Nucleon decay as a probe for grand unified theories

From the viewpoint of chirality structure in the effective interactions

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Based on an ongoing work with Koichi Hamaguchi, Hor Shihwen and Natsumi Nagata

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Introduction

- Nucleon decay lifetimes can now be predicted with much accuracy thanks to the advancement of lattice QCD computation. →Aoki-san's talk

- Given the upcoming nucleon decay experiments such as Hyper-K, DUNE, JUNO, …, it is worth reconsidering the extent to which nucleon decay can probe new physics.

- In this talk, we focus on the strangeness-conserving four-fermi nucleon decay operators, and see

  - The relations between their chirality structure and the ratios of partial decay widths

  - Applications to the minimal SU(5) GUT with high-scale SUSY

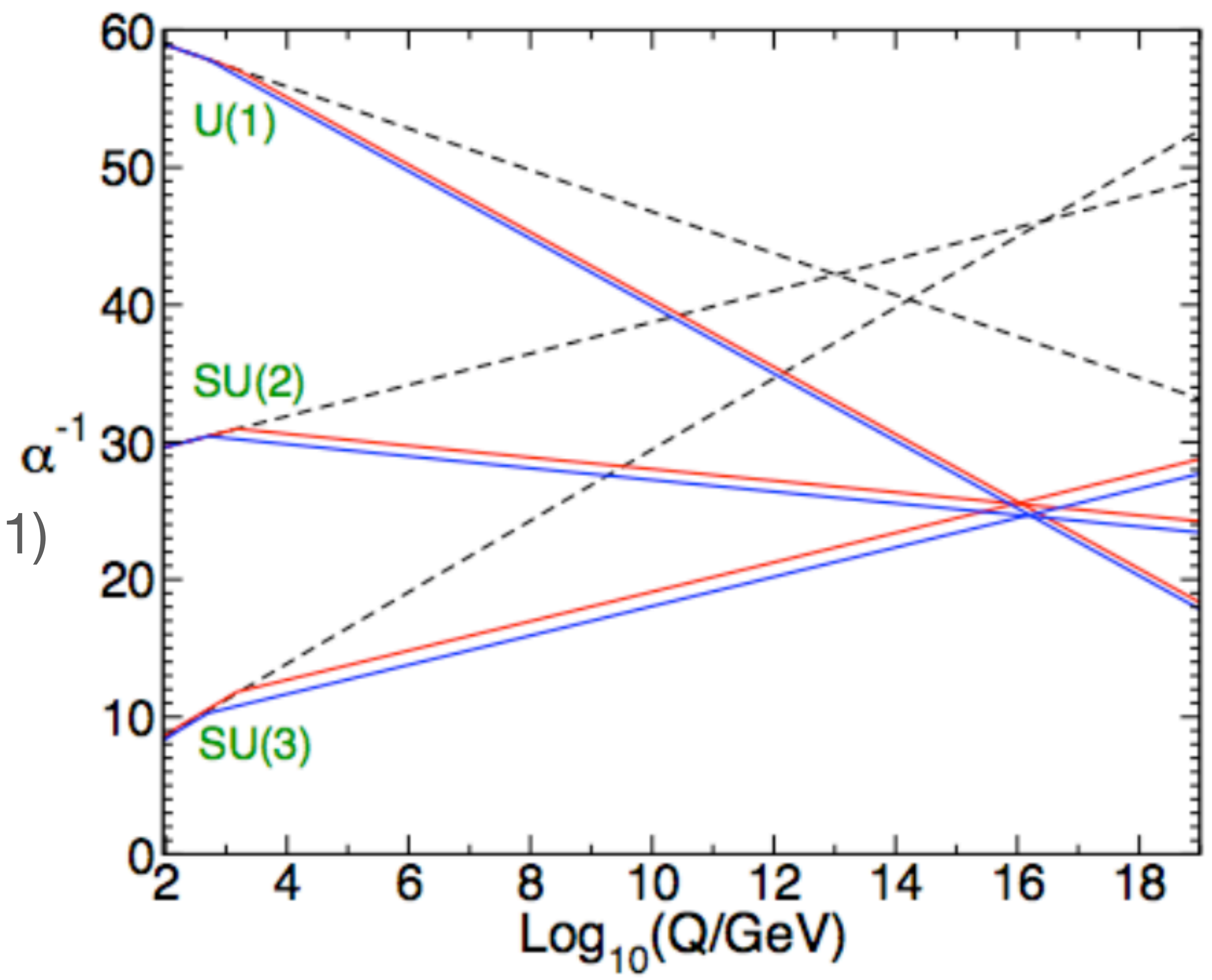
Grand unified thoeries (GUTs)

H. Georgi and S. L. Glashow (1974)

- Gauge coupling unification
  - $SU(5) \to SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$
  - SUSY improves the accuracy of unification (SUSY GUTs).
    S. Dimopoulos and H. Georgi (1981), N. Sakai (1981)
- Quark-lepton unification

$$\mathbf{10}_i = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u^\dagger_{Ri3} & -u^\dagger_{Ri2} & u^1_{Li} & d^1_{Li} \\ -u^\dagger_{Ri3} & 0 & u^\dagger_{Ri1} & u^2_{Li} & d^2_{Li} \\ u^\dagger_{Ri2} & -u^\dagger_{Ri1} & 0 & u^3_{Li} & d^3_{Li} \\ -u^1_{Li} & -u^2_{Li} & -u^3_{Li} & 0 & e^\dagger_{Ri} \\ -d^1_{Li} & -d^2_{Li} & -d^3_{Li} & -e^\dagger_{Ri} & 0 \end{pmatrix}, \qquad \bar{\mathbf{5}}_i = \begin{pmatrix} d^\dagger_{Ri1} \\ d^\dagger_{Ri2} \\ d^\dagger_{Ri3} \\ e_{Li} \\ -\nu_{Li} \end{pmatrix}$$

- Charge quantization
- Experimental signature: **nucleon decay**

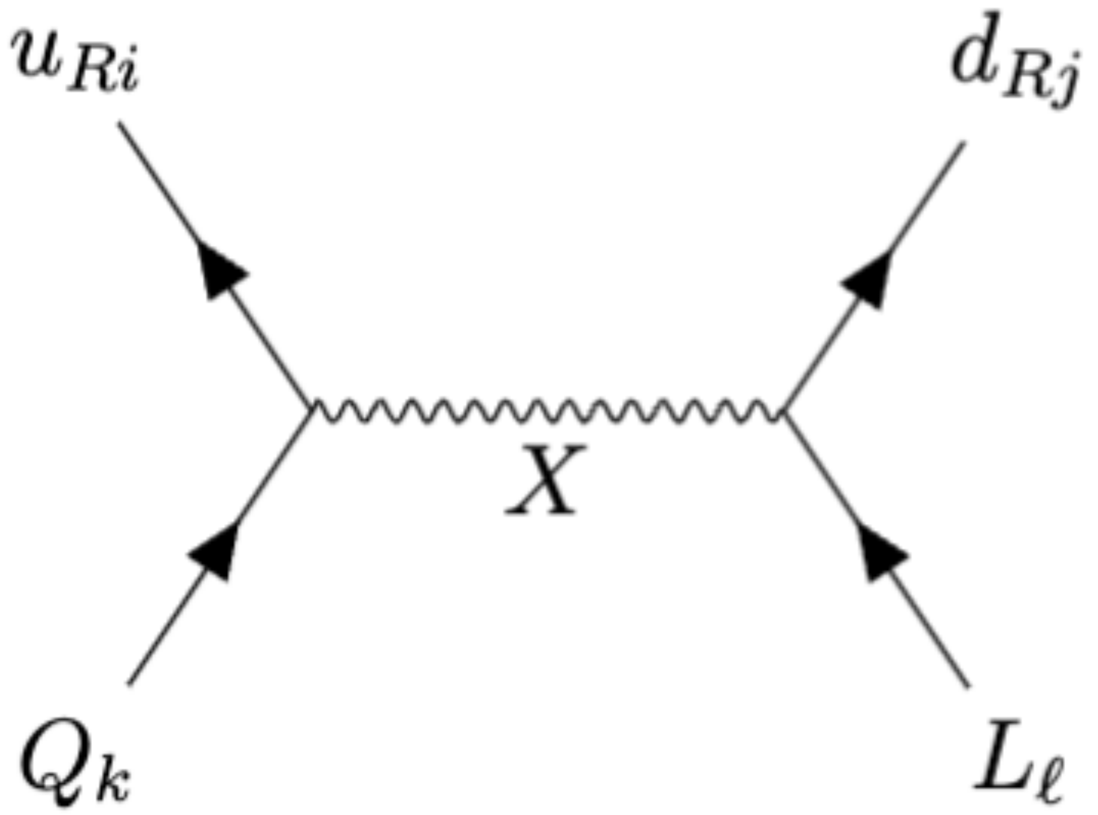


Plot from P. Langacker (1981)

Nucleon decay

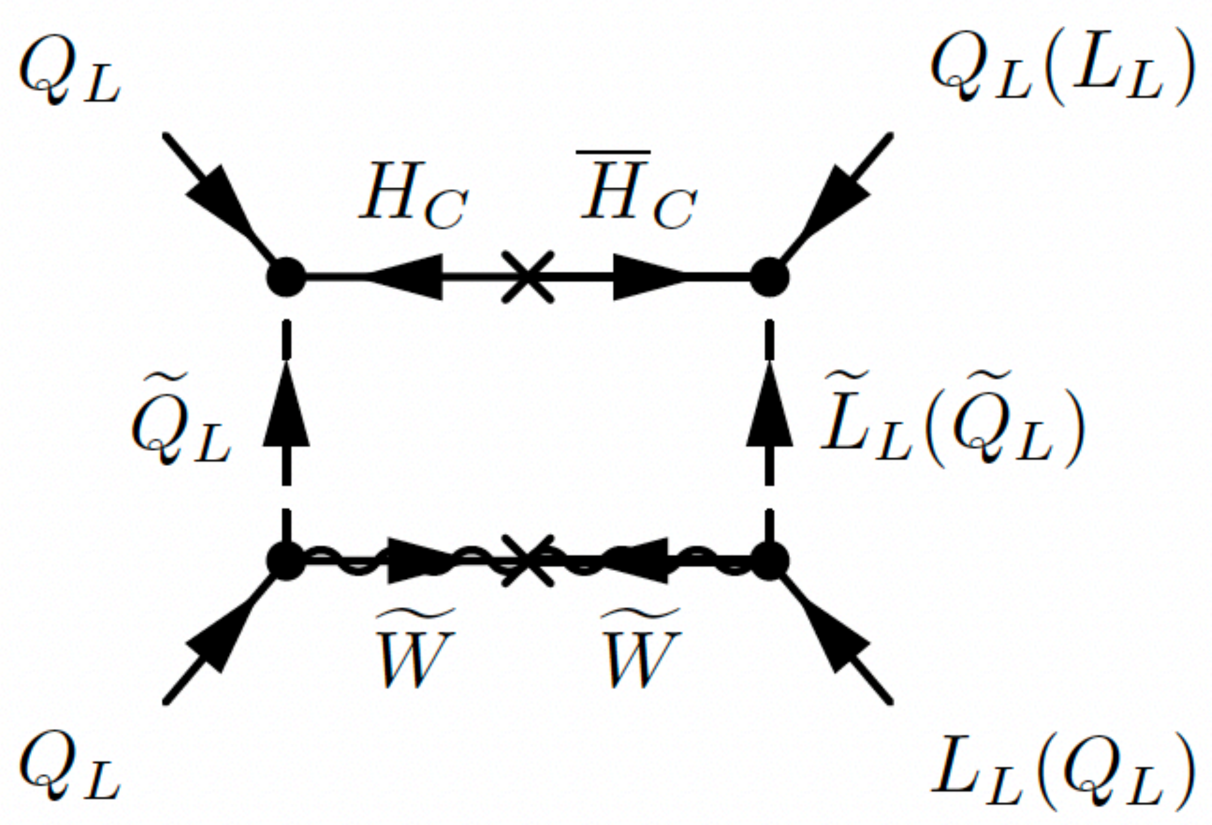
- In non-SUSY GUTs, the decay is typically determined by the exchange of the GUT gauge boson. H. Georgi and S. L. Glashow (1974)

- In SUSY GUTs, the exchange of a color-triplet Higgs and SUSY particles is also important. S. Weinberg (1982), N. Sakai and T. Yanagida (1982)



Dim-6

$$\propto \frac{1}{M_X^2}$$



Dim-5

$$\propto \frac{m_{\tilde{g}}}{M_{H_C} M_{\rm SUSY}^2}$$

- $m_{\tilde{g}}$: gaugino/higgsino mass
- Sfermions are assumed to have $M_{\rm SUSY}$

Can we distinguish those mechanisms using the input of nucleon decay experiments?

Plan of the talk

- Introduction
- Effective interactions for nucleon decay
- The chirality structure and the ratios of partial decay widths
- Application to the SUSY SU(5) GUT
- Summary

Effective interactions for nucleon decay

- Given the stringent limits on nucleon lifetimes, the new physics scale is expected to be decoupled from the electroweak scale.
  - Captured by $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ invariant operators
  - Lowest-order: four-Fermi interactions

S. Weinberg (1979)
F. Wilczek and A. Zee (1979)
L. F. Abbott and M. B. Wise (1980)

$$\mathcal{O}^{(1)}_{ijkl} = \epsilon_{abc}\epsilon^{\alpha\beta}(u^a_{Ri}d^b_{Rj})(Q^c_{Lk\alpha}L_{Ll\beta}) \ ,$$
$$\mathcal{O}^{(2)}_{ijkl} = \epsilon_{abc}\epsilon^{\alpha\beta}(Q^a_{Li\alpha}Q^b_{Lj\beta})(u^c_{Rk}e_{Rl}) \ ,$$
$$\mathcal{O}^{(3)}_{ijkl} = \epsilon_{abc}\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}(Q^a_{Li\alpha}Q^b_{Lj\gamma})(Q^c_{Lk\delta}L_{Ll\beta}) \ ,$$
$$\mathcal{O}^{(4)}_{ijkl} = \epsilon_{abc}(u^a_{Ri}d^b_{Rj})(u^c_{Rk}e_{Rl}) \ ,$$

i, j, k, l: flavor index

Chirality:

- $\mathcal{O}^{(1)}, \mathcal{O}^{(2)}$ → Mixed-type
- $\mathcal{O}^{(3)}, \mathcal{O}^{(4)}$ → Pure-type

- $$\mathcal{L}_{\rm SM,eff} = \sum_{I,ijkl} C^{ijkl}_{(I)}\mathcal{O}^{(I)}_{ijkl} + \text{h.c.}$$

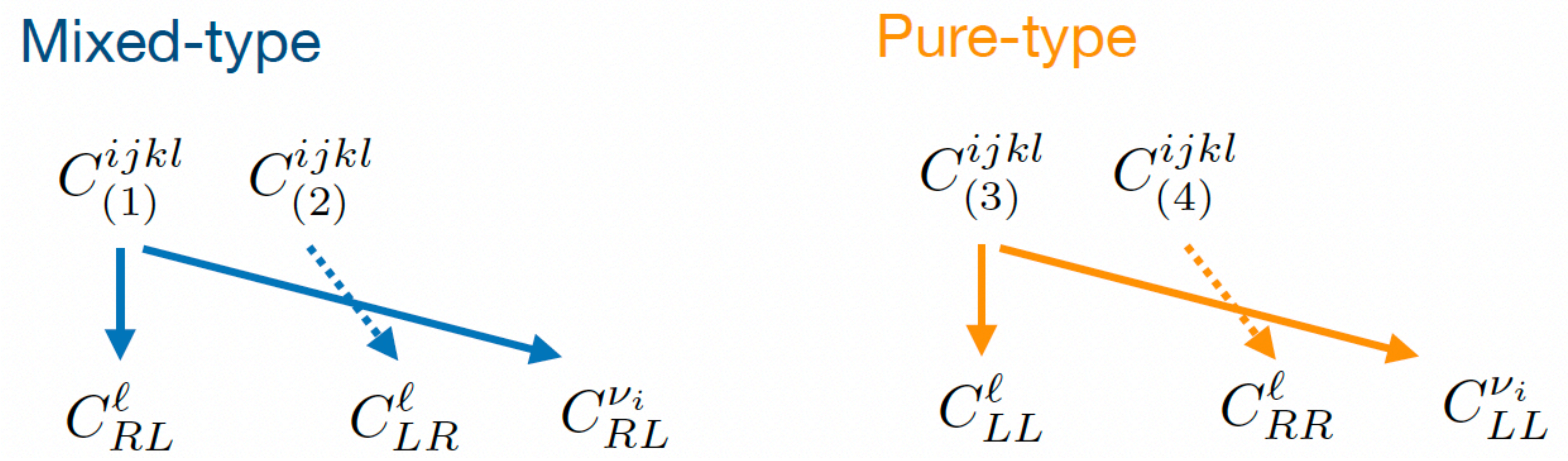
$C^{ijkl}_{(I)}$: Wilson coefficients for those operators

Effective interactions for nucleon decay

- Below the electroweak scale, $\mathcal{O}^{(I)}_{ijkl}$ are matched to $SU(3)_c \times U(1)$ invariant operators in the mass basis:

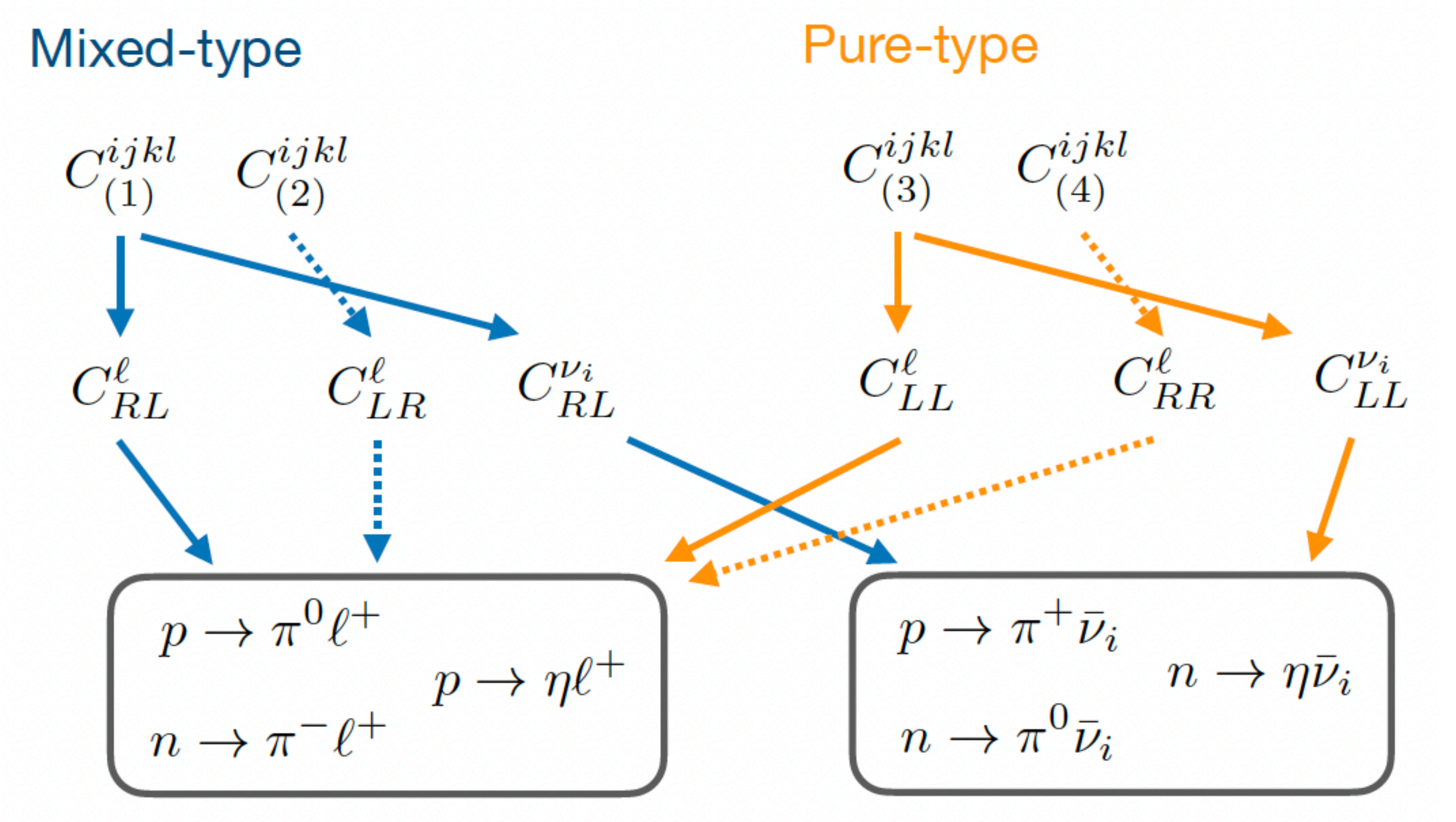
$$\begin{aligned}\mathcal{L}_{\rm eff} = {} & C^{\ell}_{RL}\left[\epsilon_{abc}(u^a_R d^b_R)(u^c_L \ell_L)\right] + C^{\ell}_{LR}\left[\epsilon_{abc}(u^a_L d^b_L)(u^c_R \ell_R)\right] \\ & + C^{\ell}_{LL}\left[\epsilon_{abc}(u^a_L d^b_L)(u^c_L \ell_L)\right] + C^{\ell}_{RR}\left[\epsilon_{abc}(u^a_R d^b_R)(u^c_R \ell_R)\right] \\ & + C^{\nu_i}_{RL}\left[\epsilon_{abc}(u^a_R d^b_R)(d^c_L \nu_{Li})\right] + C^{\nu_i}_{LL}\left[\epsilon_{abc}(u^a_L d^b_L)(d^c_L \nu_{Li})\right] + \text{h.c.}\end{aligned}$$

$\ell = e, \mu$



Effective interactions for nucleon decay

- Possible decay channels and their relation to the effective operators



$\ell = e, \mu$

- Some of them are related via SU(2) isospin

$$\Gamma(n \to \pi^0 \bar{\nu}) = \frac{1}{2}\Gamma(p \to \pi^+ \bar{\nu})$$

$$\Gamma(n \to \pi^- \ell^+) = 2\Gamma(p \to \pi^0 \ell^+)$$

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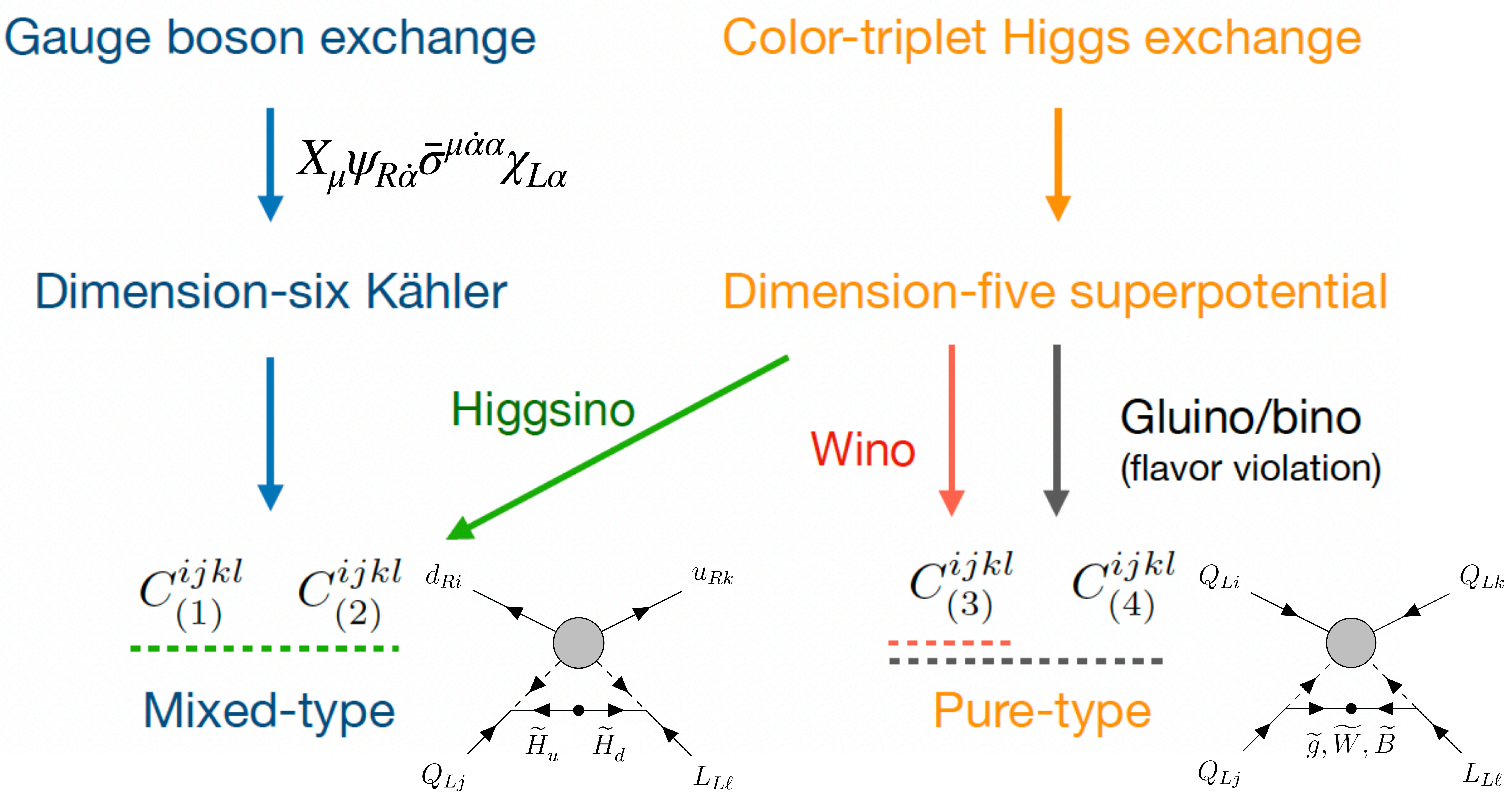
Why focus on chirality?

K. Hamaguchi, H. Shihwen, N. Nagata and **HT** in preparation

- The chirality structure is intimately linked to the underlying mechanisms.



The direct method

Y. Aoki et al. (2017)
J. S. Yoo, et al. (2022)

- To calculate the decay rate, we need to evaluate hadron matrix elements. We use the values directly obtained by the lattice simulation.

I	0		1	
$\chi\chi'$	LR	LL	LR	LL
$W_{p\pi^+\nu}$, $\sqrt{2}W_{p\pi^0\ell}$, $\sqrt{2}W_{n\pi^0\nu}$, $W_{n\pi^-\ell}$	-0.159	0.151	0.169	-0.134
$W_{p\eta\ell}$, $W_{n\eta\nu}$	0.006	0.113	0.044	-0.044

- $W^{RR} = W^{LL}$, $W^{RL} = W^{LR}$
- In units of GeV^2

$$\langle \Pi(\boldsymbol{p}) | \mathcal{O}_{\chi\chi'} | N(\boldsymbol{k}) \rangle = P_{\chi'} \left[W_0^{\mathcal{O}}(q^2) + \frac{\not{q}}{m_N} W_1^{\mathcal{O}}(q^2) \right] u_N(\boldsymbol{k})$$

$$\mathcal{O}_{\chi\chi'} = \epsilon_{abc}(q_\chi^a q_\chi'^b) q_{\chi'}''^c$$

- $\mathcal{O}(m_\ell / m_p)$ → only relevant for anti-muon

Chirality structure and branching fractions

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- In the absence of the pure-type operators, i.e., $C^{\ell}_{LL} = C^{\ell}_{RR} = 0$ and neglecting the charged lepton's mass, i.e., m_ℓ =0,

$$\Gamma(N \to \Pi \ell^+) = \frac{m_N}{32\pi}\left(1 - \frac{m_\Pi^2}{m_N^2}\right)^2 |W^{LR}_{N\Pi\ell,0}|^2 \left[|C^{\ell}_{RL}|^2 + |C^{\ell}_{LR}|^2\right]$$

$p \to \eta e^+$ and $p \to \pi^0 e^+$ share the same factor which depends on UV.

- The ratios $\Gamma(p \to \eta \ell^+)/\Gamma(p \to \pi^0 \ell^+)$ are solely determined by low-energy quantities. The same applies to the absence of the mixed type.

$$\frac{\Gamma(p \to \eta e^+)}{\Gamma(p \to \pi^0 e^+)} = \frac{(1 - m_\eta^2/m_p^2)^2}{(1 - m_\pi^2/m_p^2)^2} \cdot \frac{|W^{XX'}_{p\eta\ell}|^2}{|W^{XX'}_{p\pi^0\ell}|^2} \simeq \begin{cases} 0.0013 & \text{(Mixed only)} \\ 0.51 & \text{(Pure only)} \end{cases}$$

- What happens when both types are present, or $m_\ell \neq 0$? Can we extract a model-dependence (= the size of the Wilson coefficients) from the ratios?

… Yes!

Positron channels: $\Gamma(p \to \eta e^+)/\Gamma(p \to \pi^0 e^+)$

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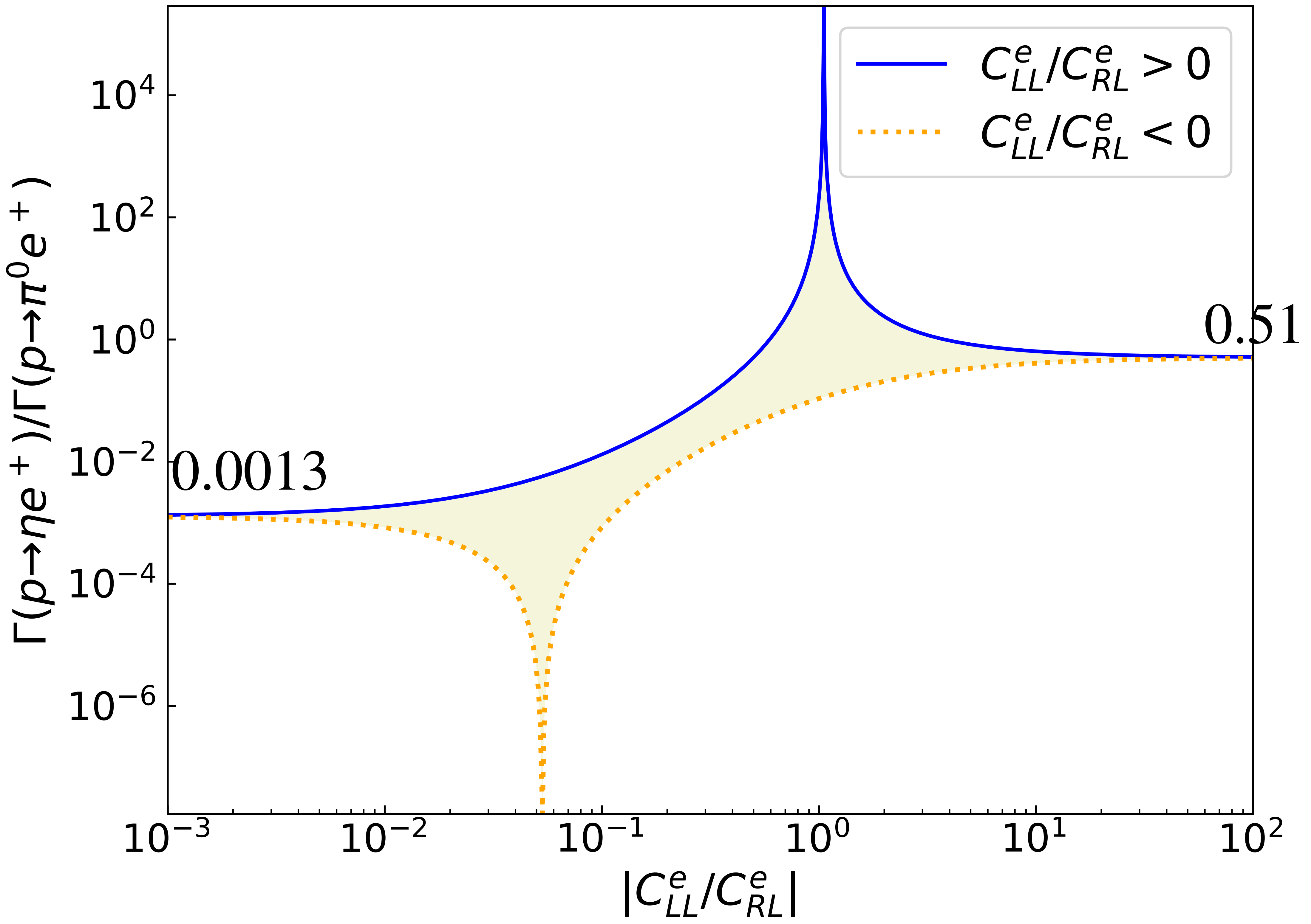

- $C^e_{RR} = C^e_{LR} = 0,\ m_e = 0$
- $|C^e_{LL}/C^e_{RL}| \ll 1$ and $|C^e_{LL}/C^e_{RL}| \gg 1$ approach orders of magnitude different values.
- Peaks at
  - $|C^e_{LL}/C^e_{RL}| \simeq 1$ for $C^e_{LL}/C^e_{RL} > 0$
  - $|C^e_{LL}/C^e_{RL}| \simeq 0.1$ for $C^e_{LL}/C^e_{RL} < 0$

- Not just the mixed- or pure-type only scenarios, but also the presence of both types can be probed.

Positron channels: $\Gamma(p \to \eta e^+)/\Gamma(p \to \pi^0 e^+)$

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- To understand the behavior, let us use the expressions calculated from chiral lagrangian (i.e., the so-called indirect method M. Claudson, M. B. Wise, and L. J. Hall (1982))

$$\Gamma(p \to \pi^0 e^+) = \frac{m_p}{32\pi}\left(1 - \frac{m_\pi^2}{m_p^2}\right)^2 \frac{(1+D+F)^2\alpha^2}{2f^2}\left[|C^e_{RL} - C^e_{LL}|^2 + |C^e_{LR} - C^e_{RR}|^2\right]$$

$$\Gamma(p \to \eta e^+) = \frac{m_p}{32\pi}\left(1 - \frac{m_\eta^2}{m_p^2}\right)^2 \frac{\alpha^2}{6f^2}\Big[|(1+D-3F)C^e_{RL} + (3-D+3F)C^e_{LL}|^2$$
$$+ |(1+D-3F)C^e_{LR} + (3-D+3F)C^e_{RR}|^2\Big] ,$$

- $p \to \pi^0 e^+$ is suppressed when $C^e_{LL}/C^e_{RL} = C^e_{RR}/C^e_{LR} = 1$
- $p \to \eta e^+$ is suppressed when $C^e_{LL}/C^e_{RL} = C^e_{RR}/C^e_{LR} = -(1+D-3F)/(3-D+3F) \simeq -0.1$

Anti-muon channels: $\Gamma(p \to \eta\mu^+)/\Gamma(p \to \pi^0\mu^+)$

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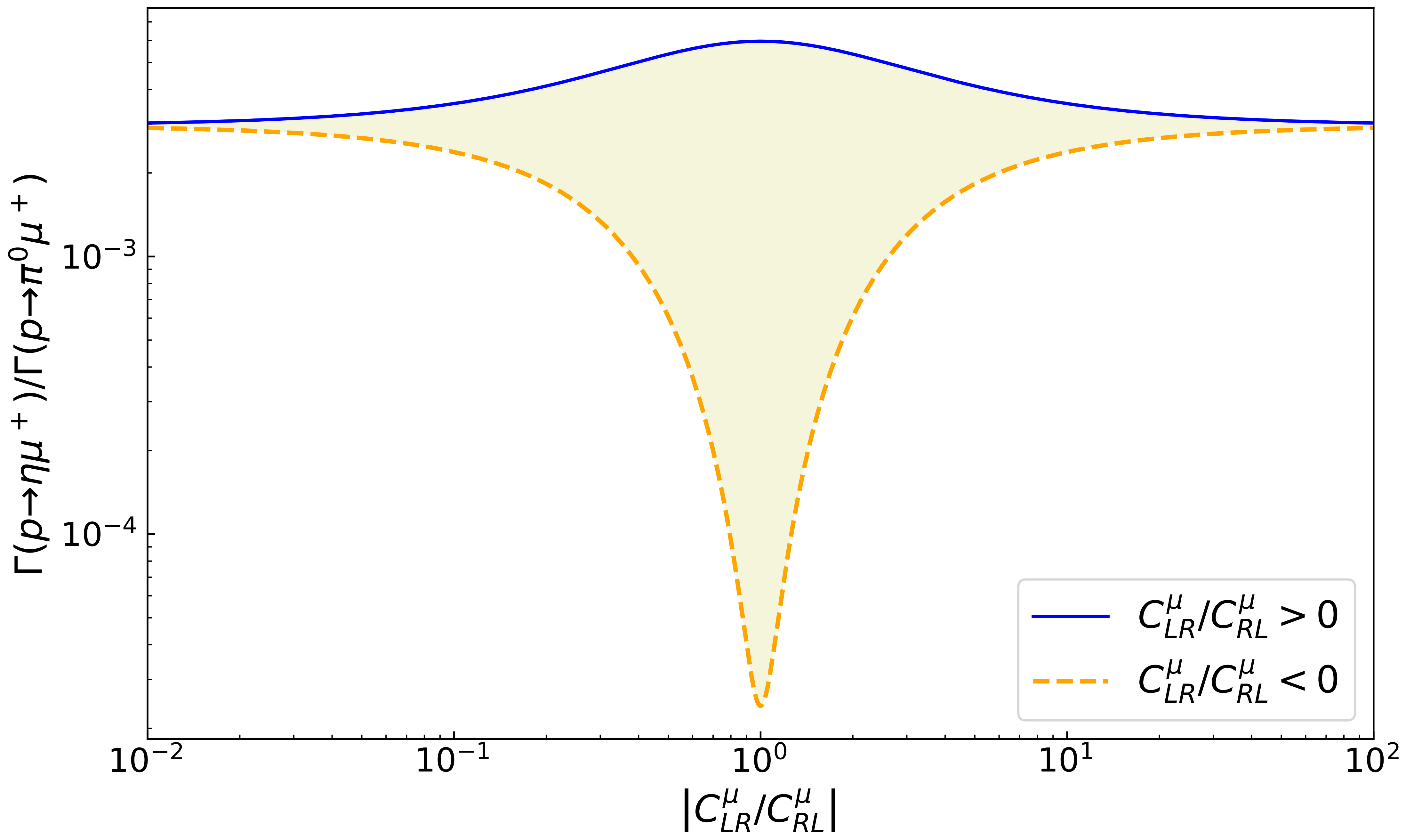
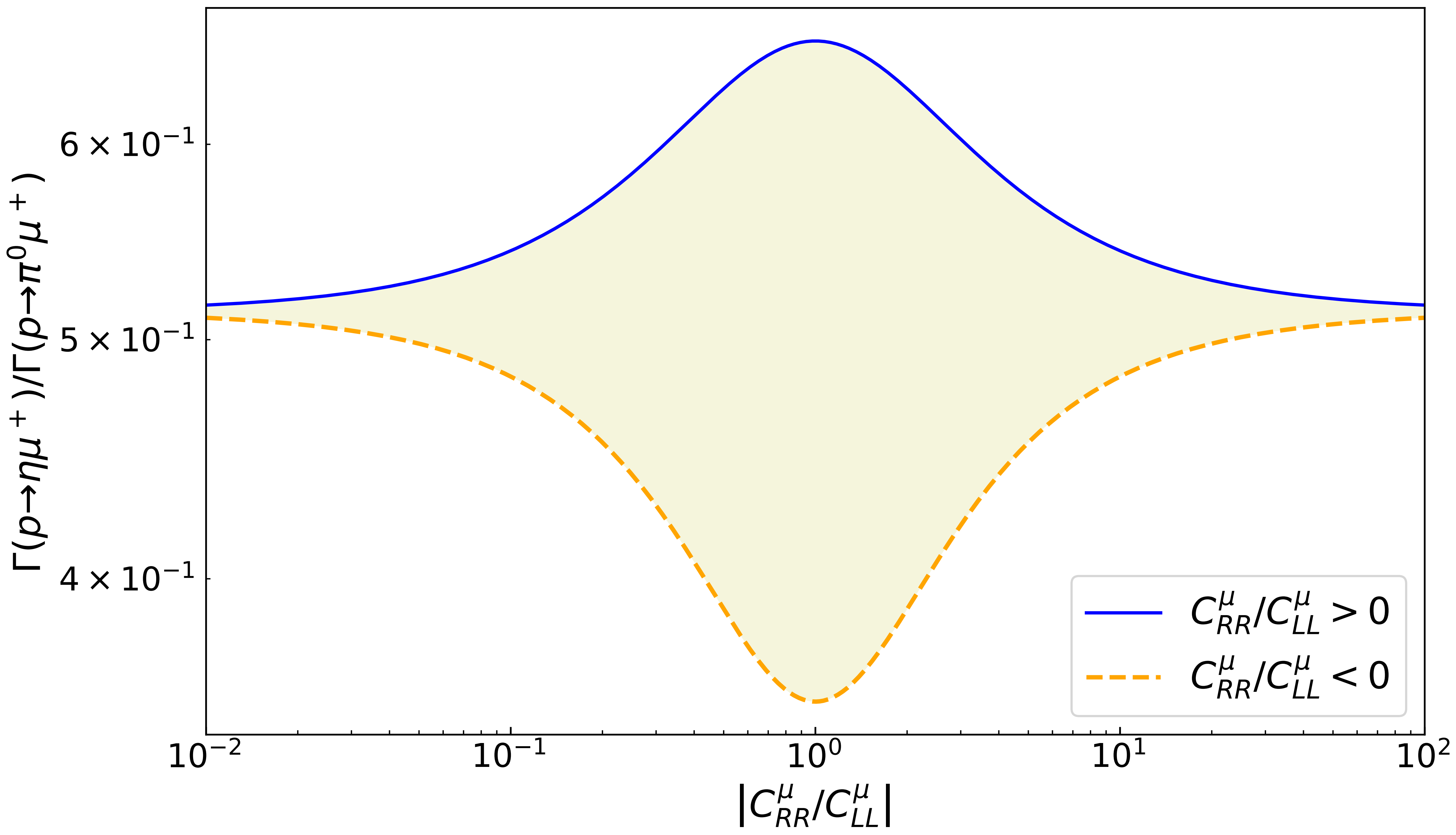
- Let us next explore the role of m_μ

Mixed-only



Pure-only



- The effect of m_μ is larger for the mixed-only case.

Anti-muon channels: $\Gamma(p \to \eta\mu^+)/\Gamma(p \to \pi^0\mu^+)$

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- For the mixed-only case, up to $\mathcal{O}(m_\mu/m_p)$,

$$\Gamma(p \to \Pi\mu^+) = \frac{m_p}{32\pi}\left(1 - \frac{m_\Pi^2}{m_p^2}\right)^2 \left(W^{LR}_{p\Pi\ell,0}\right)^2 \left[|C^\mu_{RL}|^2 + |C^\mu_{LR}|^2\right] ,$$
$$\times \left[1 + \frac{4m_\mu}{m_p}\left\{\frac{W^{LR}_{p\Pi\ell,1}}{W^{LR}_{p\Pi\ell,0}} + \left(1 - \frac{m_\Pi^2}{m_p^2}\right)^{-1}\right\}\frac{\mathrm{Re}(C^\mu_{RL}C^{\mu *}_{LR})}{|C^\mu_{RL}|^2 + |C^\mu_{LR}|^2}\right]$$

  - The source of the dependence on $|C^e_{LR}/C^e_{RL}|$
  - This effect is enhanced when $|W^{LR}_{p\Pi\mu,1}| \gg |W^{LR}_{p\Pi\mu,0}|$

I	0		1	
$\chi\chi'$	LR	LL	LR	LL
$W_{p\pi^+\nu}$, $\sqrt{2}W_{p\pi^0\ell}$, $\sqrt{2}W_{n\pi^0\nu}$, $W_{n\pi^-\ell}$	−0.159	0.151	0.169	−0.134
$W_{p\eta\ell}$, $W_{n\eta\nu}$	0.006	0.113	0.044	−0.044

Form factors obtained by lattice QCD in units of GeV^2

Y. Aoki et al. (2017)
J. S. Yoo, et al. (2022)

Other channels

- The ratio of neutrino channels is related to that of charged leptons in either mixed- or pure-only cases.

$$\Gamma(N \to \Pi \bar{\nu}) = \frac{m_N}{32\pi}\left(1 - \frac{m_\Pi^2}{m_N^2}\right)^2 \left|W_{N\Pi\nu,0}^{RL}\right|^2 \sum_i |C_{RL}^{\nu_i}|^2$$

$$\frac{\Gamma(n \to \eta\bar{\nu})}{\Gamma(n \to \pi^0\bar{\nu})} = \frac{(1 - m_\eta^2/m_n^2)^2}{(1 - m_\pi^2/m_n^2)^2} \cdot \frac{|W_{n\eta\nu}^{\chi\chi'}|^2}{|W_{n\pi^0\nu}^{\chi\chi'}|^2} \simeq \frac{\Gamma(p \to \eta e^+)}{\Gamma(p \to \pi^0 e^+)}$$

- Due to the presence of more Wilson coefficients (=six for the neutrino ratio), it is difficult to extract information about the Wilson coefficients on generic grounds. The same applies to $\Gamma(n \to \pi^0\bar{\nu})/\Gamma(p \to \pi^0\ell^+)$.

- Those ratios can still become powerful probes when considering specific UV models.

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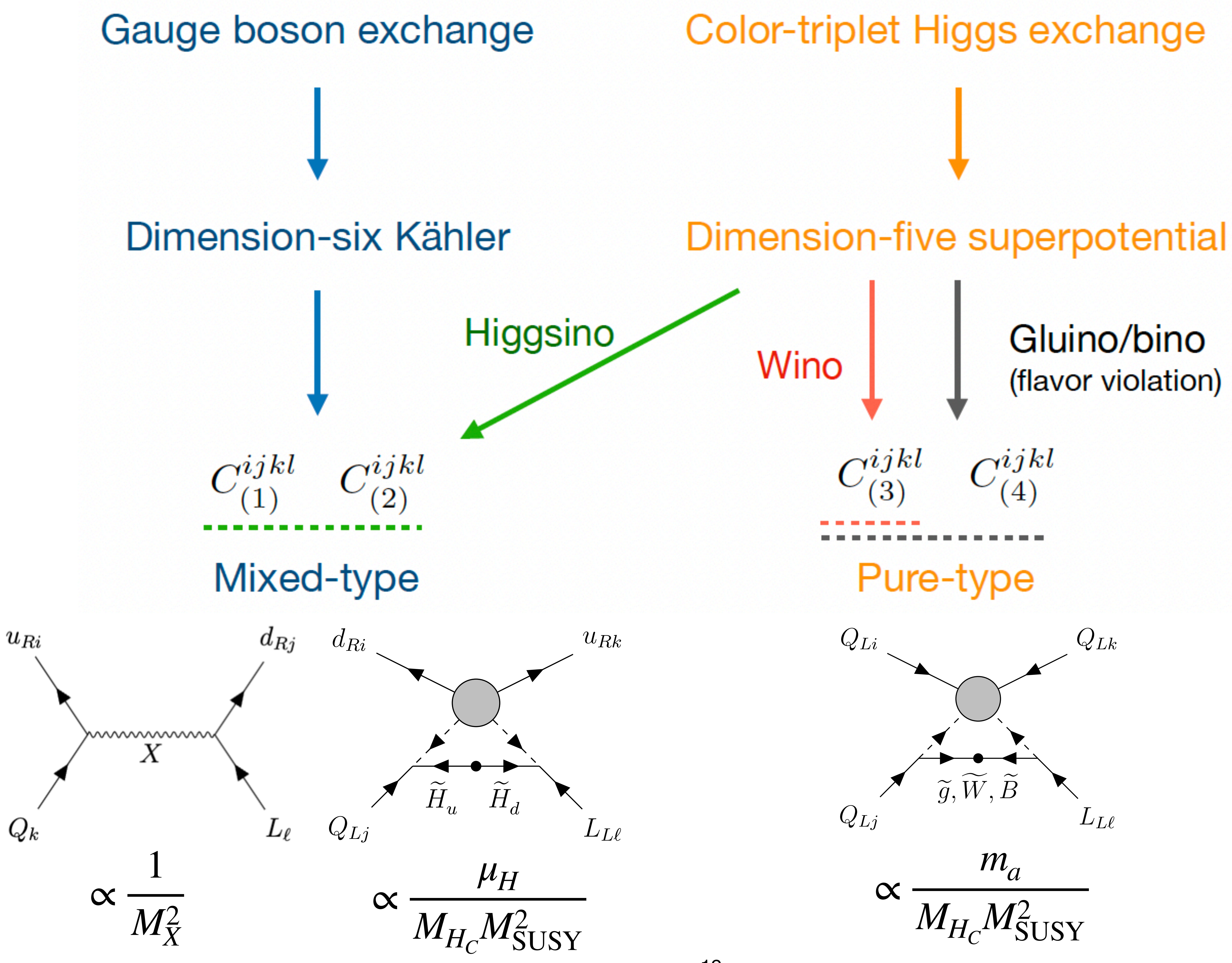
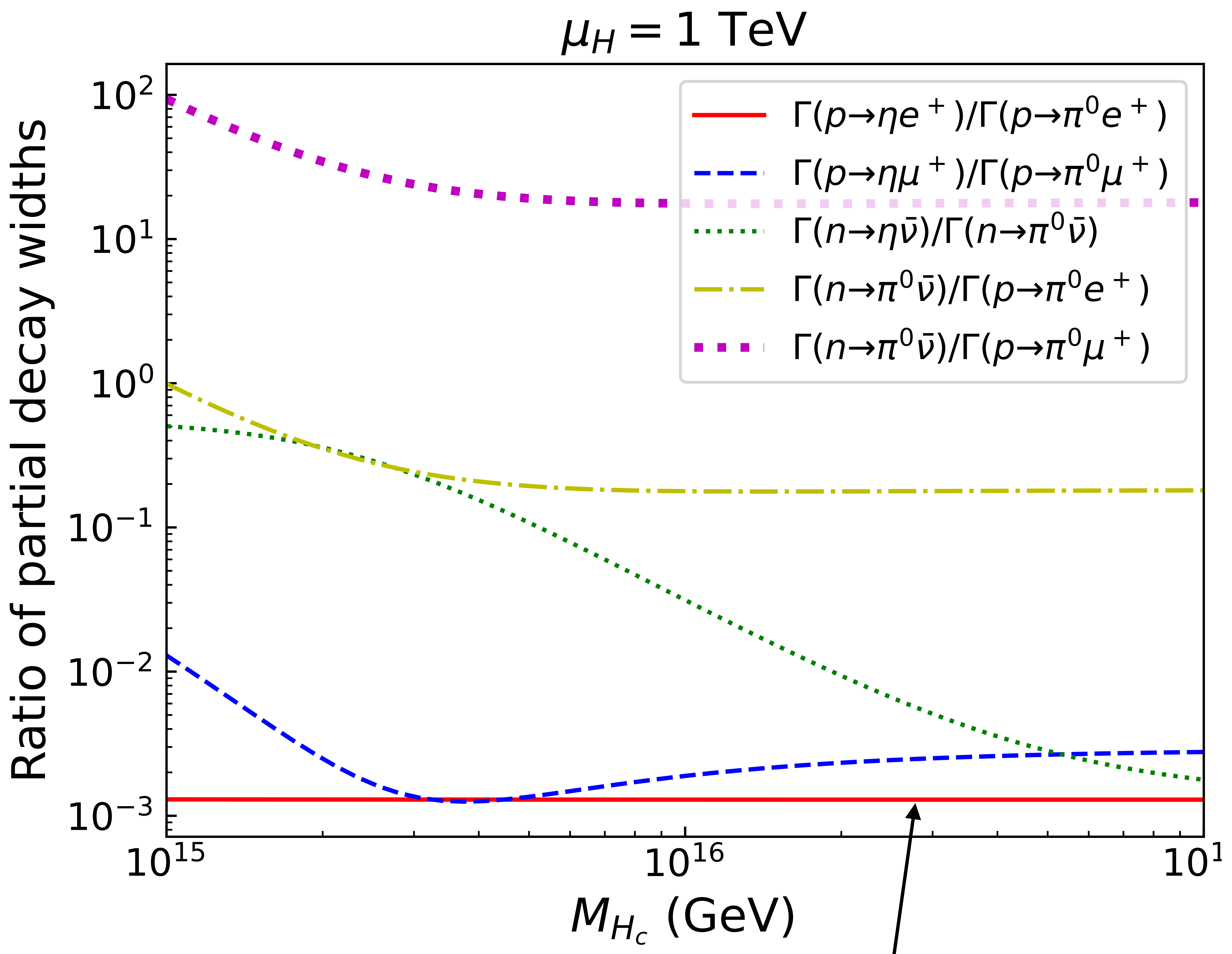
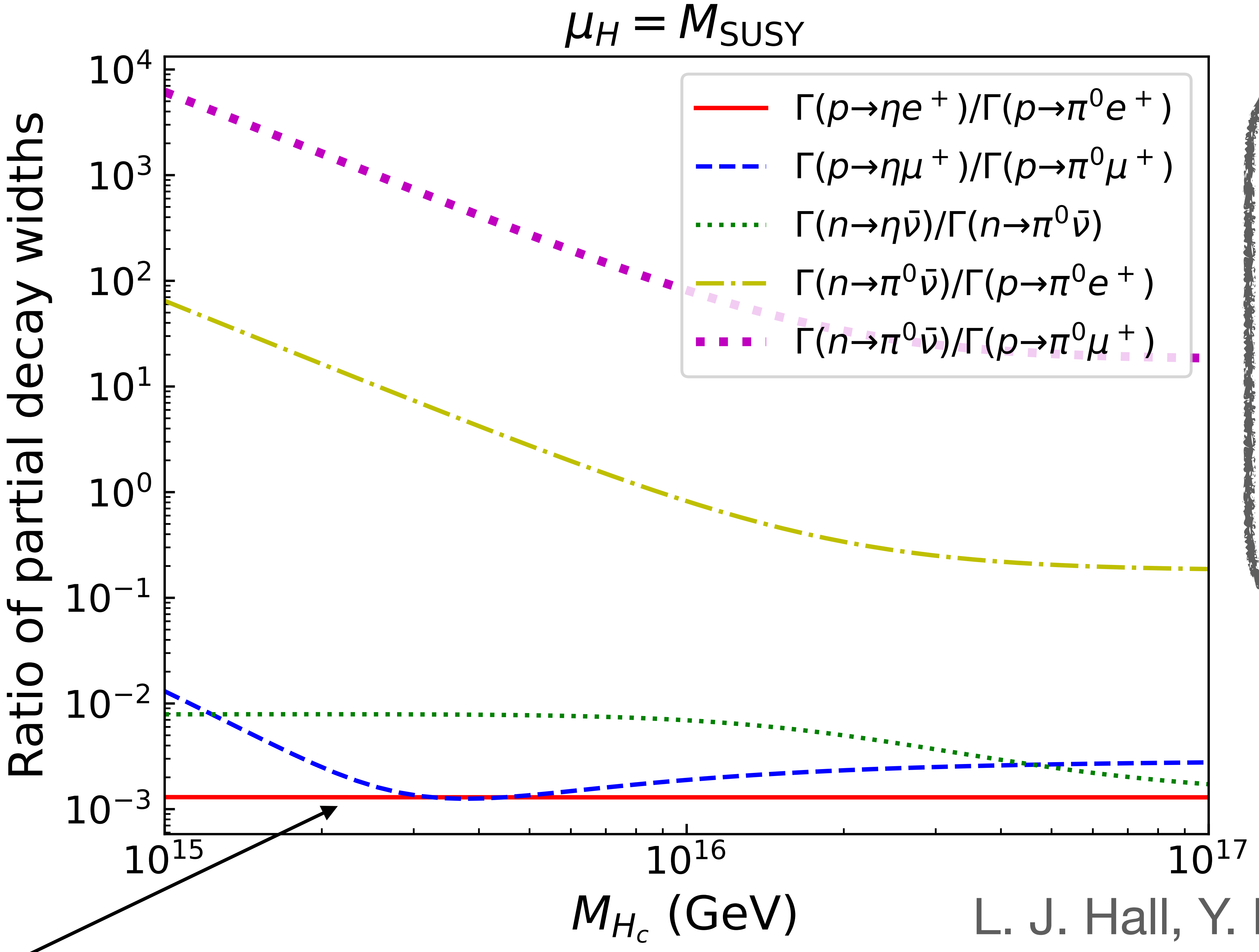
Summary so far about the chirality structure



The minimal SU(5) with high-scale SUSY

K. Hamaguchi, H. Shihwen, N. Nagata and **HT** in preparation





- $M_X = 10^{16}$ GeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\rm SUSY} = 100$ TeV
- $\tan\beta = 3$

Mini-split type mass spectrum

L. J. Hall, Y. Nomura and S. Shirai (2012)
M. Ibe, S. Matsumoto and T. T. Yanagida (2012)

$\Gamma(p \to \eta e^+)/\Gamma(p \to \pi^0 e^+)$: gauge-boson exchange (mixed-type) dominates.

The minimal SU(5) with high-scale SUSY

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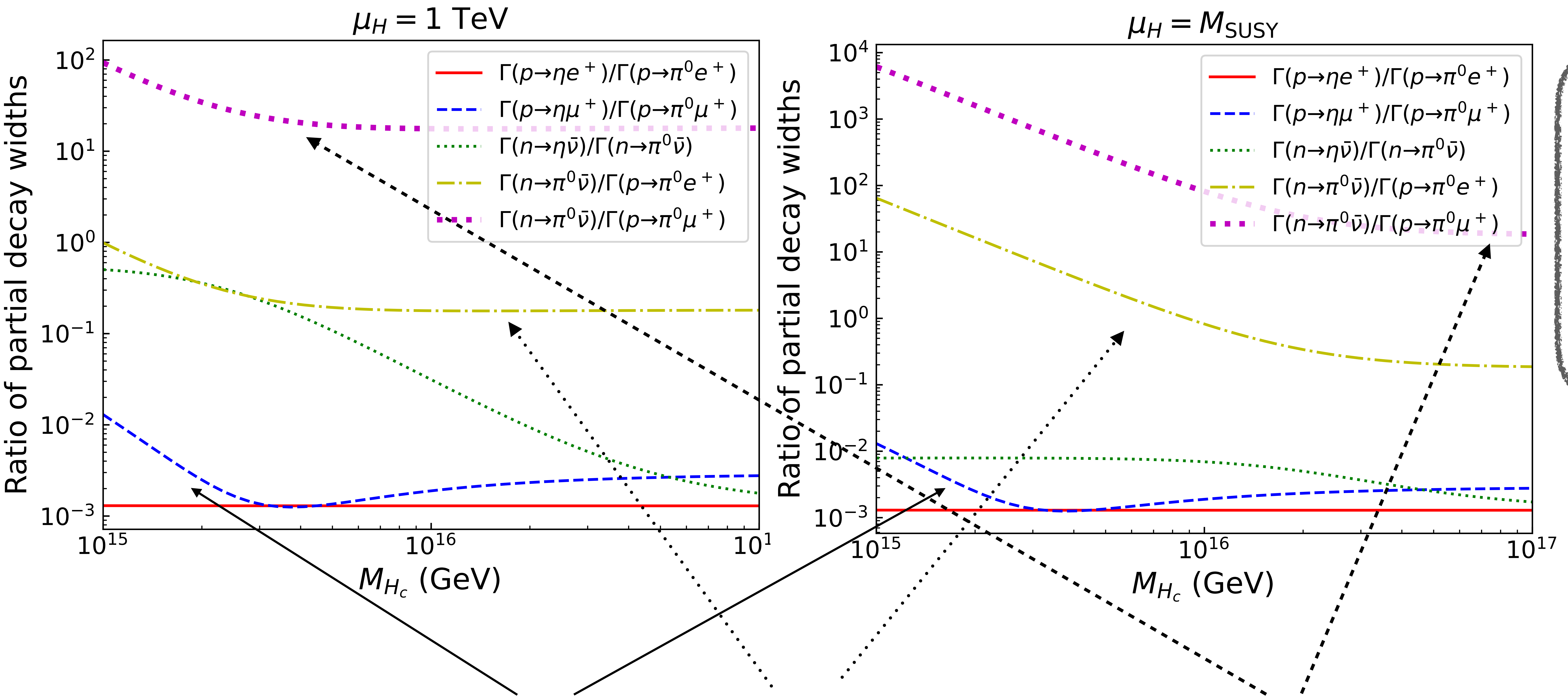

- $M_X = 10^{16}$ GeV
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$\Gamma(p\to\eta\mu^+)/\Gamma(p\to\pi^0\mu^+)$, $\Gamma(n\to\pi^0\bar{\nu})/\Gamma(p\to\pi^0 e^+)$, $\Gamma(n\to\pi^0\bar{\nu})/\Gamma(p\to\pi^0\mu^+)$:

wino-exchange contributes (pure-type) as M_{H_C} decreases.

The minimal SU(5) with high-scale SUSY

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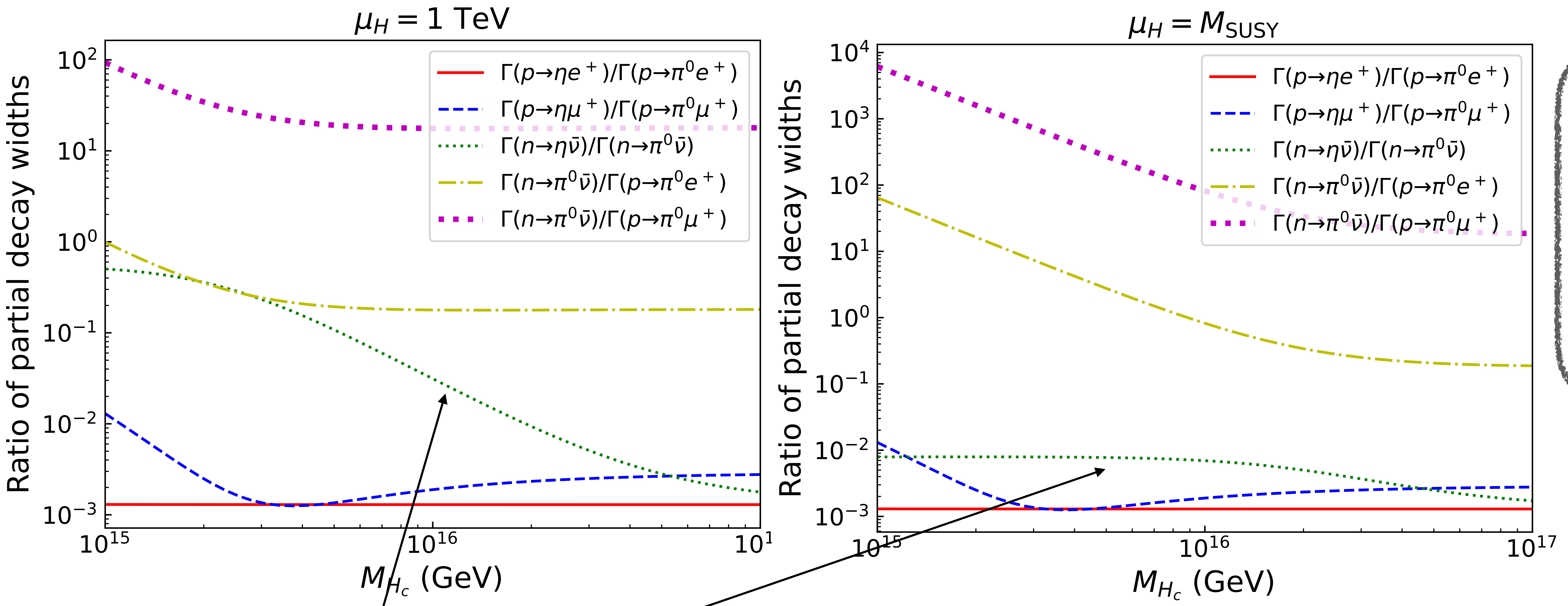

- $M_X = 10^{16}$ GeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\rm SUSY} = 100$ TeV
- $\tan\beta = 3$

$\Gamma(n \to \eta\bar{\nu})/\Gamma(n \to \pi^0\bar{\nu})$:

The left figure ($\mu_H = 1$ TeV) reflects a larger contribution of the wino-exchanging process (pure-type), while the higgsino or gauge-boson exchange (mixed) dominates more in the right ($\mu_H = 1$ PeV).

The minimal SU(5) with high-scale SUSY

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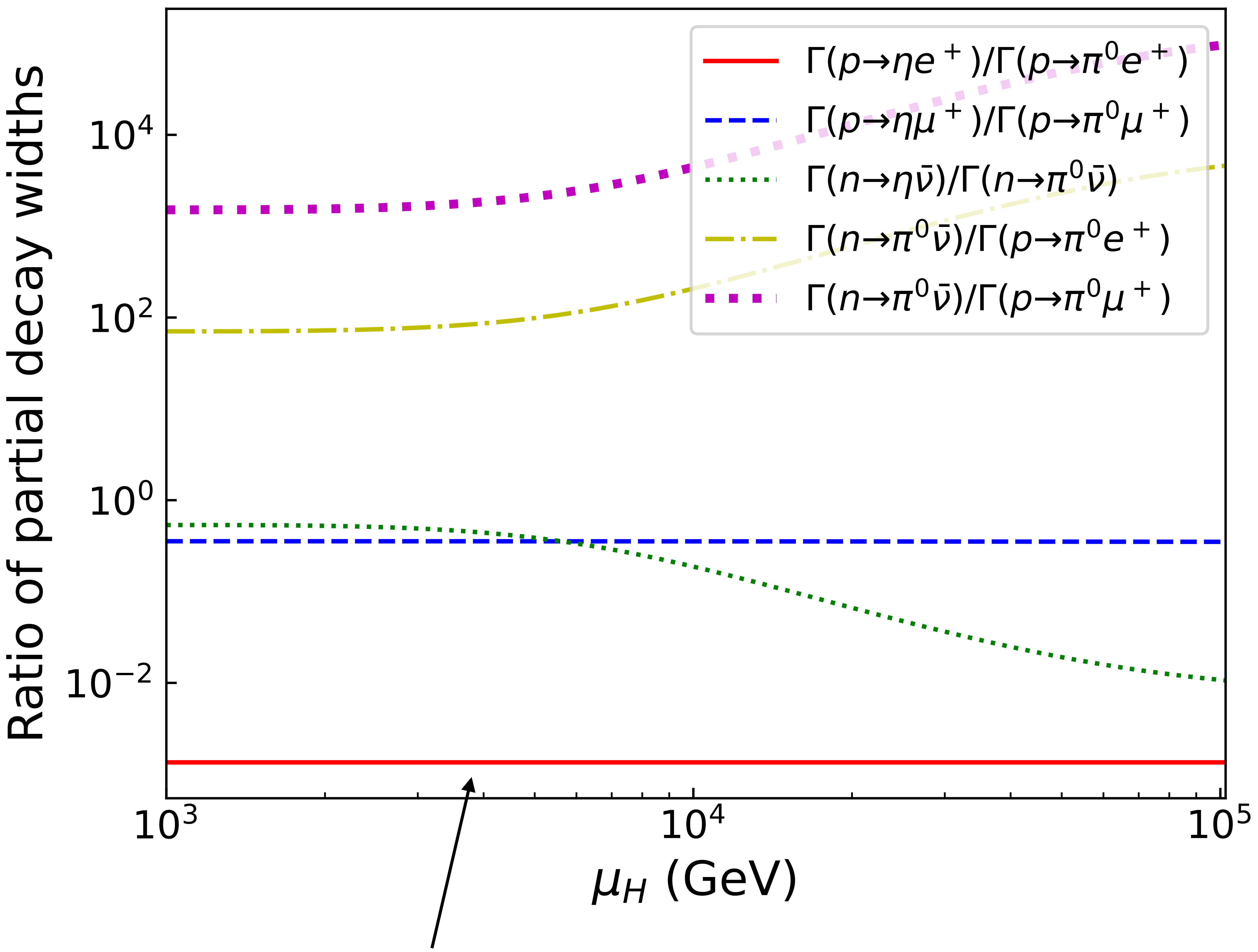

- $M_X = 10^{17}$ GeV
- $M_{H_C} = 10^{16}$ GeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\rm SUSY} = 100$ TeV
- $\tan\beta = 3$

$\Gamma(p \to \eta e^+)/\Gamma(p \to \pi^0 e^+)$: gauge-boson exchange (mixed-type) dominates even when the GUT gauge boson is heavy.

The minimal SU(5) with high-scale SUSY

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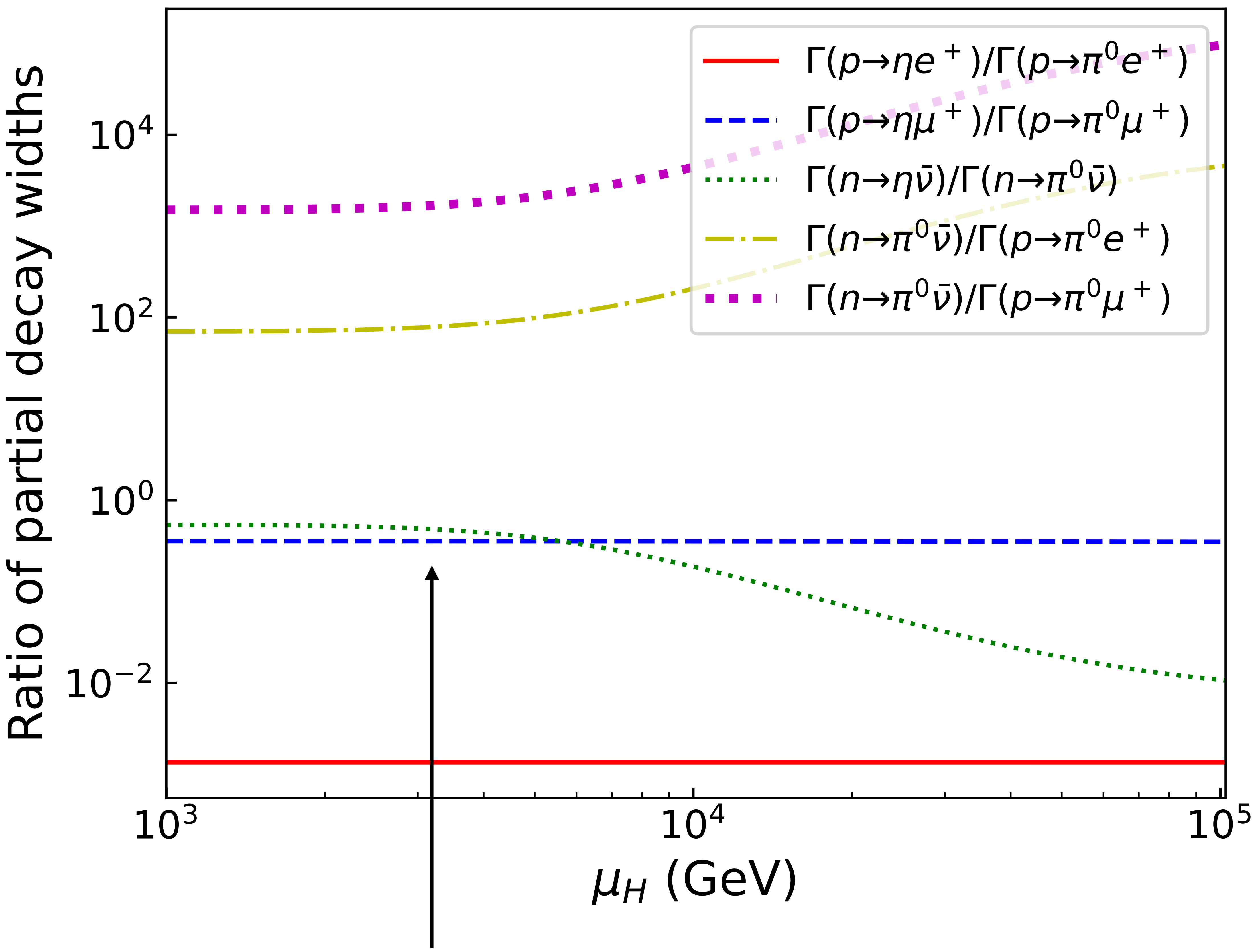

- $M_X = 10^{17}$ GeV
- $M_{H_C} = 10^{16}$ GeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\rm SUSY} = 100$ TeV
- $\tan\beta = 3$

$\Gamma(p \to \eta\mu^+)/\Gamma(p \to \pi^0\mu^+)$: wino contribution (mixed-type) dominates regardless of the higgsino mass.

→ **this ratio is sensitive to dim-5 wino v.s. the GUT gauge boson competition.**

The minimal SU(5) with high-scale SUSY

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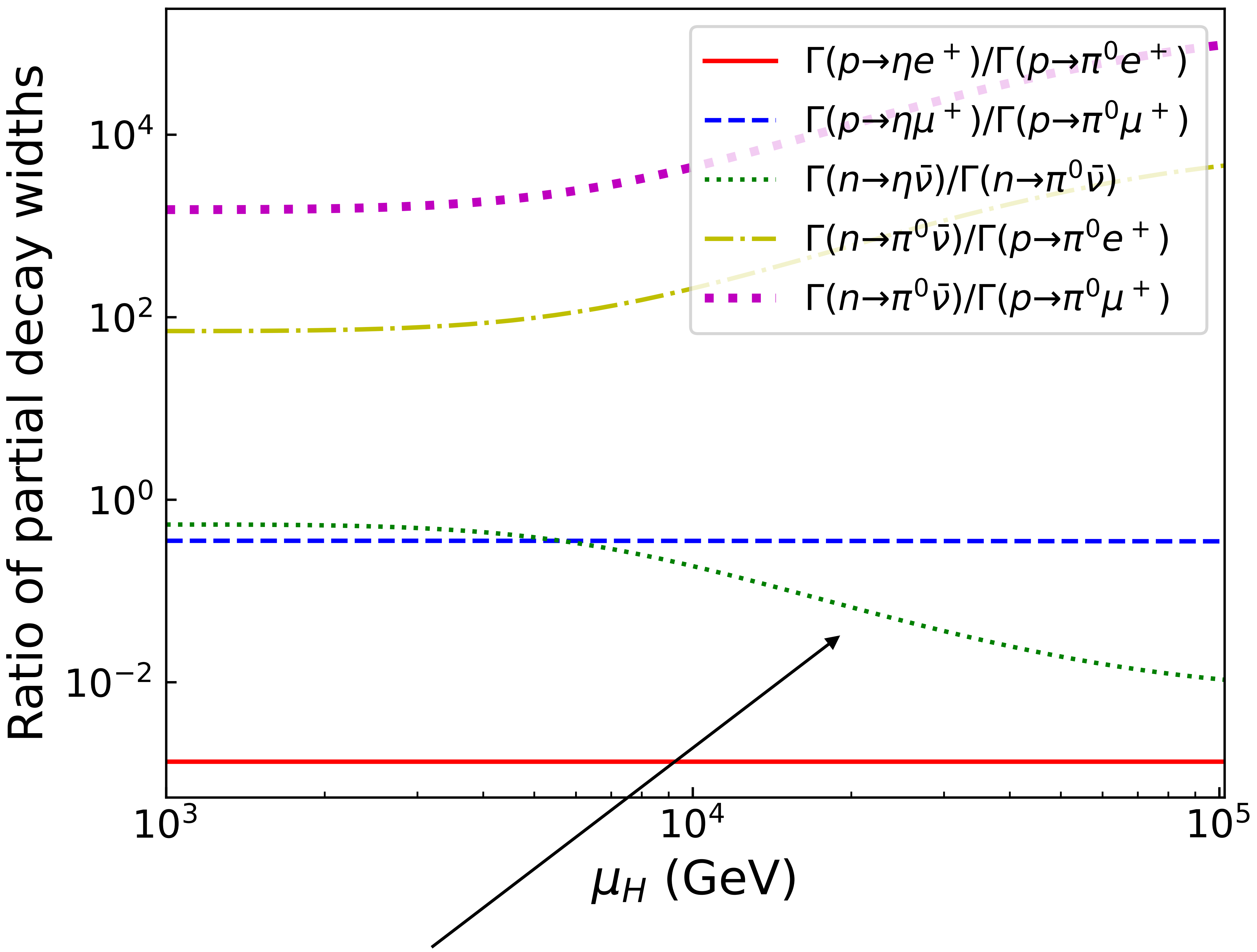

- $M_X = 10^{17}$ GeV
- $M_{H_C} = 10^{16}$ GeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\rm SUSY} = 100$ TeV
- $\tan\beta = 3$

$\Gamma(n \to \eta\bar{\nu})/\Gamma(n \to \pi^0\bar{\nu})$: higgsino contribution (mixed-type) becomes larger as its mass increases.

→ **this ratio is most useful to discriminate the higgsino contribution from that of wino.**

Sfermion flavor violation and ratios

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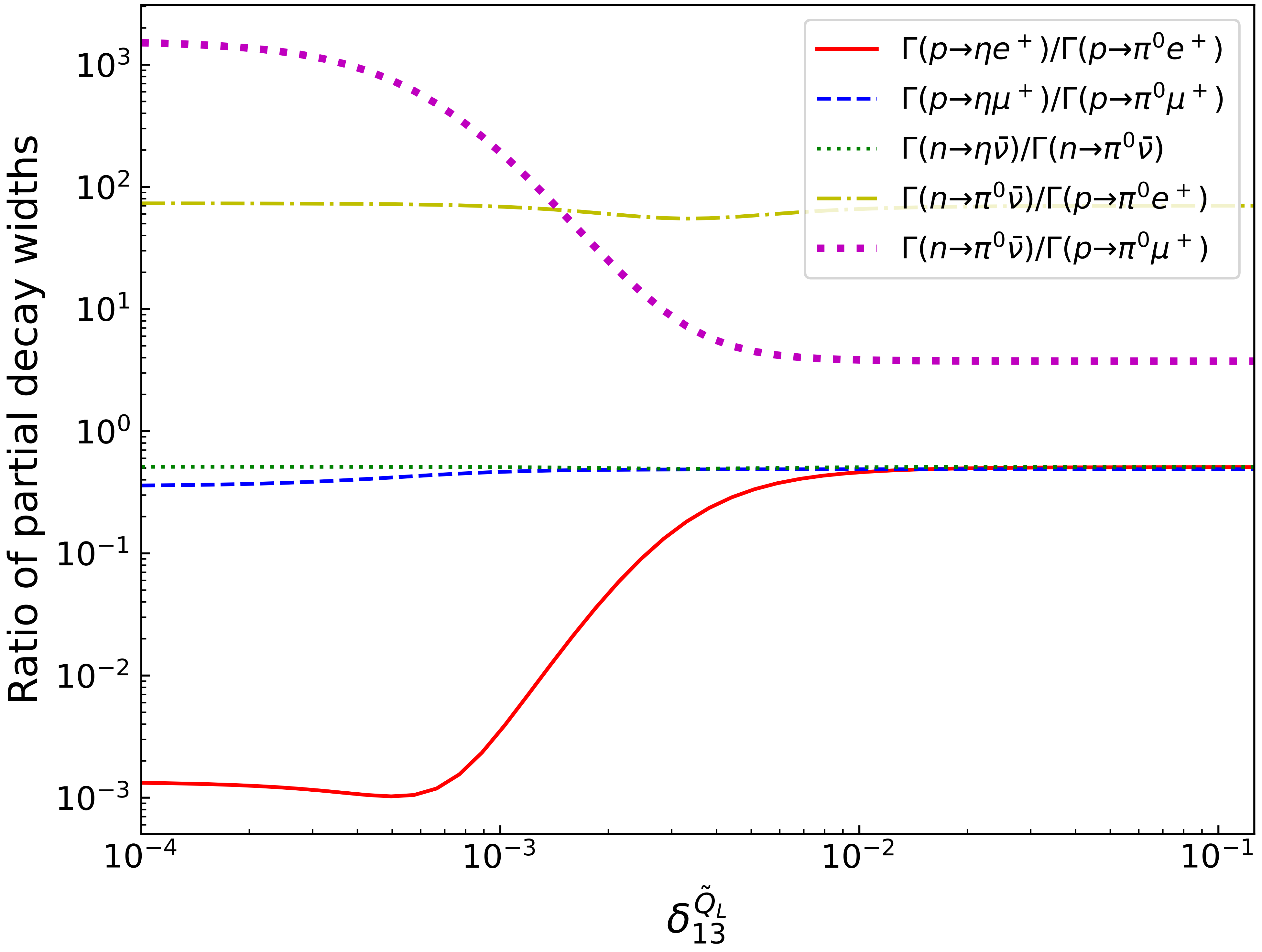
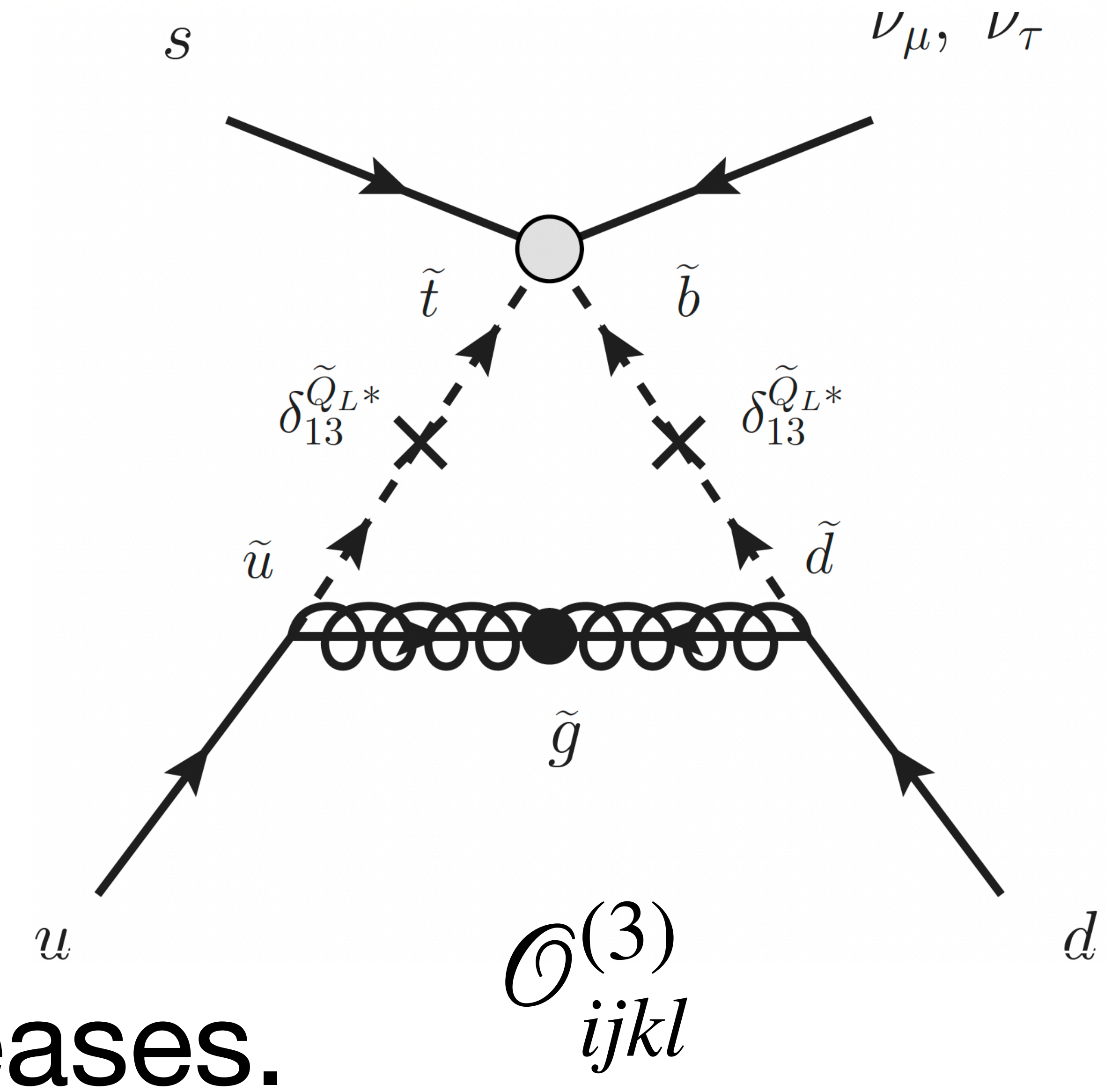


- $M_X = 10^{17}$ GeV
- $M_{H_C} = 10^{16}$ GeV
- $M_1 = 5$ TeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\rm SUSY} = 100$ TeV
- $\tan\beta = 3$
- $\mu_H = 200$ GeV

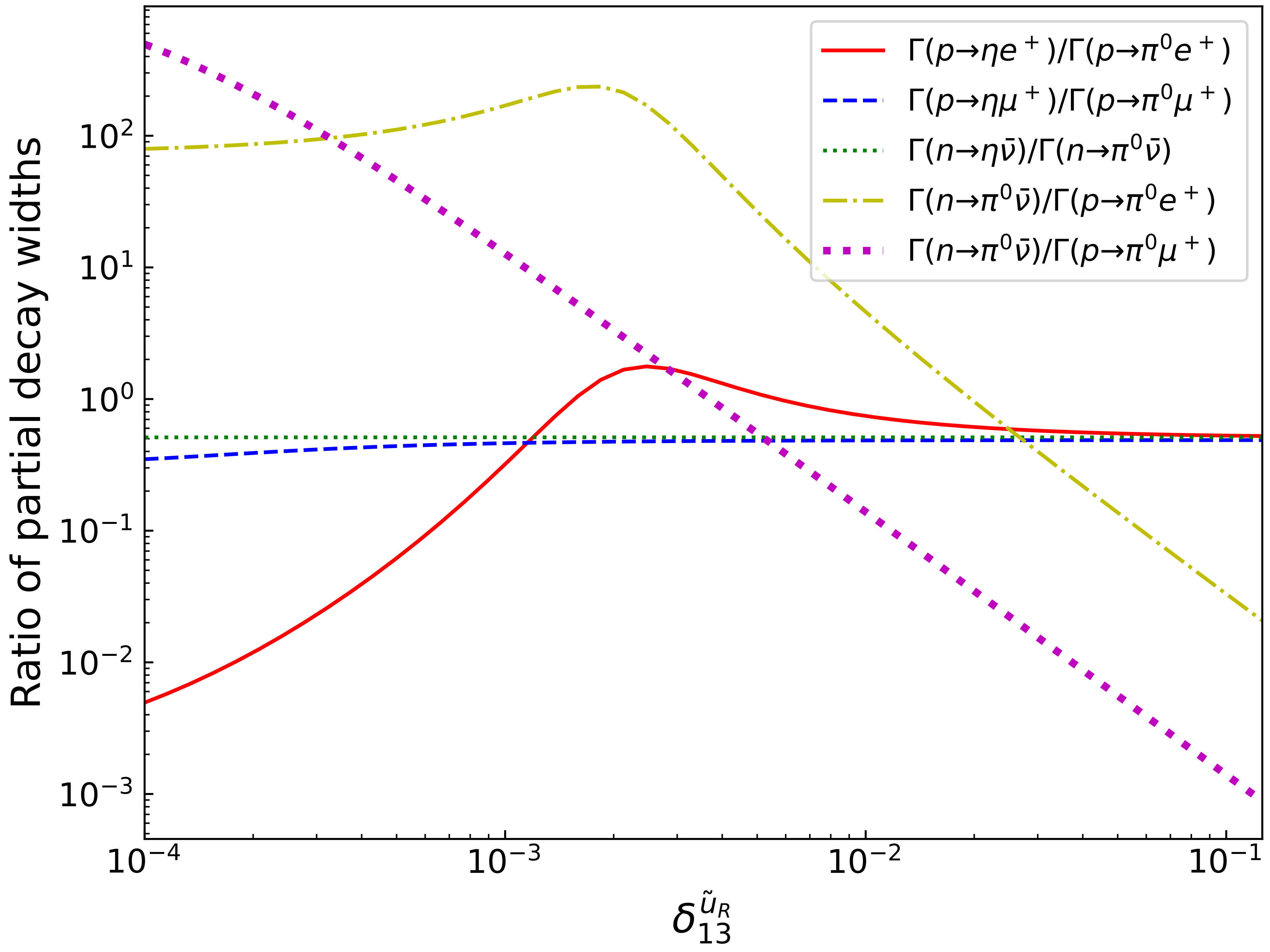
$$\widetilde{m}^2_{\tilde{f}} = M^2_{\rm SUSY} \begin{pmatrix} 1 & \delta^{\tilde{f}}_{12} & \delta^{\tilde{f}}_{13} \\ \delta^{\tilde{f}*}_{12} & 1 & \delta^{\tilde{f}}_{23} \\ \delta^{\tilde{f}*}_{13} & \delta^{\tilde{f}*}_{23} & 1 \end{pmatrix}$$



- In this case, the gluino exchange dominates as $\delta_{13}^{\tilde{Q}_L}$ increases.
- $\Gamma(n \to \pi^0 \bar{\nu})/\Gamma(p \to \pi^0 \mu^+)$ is a useful probe of this scenario.

Sfermion flavor violation and ratios

K. Hamaguchi, H. Shihwen, N. Nagata and **HT** in preparation



- $M_X = 10^{17}$ GeV
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- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\rm SUSY} = 100$ TeV
- $\tan\beta = 3$
- $\mu_H = 200$ GeV

$$\widetilde{m}^2_{\tilde{f}} = M^2_{\rm SUSY}\begin{pmatrix} 1 & \delta^{\tilde{f}}_{12} & \delta^{\tilde{f}}_{13} \\ \delta^{\tilde{f}*}_{12} & 1 & \delta^{\tilde{f}}_{23} \\ \delta^{\tilde{f}*}_{13} & \delta^{\tilde{f}*}_{23} & 1 \end{pmatrix}$$

- Unlike $\delta^{\tilde{Q}_L}_{13}$, $\delta^{\tilde{u}_R}_{13}$ contribute to $\mathcal{O}^{(4)}_{ijkl}$ → only charged-lepton modes are affected
- $\Gamma(n\to\pi^0\bar{\nu})/\Gamma(p\to\pi^0\ell^+)$ is a useful probe of this scenario.

Summary

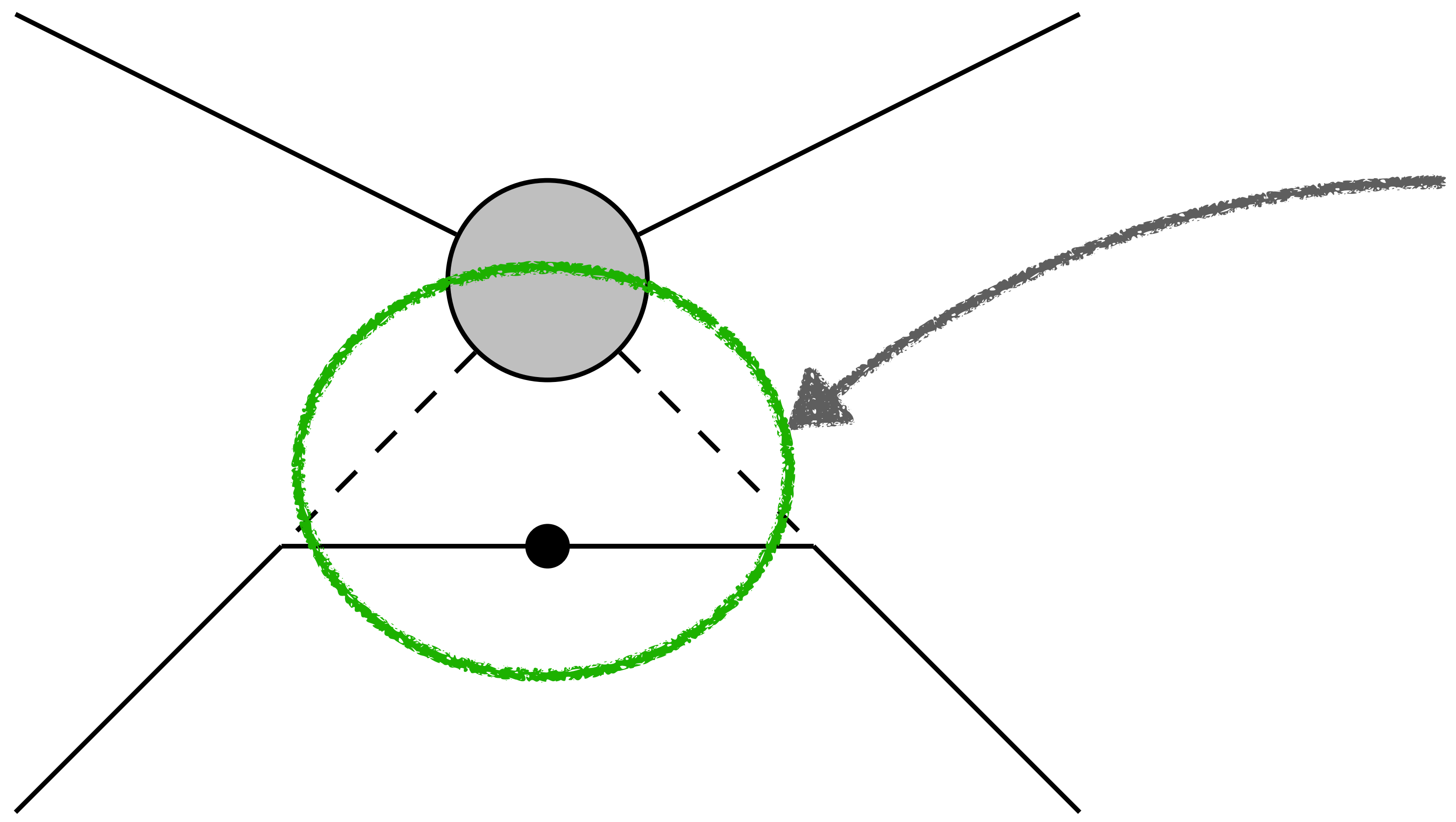
- The chirality structure of the nucleon decay effective interactions reflects its underlying mechanism.
- The ratios of partial decay widths can reveal the chirality structure. It is therefore important to measure various channels.
- In the minimal SU(5) with high-scale SUSY,
  - $\Gamma(p \to \eta e^+)/\Gamma(p \to \pi^0 e^+)$: determined by the GUT gauge boson.
  - $\Gamma(p \to \eta \mu^+)/\Gamma(p \to \pi^0 \mu^+)$: useful to distinguish the wino/gauge-boson exchange.
  - $\Gamma(n \to \eta \bar{\nu})/\Gamma(n \to \pi^0 \bar{\nu})$: most useful to observe the effect of higgsino exchange.
- Sfermion flavor violation can be probed by $\Gamma(n \to \pi^0 \bar{\nu})/\Gamma(p \to \pi^0 \ell^+)$.

Backup

SUSY dimension-five nucleon decay operators

P. Nath and R. L. Arnowitt (1988)
J Hisano, H. Murayama and T. Yanagida (1993)

- Higgsino/gaugino-dependence



- $m_{\tilde{g}}$: higgsino or gaugino mass
- m_i: sfermion mass

Loop function:

$$F(M, m_1, m_2) \equiv \int \frac{d^4p}{\pi^2} \frac{i}{(\not{p} - M)(p^2 - m_1^2)(p^2 - m_2^2)}$$

$$= \frac{M}{m_1^2 - m_2^2} \left(\frac{m_1^2}{m_1^2 - M^2} \ln \frac{m_1^2}{M^2} - \frac{m_2^2}{m_2^2 - M^2} \ln \frac{m_2^2}{M^2} \right)$$

$$\to \frac{M}{m_1^2},$$

When $M \ll m_1 \sim m_2$

The dimension-five contribution is proportional to $m_{\tilde{g}}/(M_{H_C} M_{\rm SUSY}^2)$

Current bounds and future prospects

Decay Mode	Current [years]	HK sensitivity [years]
$p \to \pi^0 e^+$	2.4×10^{34} [25]	7.8×10^{34} [11]
$p \to \pi^0 \mu^+$	1.6×10^{34} [25]	7.7×10^{34} [11]
$p \to \eta e^+$	1.0×10^{34} [26]	4.3×10^{34} [11]
$p \to \eta \mu^+$	4.7×10^{33} [26]	4.9×10^{34} [11]
$p \to \pi^+ \bar{\nu}$	3.9×10^{32} [27]	
$n \to \pi^- e^+$	5.3×10^{33} [26]	2.0×10^{34} [11]
$n \to \pi^- \mu^+$	3.5×10^{33} [26]	1.8×10^{34} [11]
$n \to \pi^0 \bar{\nu}$	1.1×10^{33} [27]	
$n \to \eta \bar{\nu}$	1.6×10^{32} [28]	

[11]: Hyper-Kamiokande Collaboration [arXiv: 1805.04163]

[25]: Super-Kamiokande Collaboration [arXiv: 2010.16098]

[26]: Super-Kamiokande Collaboration [arXiv: 1705.07221]

[27]: Super-Kamiokande Collaboration [arXiv: 1305.4391]

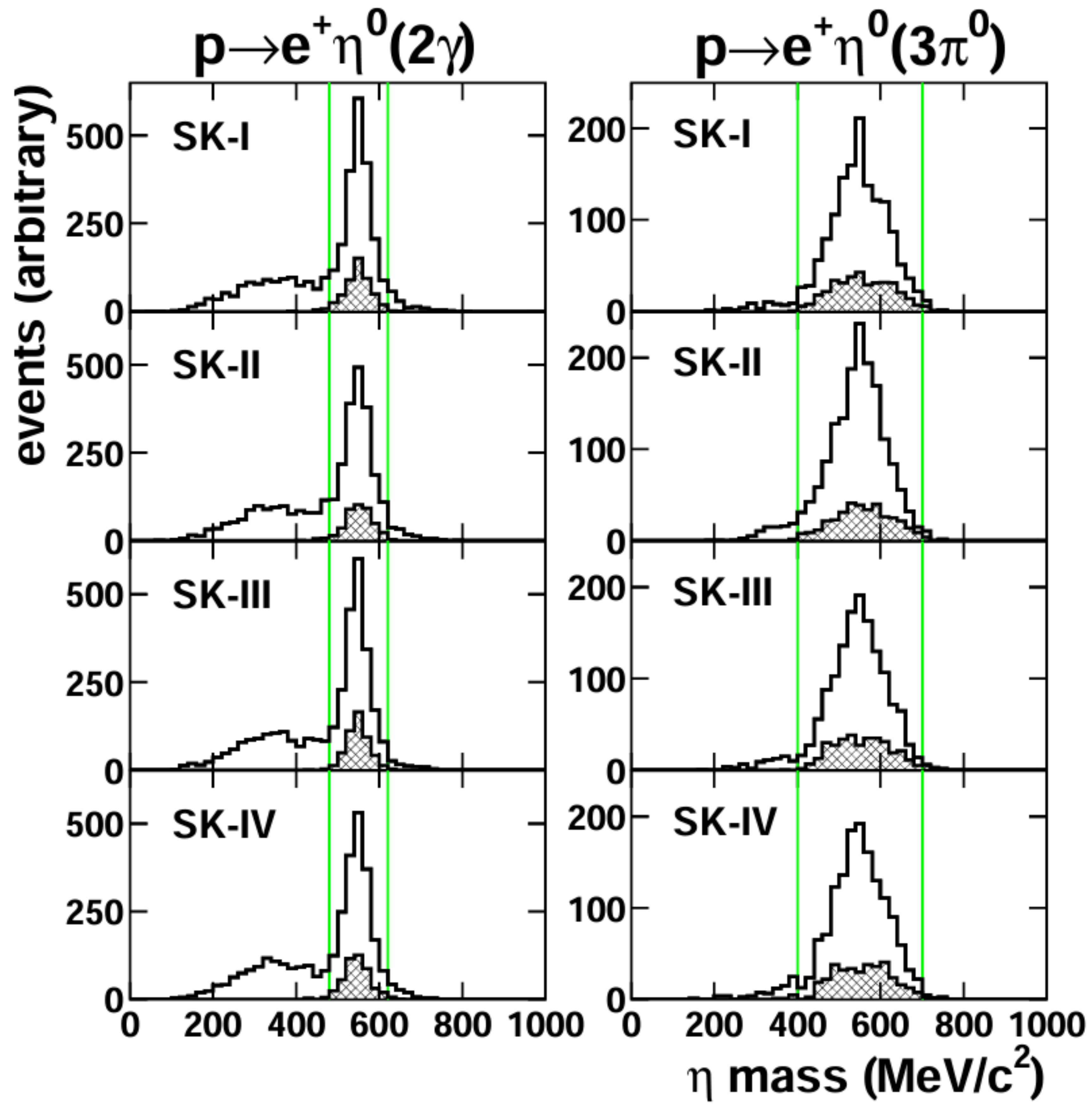
[28]: C. McGrew et al. (1999)

- Units: 10^{33} years
- 90% CL
- 1.9 Megaton-year exposure is assumed for the prospect.

Event Reconstruction

Super-Kamiokande Collaboration [arXiv: 1705.07221]

- Eta mass reconstruction at SK



- Left: $\eta \to 2\gamma$, branching ratio = 39%
- Right: $\eta \to 3\pi^0$, branching ratio = 33%
- Open histogram: Monte-Carlo events
- Hatched histogram:

  - left: true $\eta \to 2\gamma$
  - right: true $\eta \to 3\pi^0$

Effective interactions for nucleon decay

S. Weinberg (1979)
D. V. Nanopoulos and S. Weinberg (1979)

- Assume the tree-level exchange of a scalar or vector boson
  - Vector boson V_μ can induce $\mathcal{O}^{(1)}_{ijk\ell}$, $\mathcal{O}^{(2)}_{ijk\ell}$

    Renormalizable interaction: $V_\mu \psi_{R\dot{\alpha}} \bar{\sigma}^{\mu\dot{\alpha}\alpha} \chi_{L\alpha}$

    ψ_R include the conjugate of ψ_L
  - Scalar boson S can induce $\mathcal{O}^{(1)}_{ijk\ell}$, $\mathcal{O}^{(2)}_{ijk\ell}$, $\mathcal{O}^{(3)}_{ijk\ell}$, $\mathcal{O}^{(4)}_{ijk\ell}$.

    Renormalizable interactions: $S(\psi_L \chi_L)$, $S(\psi_R \chi_R)$

$$\mathcal{O}^{(1)}_{ijk\ell} \equiv \epsilon_{abc}\epsilon_{\alpha\beta}\left(u^a_{Ri} d^b_{Rj}\right)\left(Q^{c\alpha}_k L^\beta_\ell\right),$$

$$\mathcal{O}^{(2)}_{ijk\ell} \equiv \epsilon_{abc}\epsilon_{\alpha\beta}\left(Q^{a\alpha}_i Q^{b\beta}_j\right)\left(u^c_{Rk} e_{R\ell}\right),$$

$$\mathcal{O}^{(3)}_{ijk\ell} \equiv \epsilon_{abc}\epsilon_{\alpha\beta}\epsilon_{\gamma\delta}\left(Q^{a\alpha}_i Q^{b\gamma}_j\right)\left(Q^{c\delta}_k L^\beta_\ell\right),$$

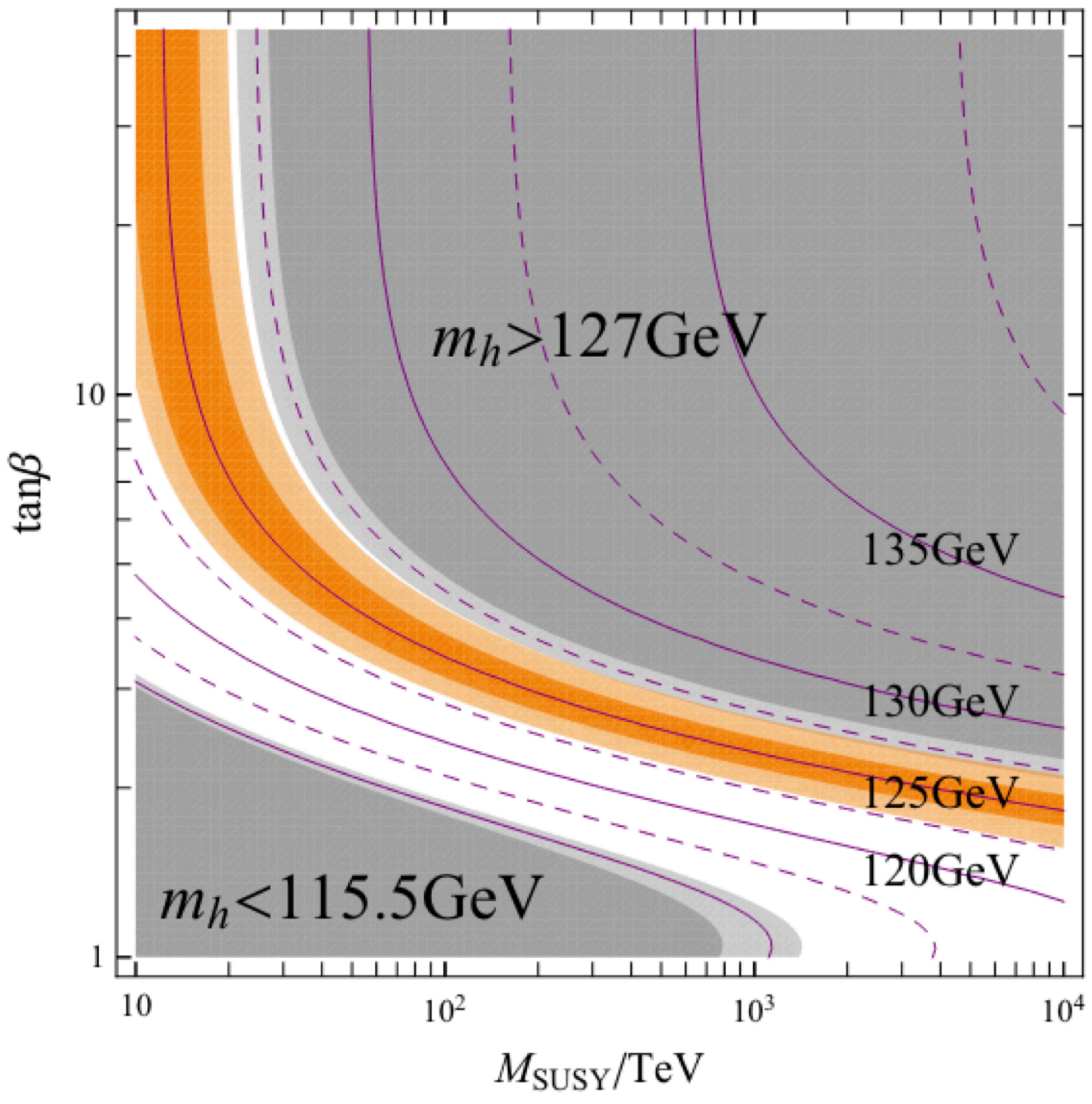
$$\mathcal{O}^{(4)}_{ijk\ell} \equiv \epsilon_{abc}\left(u^a_{Ri} d^b_{Rj}\right)\left(u^c_{Rk} e_{R\ell}\right),$$

- In non-SUSY GUTs, the gauge boson exchange typically dominates: mixed-type
- In SUSY-GUT, the one-loop contribution can be significant: both can contribute

High-scale SUSY

M. Ibe, S. Matsumoto, T. T. Yanagida (2012)

- Relation between SUSY-breaking scale and $\tan\beta$



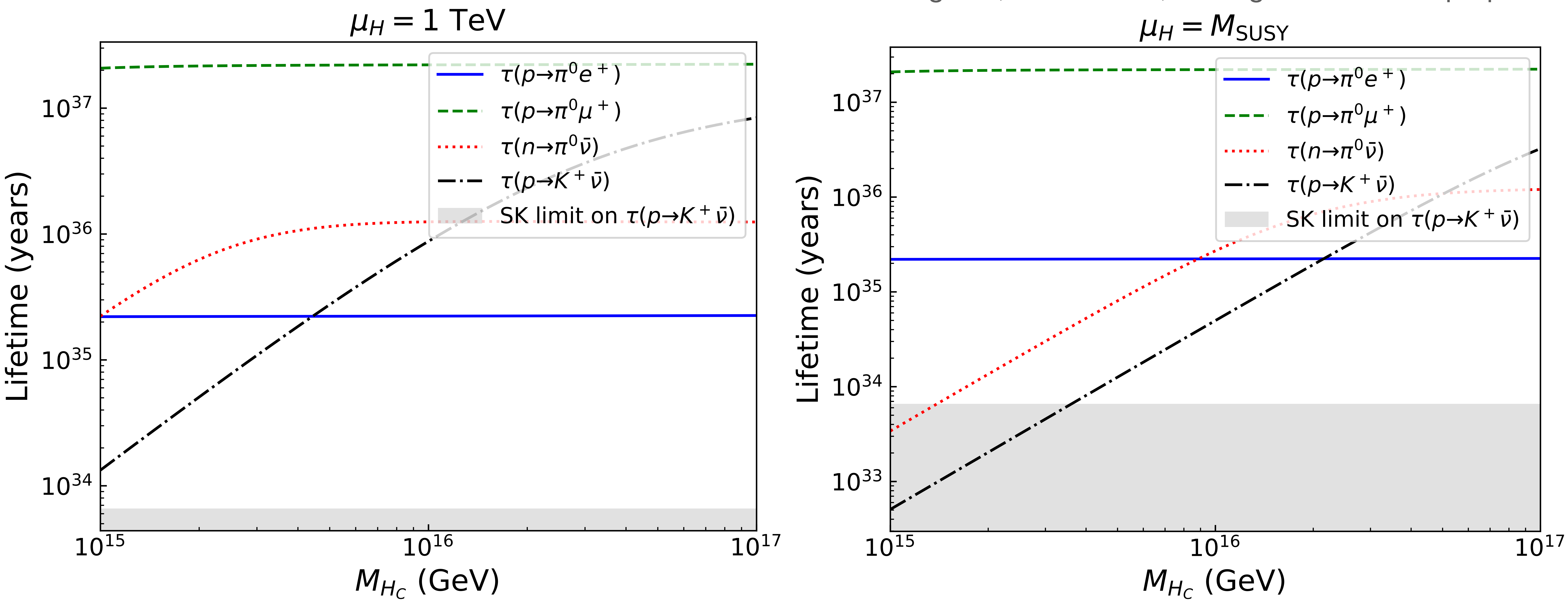
For $M_{\rm SUSY} = 10^2$ TeV,

$\tan\beta \sim 3$ is needed to reproduce

the correct Higgs mass.

The minimal SU(5) with high-scale SUSY

K. Hamaguchi, H. Shihwen, N. Nagata and **HT** in preparation



- $M_X = 10^{16}$ GeV, $M_2 = 1$ TeV, $M_3 = 10$ TeV, $M_{\rm SUSY} = 100$ TeV, $\tan\beta = 3$

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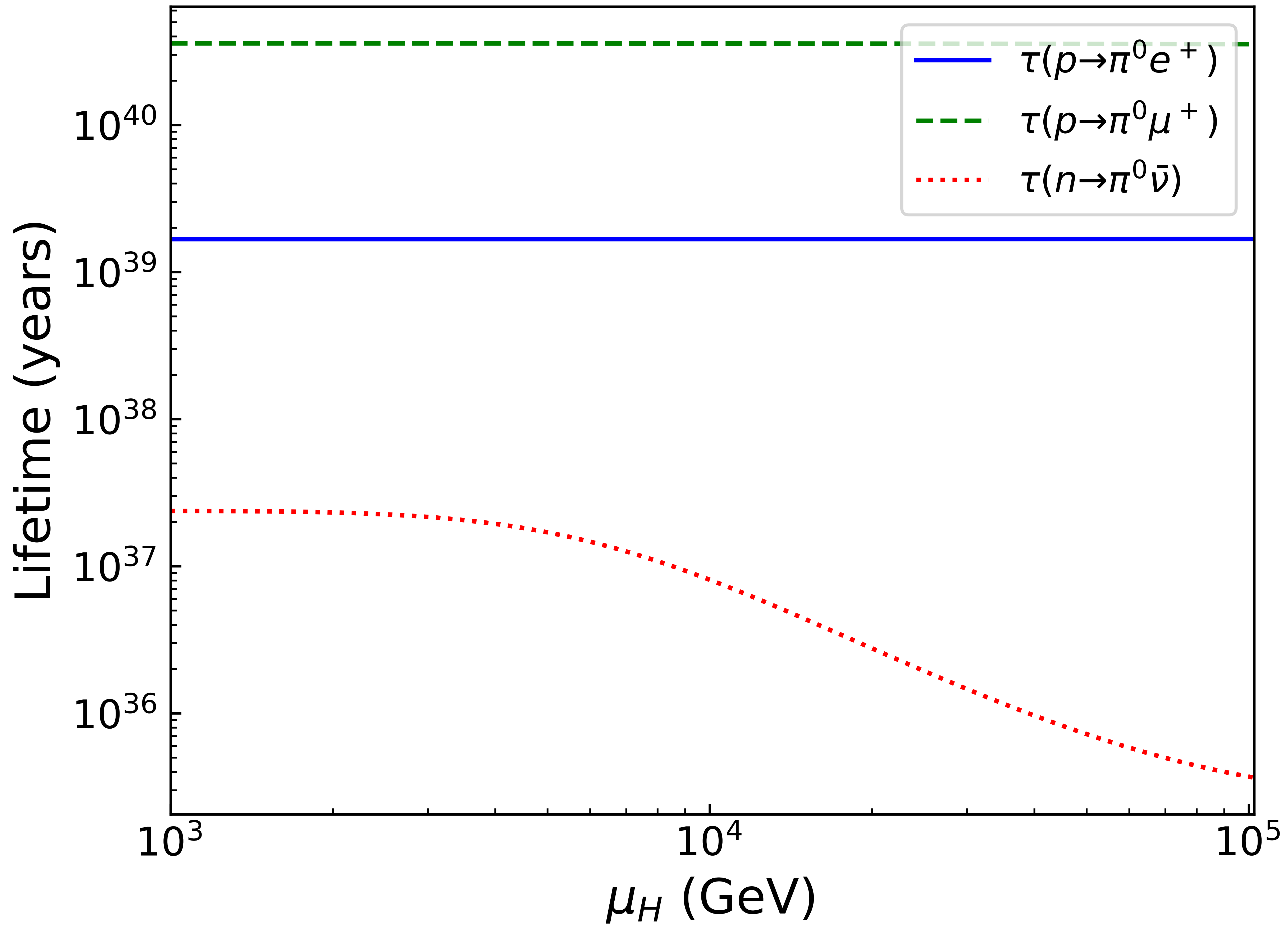
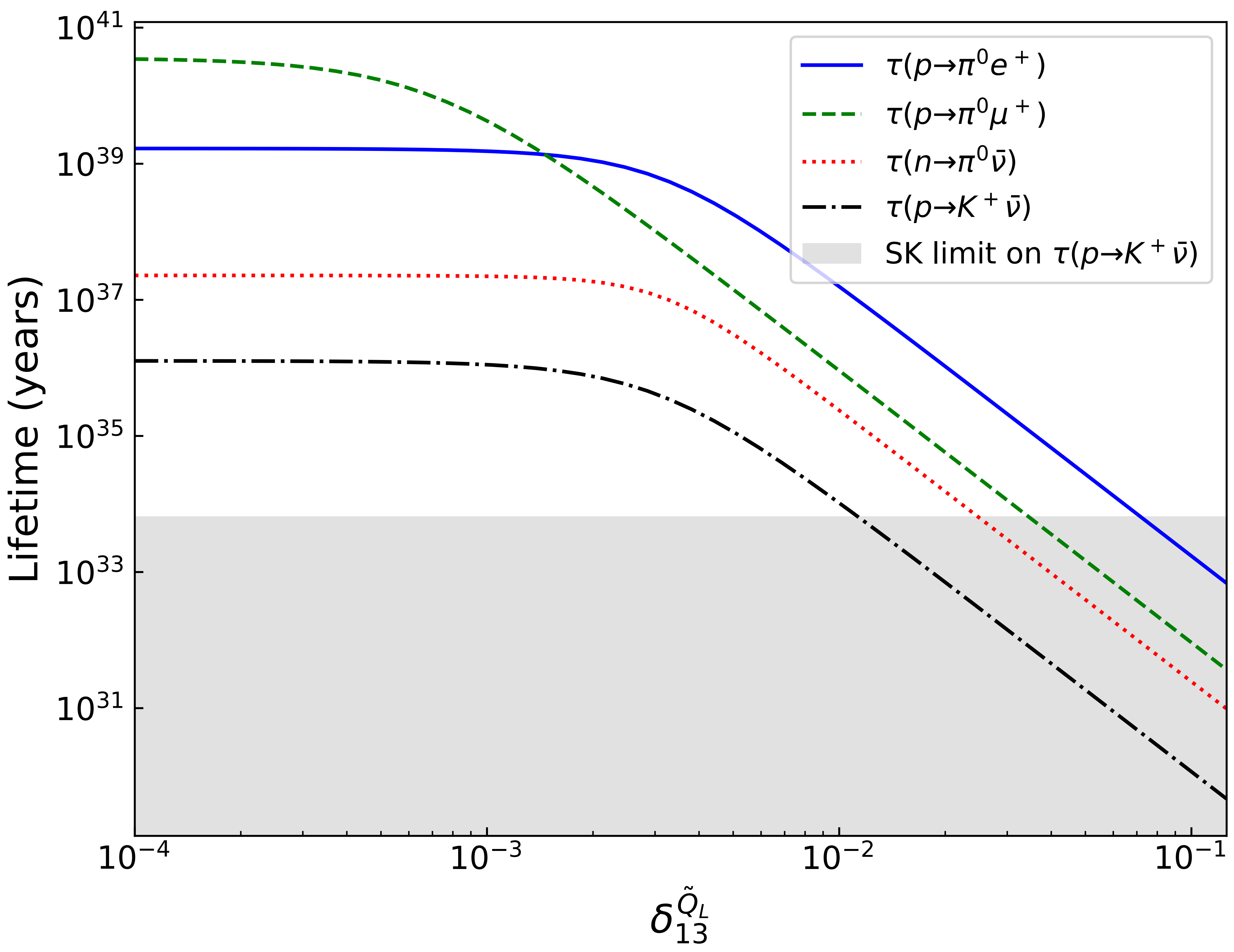


- $M_X = 10^{17}$ GeV, $M_{H_C} = 10^{16}$ GeV, $M_2 = 1$ TeV, $M_3 = 10$ TeV, $M_{\rm SUSY} = 100$ TeV, $\tan\beta = 3$

Sfermion flavor violation

K. Hamaguchi, H. Shihwen, N. Nagata and **HT** in preparation



In the presence of flavor violation,

- various channels could be accessible by upcoming experiments, or
- even ruled out by current experimental limits for large $\delta_{13}^{\tilde{f}}$

→ can readily be avoided if $M_{\rm SUSY}$ or M_{H_C} is larger

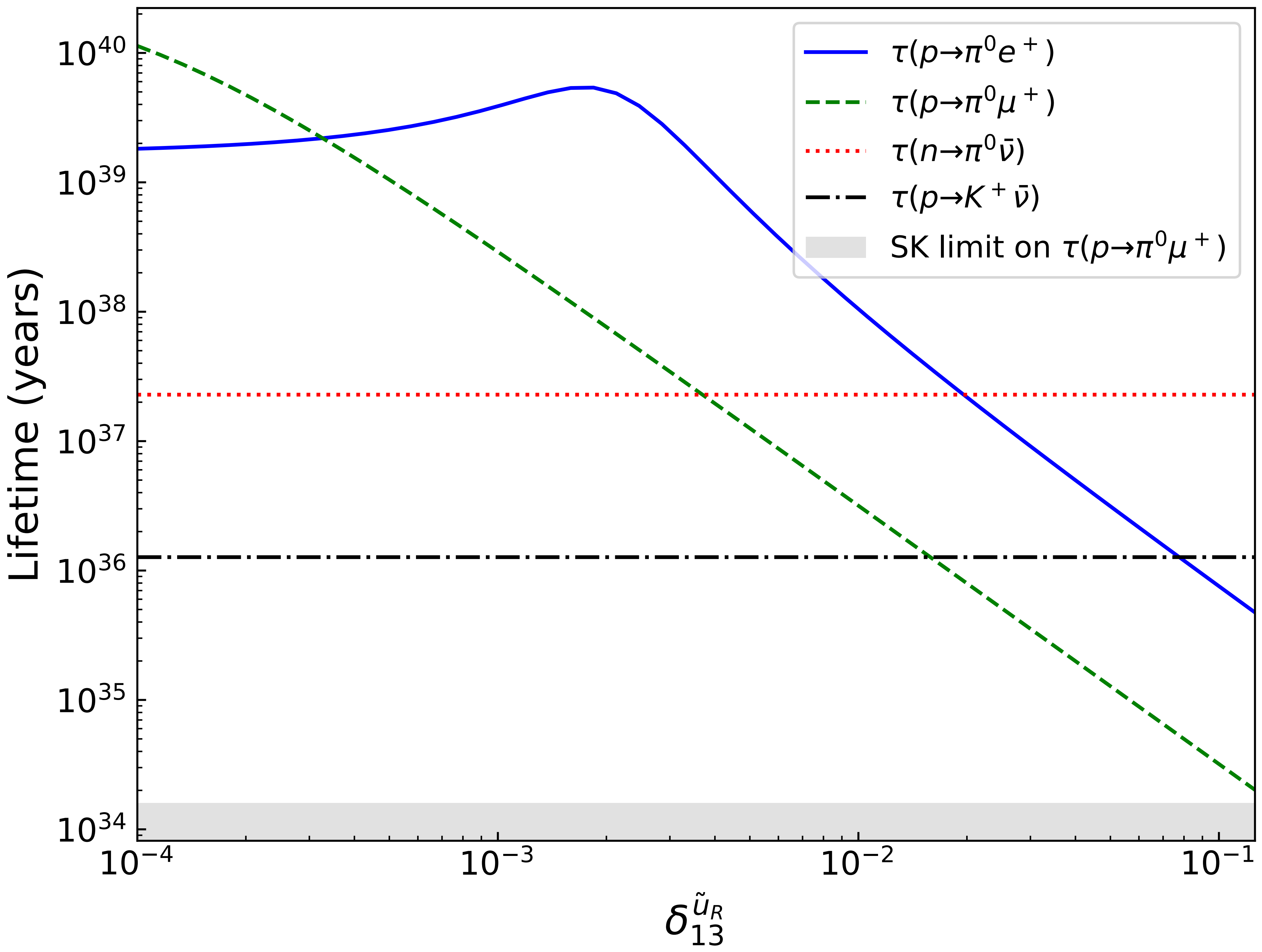
$$\widetilde{m}_{\tilde{f}}^2 = M_{\rm SUSY}^2 \begin{pmatrix} 1 & \delta_{12}^{\tilde{f}} & \delta_{13}^{\tilde{f}} \\ \delta_{12}^{\tilde{f}*} & 1 & \delta_{23}^{\tilde{f}} \\ \delta_{13}^{\tilde{f}*} & \delta_{23}^{\tilde{f}*} & 1 \end{pmatrix}$$

$M_X = 10^{17}$ GeV, $M_{H_C} = 10^{16}$ GeV, $\mu_H = 200$ GeV,

$M_1 = 5$ TeV, $M_2 = 1$ TeV, $M_3 = 10$ TeV, $M_{\rm SUSY} = 100$ TeV, $\tan\beta = 3$

Sfermion flavor violation

K. Hamaguchi, H. Shihwen, N. Nagata and **HT** in preparation



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$$\widetilde{m}^2_{\tilde{f}} = M^2_{\rm SUSY}\begin{pmatrix} 1 & \delta_{12}^{\tilde{f}} & \delta_{13}^{\tilde{f}} \\ \delta_{12}^{\tilde{f}*} & 1 & \delta_{23}^{\tilde{f}} \\ \delta_{13}^{\tilde{f}*} & \delta_{23}^{\tilde{f}*} & 1 \end{pmatrix}$$

Uncertainties in hadron matrix elements

- As we have seen in Aoki san's talk, the uncertainties in the nucleon decay matrix elements obtained by the lattice QCD calculation are currently $\sim 10\,\%$.

- **We expect the uncertainties to be significantly reduced if, instead of the matrix elements themselves, the ratios of them are estimated directly by the calculation.** This is because each matrix element is not independent, but correlated with each other.

- At leading order in the chiral perturbation,

$$W^{LR}_{p\pi^+\nu,0} = \frac{1+D+F}{f}\alpha \ , \qquad W^{LR}_{p\pi^+\nu,1} = -\frac{2(D+F)}{f}\alpha \ ,$$

$$W^{LL}_{p\pi^+\nu,0} = \frac{1+D+F}{f}\beta \ , \qquad W^{LL}_{p\pi^+\nu,1} = -\frac{2(D+F)}{f}\beta \ ,$$

$$W^{LR}_{p\eta\ell,0} = -\frac{1+D-3F}{\sqrt{6}f}\alpha \ , \qquad W^{LR}_{p\eta\ell,1} = \frac{2(D-3F)}{\sqrt{6}f}\alpha \ ,$$

$$W^{LL}_{p\eta\ell,0} = \frac{3-D+3F}{\sqrt{6}f}\beta \ , \qquad W^{LL}_{p\eta\ell,1} = \frac{2(D-3F)}{\sqrt{6}f}\beta \ ,$$

$$\langle 0|\epsilon_{abc}(u^a_R d^b_R)u^c_L|p\rangle \equiv \alpha P_L u_p \ ,$$

$$\langle 0|\epsilon_{abc}(u^a_L d^b_L)u^c_L|p\rangle \equiv \beta P_L u_p \ ,$$

$$\alpha = -0.01257(111)\ \mathrm{GeV}^3 \ ,$$
$$\beta = 0.01269(107)\ \mathrm{GeV}^3 \ .$$

$\alpha \simeq -\beta$ to $\sim 1\,\%$

If we take their ratios, f, α, β will cancel out.

Comments on $W^{\chi\chi'}_{p\eta\ell,1}$

- In previous lattice simulations, the values of $W^{\chi\chi'}_{p\eta\ell,1}(0)$ were not estimated.

However, the following combination is evaluated in Y. Aoki et al. (2017).

$$W^{\chi\chi'}_{p\eta\ell,\mu} \equiv W^{\chi\chi'}_{p\eta\ell,0} + \frac{m_\mu}{m_N} W^{\chi\chi'}_{p\eta\ell,1}$$

- We extract $W^{\chi\chi'}_{p\eta\ell,1}(0)$ from $W^{\chi\chi'}_{p\eta\ell,\mu}(m_\mu^2)$ by using $W^{\chi\chi'}_{p\eta\ell,0}(0)$, neglecting m_μ^2

dependence.