New Vacuum Solutions and Black Strings in the 4D Standard Model

Yu Hamada[1], Yuta Hamada[2,3], **Hayate Kimura**[3]

[1]DESY, [2]KEK, [3]SOKENDAI

PPP2024, Aug. 22, 2024

Short summary

String theory $\xrightarrow{\text{compactify}}$ enomous lower dimensional vacua (String landscape)

The same is true for standard model : SM landscape

Ex : compactified on S^1 $\rightarrow$ stabilized by Casimir energy of SM particles

On the other hand,

this S^1 vacuum is reproduced near the horizon of "quantum" black string in our 4D world

Sec. 1∼3 are the review of [Arkani-Hamed et. al. 07]

We construct this black string solution
by numerical calculation

Compactified SM

The action for SM+GR

Our starting point is 4D SM+GR action :

$$S = \int d^4x \sqrt{-g^{(4)}} \left(\frac{1}{2} M_4^2 \mathcal{R}^{(4)} - \Lambda_4 + \dots \right)$$

$\mathcal{R}^{(4)}$: 4D Ricci scalar M_4 : 4D Planck mass

Λ_4 : 4D cosmological constant

. . . include other fermions, gauge fields in SM

Kaluza Klein mechanism for 4D metric

Compactify one spacial dimention on S^1 by Kaluza-Klein mechanism.

The metric components are decomposed :

$$g_{\mu\nu}^{(4)} \rightarrow g_{ij}^{(3)}, A_i, R \ (i,j = 0,1,2)$$

R : S^1 radius in 4D → scalar in 3D (radion)

integrate out the compact coordinate x_3 → effective 3D action :

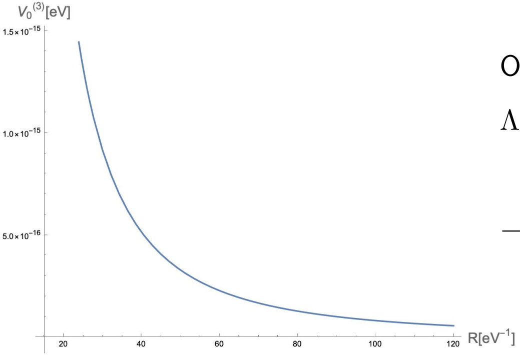
$$S = \int d^3x \sqrt{-g^{(3)}} \left[\frac{1}{2} r M_4^2 \mathcal{R}^{(3)} - r M_4^2 \left(\frac{\partial R}{R} \right)^2 - \frac{r M_4^2}{8} \left(\frac{2\pi R}{r} \right)^2 R^2 F_{ij} F^{ij} - \frac{r^3 \Lambda_4}{(2\pi R)^2} + \dots \right]$$

Radion potential ?

Classical potential for the radion is the cosmological const. term :

$$V_0^{(3)}(R) = \frac{r^3\Lambda_4}{(2\pi R)^2}$$

The radion field (moduli of S^1) can not be stabilized.



On the other hand ...

Λ_4 is sufficiently small

$(\sim 3.25 \times 10^{-11}\mathrm{eV}^4)$

$\longrightarrow$ quantum collection should be taken into account

Radion Stabilization

Casimir energy as radion potential

(constant) expectation values for scalars with quantum correction

= the stationary point of the effective potential

1-loop free fields correction for effective potential is following :

$$V^{(4)} = \underbrace{V_0^{(4)}}_{\text{classical potential}} \underbrace{-\frac{i}{2}\sum_{\text{particle}}(-1)^{2s_p} n_p \int \frac{d^4k}{(2\pi)^4}\ln\left(m_p^2+k^2\right)}_{\text{Casimir potential } \equiv V_1^{(4)}}$$

Note that one dimention is compactified

$\Longrightarrow$ the momentum is quantized as $\dfrac{2\pi n}{R}$ and $\int \to \sum$

$\Longrightarrow$ R appear in $V_1^{(4)}$: radion potential

Boundary condition

The sum is evaluated as following :

$$V_1^{(4)} = -\sum_{\text{particle}} (-1)^{2s_p} n_p \frac{m_p^4}{2\pi^2} \sum_{n=1}^{\infty} \frac{\cos(2\pi n\theta_p)}{(2\pi R m_p n)^2} K_2(2\pi R m_p n)$$

θ_p is the parameter which characterize the boundary conditions of fields :

$$\psi_p(x_3 + 2\pi R) = e^{i2\pi\theta_p}\psi_p(x_3)$$

Possible θ_p reflects symmetry of the field :

ψ_p has $U(1)$ symmetry $\Longrightarrow$ θ_p can be arbitrary value

gauge boson, graviton $\Longrightarrow$ periodic ($\theta_p = 0$)

Stabilized radion ; new SM vacuum solution

particle with mass m_p

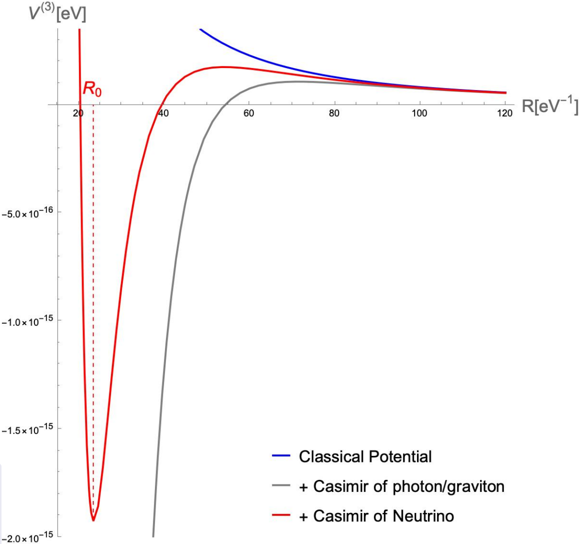
$\rightarrow$ Casimir potential is relevant for $R < 1/m_p$

Casimir energy of (massless Bosons : photon, graviton) + (Majorana Neutorinos)

$\downarrow$

New Vacuum Solution

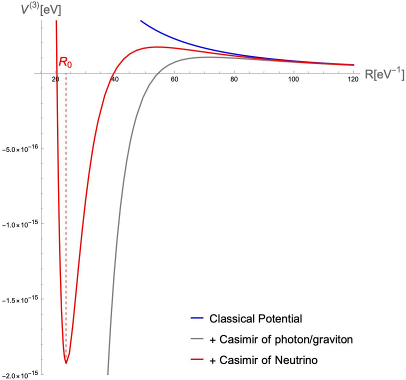
$(S^1 \times AdS_3)$!



Assumptions for stabilization

In previous solution, the assumption :

1. Neutrino spectrum is normal hierarchy and lightest Neutrino is massless
   - The existence and type (dS, AdS) of solution is sensitive to Neutrino mass and dof
   - Casimir of the other particle is not relevant in this scale



2. The boundary condition of Majorana fields is periodic instead of anti-periodic

$\Longrightarrow$ fields feel $\mathbb{Z}_2$ flux on S^1 (cf. Aharonov-Bhom effect)

Interpolating Black Brane

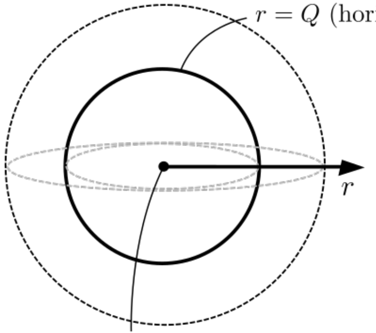
Interpolating Black Brane

The important perspective :

Extremal charged black brane interpolate between non-compact spacetime and compact spacetime which is compactified by its charge flux

Then, is there any black brane which interpolate with previous vacuum?

Example : Reissner-Nordström BH



$$ds^2 = -\left(1 - \frac{Q}{r}\right)^2 dt^2 + \left(1 - \frac{Q}{r}\right)^{-2} dr^2 + r^2 d\Omega^2$$

$$\downarrow \ \xi = r - Q \ll Q$$

$$ds^2 \sim -\left(\frac{\xi}{Q}\right)^2 dt^2 + \left(\frac{Q}{\xi}\right)^2 d\xi^2 + Q^2 d\Omega^2$$

$$: S^2 \times AdS_2$$

$S^2 \times AdS_2$: Vacuum solution stabilized by electric flux Q

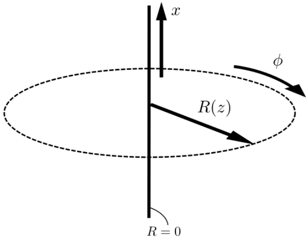
(potential for moduli : $F_{\mu\nu}F^{\mu\nu}$)

Analogy to the new vacuum

Our new vacuum is only one spacial dimension is compactified

$\Longrightarrow$ consider an object extended 1D from the BH : Black String

$\mathbb{Z}_2$ charged Black String should reproduce the new $S^1 \times AdS_3$ vacuum near its horizon



metric ansatz :

$$ds^2 = A^2(z)(-dt^2 + dx^2) + dz^2 + R^2(z)d\phi^2$$

What we have to do :

Solve Einstein eq. with $T^{\mu\nu} = \dfrac{-2\pi}{\sqrt{-g^{(4)}}} \dfrac{\delta}{\delta g^{(4)}_{\mu\nu}} \int dx^4 \sqrt{-g^{(4)}}(-\Lambda_4 - V_1^{(4)})$

Einstein equation

Einstein equations are written as following :

$$\frac{A'}{A} = -\frac{R'}{R} - \sqrt{\frac{R'^2}{R^2} - \frac{V_1^{(4)} + \Lambda_4}{M_4^2}}, \quad R'' + \gamma R' = -\partial_R U(R)$$

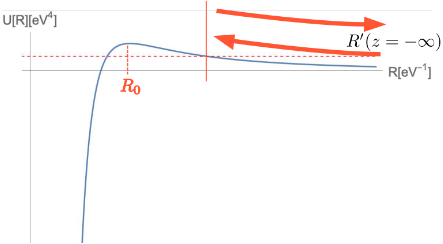
$$\left(\partial_R U(R) = \frac{R}{M_4^2}\left(V_1^{(4)} + \Lambda_4 - \frac{1}{2} R \partial_R V_1^{(4)}\right), \quad \gamma = -2\left(\frac{R'}{R} + \sqrt{\frac{R'^2}{R^2} - \frac{V_1^{(4)} + \Lambda_4}{M_4^2}}\right)\right)$$

$*$ For simplicity, discuss the $\Lambda_4 = 0$ case

R EOM : classical mechanical EOM

with friction γ and potential $U(R)$

$\Longrightarrow$ shooting problem

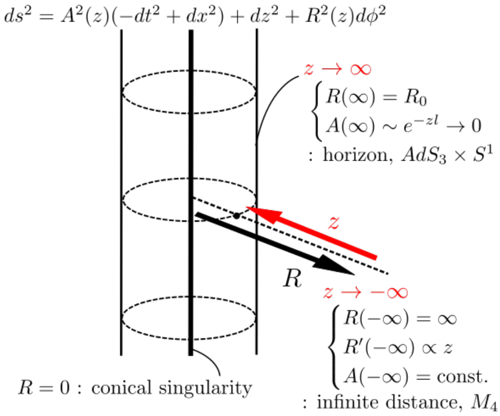


Extremal black string solution ?

For A,

- consistent with expected boundary condition at $z \to -\infty$:
  $A'|_{z=-\infty} = 0, \ R|_{z=-\infty} \propto z$

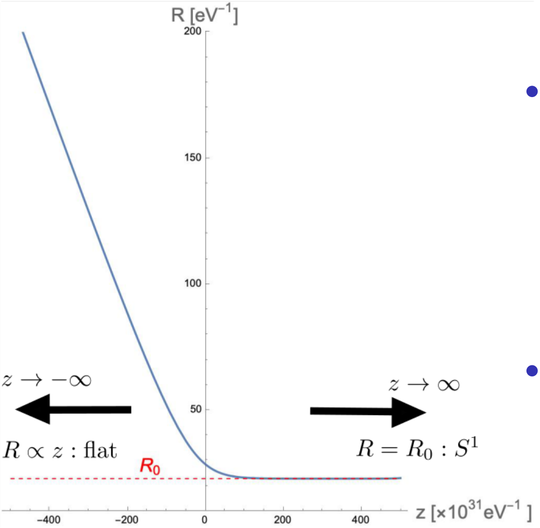
- When $R \equiv R_0$,
  $A \sim e^{-z\sqrt{V_1^{(4)}(R_0)/M_4}}$:
  $AdS_3 \times S^1$



"Stopping solution" of R EOM should be extremal solution

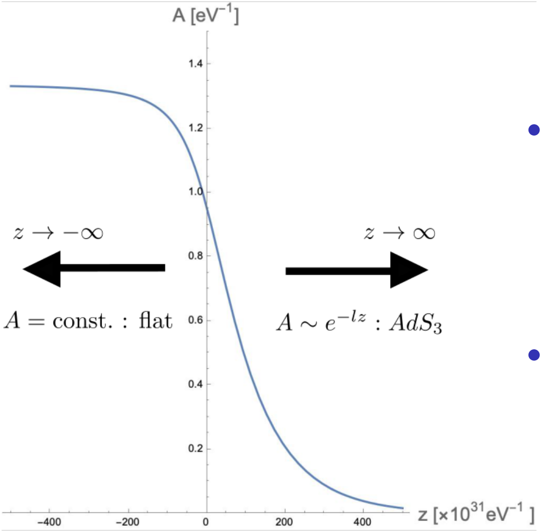
Numerical Calculation

$\Lambda_4 = 0$ case : $R(z)$



- This solution indeed interpolates the flat part ($z \to -\infty$) and the part R is constant (: S^1 is decoupled from ds^2) ($z \to \infty$)
- $|R'(-\infty)| \sim 4.3 \times 10^{-32}$
$$\sim \frac{m_\nu}{M_4}$$

$\Lambda_4 = 0$ case : $A(z)$



- This solution indeed interpolates flat part and AdS part : when $z \to \infty$, $A \sim e^{-lz}$, $l \sim 8.4 \times 10^{-34}[\text{eV}]$
- $A \to 0$ when $z \to \infty$: $z \to \infty$ corresponds to the horizon

Conclusion

Conclusion

- By using of Casimir energy, we can obtain the new vacuum solution in SM : SM vacuum is not unique, but there is SM Landscape
- The compact SM vacuum is reproduced near the horizon of a 4D black string

Future direction

- asymptotically dS_4
- non-extremal black string
- cosmological argument ?
- $E_8 \times E_8$ heterotic SUGRA