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CALCULATION SCHEME OF PHOTON-MEDIATED HIGGS PRODUCTION AT $\mu^+\mu^+$ COLLIDERS

1. INTRODUCTION: WHY MUON COLLIDERS?

	Hadrons (e.g., p p)	Electron – Positron	Muons
High Energy	✓	✗	✓
Full Energy Accessible	✗	✓	✓
Form Factor	Circular	Circular / Linear	Circular
Clean Background / Precision	✗	✓	✓
Efficient Vector Boson Fusion (VBF)	✗	✗	✓

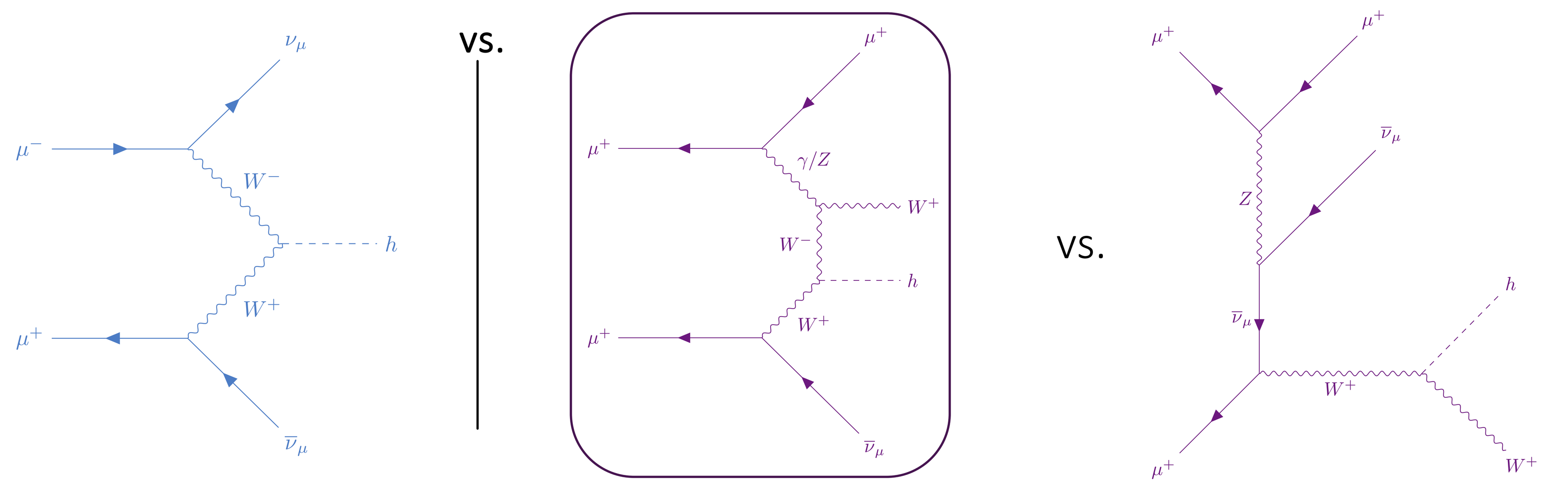
What kind of muon colliders?

	$\mu^+\mu^-$	μ^+e^-	$\mu^+\mu^+$
Electrically Neutral	✓	✓	✗
Vector Boson Fusion (VBF) w/o s-Channel Background (e.g., W^+W^-, $q\bar{q}$)	✗	✓	✓
Special Advantage	Resonant s-Channel Production	Flavor Physics	μ^+ Cooling Technology Already Exsists!
Disadvantage	μ^- Cooling Technology Needed	Lower Energy	Suppression From Extra Vertex…?

See, e.g.: The muon Smasher's guide; Al Ali et al.; 2021
µTRISTAN; Hamada, Kitano, Matsudo, Takaura, Yoshida; 2022

2. THE TOTAL CROSS SECTION OF $\mu^+\mu^+ \to \mu^+\bar{\nu}_\mu W^+ h$

- Higgs production: precision measurements of Higgs couplings, understanding electroweak symmetry breaking, determine Higgs potential, dark matter (?), etc.
- VBF generally enhanced as $\sim \log(s/m_V^2)$, but need extra coupling!
- VBF: large number of produced Higgs in $\mu^+\mu^-$, and e^+e^- collisions; for $\mu^+\mu^+$, need $\gamma/Z \longrightarrow W^+W^-$ splitting, or ZZ-fusion (suppressed)



So what is the problem?

- Collinear emission of photon $\longrightarrow$ collinear divergence (regulated by muon mass)
- Numerical instabilities in num. phase-space integral of event generator MadGraph due to massless photon and massless (or very light) muon
- Equivalent Photon (Weizsäcker-Williams) approximation (EPA) for collinear photons
- $\Longrightarrow$ New Calculation Scheme by splitting the phase-space integral
- Use approximations in respectively accurate regions

$$\sigma = \sigma\Big|_{p_T < p_T^{(\mathrm{cut})}} + \sigma\Big|_{p_T \geq p_T^{(\mathrm{cut})}}$$

$$\simeq \sigma_{\mathrm{EPA}}\Big|_{p_T < p_T^{(\mathrm{cut})}} + \sigma_{\mathrm{MG}}\Big|_{p_T \geq p_T^{(\mathrm{cut})}}$$

Approximate muons as massless

See, e.g.: Improving the Weizsacker-Williams approximation in electron-proton collisions; Frixione, Mangano, Nason, Ridolfi; 1993
The automated computation of tree-level and next-to-leading order differential cross-sections, and their matching to parton shower simulations; Alwall et al.; 2014
Similar procedure "matrix element matching" in: The Effective Vector Boson approximation in High-Energy Muon Collisions; Ruiz, Costantini, Maltoni, Mattelaer; 2022

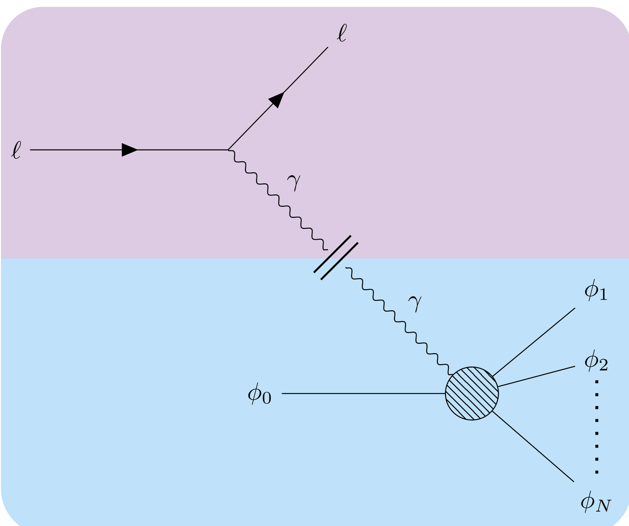
EPA (low-p_T) Cross Section

Intermediate photon quasi on-shell $\longrightarrow$ factorization!

$$\sigma = \sigma_{\mathrm{EPA}} + \Delta\sigma_{\mathrm{non-factorizable}}(q^2)$$

$$= \int \mathrm{d}x \left(\sigma_\gamma(xs)\, f_{\gamma/\ell}(x) + \mathcal{O}(1/x) \right)$$

Partonic cross section — Parton Distribution Function (PDF)



- Relate momentum transfer squared q^2 to transverse mom. via $p_T^2 = -q^2(1-x) - m_\ell^2 x^2$

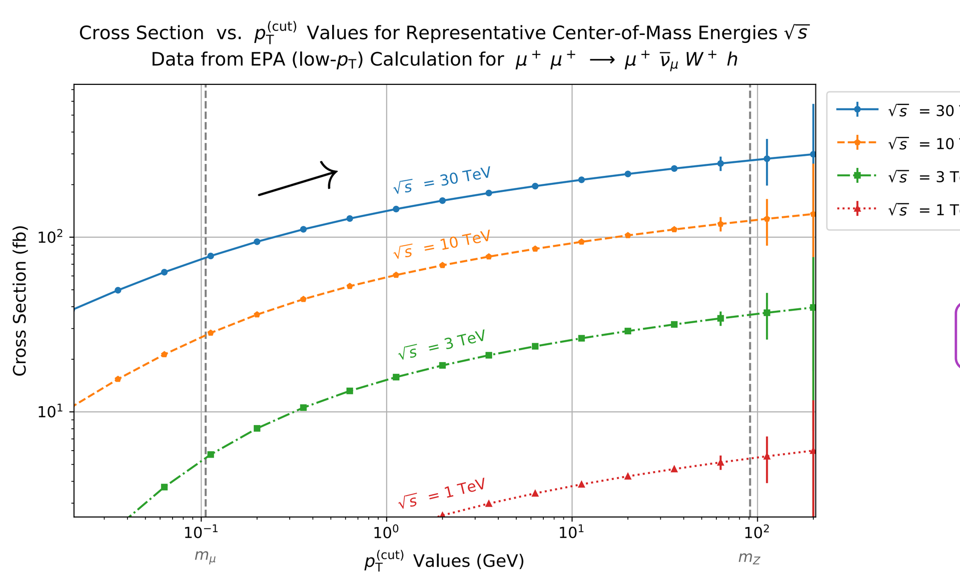
$$\Longrightarrow f_{\gamma/\ell}^{p_T^{(\mathrm{cut})}}(x) = \frac{\alpha}{2\pi}\left[-\frac{2(1-x)}{x} \frac{\left(p_T^{(\mathrm{cut})}\right)^2}{\left(p_T^{(\mathrm{cut})}\right)^2 + m_\ell^2 x^2} + \frac{1+(1-x)^2}{x} \log \frac{\left(p_T^{(\mathrm{cut})}\right)^2 + m_\ell^2 x^2}{m_\ell^2 x^2} \right]$$

- Relative error controlled by $\dfrac{\left(p_T^{(\mathrm{cut})}\right)^2}{s_{\min}} = \dfrac{\left(p_T^{(\mathrm{cut})}\right)^2}{(m_h + m_W)^2}$

For MadGraph (high-p_T) cross section, rel. error associated with massless approx. controlled by:

$$\frac{m_\mu^2}{\left(p_T^{(\mathrm{cut})}\right)^2} \left[\log \frac{s}{\left(p_T^{(\mathrm{cut})}\right)^2} \log \frac{s}{s_{\min}} - \frac{2}{3} \left(\log \frac{s}{s_{\min}} \right)^2 \right]^{-1}$$

Results for Higgs Production



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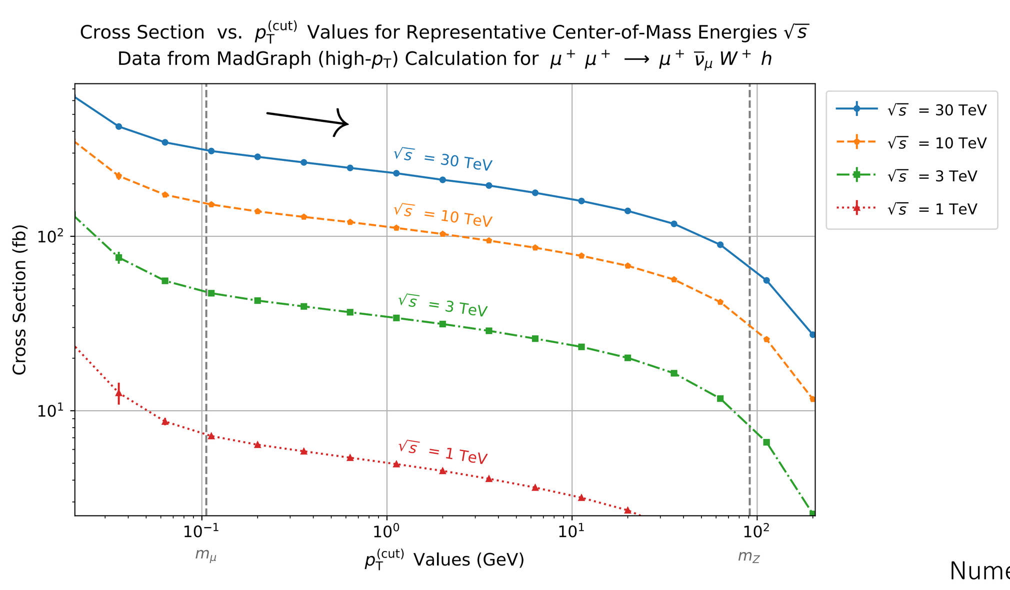

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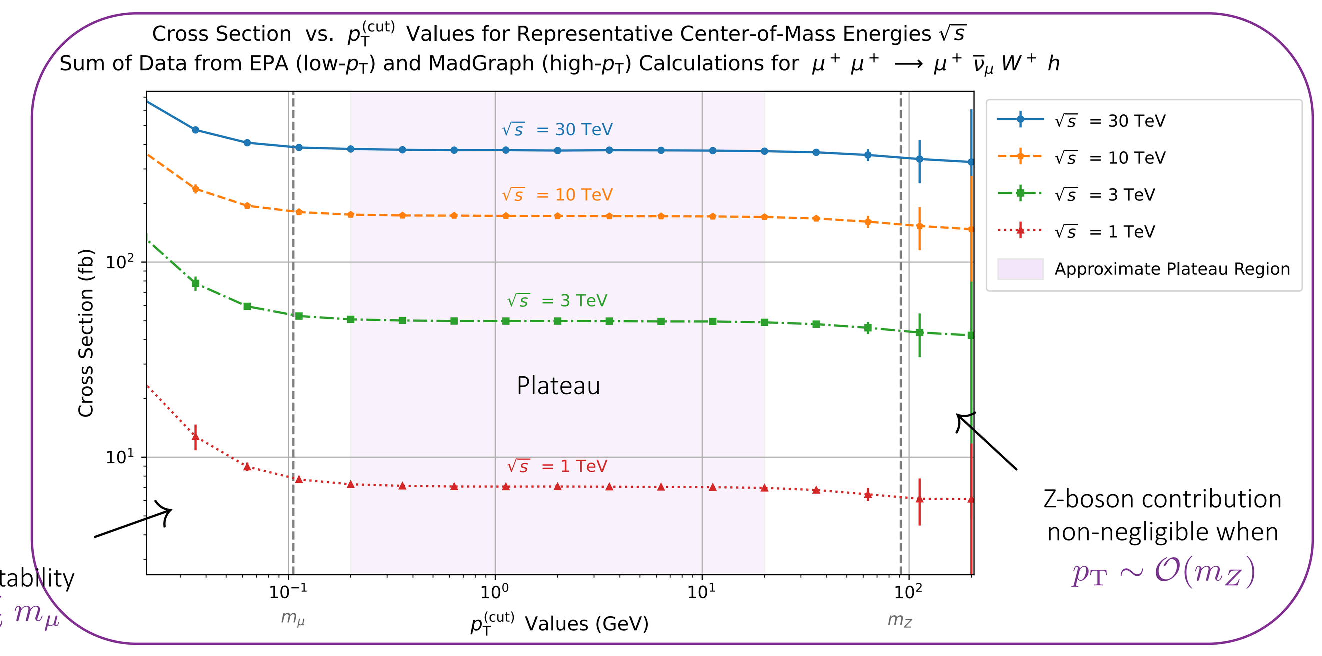
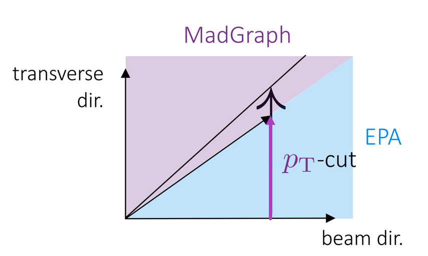
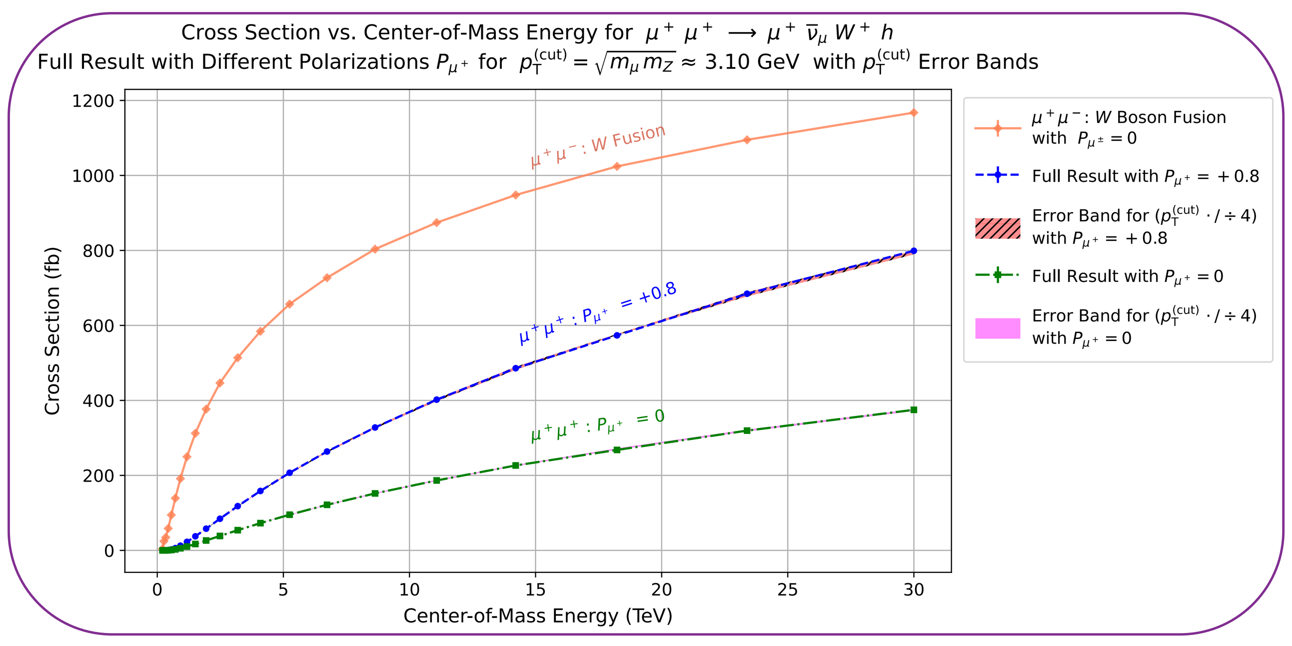




- Increasing $p_T^{(\mathrm{cut})}$ increases phase-space covered by EPA (low-p_T) $\Longrightarrow$ cross section increases
- Vice-versa for the MadGraph (high-p_T) cross section

- Too-small $p_T^{(\mathrm{cut})}$: numerical instability in MadGraph (even with finite m_μ)
- Too-large $p_T^{(\mathrm{cut})}$: Z-boson contribution and non-coll. div. diagrams missed in EPA



- Small errors associated with choice of $p_T^{(\mathrm{cut})}$ (< 1%)
- Optimal value physically well-motivated
- Created semi-automatic implementation in MadGraph

Conclusion

- Flat distribution $\Longrightarrow$ independent of p_T-cut $\Longrightarrow$ scheme works well!
- Calculation scheme applicable to any process with intermediate, collinear photon!
- Enhanced cross section from polarization due to larger W couplings
- Ryoto's poster: enhancement due to $(\log s)^3$ growth, in contrast to $\log s$ behavior for $\mu^+\mu^-$!
- $\mu^+\mu^+$ cross section not much smaller than for leading $\mu^+\mu^-$ process, only about factor of 2 at 10 TeV energies, despite naïve expectation of $\alpha \sim \mathcal{O}(1/100)$ suppression!
- Findings for higher-order cross section also important for $\mu^+\mu^-$ and e^+e^- colliders
- Comparable cross section + available cooling technology…

$\Longrightarrow$ $\mu^+\mu^+$-colliders are great Higgs factories!