Wave optics of gravitational waves lensed by a Kerr black hole

Kei-ichiro Kubota in collaboration with Shun Arai, Shinji Mukohyama, and Hayato Motohashi

Yukawa Institute for Theoretical Physics, Kyoto University

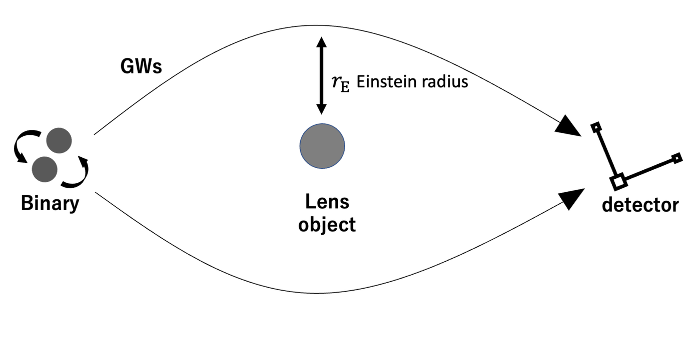
submitted in arXiv:2408.03289

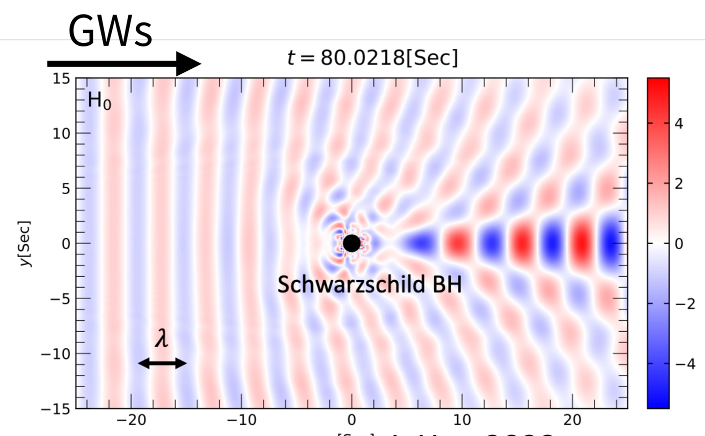
Introduction

- Wave effects are important in the propagation of gravitational waves since the wavelength is as long as astrophysical objects.

$M \gg \lambda$ $(M\omega \gg 1)$
- Behave as particles
- Geometrical optics approximation
- Solve null geodesic

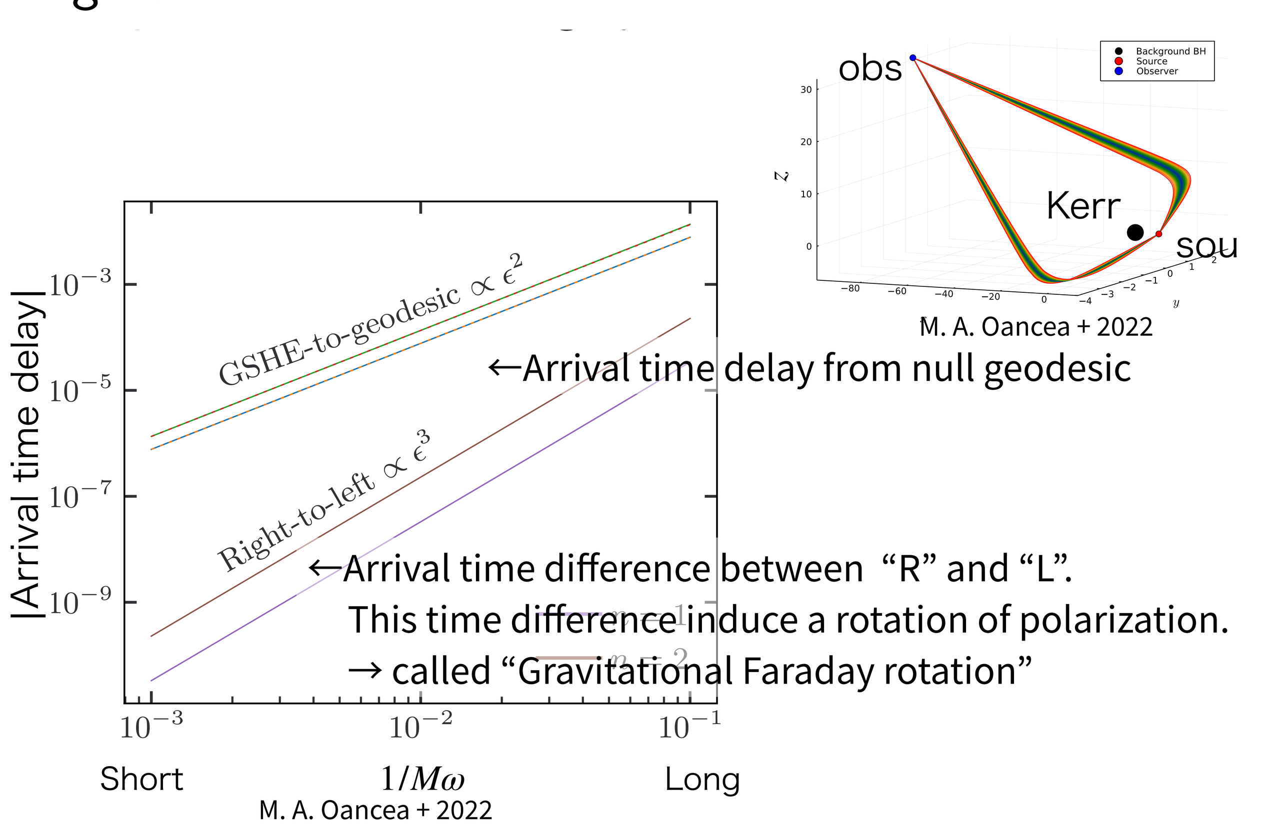
$M < \lambda$ $(M\omega < 1)$
- Diffraction is importnat
- Geometrical optics approximation is not valid
- Need calculation with wave effect





J. He+ 2022

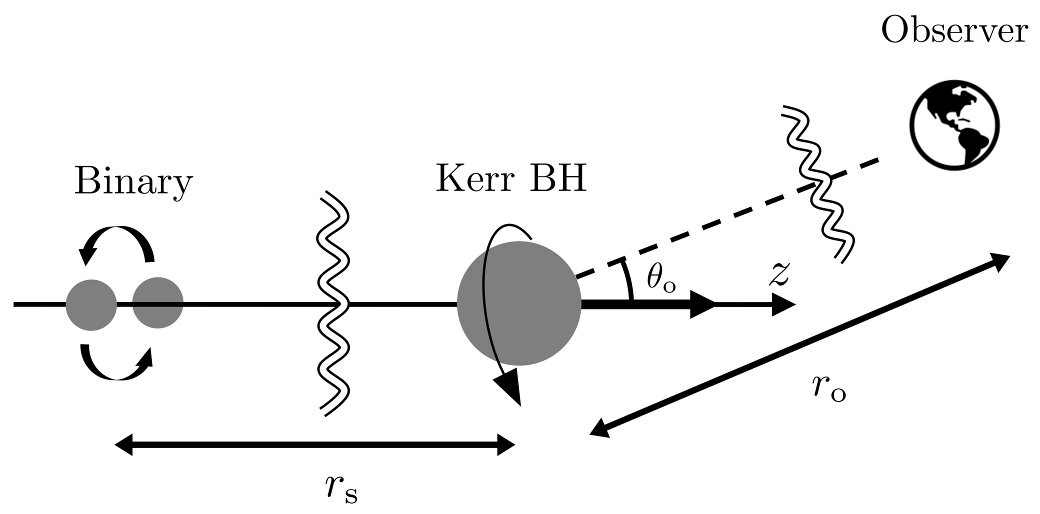
- Additionally, recent papers show that the effect of the spin of gravitational waves spin appears as a correction to the short wavelength limit. Thus the effect is more pronounced for longer wavelengths.



- However, the propagation taking both the spin effect and wave effect into account has been poorly studied.

Set up

- An equal mass circular binary and a Kerr black hole are on z-axis.
- The rotation axis of the binary orbit is aligned with z-axis to see the difference in the scattering of left-handed and right-handed circular polarized gravitational waves.
- We consider two cases.
  - **Co-rotating**: The binary rotates in the same direction as the Kerr BH.
  - **Counter-rotating**: The binary rotates in the opposite direction to the Kerr BH.



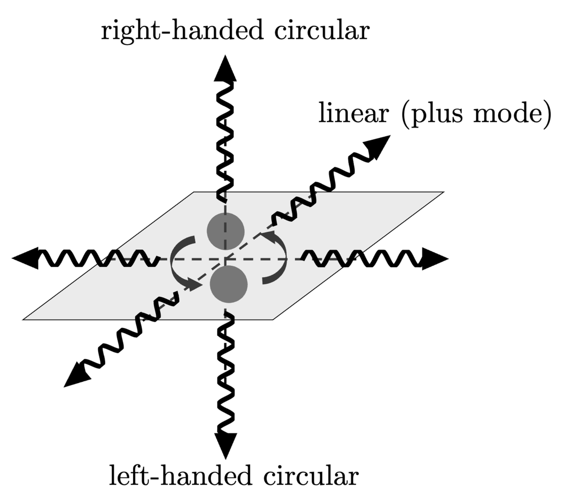
- We numerically solve the Teukolsky equation, which is the equation for the null tetrad component of the Weyl scalar,

$$\Psi_4 = -C_{\mu\nu\rho\sigma} n^\mu \bar{m}^\nu n^\rho \bar{m}^\sigma \simeq \frac{1}{8}(\partial_t - \partial_r)^2 (h_+ - ih_\times) \text{ for } r \to \infty.$$

Polarization of GWs emitted from circular binary

Gravitational wave sourced by a binary are given by

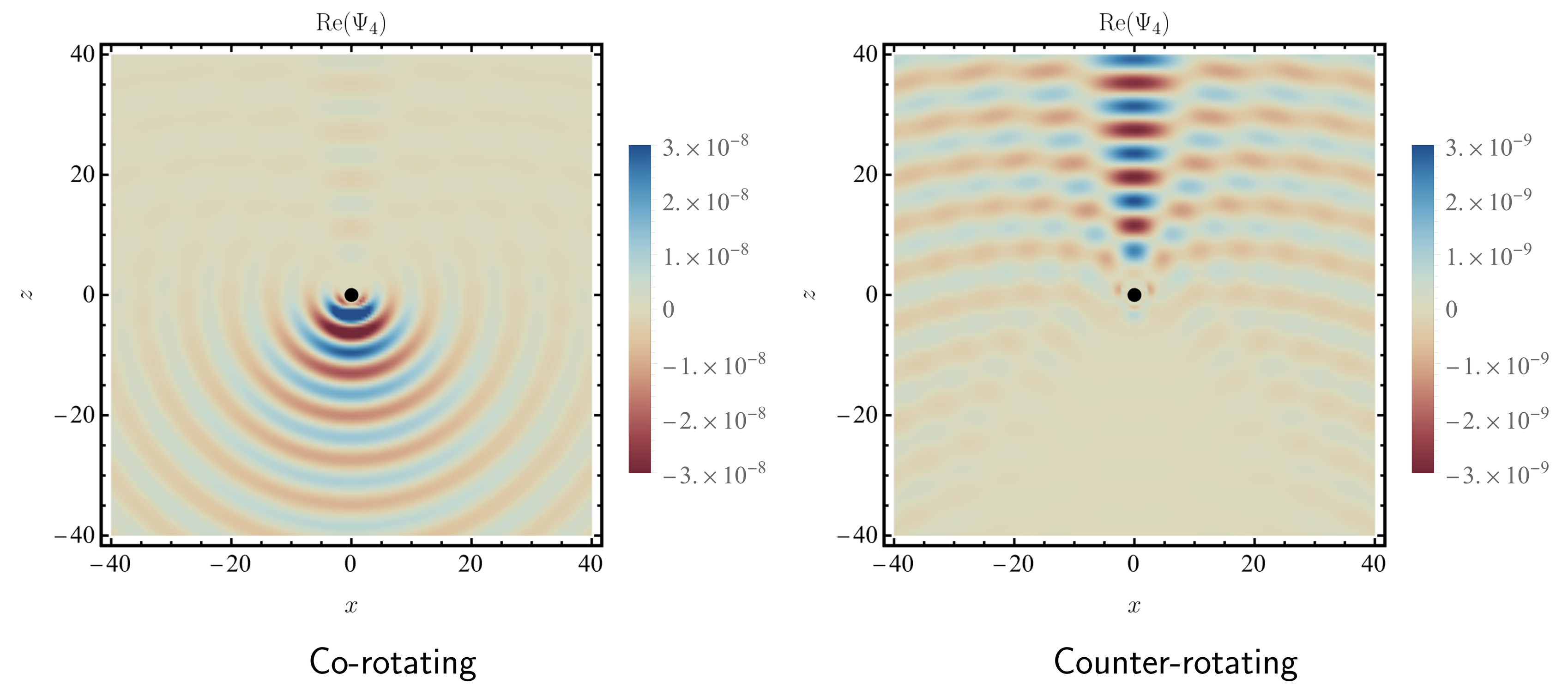
$$\begin{cases} h_+ = \dfrac{2\mu\omega_s^2 d^2}{r}\dfrac{1+\cos^2\theta}{2}\cos(2\omega_s(t-r)+2\varphi) \\ h_\times = \dfrac{2\mu\omega_s^2 d^2}{r}\cos\theta\sin(2\omega_s(t-r)+2\varphi), \end{cases}$$



✓ Gravitational waves emitted in the z-axis direction are circularly polarized.

Results

Co-rotating v.s. Counter-rotating

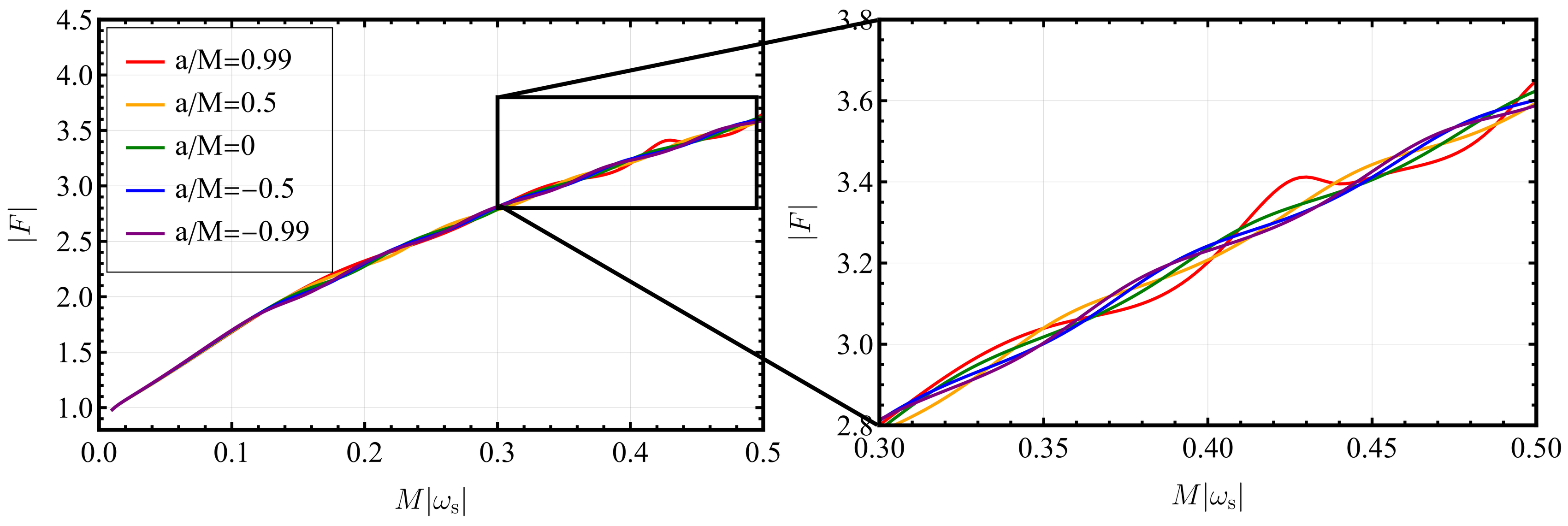


The density plot of $\mathrm{Re}(\Psi_4)$ when the superradiance occur. The density plot of $\mathrm{Re}(\Psi_4)$ for the source with $M\omega_s = 4/10$ at $(r_s, \theta_s) = (100M, \pi)$. The Kerr parameter is $a/M = 99/100$ (left) and $a/M = -99/100$ (right). The mass of the binary is $\mu = 10^{-4}M$.

✓ The superradiance occurs only for prograde backward scattering.

Amplification factor

Amplification factor $F := \Psi_4^{\text{lensed}}/\Psi_4^{\text{unlensed}}$ quantifies how much brighter the source appears due to the gravitational lensing effect.

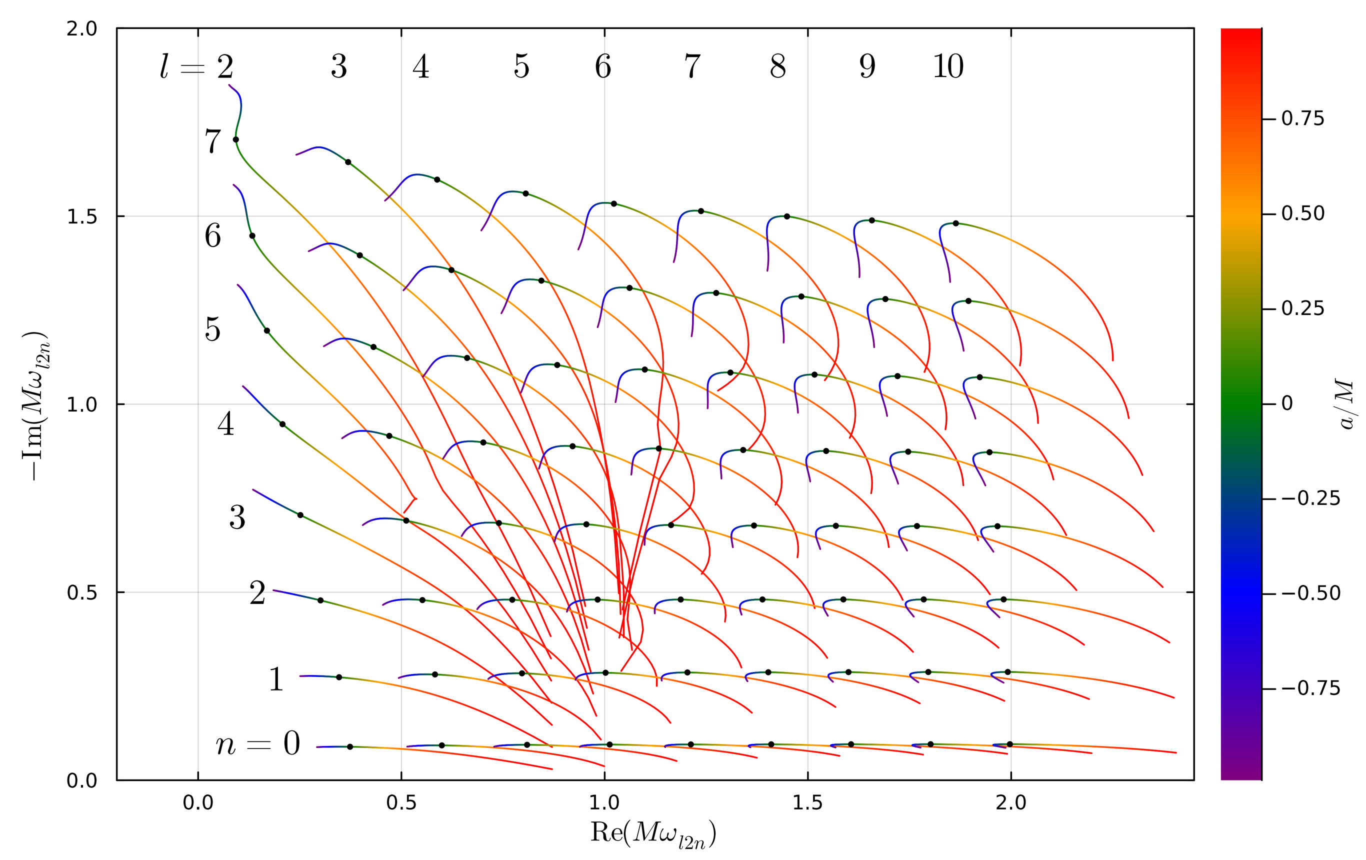


The absolute value of the amplification factor for $a/M = 0.99$ (red), 0.5 (orange), 0 (green), -0.5 (blue), and -0.99 (purple).

✓ As the frequency decreases, lensed Ψ_4 approaches unlensed one.
✓ The frequency- and spin-dependent small-period oscillation appear.

Relation to quasi-normal mode

- The small-period oscillation is related to the quasi-normal mode
  - The oscillation is caused by the interference between the direct ray and winding ray passing through the photon ring.
  - The quasi-normal modes are regarded as slowly leaking modes from the photon ring.
- The imaginary part of the frequency of the quasi-normal mode is associated with the energy dissipation rate of the null congruence of the photon ring.
  → As the absolute value of the imaginary part decreases, the winding ray is less dissipative, and thus the small-period oscillation becomes enhanced.



✓ The spin-dependence of the small-period oscillation can be explained from the view of the quasi-normal mode.