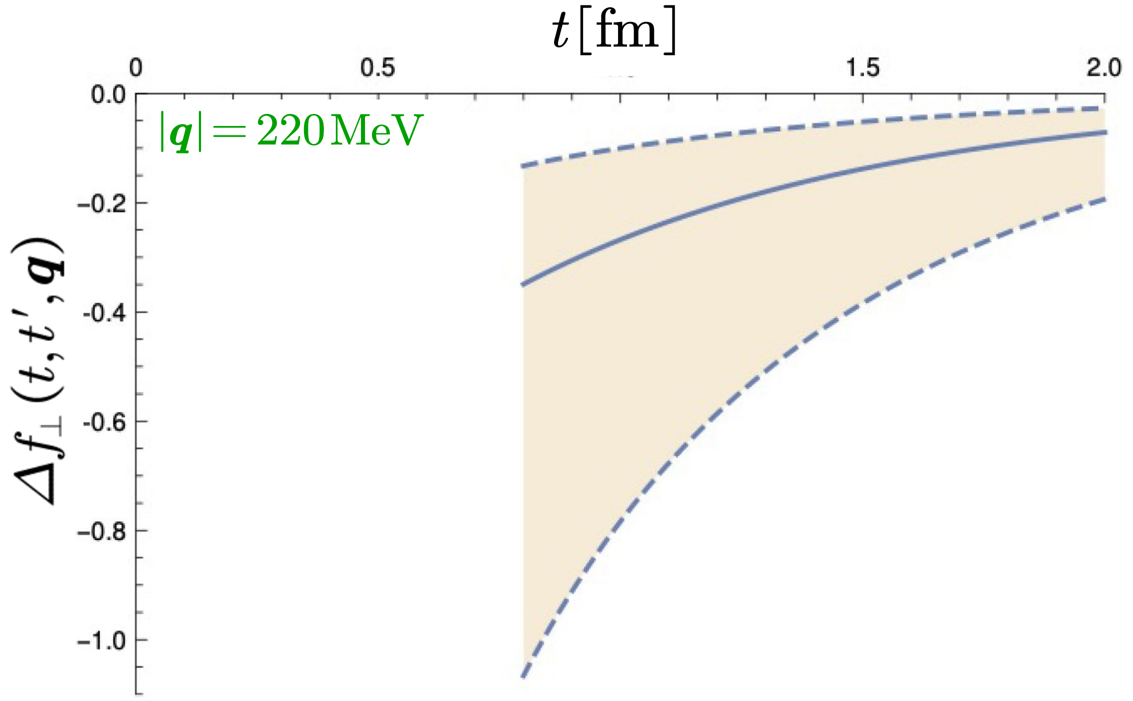
- LEC
  - LO: $f = f_\pi$, $g_{B^*B\pi}$ = 0.5
  - NLO: β, β′ : unknown

$$-\Lambda_\chi^{-1} \le \beta,\ \beta' \le \Lambda_\chi^{-1}$$

- source-sink separation t = 1.3 fm

⇒ $B^*\pi$ contamination to $M_{B,\mathrm{eff}}$, f_B, $B\to\pi$ FFs, ...

possibly large effects in $f_\perp$ (!)

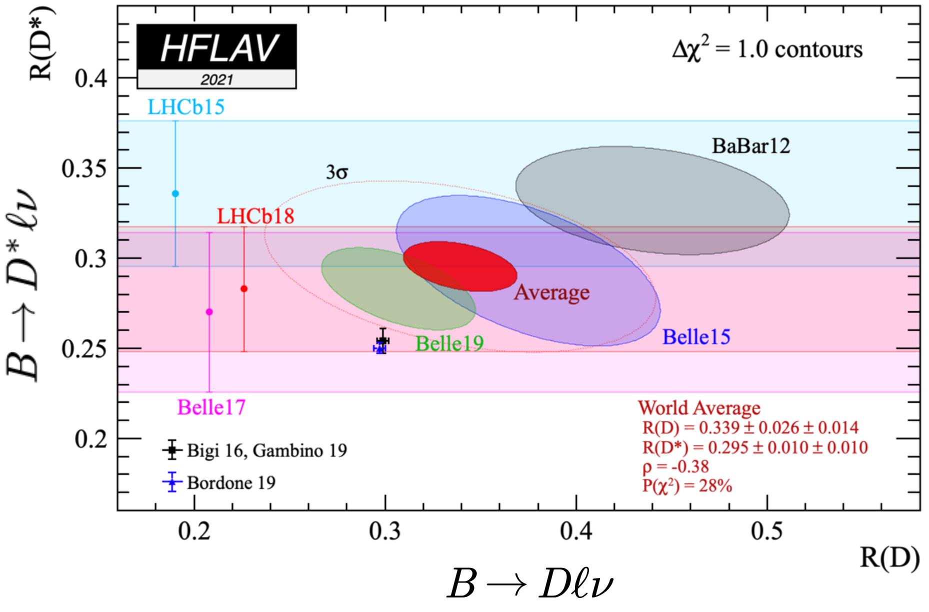


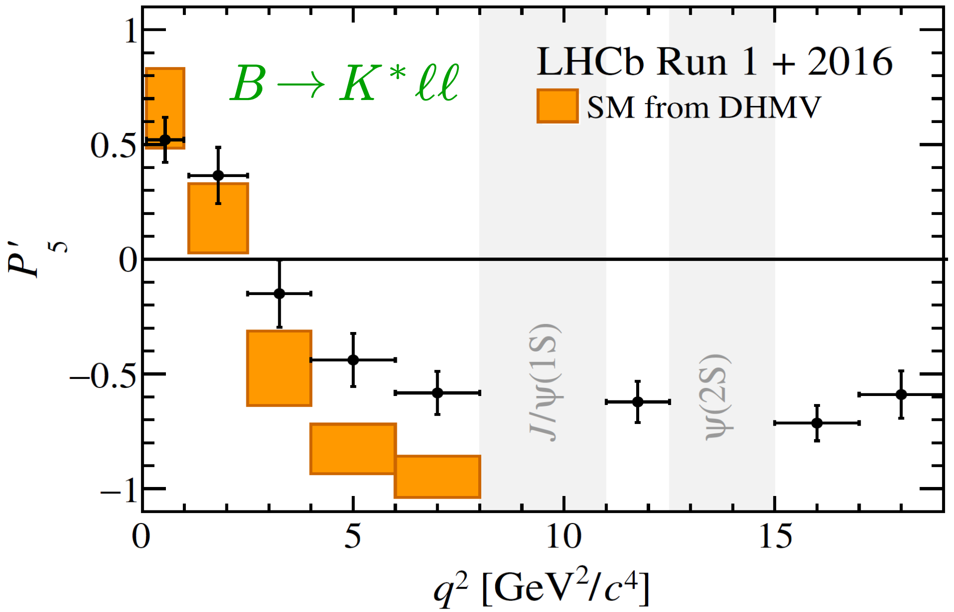
- depending on NLO LECs
- smearing may help ?

⇒ how to calculate NLO LECs from 3pt func

B anomalies

tantalizing 2-4 σ tensions b/w the SM and experiments





• • •

and else

lepton flavor universality violation (LFUV)

$$R(D^{(*)}) = \frac{\Gamma(B\to D^{(*)}\tau\nu)}{\Gamma(B\to D^{(*)}\{e,\mu\}\nu)}$$

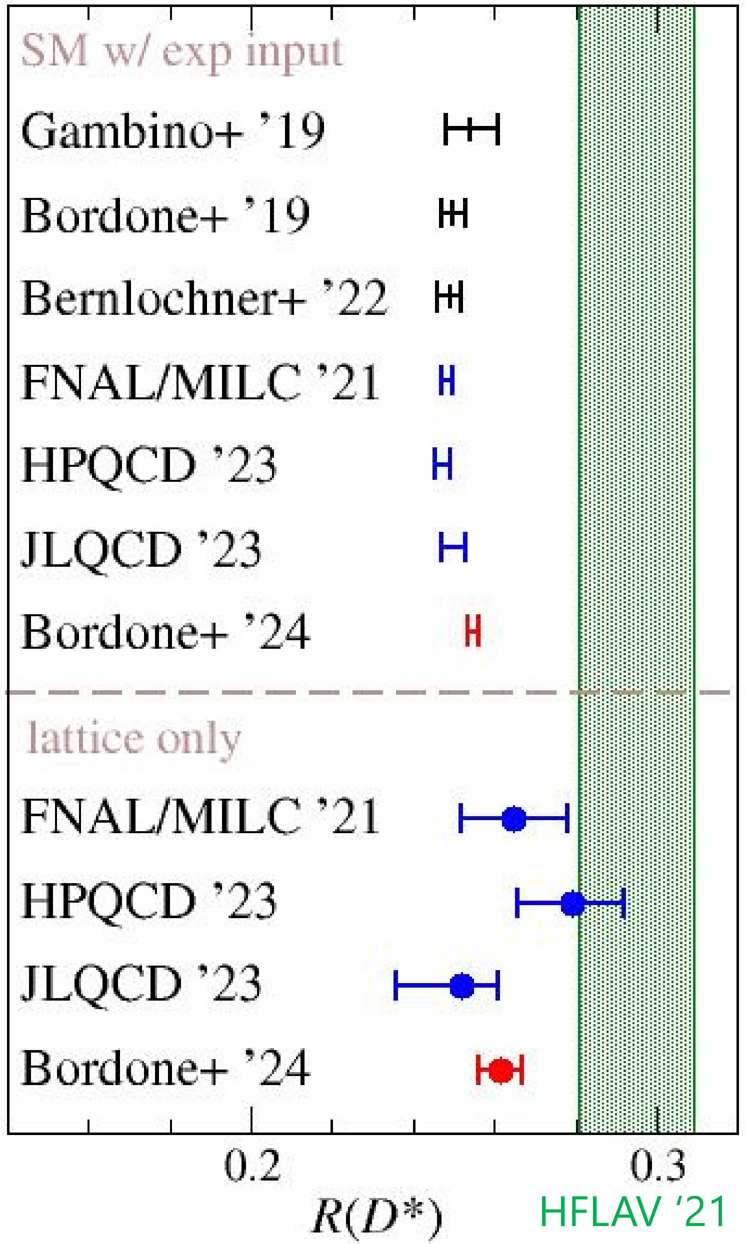
but "SM" values is not purely theoretical

$b\to s\ell\ell,\ s\nu\nu$ FCNC induced

$$B_s\to\mu\mu,\ \ B\to K\ell\ell,\ \ B\to K\nu\bar{\nu},\ \cdots\ \cdots$$

we need more careful/detailed studies and other hints to clarify new physics

$R(D^*)$



$R(D^*) = \Gamma(B \to D^* \tau\nu)/\Gamma(B \to D^* \ell\nu) \quad (\ell = e, \mu)$

"SM value" w/ experimental input for FFs [Belle '18 [+BarBar 19]]

– lattice $f(w=1)$ + exp FFs $\Rightarrow$ 3% accuracy, 2.6σ from $R(D^*)_{exp}$

– lattice $f(1)$ + HQET + exp FFs $\Rightarrow$ 1%, 3.1σ

– lattice + exp FFs $\Rightarrow$ $\leq$1%, $\geq$3.1σ

"SM" w/ experimental input [Belle '23, Belle II '23] $\Rightarrow$ 0.6%, 2.8σ

pure SM value only from lattice FFs

– individual : ~ 5%, $\leq$2.3σFFs

pure SM value using all lattice FFs

– ~ 2%, $\leq$2.2σFFs

phase factor $(w^2\text{-}1)^{1/2}$, $w_{\max,\tau} < w_{\max,\ell}$ $\Leftrightarrow$ extension to large w

$B \to D^{**} \ell \nu$

"1/2 vs 3/2 puzzle"

excited state D^{**}

	s_l
$D^{**}_{1/2} = D_0^*, D_1^*$	1/2
$D^{**}_{3/2} = D_1, D_2^*$	3/2

$B \to D^{**} \ell \nu$

$B \to D^{**}(\to D^{(*)}\pi)\,\ell\nu$

- important background for $B \to D^{(*)}\ell\nu$, $\pi\ell\nu$ による
- 15% of $B \to X_c \ell\nu$ $\Rightarrow$ may serve as a probe of NP

Uraltsev 's sum rule on FFs @ w=1 $\Rightarrow$ and "1/2 vs 3/2 puzzle"

for m_Q=∞, w=1

$\tau_{1/2}(1) < \tau_{3/2}(1)$

from experiments

ALEPH, DELPHI, D0

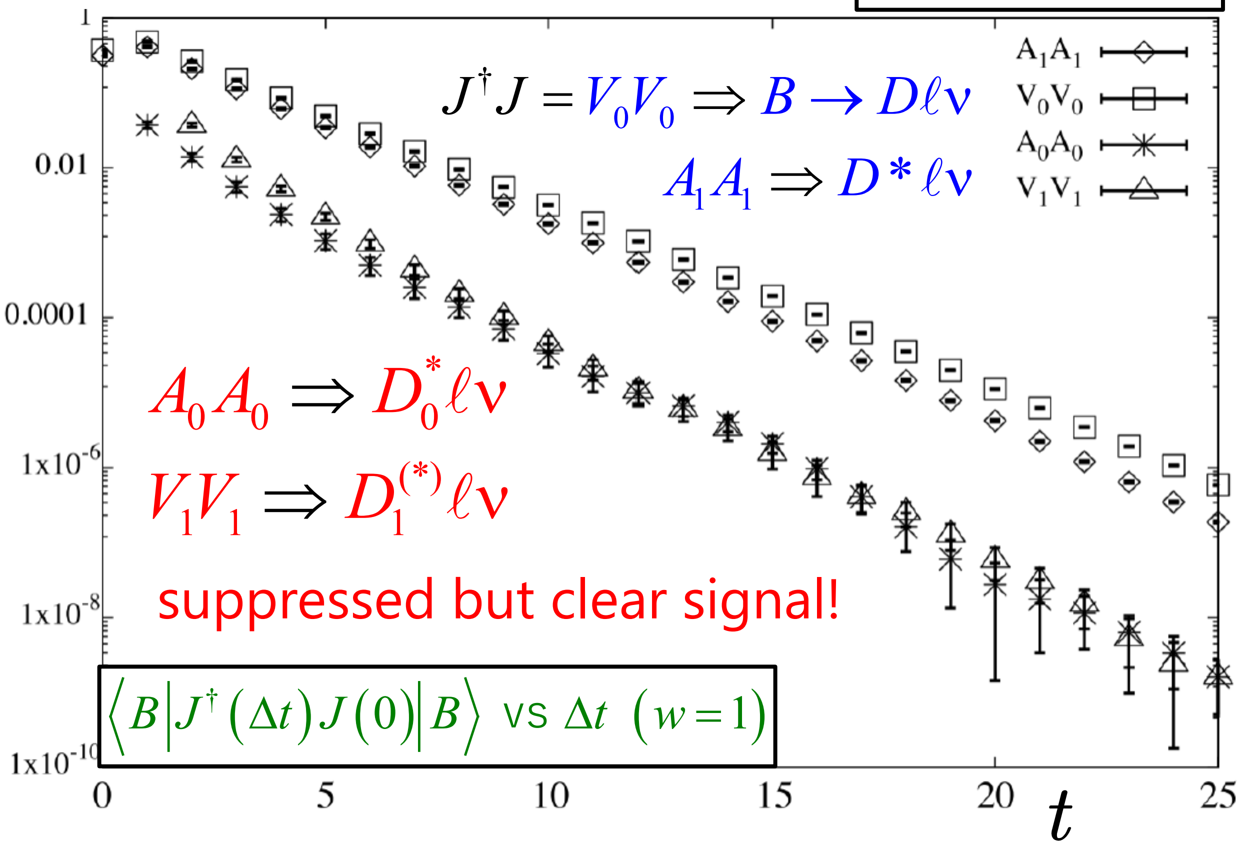
$$\Gamma\left(B \to D^{**}_{1/2}\ell\nu\right) \ll \Gamma\left(B \to D^{**}_{3/2}\ell\nu\right) \quad \Leftrightarrow \quad \Gamma\left(B \to D^{**}_{1/2}\ell\nu\right) \approx \Gamma\left(B \to D^{**}_{3/2}\ell\nu\right)$$

lattice study of relevant FFs ($\tau_{\{1/2,3/2\}}$) may provide hints

$B \to D^{**}\ell\nu$

Bailas+ (JLQCD) @ Lattice '19

$m_b = 2.4 m_c, \quad w = 1$



@ w=1

$$e.g. \ \langle D^*(\varepsilon,0)|V^\mu|B(0)\rangle = iM_B M_{D^*}\epsilon^{\mu\nu\rho\sigma}\varepsilon_\nu^*\delta_\rho^4\delta_\sigma^4 = 0$$

Iassume that even higher state are negligible...

$$\tau_{1/2}(1) = 0.45(7)_{\text{stat}}, \quad \tau_{3/2}(1) = 0.39(6)_{\text{stat}}$$

$$\tau_{1/2}(1) \approx \tau_{3/2}(1)$$

consistent w experiment (!)

Blossier et al. '09, vs LQCD @ $m_Q = \infty$

$\tau_{1/2}(1) = 0.38(5), \tau_{3/2}(1) = 0.53(3)$

- Hu+ (JLQCD) @ Lattice '24: $\tau_{1/2}$ = 0.16(5) ⇔ $\tau_{2/3}$ = 0.28(4) @ $m_{b,\text{phys}}$
- $1/m_b^n$ corrections are key to be consistent w/ experiments (?) - need more simulations
- interesting to study D^{**} at non-zero recoils

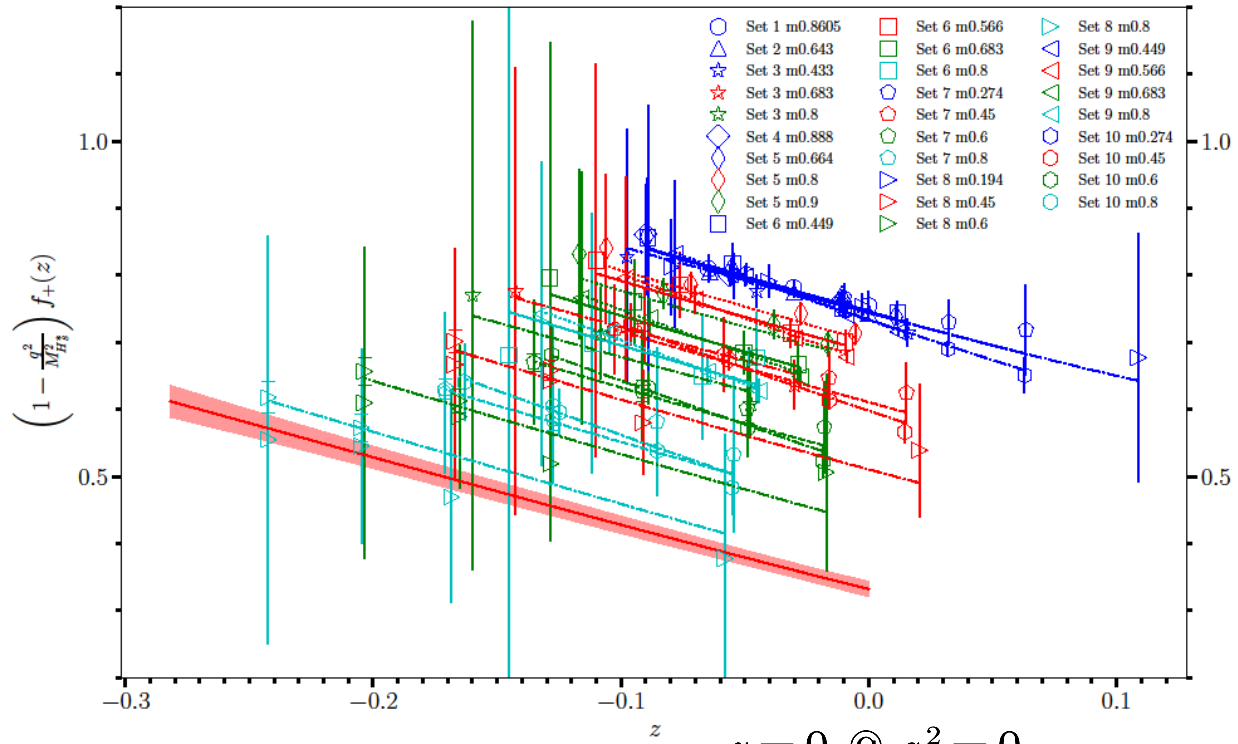
$B \to K\ell\ell$、$K\nu\nu$

$$\langle K(p')|V_\mu|B(p)\rangle = \left\{P - \frac{\Delta M^2}{q^2} q\right\}_\mu f_+(q^2) + \frac{\Delta M^2}{q^2} q_\mu f_0(q^2) \quad \langle K(p')|T_{k0}|B(p)\rangle = \frac{2iM_B p_{K,k}}{M_B + M_K} f_T(q^2)$$

HPQCD:2207.12468, 2207.13371

- gold-plated
- share FFs
- physical *ud*, *s*; *b* w/ $m_b/m_{b,\text{phys}}$ = 0.85

  ⇔ HPQCD '13, Fermilab/MILC '15
- full q^2 region
- 4-7% uncertainty, dominated by stat. error



$z = 0$ @ $q^2 = 0$

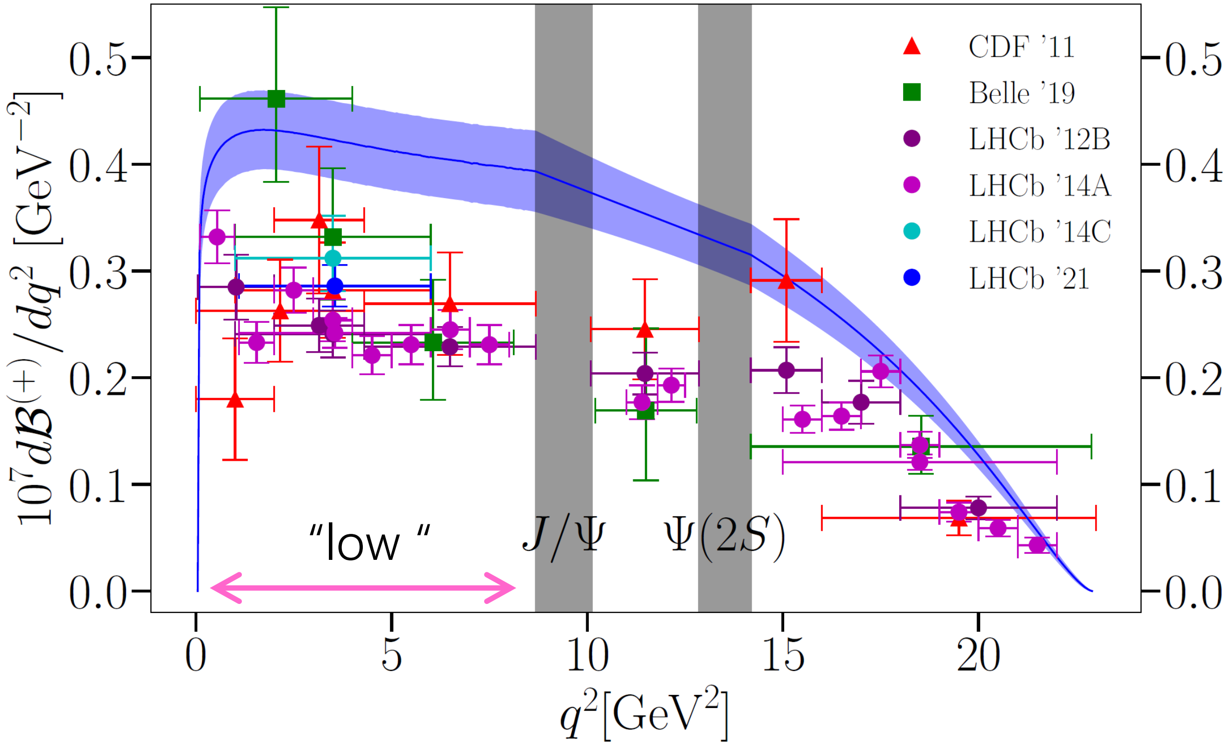
- $B \to K^*\ell\ell$, $B \to K^*\nu\nu$: non gold-plated

  ⇔ framework for $B \to \rho\ell\nu$ may be used

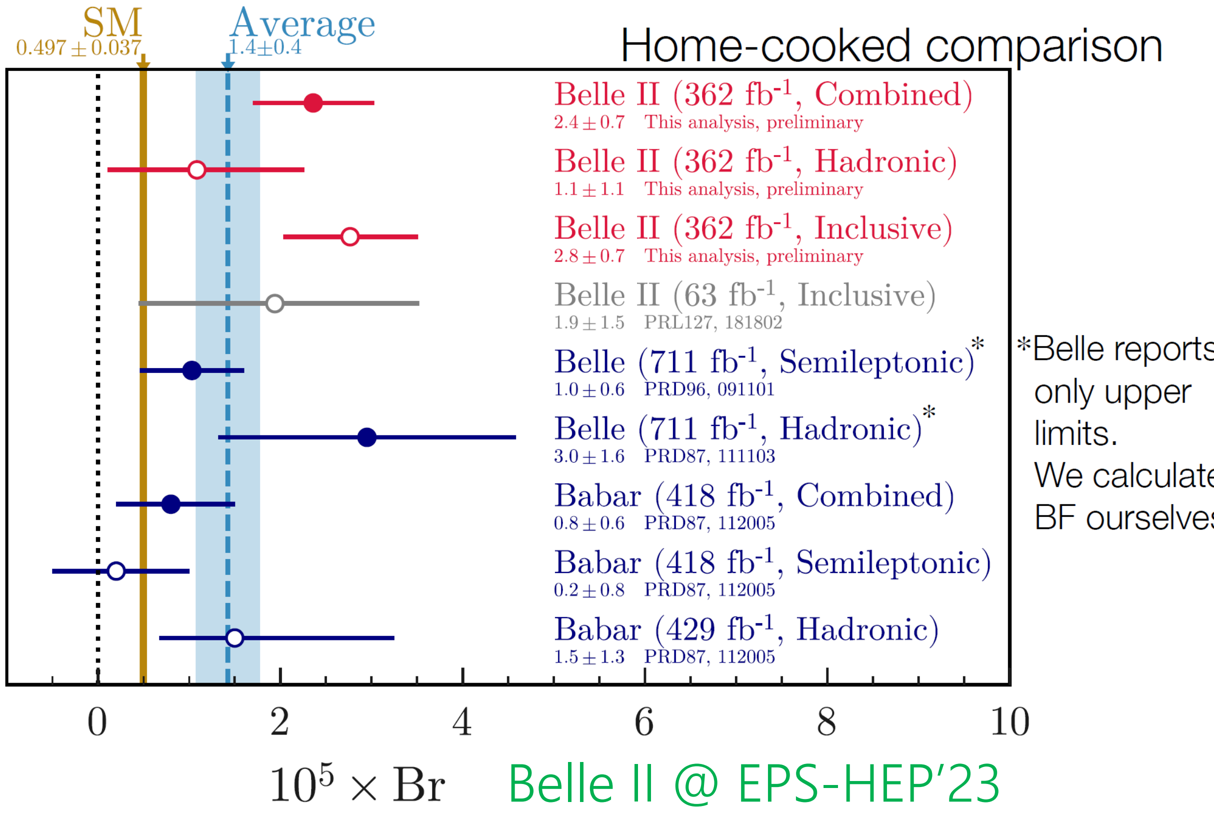
$$\left(1 - \frac{q^2}{M^2_{B_s^*}}\right) f_+(z) \quad \text{vs} \quad z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+}}{\sqrt{t_+ - q^2} + \sqrt{t_+}}$$

$B \to K\ell\ell$、$K\nu\nu$

$B \to K\ell\ell$



$B^+ \to K^+ \nu\nu$



*Belle reports only upper limits. We calculate BF ourselves

Belle II @ EPS-HEP'23

- Fermilab/MILC '15 ≈ 2σ tension
  ⇒ 4.7σ (vs LHCb '21 @ low "q2")
- significant shift in tensor couplings from SM

$$C_9 (\bar{s}\gamma_\mu P_L b)\left(\bar{\ell}\gamma_\mu \ell\right), \;\; C_{10} (\bar{s}\gamma_\mu P_L b)\left(\bar{\ell}\gamma_\mu\gamma_5 \ell\right),$$

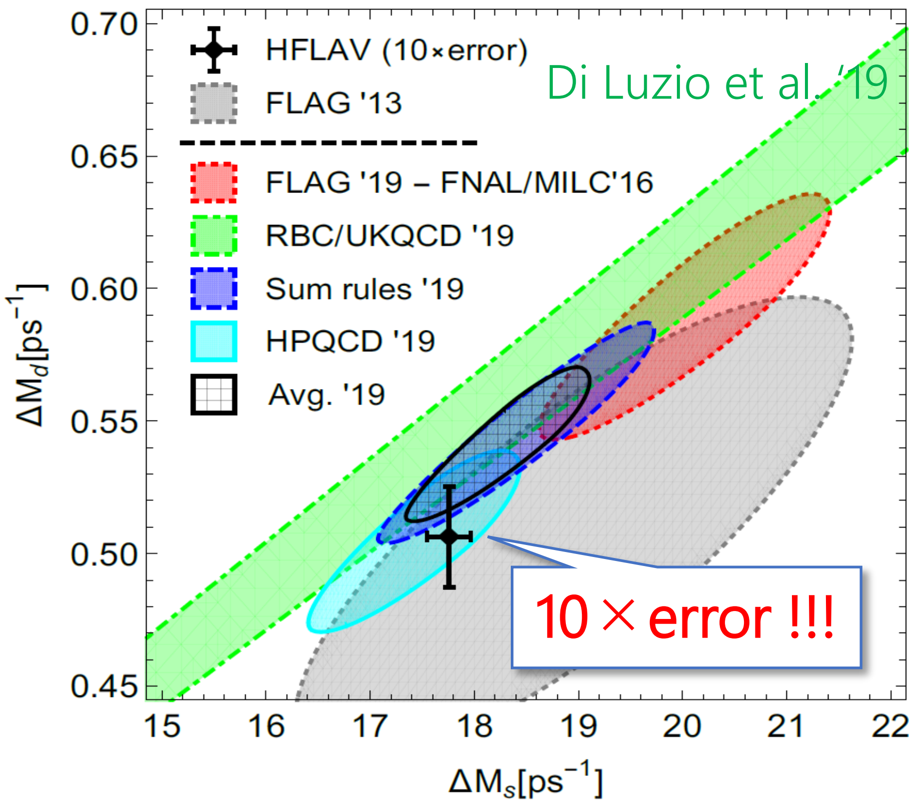
- Belle II : 1st observation (2.7σ from SM?, 5GeV DM
- $\mathcal{B}_{SM}$ = 4.97(37) × 10^{-6} [7.5%]
  ⇔ 11% @ Belle II 50 ab^{-1}

$B_{(s)}$中間子混合

質量固有状態の質量差 ⇔ bag parameter

$$\Delta M_{(s)} = \frac{G_F^2}{16\pi^2}\frac{m_W^2}{M_{B_{(s)}}} S_{0,\mathrm{EW}}\,\eta_{2B,\mathrm{pQCD}} \left|V_{tb}^* V_{td(s)}\right|^2 \left\langle \bar{B}_q \left| Q_1 \right| B_q \right\rangle \quad \left\langle \bar{B}_q \left| Q_1 \right| B_q \right\rangle = \frac{8}{3} M_{B_{(s)}}^2 f_{B_{(s)}}^2 B_{B_{(s)}}$$

$$Q_1 = VV + AA$$



– 2σ tension b/w Fermilab/MILC '19 vs HPQCD '19

+ 演算子混合 Q_2 = VV-AA

– これ以降、格子QCDの進展無し

– 実験は各段に高精度

– JLQCD+RBC/UKQCD @ US+UK

+ domain-wall ⇒ ~~演算子混合~~

+ JLQCD : fine lattices w/ $a^{-1} \lesssim 4.5$GeV

+ RBC/UKQCD : physical M_π (⇒$K\to\pi\pi$)