

# ***Black Hole Complementarity and Quantum Gravity***

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iTHEMS



# Two pillars of modern physics

## Quantum mechanics

$$i\hbar \frac{\partial}{\partial t} |\Psi\rangle = \mathcal{H} |\Psi\rangle$$

## General relativity

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G_N}{c^4} T_{\mu\nu}$$



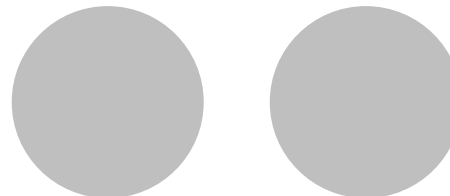
photos: Nobel Foundation archive

not get along well

No problem in “usual” circumstances (below,  $c = \hbar = 1$ )

$p$      $e^-$   
•    •

$$G_N \frac{m_p m_e}{r^2} \sim 10^{-39} \frac{e^2}{4\pi\epsilon_0 r^2}$$



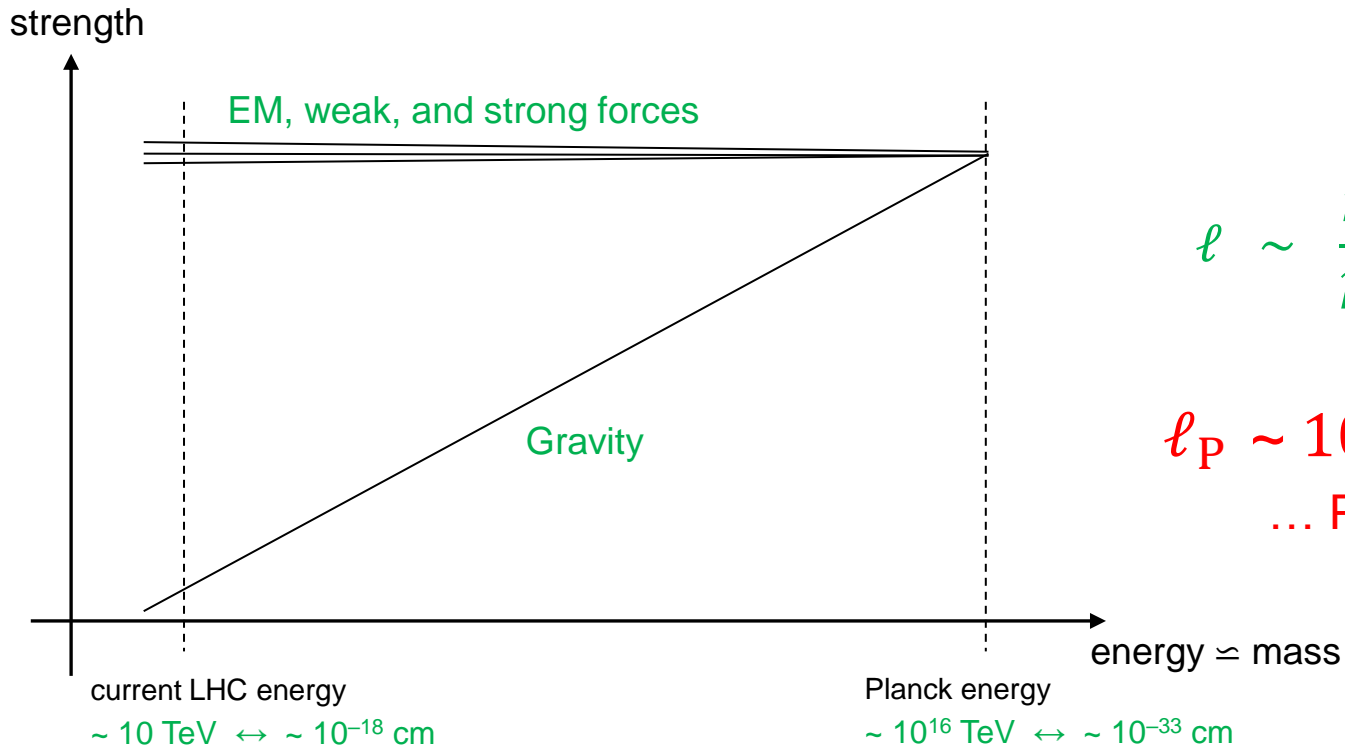
macroscopic body  
( $N \gg 1$  particles)

quantum interference  $\sim \epsilon^N \ll 1$

→ A “patchwork” is enough.

# Interesting things occur in “unusual” situations

cf.  $v \gg v_{\text{usual}}$  in Newtonian mechanics



$$\ell \sim \frac{\hbar}{p} \sim \frac{\hbar}{E/c}$$

$$\ell_P \sim 10^{-33} \text{ cm}$$

... Planck length

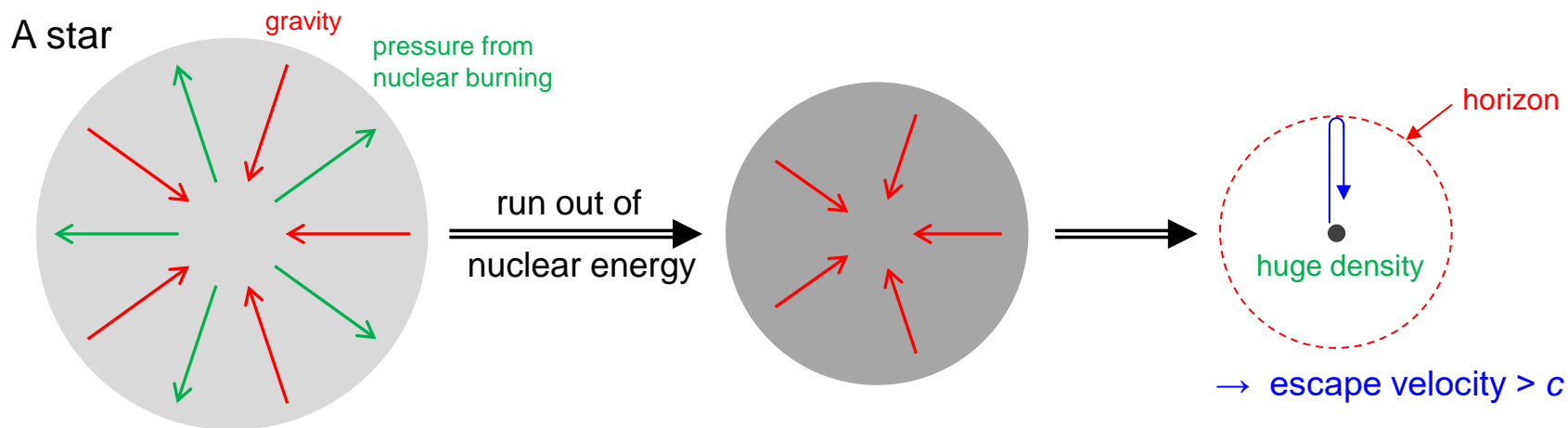
At  $\sim \ell_P$ , theoretical control of quantum field theory  
(point particles in continuous spacetime) is lost.

→ string theory

There is a problem that the current formulation  
of string theory cannot (directly) address. (today's theme)

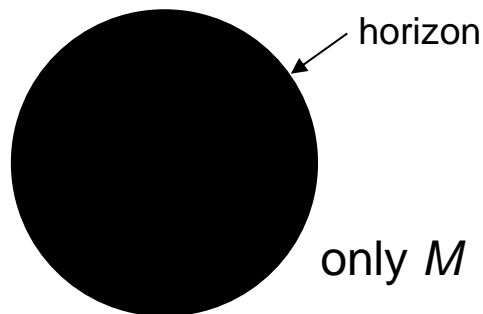
Unusual situations can occur at long distances:

## Black Holes



The interior cannot be seen *from the outside*.

— “No hair” theorem



⇒ What are the quantum properties?

# Black Hole Thermodynamics

# A puzzle

Another pillar of modern physics

Statistical mechanics

$$S = k_B \ln W$$

(below,  $k_B = 1$ )

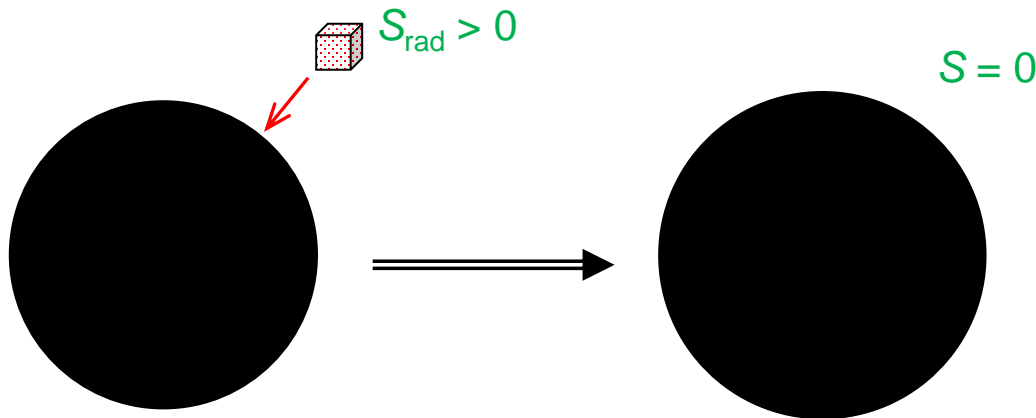


photo: Univ. Frankfurt

for any system  
for all practical purposes

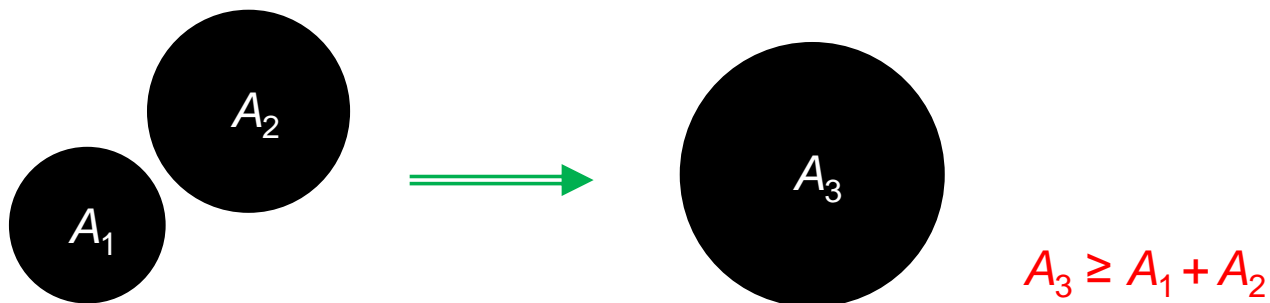
$$\Delta S \geq 0$$

What happens if matter falls into a black hole?



...  $\Delta S < 0$  !?

# A peculiar property of BHs in general relativity



## A proposal [Bekenstein, 1973]



photo: APS

The entropy of a BH is proportional to its horizon area.

$$S_{\text{BH}} = \frac{A}{4G_{\text{N}}}$$

Note:  $G_{\text{N}} = \ell_{\text{P}}^2 \sim (10^{-33} \text{ cm})^2 \rightarrow$  huge entropy  
 e.g. A solar mass BH has  $S \sim 10^{78}$  while the sun has  $\sim 10^{60}$ .

Indeed,  $\Delta \left( \frac{A}{4G_{\text{N}}} + S_{\text{matter}} \right) \geq 0$

Does this make sense?

$$\frac{A}{4G_{\text{N}}} = 4\pi G_{\text{N}} M^2 \rightarrow \frac{\partial S}{\partial E} = \frac{1}{T} \rightarrow \text{finite temperature}$$

Doesn't a BH only absorb stuff?

# Black holes radiate [Hawking, 1974]



photo: NASA

The horizon is “smooth.”



Quantum mechanical effect

There must be radiation corresponding to  $T_H \sim \frac{1}{8\pi M G_N}$ .

Hawking temperature



# Black holes radiate [Hawking, 1974]



photo: NASA

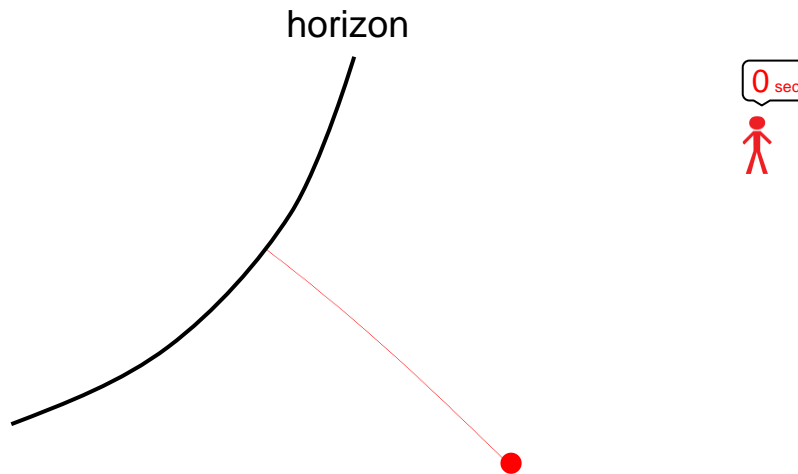
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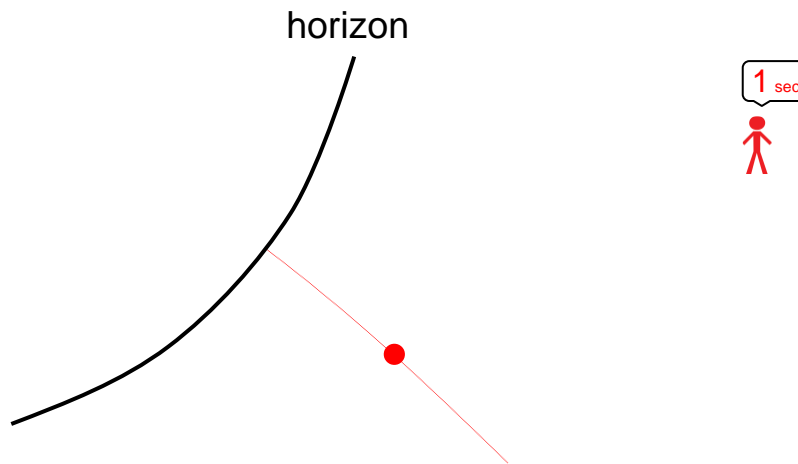
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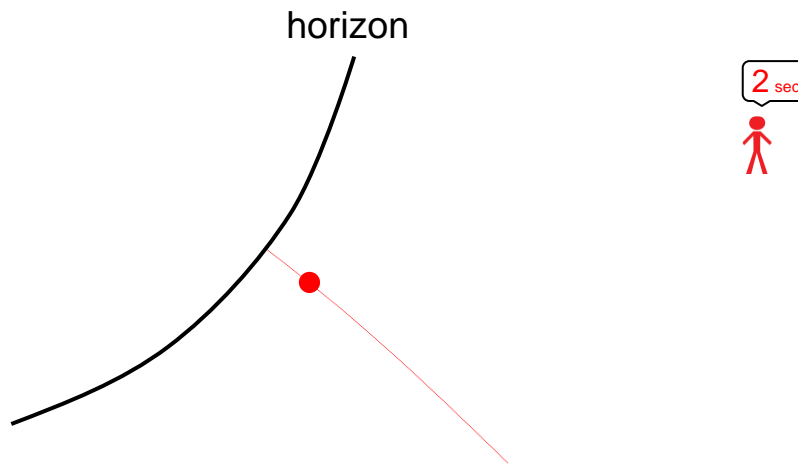
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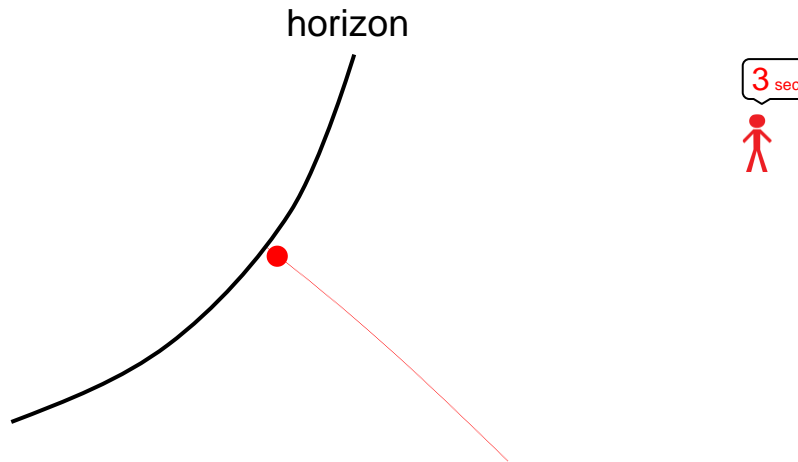
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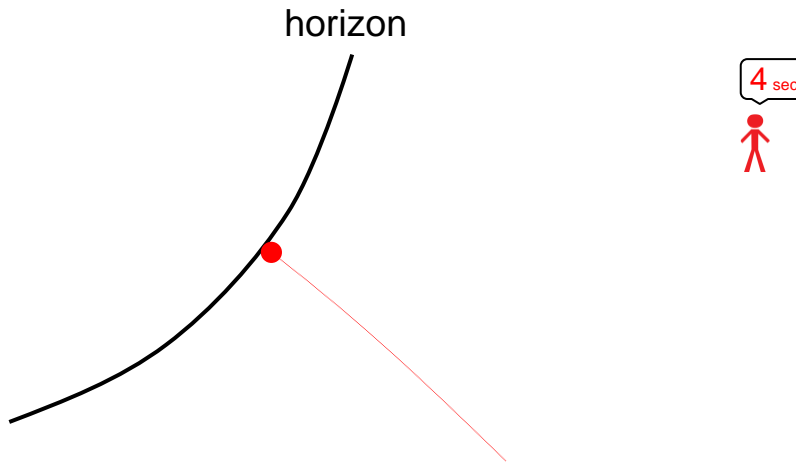
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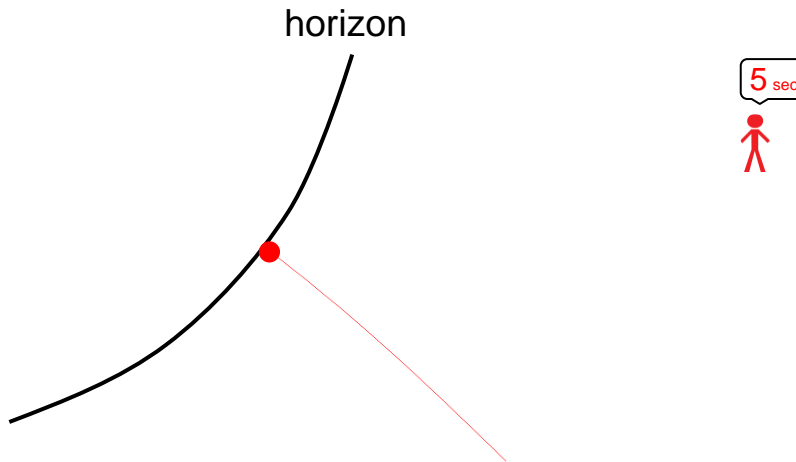
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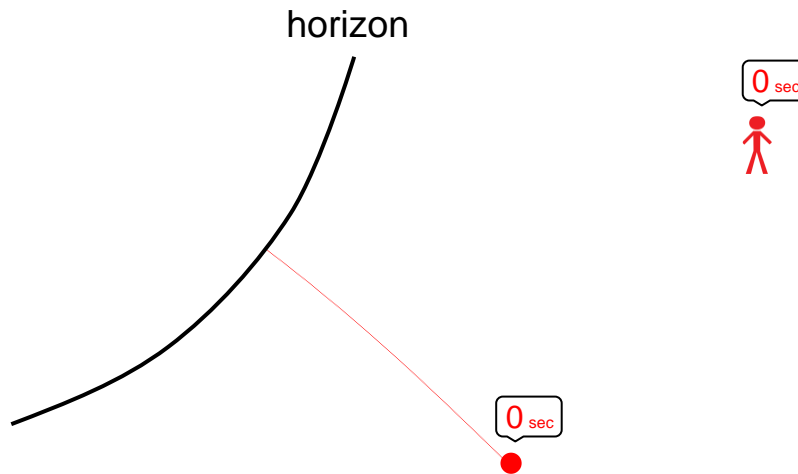
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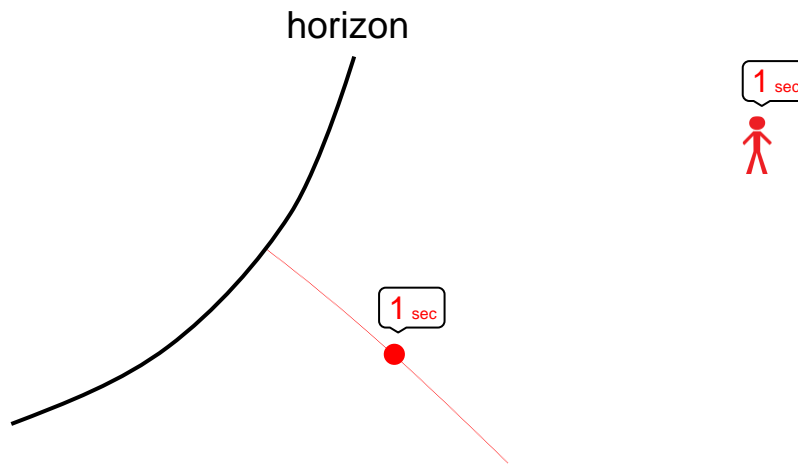
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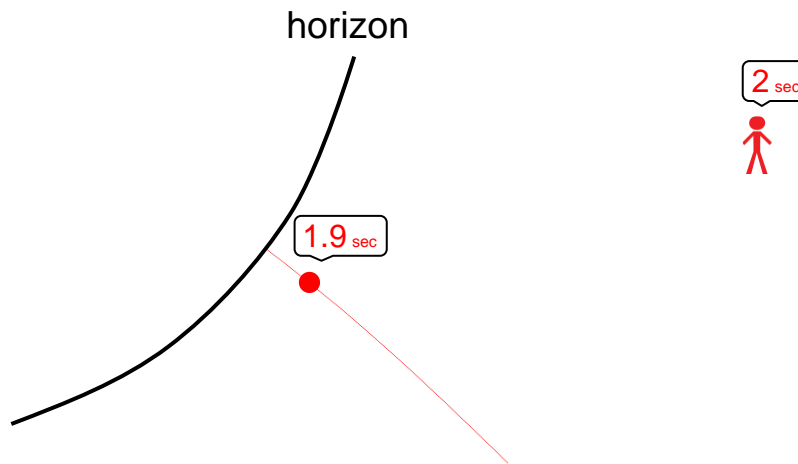
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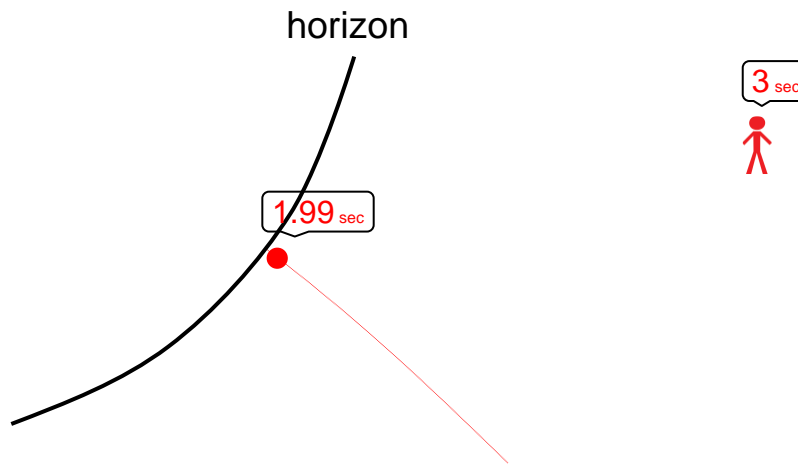
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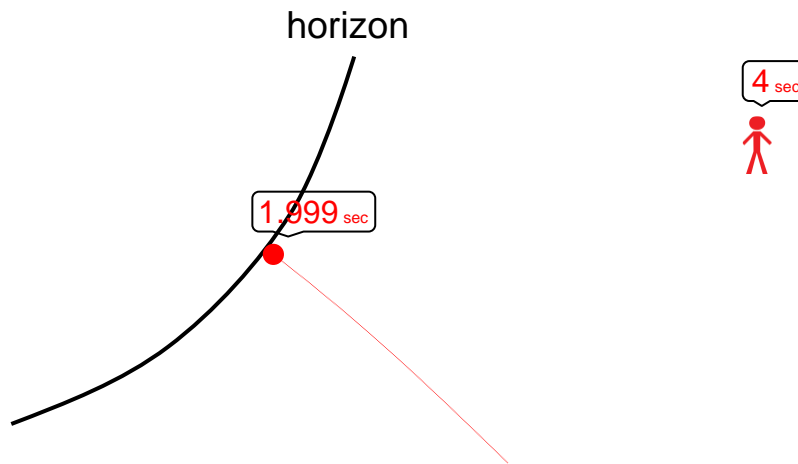
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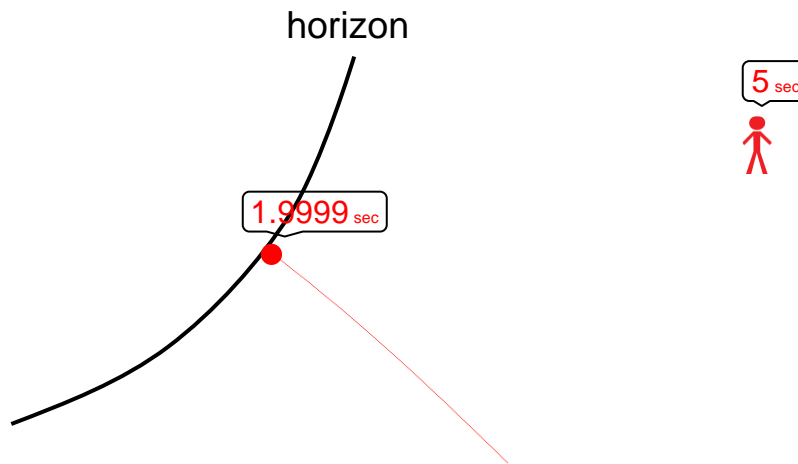
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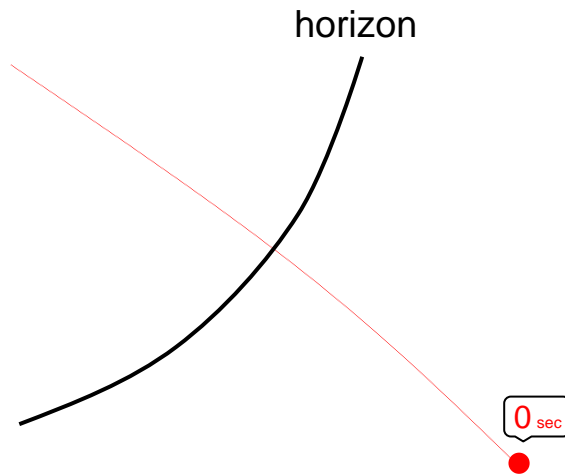
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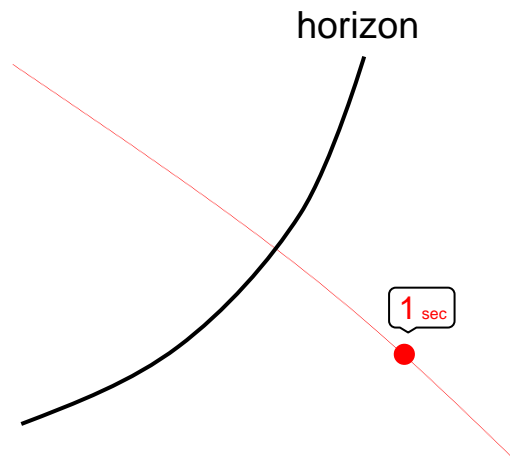
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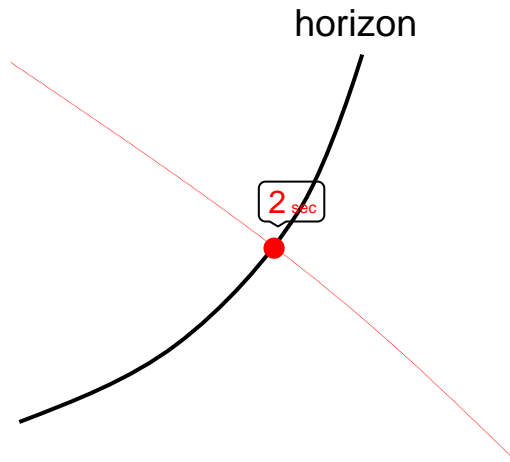
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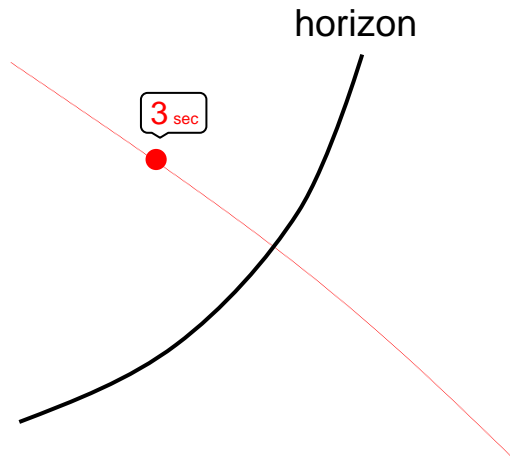
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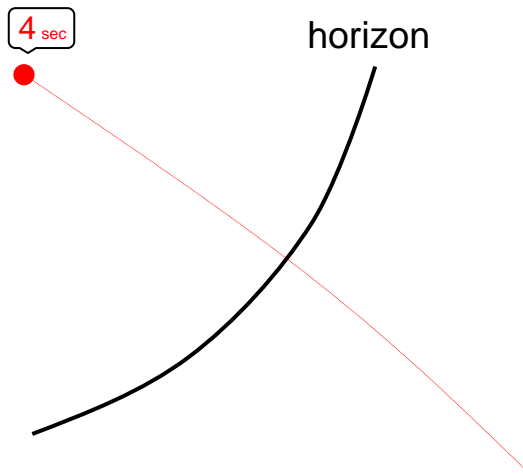
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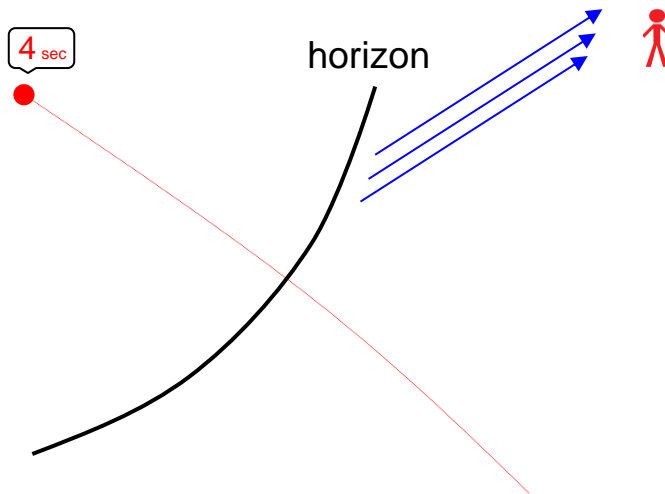
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Hawking temperature

BHs are thermodynamic objects.

→ Spacetime is composed of microscopic d.o.f.s!



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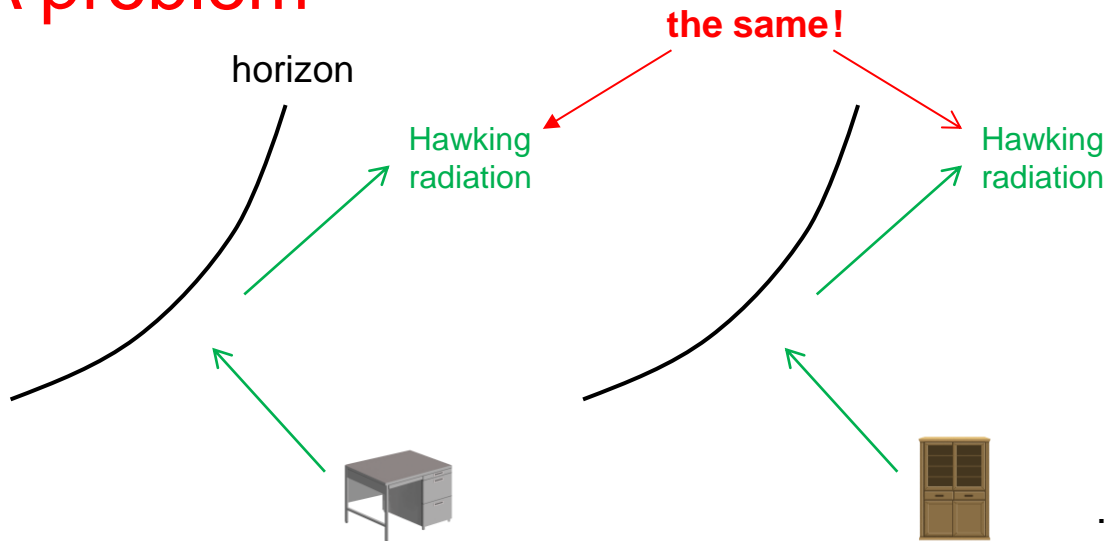
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Hawking temperature

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## A problem



The time evolution  
is **not** one-to-one!  
(not unitary)

... (the original form of)  
**BH information problem**

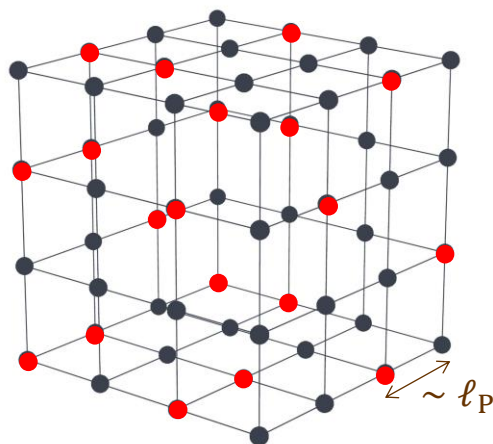
# Holography

## A clue comes from the BH physics itself

A BH is the highest entropy state of the region,

$$\text{and still } S \propto \frac{A}{\ell_P^2}$$

Strange!



$$S \sim \ln 2^{V/\ell_P^3} \propto \frac{V}{\ell_P^3} \gg \frac{A}{\ell_P^2}$$

$$(\ell_P \sim 10^{-33} \text{ cm})$$

The concept that spacetime exists down to  $\sim \ell_P$  is an illusion!

→ suggests that there is a formulation of quantum gravity  
in spacetime **one less dimension** than the naïve one.

Sounds crazy?

# AdS/CFT correspondence [Maldacena, 1997]

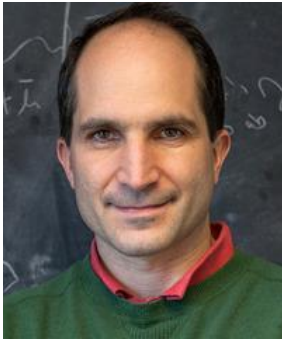
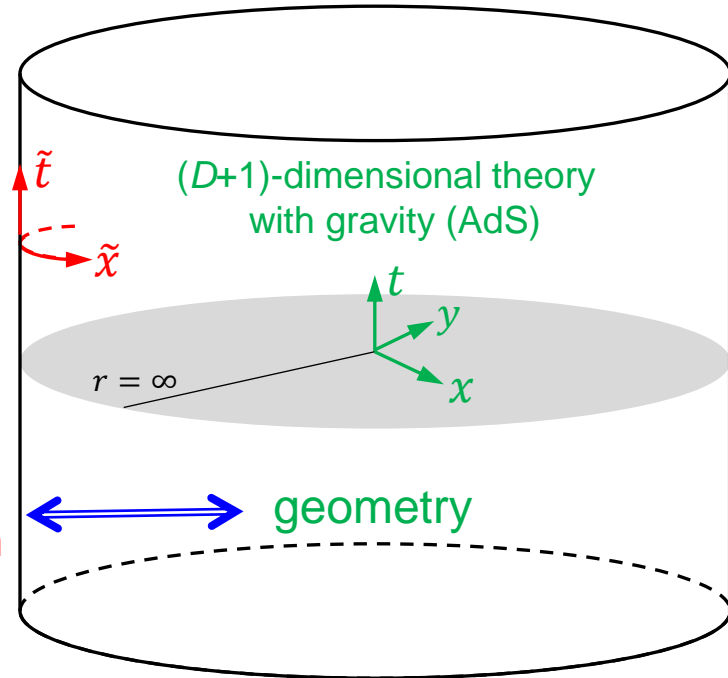
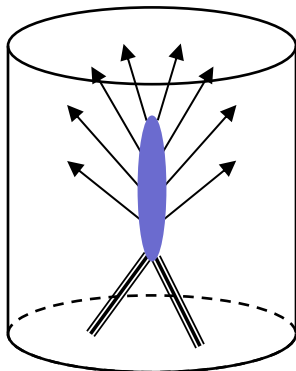


photo: IAS

$D$ -dimensional theory  
without gravity (CFT)



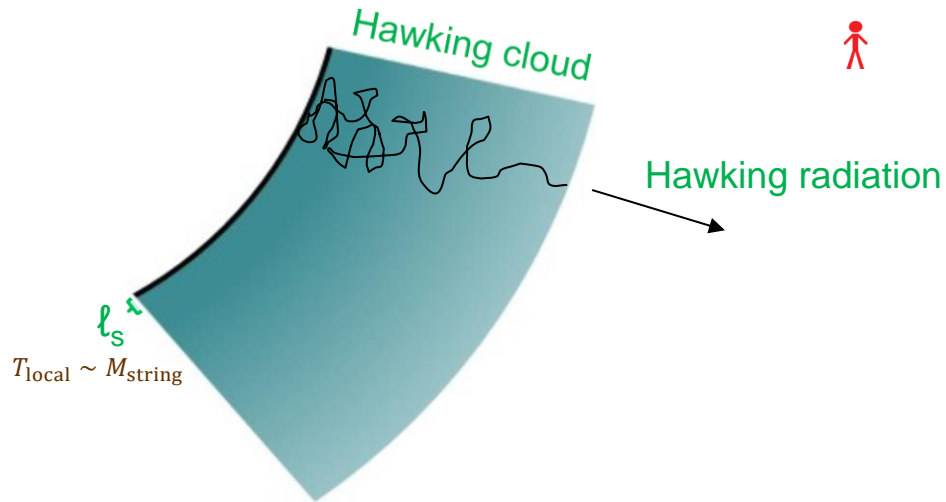
BH evolution **must be** unitary.



=

A process in non-gravitational  
(unitary) theory

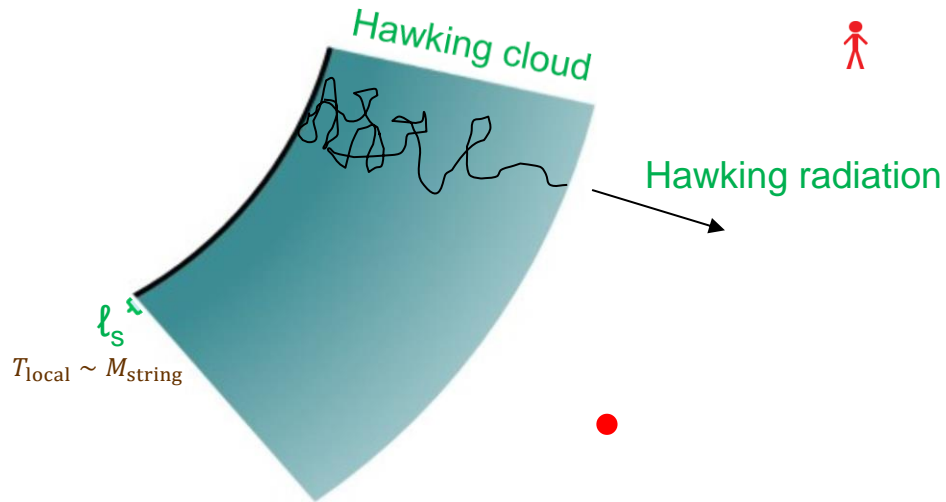
# BH at the quantum level



The horizon behaves  
as the surface of regular material.

... no issue with unitarity

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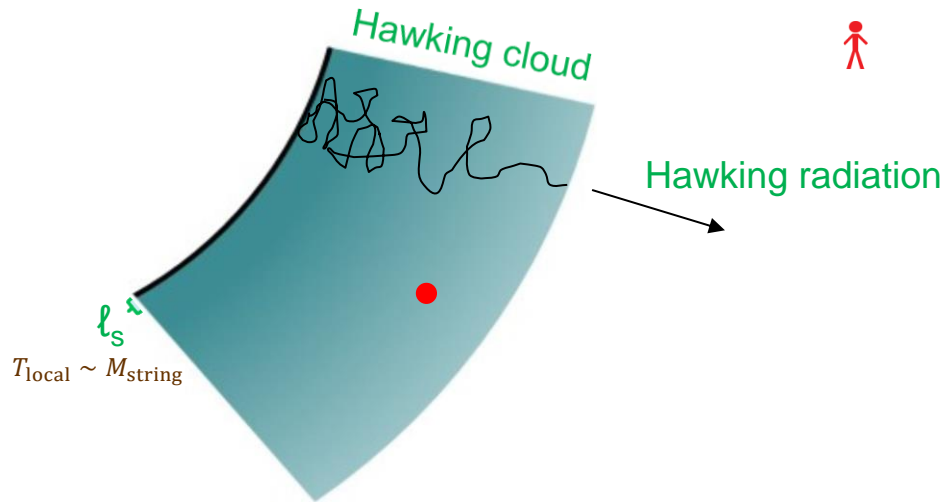


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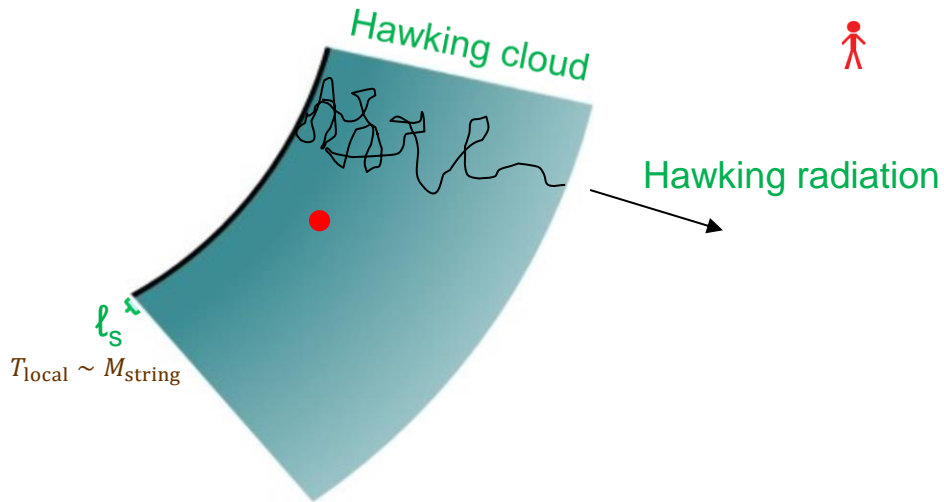
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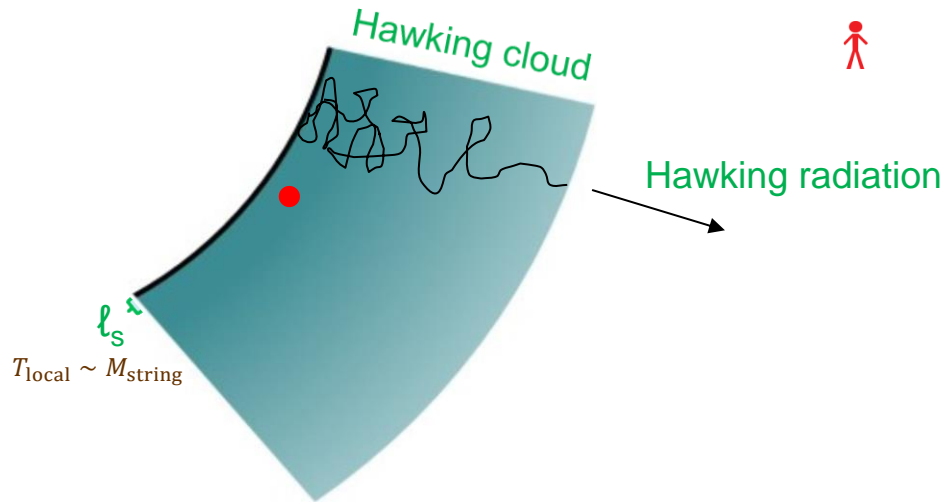
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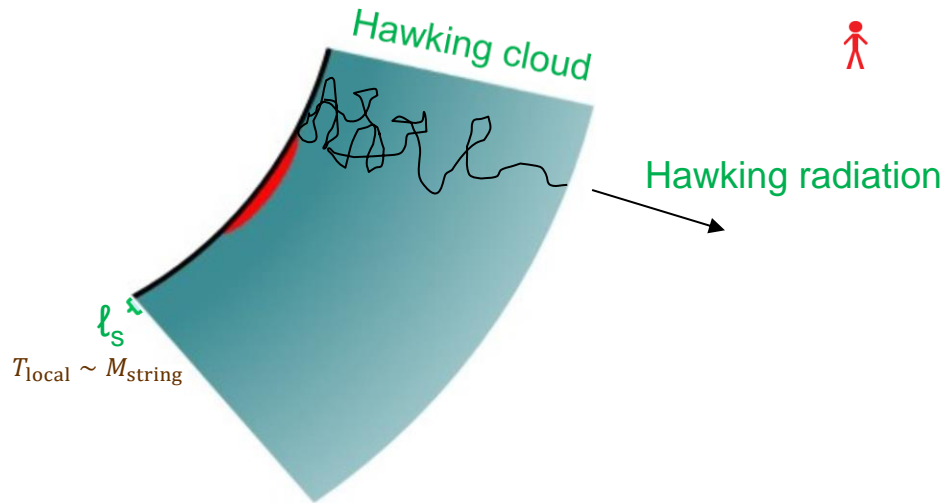
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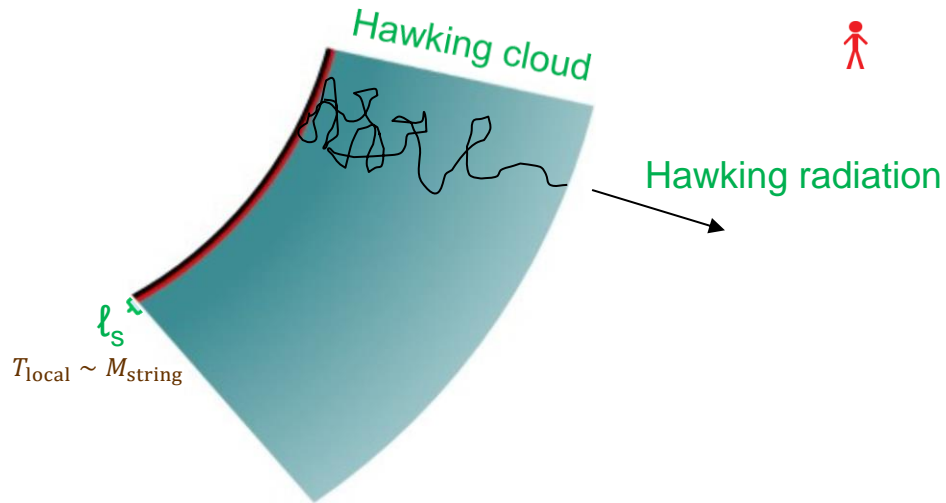
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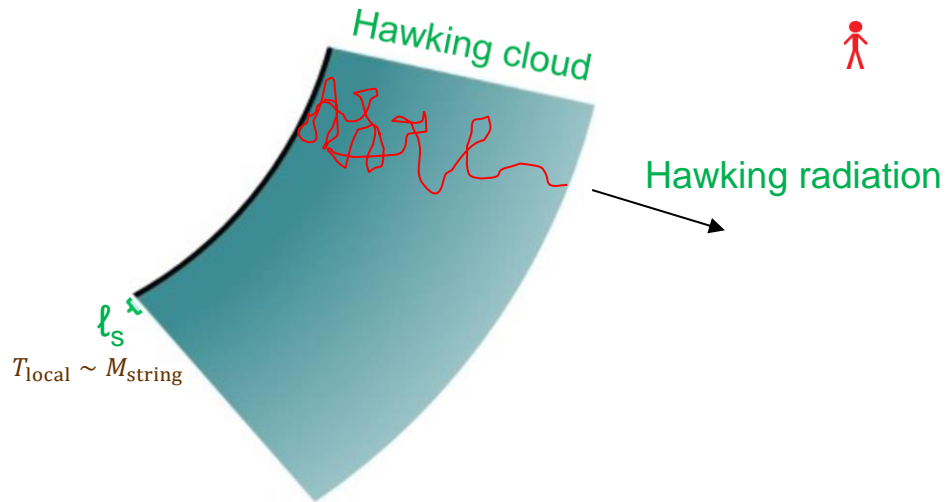
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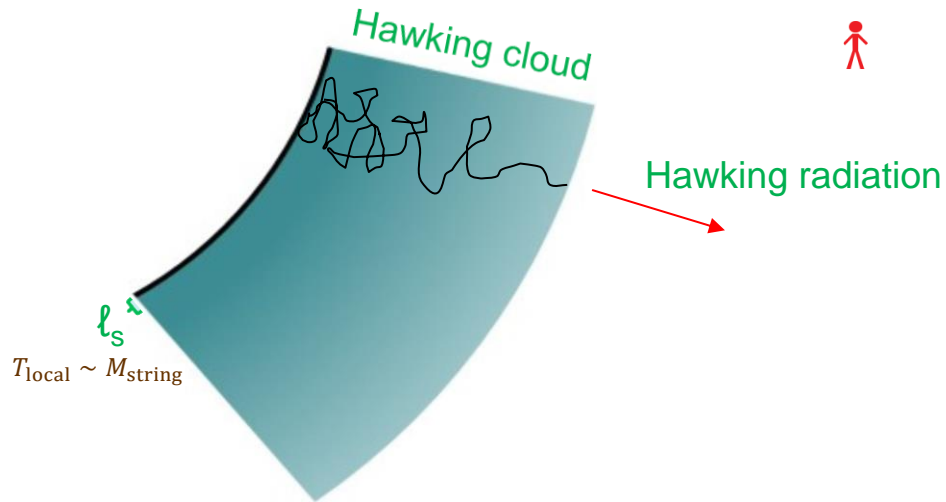
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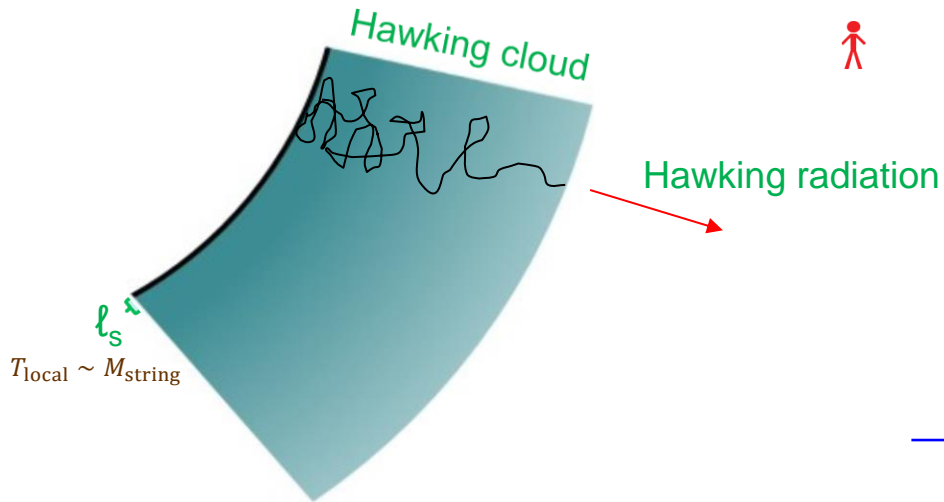
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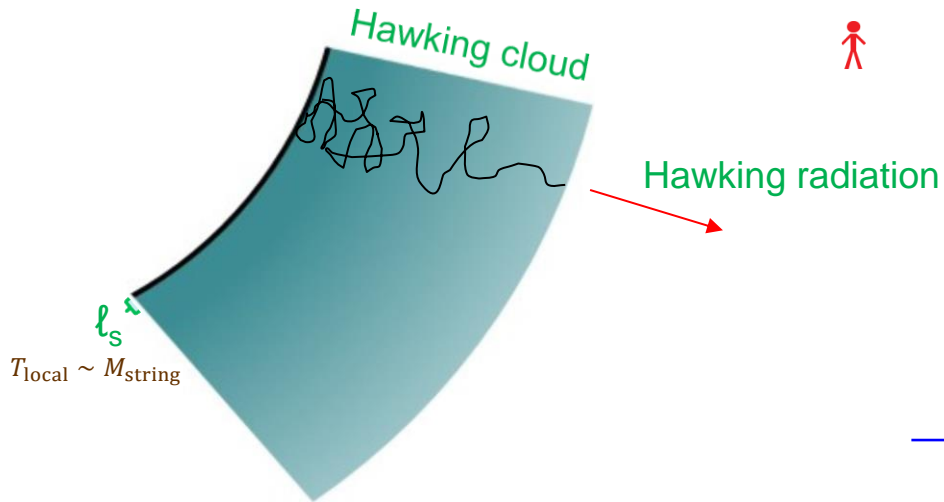
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→ What about the interior?



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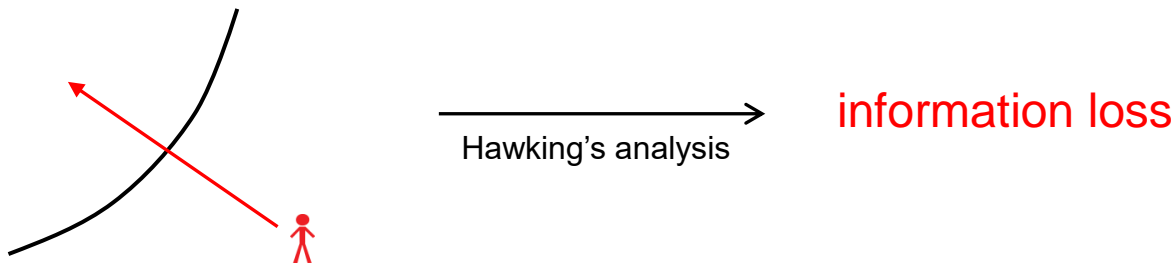


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→ What about the interior?

## Alternatively

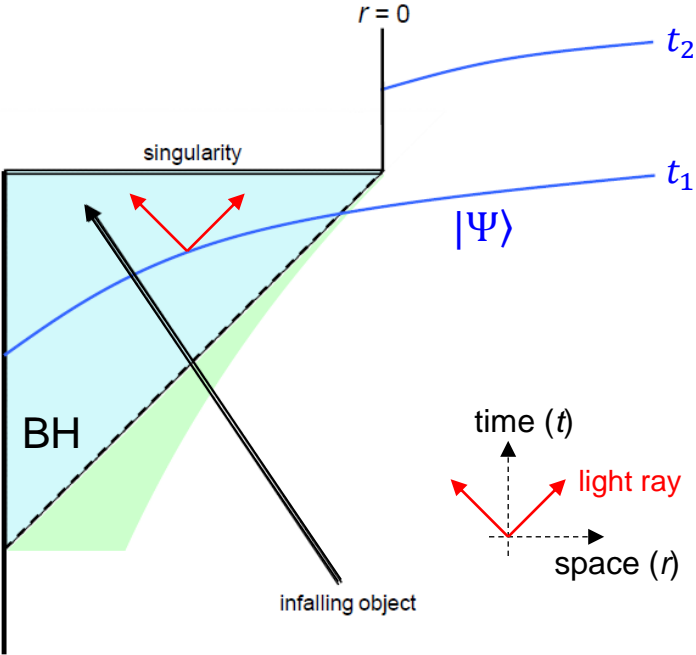


→ What was wrong with Hawking's analysis?

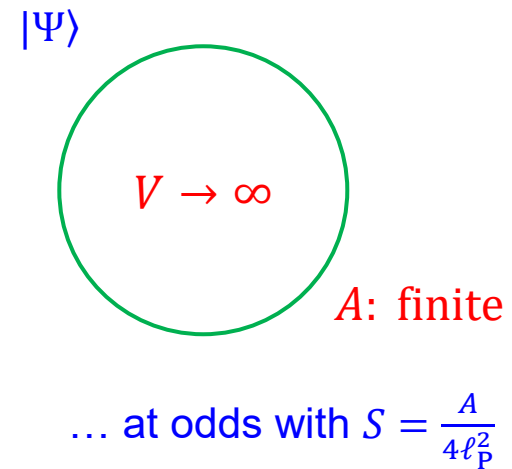
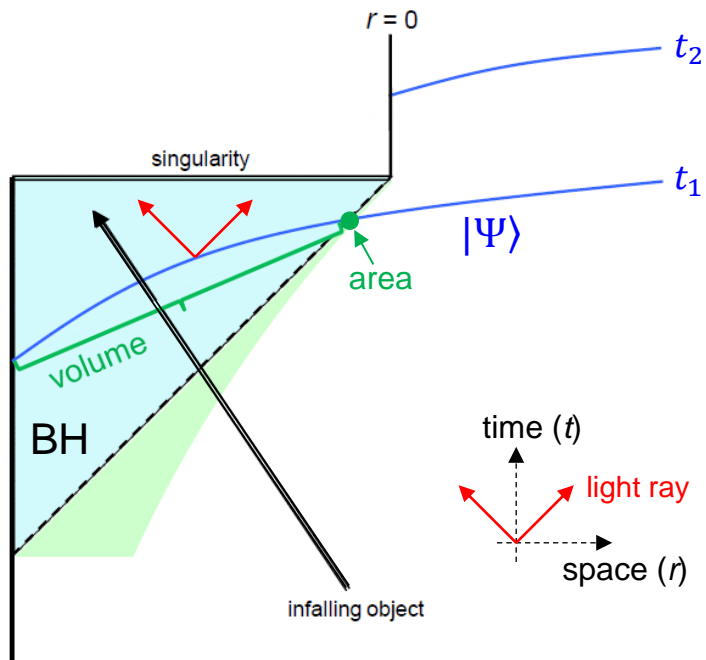
# Recent Progress I

— replica wormholes —

# Start with “global spacetime”

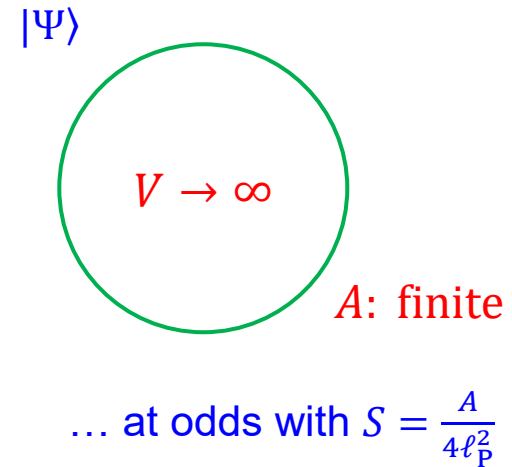
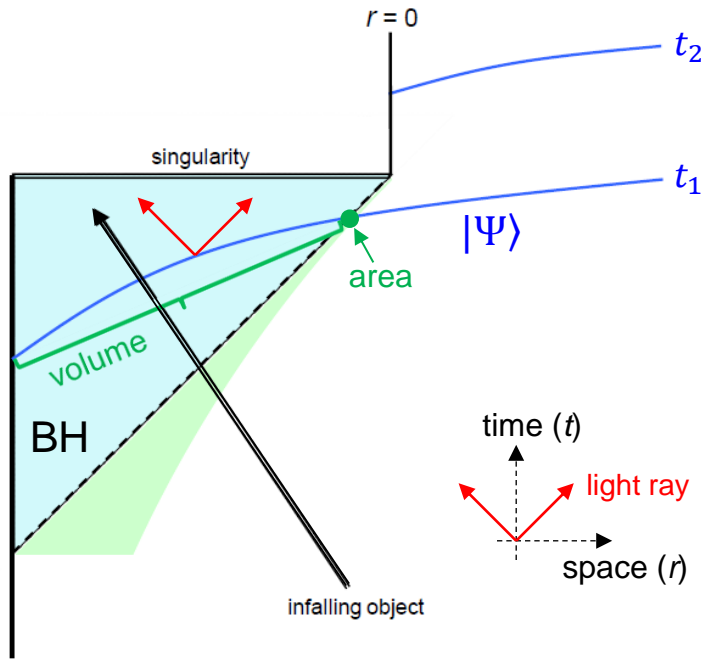


# Start with “global spacetime”



Hugely redundant!

# Start with “global spacetime”



Hugely redundant!

$$\langle \Psi_1 | \Psi_2 \rangle = 0 \quad \longrightarrow \quad \langle \Psi_1 | \Psi_2 \rangle \sim e^{-\frac{S}{2}}$$

semiclassical  
(QFT in curved spacetime)

quantum gravity

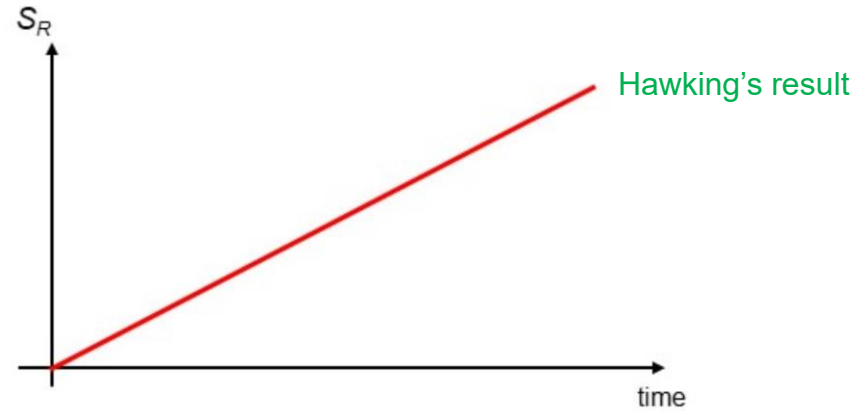
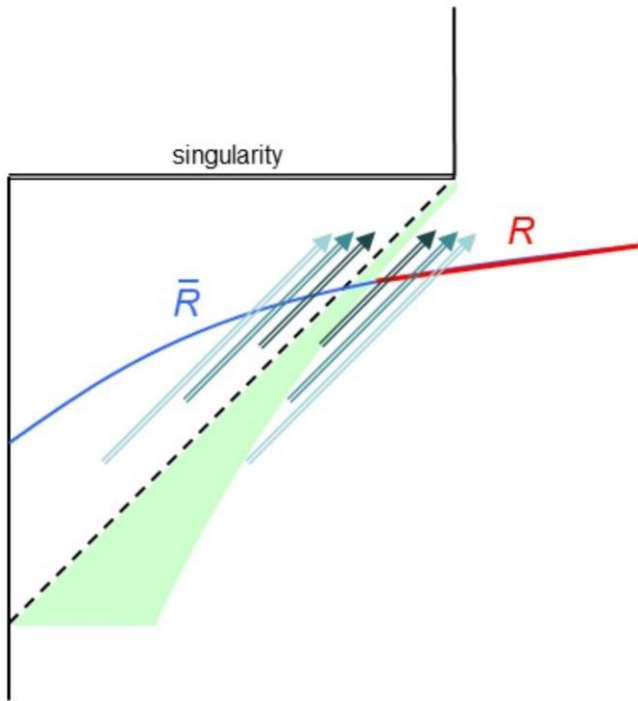
... only  $e^S$  independent states

$$|\Psi\rangle = \sum_{i=1}^{e^S} c_i |\psi_i\rangle \quad c_i \sim e^{-\frac{S}{2}}$$

$$\langle \Psi_1 | \Psi_2 \rangle = \sum_{i=1}^{e^S} c_{1,i}^* c_{2,i} \sim e^{\frac{S}{2}} e^{-S} \sim e^{-\frac{S}{2}}$$

$\rightarrow e^{e^S}$  approximately orthogonal states

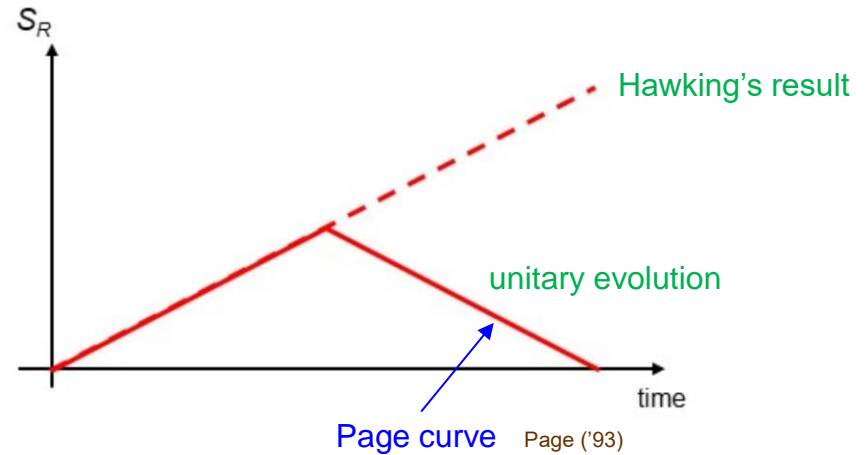
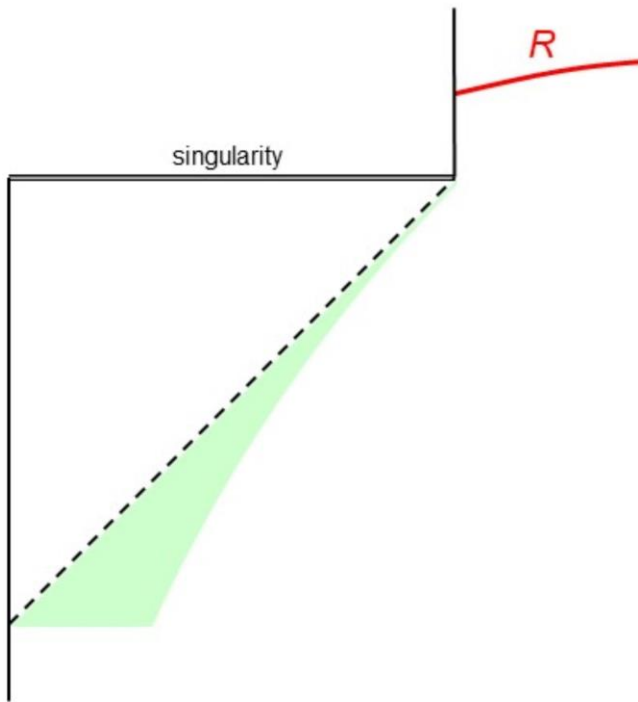
# Unitarity of Hawking evaporation



$$S_R = -\text{Tr}[\rho_R \ln \rho_R] \quad (\rho_R = \text{Tr}_{\bar{R}}|\Psi\rangle\langle\Psi|)$$

~ the # of EPR particles in  $R$  whose partners are in  $\bar{R}$

# Unitarity of Hawking evaporation



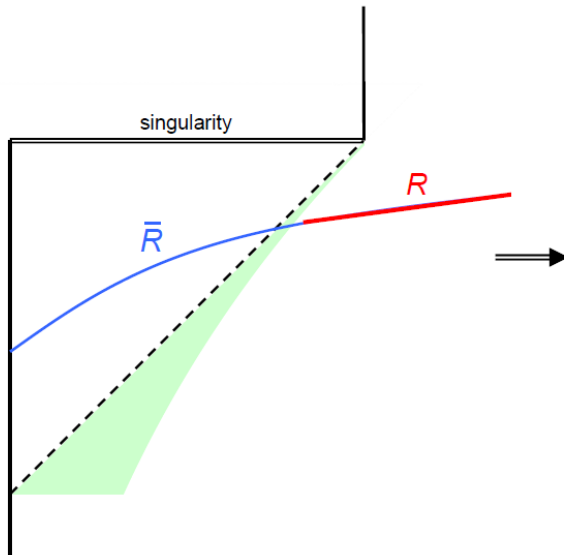
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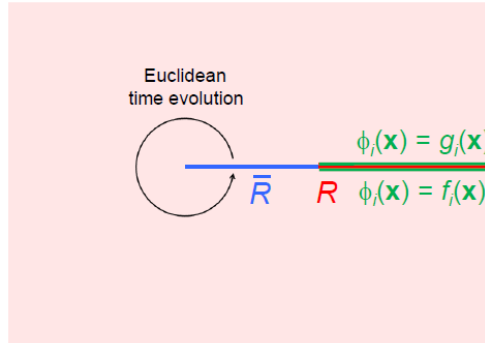
→ How to get this curve?

# Page curve from replica wormholes

Penington ('19); Almheiri, Engelhardt, Marolf, Maxfield ('19); ...  
 Penington, Shenker, Stanford, Yang ('19);  
 Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini ('19)



path integral



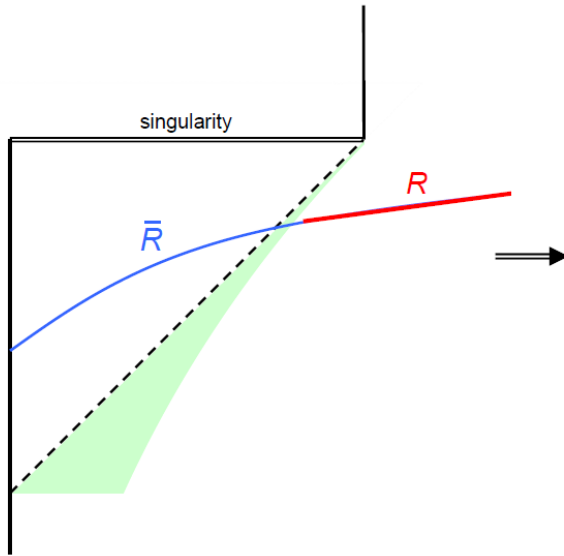
$\rightarrow \rho_R = \rho_R[f_i(\mathbf{x}), g_i(\mathbf{x})]$  ( $\sim$  coefficient of  $|g_i(\mathbf{x})\rangle\langle f_i(\mathbf{x})|$ )

$$S_R \equiv -\text{Tr}[\rho_R \ln \rho_R] = \lim_{n \rightarrow 1} \frac{1}{1-n} \ln \text{Tr}[\rho_R^n]$$

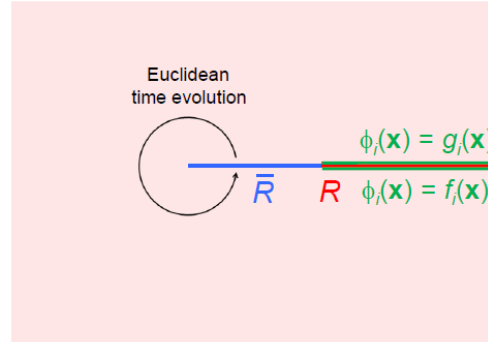


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 Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini ('19)



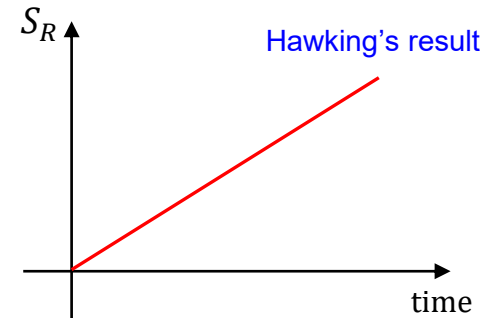
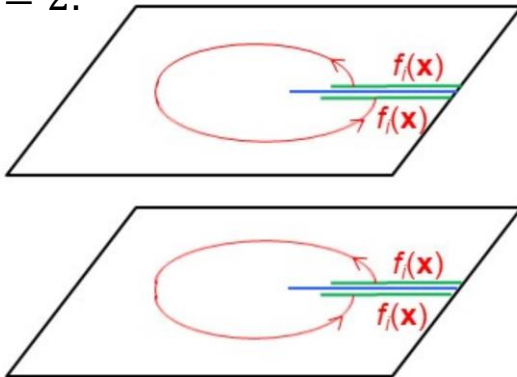
path integral



$$\rightarrow \rho_R = \rho_R[f_i(\mathbf{x}), g_f(\mathbf{x})] \quad (\sim \text{coefficient of } |g_f(\mathbf{x})\rangle\langle f_i(\mathbf{x})|)$$

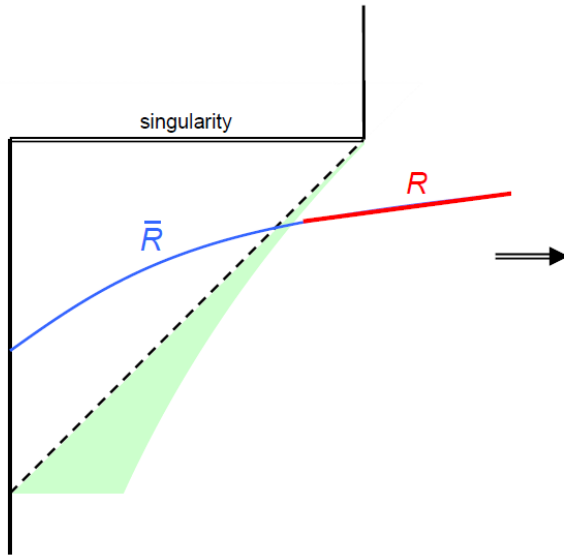
$$S_R \equiv -\text{Tr}[\rho_R \ln \rho_R] = \lim_{n \rightarrow 1} \frac{1}{1-n} \ln \text{Tr}[\rho_R^n]$$

$n = 2$ :

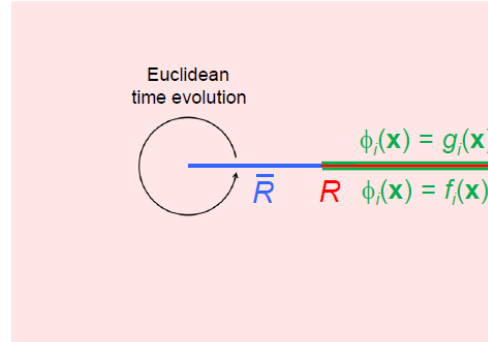


# Page curve from replica wormholes

Penington ('19); Almheiri, Engelhardt, Marolf, Maxfield ('19); ...  
 Penington, Shenker, Stanford, Yang ('19);  
 Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini ('19)



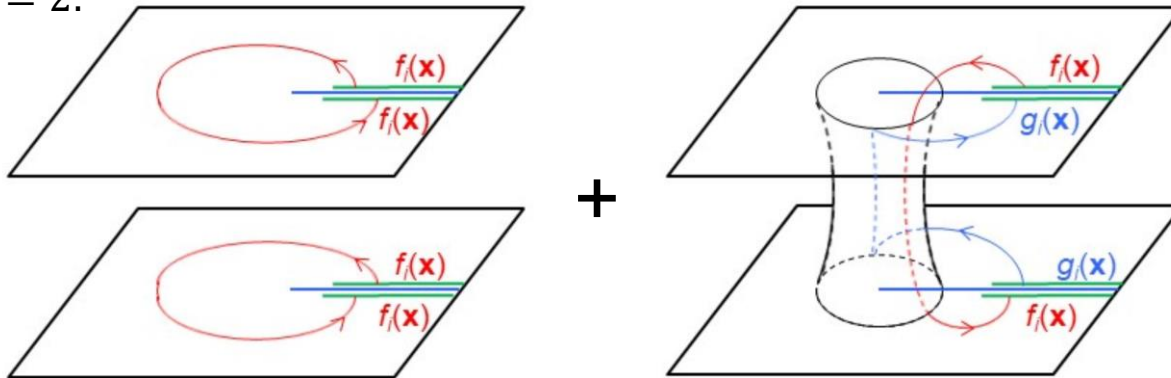
path integral



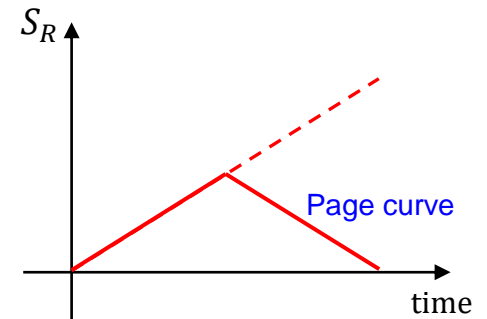
$$\rightarrow \rho_R = \rho_R[f_i(\mathbf{x}), g_i(\mathbf{x})] \quad (\sim \text{coefficient of } |g_i(\mathbf{x})\rangle\langle f_i(\mathbf{x})|)$$

$$S_R \equiv -\text{Tr}[\rho_R \ln \rho_R] = \lim_{n \rightarrow 1} \frac{1}{1-n} \ln \text{Tr}[\rho_R^n]$$

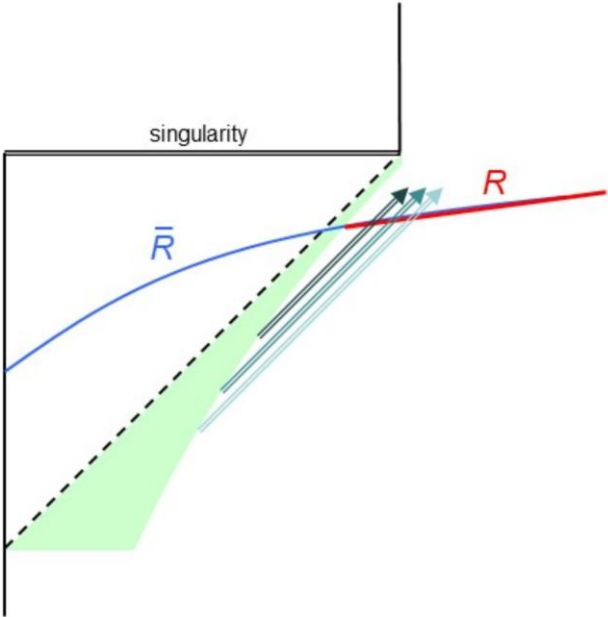
$n = 2$ :



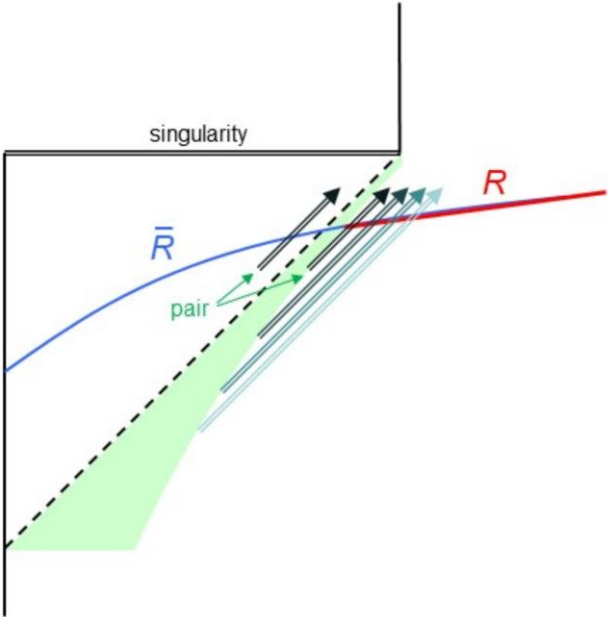
replica wormhole (nonperturbative effect)



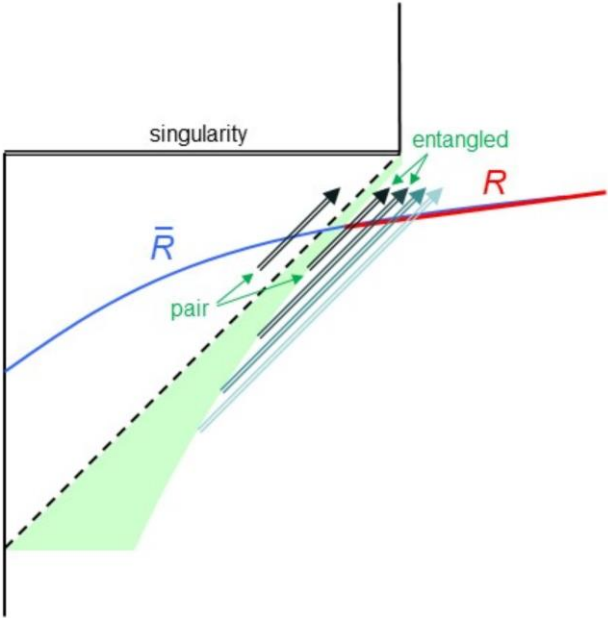
# Redundancy in the Hawking process



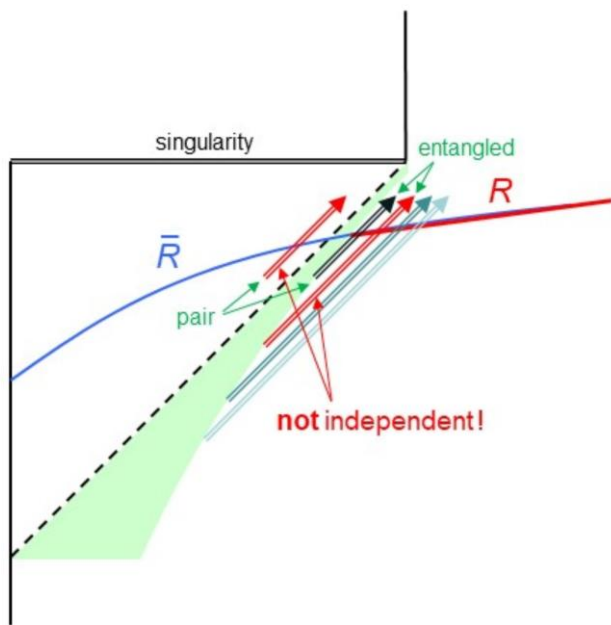
# Redundancy in the Hawking process



# Redundancy in the Hawking process



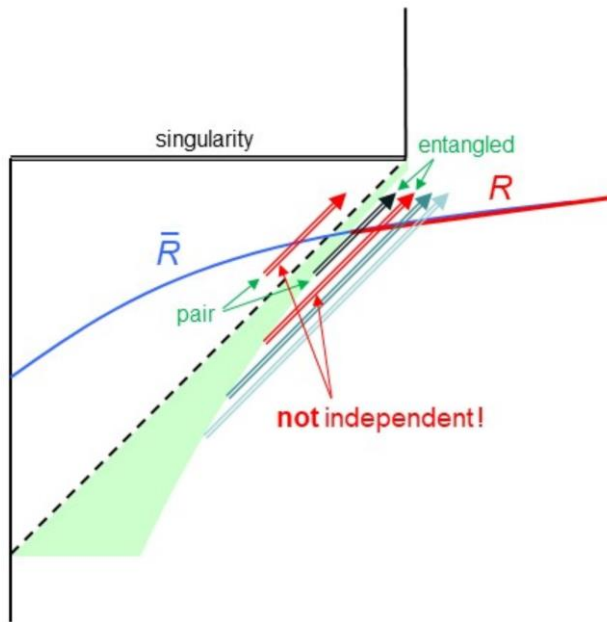
# Redundancy in the Hawking process



→ Hawking radiation emitted earlier is  
**not** independent of the interior d.o.f.s!

...; Maldacena, Susskind ('13); ...

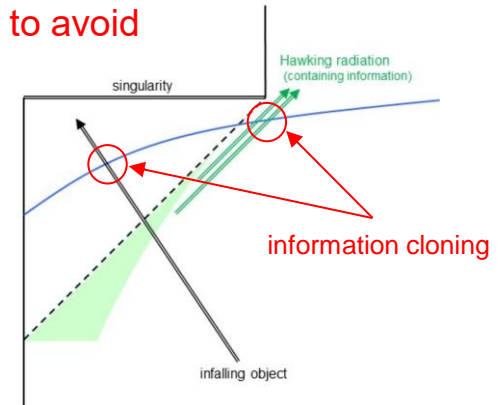
# Redundancy in the Hawking process



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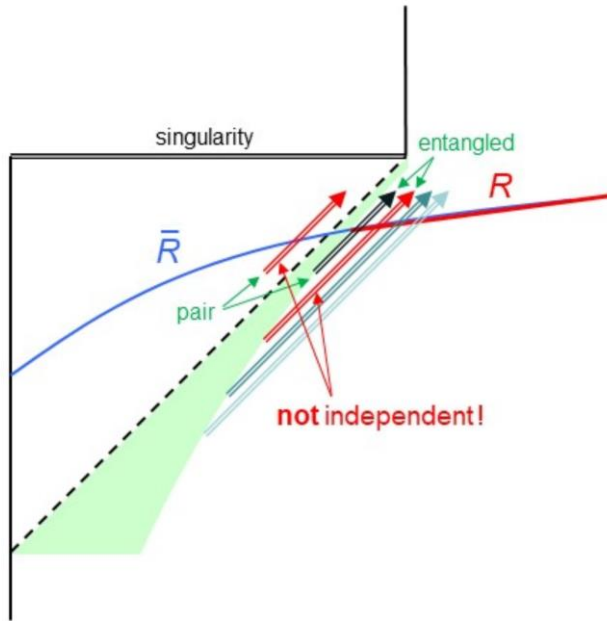
...; Maldacena, Susskind ('13); ...

- needed to avoid

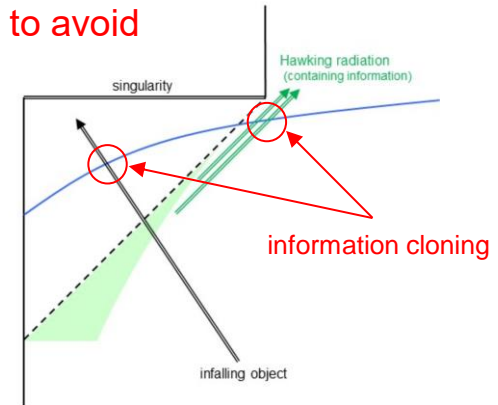


- consistent because of causality

# Redundancy in the Hawking process



- needed to avoid



- consistent because of causality

→ Hawking radiation emitted earlier is **not** independent of the interior d.o.f.s!

...; Maldacena, Susskind ('13); ...

**Global spacetime**  
(embracing the **interior**)

→  
**Replica wormholes**  
(nonperturbative effects of gravity)

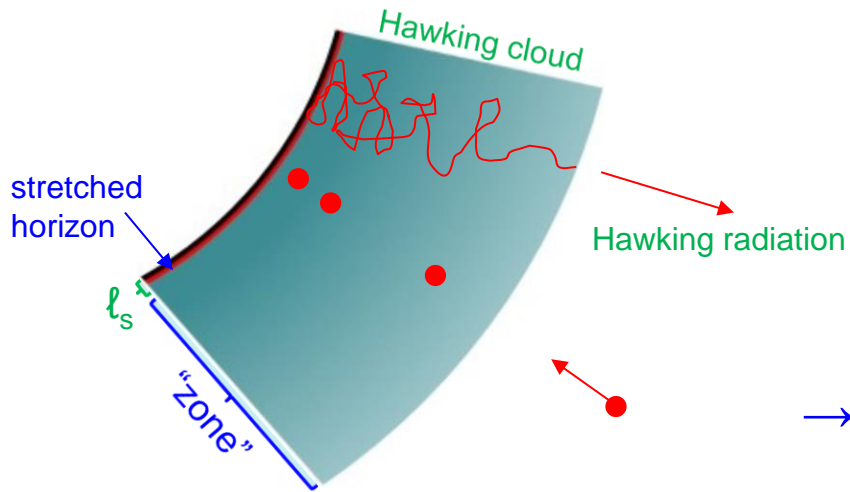
**Page curve**  
(signifying **unitarity**)



# Recent Progress II

— unitary gauge construction —

# Start with a “distant” (holographic) description



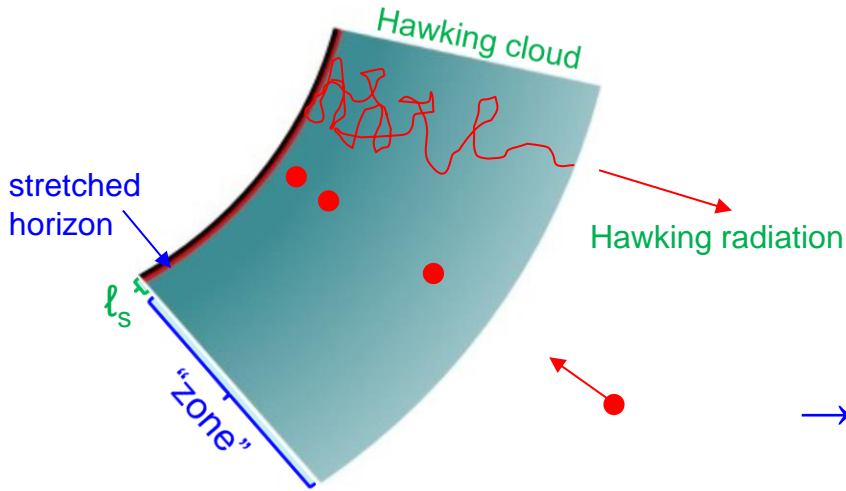
The d.o.f.s outside the horizon  
comprise the **entire** system.

→ The evolution is unitary.

→ How does the “interior” emerge?

Papadodimas, Raju ('12-'15); Verlinde, Verlinde ('12-'13);  
Y.N., Sanches, Varela, Weinberg ('12-'15); ...  
Y.N. ('19, 20)

# Start with a “distant” (holographic) description



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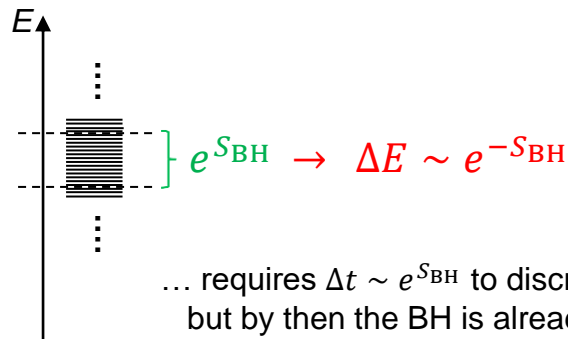
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Papadodimas, Raju ('12-'15); Verlinde, Verlinde ('12-'13);  
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Y.N. ('19, 20)

## Key features Y.N. ('19, 20)

— defining characteristics of BHs

### (I) Exponentially dense spectrum



Relevant modes:

|   |     |           |      |           |
|---|-----|-----------|------|-----------|
| { | BH  | { horizon | soft | (cloud)   |
|   |     | zone      | hard | (objects) |
| } | far |           |      |           |

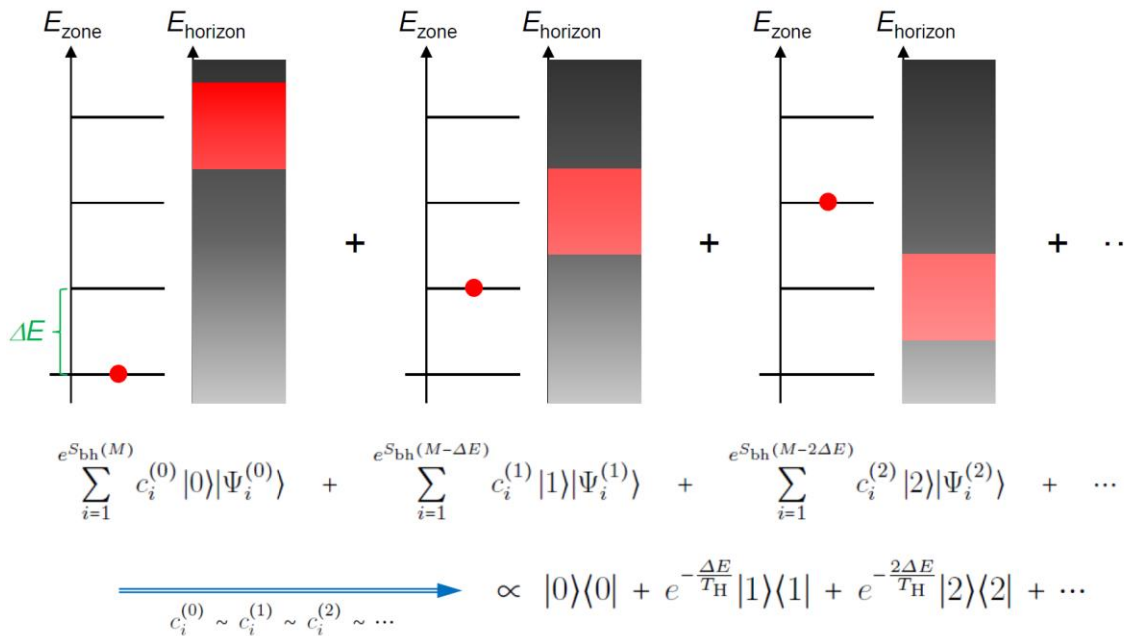
## (II) Dynamics at the stretched horizon

$$T_{\text{local}} \sim M_{\text{string}}$$

... string dynamics

- quantum chaos Maldacena, Shenker, Stanford ('15)
  - fast scrambling Hayden, Preskill ('07); Sekino, Susskind ('08)
  - universal Banks, Seiberg ('10); ...; Harlow, Ooguri ('18)
- (e.g. no global symmetry)

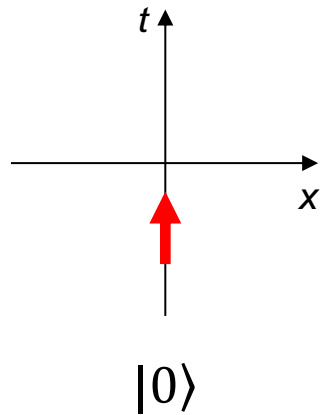
→ “ultimate” thermalization



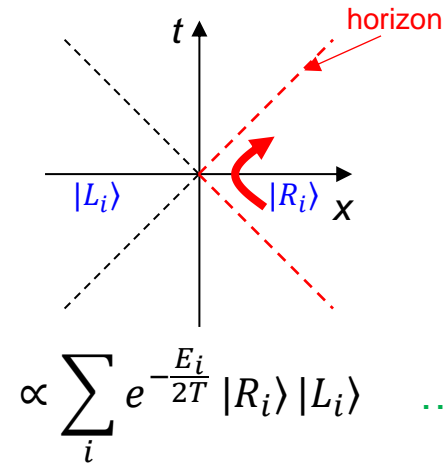
... universal across all low energy species

# Emergence of the interior

Minkowski



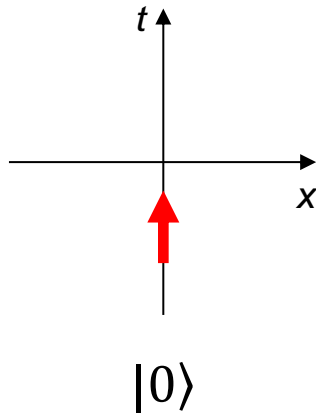
Rindler



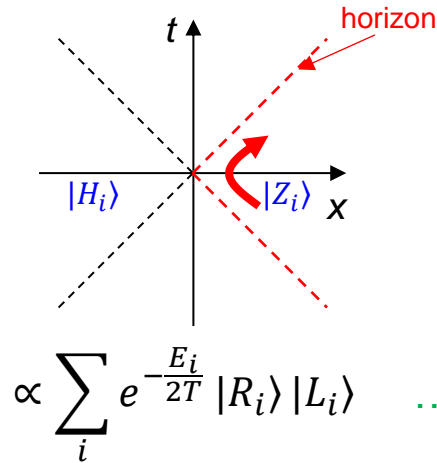
Fulling ('73); Davies ('75); Unruh ('76)

# Emergence of the interior

Minkowski



Rindler



Near empty  
Interior spacetime

(An object thrown "sees" interior spacetime)

frame change

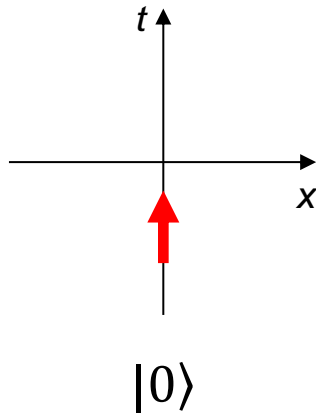
$$|\Psi_{\text{BH}}\rangle \propto \sum_i e^{-\frac{E_i}{2T_{\text{H}}}} |H_i\rangle |S_i\rangle$$

Hard mode states (pointing to  $|H_i\rangle$ )  
Soft mode states (pointing to  $|S_i\rangle$ )  
... play the role of the mirror partners  
(representing their collective excitations)

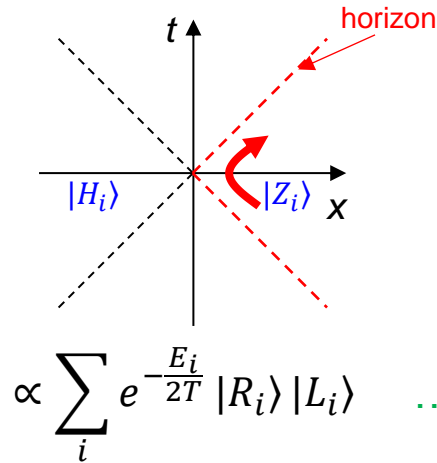
... universally thermal

# Emergence of the interior

Minkowski



Rindler



$$\propto \sum_i e^{-\frac{E_i}{2T}} |R_i\rangle |L_i\rangle \quad \dots \text{thermal state}$$

Fulling ('73); Davies ('75); Unruh ('76)

Near empty  
Interior spacetime

(An object thrown "sees" interior spacetime)

frame change

$$|\Psi_{\text{BH}}\rangle \propto \sum_i e^{-\frac{E_i}{2T_{\text{H}}}} |H_i\rangle |S_i\rangle \quad \dots \text{play the role of the mirror partners}$$

Hard mode states  
Soft mode states  
(representing their collective excitations)

... universally thermal

string dynamics

(not the case for the surface of regular material)

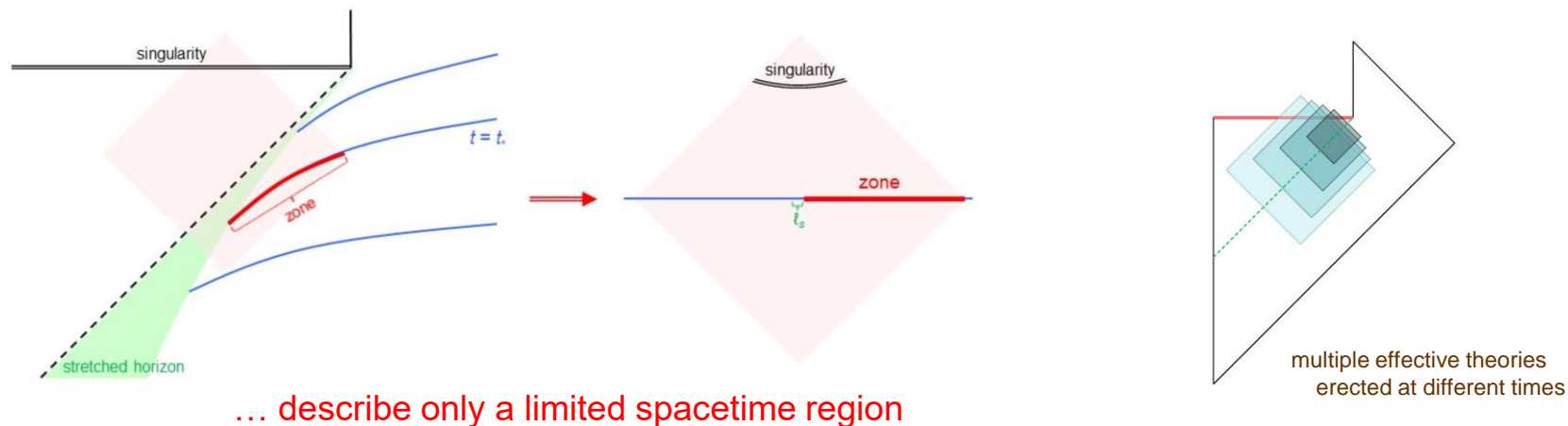
At late times, the BH is entangled with radiation

$$|\Psi_{\text{BH}}\rangle \propto \sum_i e^{-\frac{E_i}{2T_H}} |H_i\rangle |(S+R)_i\rangle \dots \text{play the role of the mirror partners}$$

Hard mode states  
Soft and far (radiation) mode states  
(representing collective excitations of these modes)

... Interior d.o.f.s involve early Hawking radiation!

Effective theory of the interior



**Distant description**  
(manifestly **unitary**)

Collective phenomena

**Interior spacetime**  
(effective emergence)

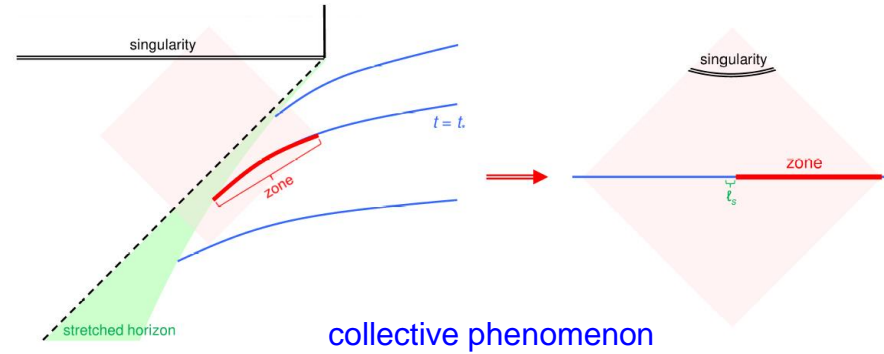
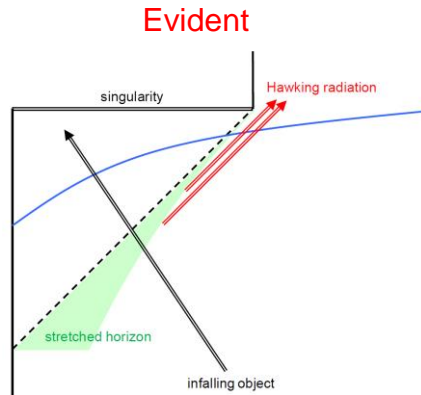


# Structure of Quantum Gravity

Global spacetime  
— General relativity —

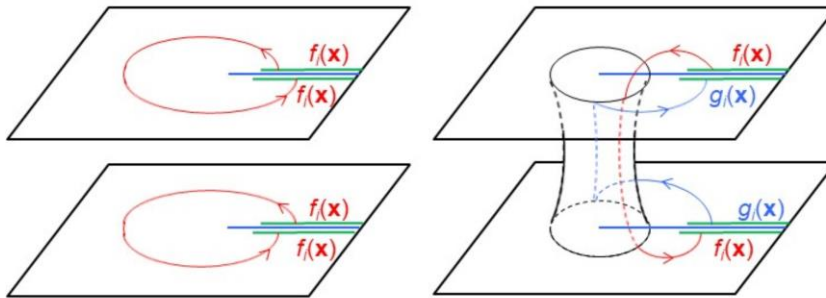
Unitary / holographic  
— Quantum mechanics —

• Interior



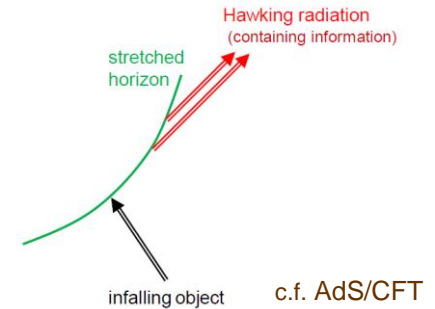
collective phenomenon

• Unitarity



replica wormholes

By construction



• Apparent violation of BH entropy

• Ensemble nature

huge interior spatial volume at late times

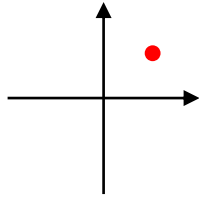
semiclassically orthogonal states  
in fact have  $\langle \Psi_1 | \Psi_2 \rangle \sim e^{-S_{\text{BH}}/2}$   
→  $e^{S_{\text{BH}}}$  states (+ null states)

Effective theory of the interior has a finite maximal volume.

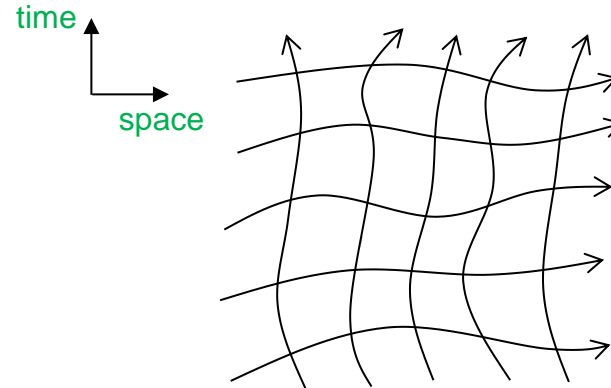
Hilbert space of dimension  $e^{S_{\text{BH}}}$  can host  $e^{S_{\text{BH}}}$  approximately orthogonal states.

# Redundancies of the description

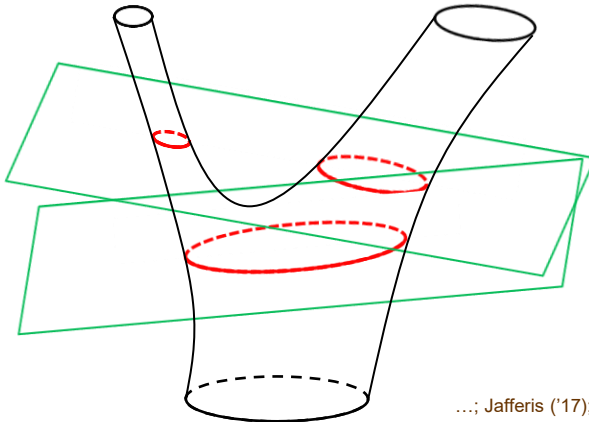
- General covariance (perturbative)



$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \leftrightarrow \begin{pmatrix} r \\ \theta \end{pmatrix} = \begin{pmatrix} \sqrt{2} \\ \pi/4 \end{pmatrix} \leftrightarrow \dots$$



- Nonperturbative redundancies



...; Jafferis ('17); Marolf, Maxfield ('20); ...

... allows for making (only) one of the two pillars manifest,  
but the theory still accommodates both of them (QM + GR).

# Summary

Black hole conundrum

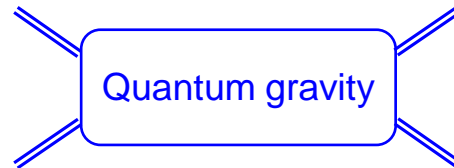


Structure of quantum gravity

⇒ Quantum mechanics & General relativity, but in a subtle manner!

High energy / Astro physics  
(Black holes, gravity, ...)

Quantum information science  
(Holography, ...)



Many-body physics (cond. matter & AMO)  
(Chaos, fast scrambling, ...)

...

## • Activities at Berkeley

### “Geoflow” collaboration

Berkeley/Stanford/Duke/Brandies/Bookhaven  
funded by DOE

Altman, Bousso, Y.N., Penington, Siddiqi, Zaletel

“From the Black Hole Conundrum to the Structure of Quantum Gravity”  
Y.N., *Mod. Phys. Lett.* **A36** (2021) 2130007 [arXiv:2011.08707 [hep-th]]

“Complementarity for a Dynamical Black Hole”  
B. Conception, Y.N., K. Ritchie, S. Weiss, arXiv:2405.15849 [hep-th]