Digital Quantum Simulation of higher-charge Schwinger model

(玄人仕様？)

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Refs:

- arXiv:2001.00485 [hep-lat],
  w/ Bipasha Chakraborty (Cambridge U.), Yuta Kikuchi (BNL → Cambridge Quantum),
  Taku Izubuchi (BNL-RIKEN BNL) & Akio Tomiya (International Prof. U. of Tech. in Osaka)
- arXiv:2105.03276 [hep-lat],
  w/ Yuta Kikuchi, Etsuko Itou (RIKEN, etc..),
  Lento Nagano (Tokyo U.) & Takuya Okuda (Tokyo U.)
- arXiv:2110.14105 [hep-th],
  w/ Yuta Kikuchi, Etsuko Itou & Yuya Tanizaki (YITP)

京都大学基礎物理学研究所
国内モレキュール型研究会「場の理論の量子計算2022」

24th, Feb., 2022

This talk is about...

Charge-q Schwinger model with topological term

1+1d QED

$$L = \frac{1}{2g^2} F_{01}^2 + \frac{\theta_0}{2\pi} F_{01} + \overline{\psi}\, \mathrm{i}\, \gamma^\mu (\partial_\mu + \mathrm{i}\, q\, A_\mu) \psi - m\, \overline{\psi} \psi$$

topological "theta term"

supposed to be difficult in the conventional Monte Carlo approach:

- real time
- ∃ sign problem even in Euclidean case when θ isn't small

Results:

[cf. Tensor Network approach:
Banuls-Cichy-Jansen-Saito '16 ,
Funcke-Jansen-Kuhn '19, etc...]

- Construction of the true vacuum
- Computation of $\langle \overline{\psi} \psi \rangle$ & consistency check/prediction

[Chakraborty-MH-Kikuchi-Izubuchi-Tomiya '20]

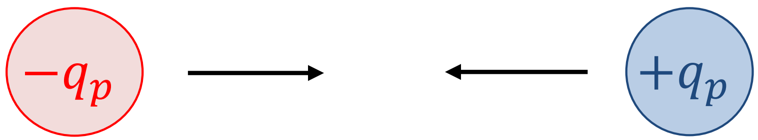
- Exploration of the screening vs confinement problem
  & negative string tension behavior for some parameters

[MH-Itou-Kikuchi-Nagano-Okuda '21] [MH-Itou-Kikuchi-Tanizaki '21]

Screening versus Confinement

Let's consider

potential between 2 heavy charged particles



Classical picture:

$$V(x) = \frac{q_p^2\, g^2}{2} x \;?$$

Coulomb law in 1+1d

||

confinement

too naive in the presence of dynamical fermions

Expectations from previous analyzes

Potential between probe charges $\pm q_p$ has been analytically computed

[Iso-Murayama '88, Gross-Klebanov-Matytsin-Smilga '95]

- massless case:

$\mu \equiv g/\sqrt{\pi}$

$$V(x) = \frac{q_p^2\, g^2}{2\mu}(1 - e^{-q\mu x})$$

screening

- massive case:

Expectations from previous analyzes

Potential between probe charges $\pm q_p$ has been analytically computed

[Iso-Murayama ’88, Gross-Klebanov-Matytsin-Smilga ’95]

- massless case:

$\mu \equiv g/\sqrt{\pi}$

$$V(x) = \frac{q_p^2\, g^2}{2\mu}\left(1 - e^{-q\mu x}\right)$$ *screening*

- massive case: [cf. Misumi-Tanizaki-Unsal ’19]

$\Sigma \equiv g e^{\gamma}/2\pi^{3/2}$

$$V(x) \sim mq\Sigma\left(\cos\left(\frac{\theta + 2\pi q_p}{q}\right) - \cos\left(\frac{\theta}{q}\right)\right) x$$ $(\mathrm{m} \ll g,\ |x| \gg 1/g\ )$

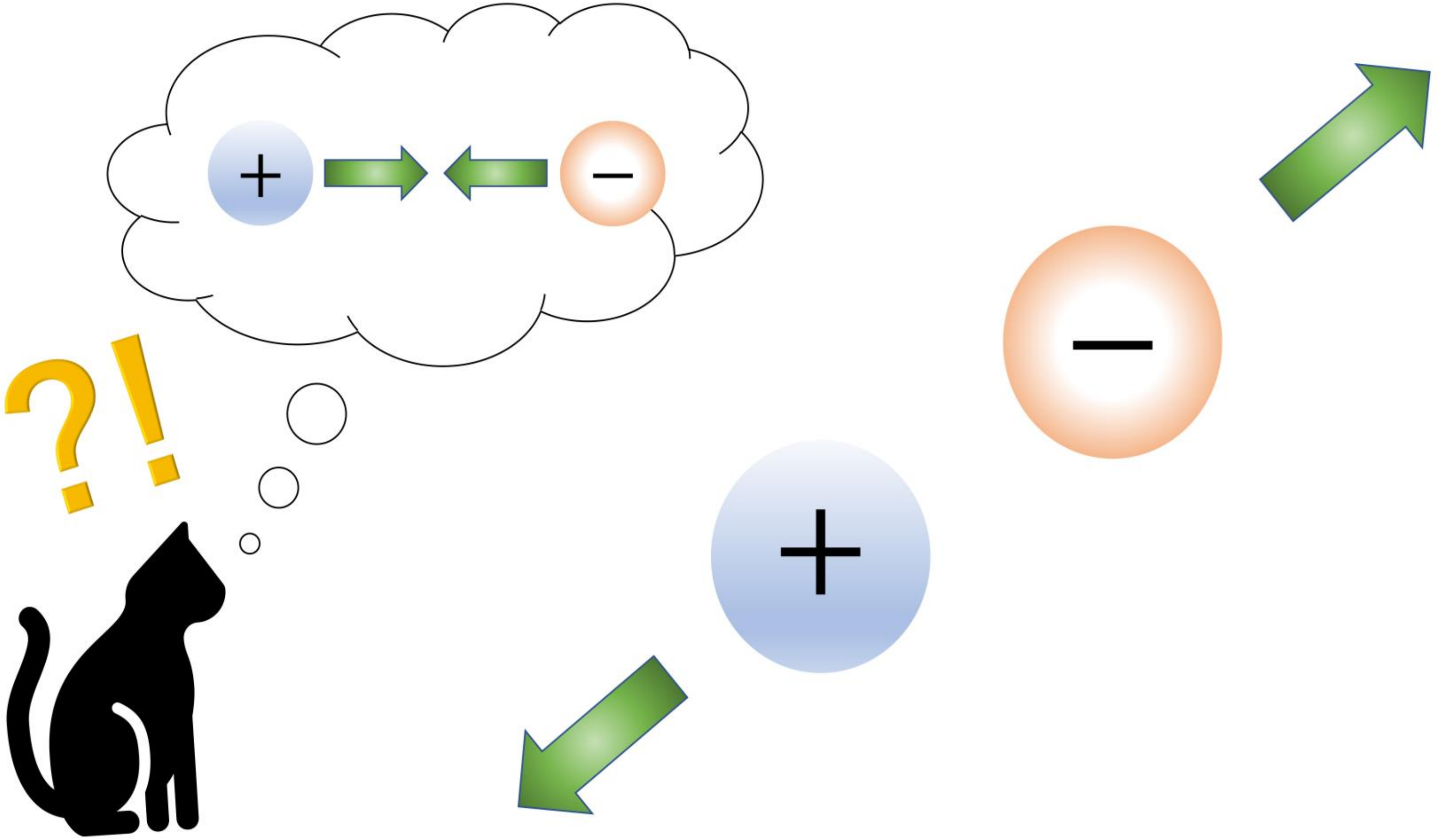
$$\begin{cases} = \mathrm{Const.} & \text{for } \mathrm{q_p/q} = \mathbf{Z} \\ \propto x & \text{for } \mathrm{q_p/q} \neq \mathbf{Z} \end{cases}$$

= Const. for $\mathrm{q_p/q} = \mathbf{Z}$ — *screening*

$\propto x$ for $\mathrm{q_p/q} \neq \mathbf{Z}$ — *confinement?*

*but sometimes **negative** slope!*

That is, as changing the parameters…



Let’s explore this aspect by quantum simulation!

Contents

“Regularization” of Hilbert space

Hilbert space of QFT is typically ∞ dimensional

⟶ Make it finite dimensional!

- Fermion is easiest (up to doubling problem)
  - Putting on spatial lattice, Hilbert sp. is finite dimensional
- scalar
  - Hilbert sp. at each site is ∞ dimensional (need truncation or additional regularization)
- gauge field (w/ kinetic term)
  - no physical d.o.f. in 0+1D/1+1D (w/ open bdy. condition)
  - ∞ dimensional Hilbert sp. in higher dimensions

Schwinger model w/ topological term

Continuum:

$$L = \frac{1}{2g^2} F_{01}^2 + \frac{\theta_0}{2\pi} F_{01} + \overline{\psi}\, \mathrm{i}\, \gamma^\mu (\partial_\mu + \mathrm{i}\, q\, A_\mu)\psi - m\, \overline{\psi}\psi$$

Taking temporal gauge $A_0 = 0$, (Π: conjugate momentum of A_1)

$$H(x) = \frac{g^2}{2}\left(\Pi - \frac{\theta_0}{2\pi}\right)^2 - \bar{\psi}\, \mathrm{i}\, \gamma^1 (\partial_1 + \mathrm{i}\, q A_1)\psi + m\bar{\psi}\psi,$$

Physical states are constrained by Gauss law:

$$0 = -\partial_1 \Pi - qg\bar{\psi}\gamma^0\psi$$

Accessible region by analytic computation

- Massive limit:

The fermion can be integrated out

&

the theory becomes effectively pure Maxwell theory w/ θ

- Bosonization:

[Coleman '76]

$$\mathcal{L} = \frac{1}{8\pi}\left(\partial_\mu \phi\right)^2 - \frac{g^2}{8\pi^2}\phi^2 + \frac{e^\gamma g}{2\pi^{3/2}} m \cos(\phi + \theta)$$

exactly solvable for $m = 0$

&

small m regime is approximated by perturbation

Sign problem in path integral formalism

In Minkowski space,

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu}^2 + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi \right] + \frac{g\theta}{4\pi} \int F \in \boldsymbol{R}$$

$$\langle \mathcal{O} \rangle = \frac{\int DAD\psi D\bar{\psi} \;\; \mathcal{O} \; e^{iS}}{\int DAD\psi D\bar{\psi} \; e^{iS}}$$

highly oscillating

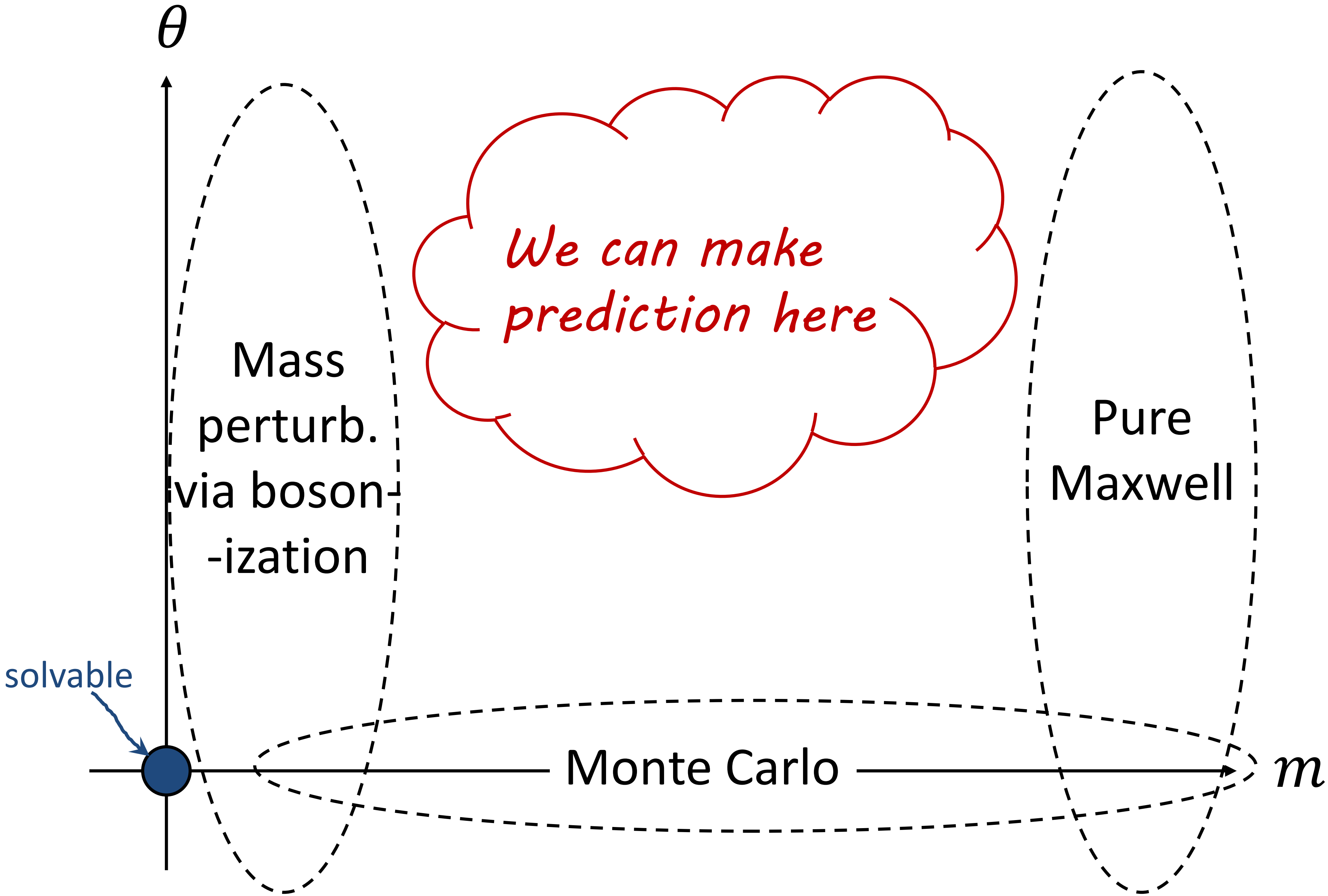
In Euclidean space,

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu}^2 + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi \right] + i \frac{g\theta}{4\pi} \int F \in \boldsymbol{C}$$

$$\langle \mathcal{O} \rangle = \frac{\int DAD\psi D\bar{\psi} \;\; \mathcal{O} \; e^{-S}}{\int DAD\psi D\bar{\psi} \; e^{-S}}$$

highly oscillating for non-small θ

Map of accessibility/difficulty



Put the theory on lattice

- **Fermion** (on site):

"Staggered fermion" [Susskind, Kogut-Susskind '75]

$$\frac{\chi_n}{a^{1/2}} \longleftrightarrow \psi(x) = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix} \begin{matrix} \rightarrow \text{odd site} \\ \rightarrow \text{even site} \end{matrix}$$

lattice spacing

Put the theory on lattice

- **Fermion** (on site): "Staggered fermion" [Susskind, Kogut-Susskind '75]

$$\frac{\chi_n}{a^{1/2}} \longleftrightarrow \psi(x) = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$$

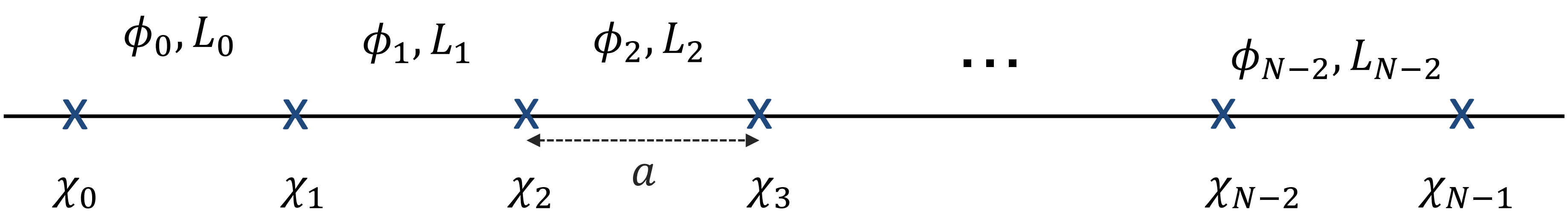
ψ_u → odd site

ψ_d → even site

a: lattice spacing

- **Gauge field** (on link):

$$\phi_n \leftrightarrow -agA^1(x), \qquad L_n \leftrightarrow -\frac{\Pi(x)}{g}$$



Lattice theory w/ staggered fermion

Hamiltonian:

$$H = J\sum_{n=0}^{N-2}\left(L_n + \frac{\theta_0}{2\pi}\right)^2 - \mathrm{i}w\sum_{n=0}^{N-2}\left[\chi_n^\dagger (U_n)^q \chi_{n+1} - \mathrm{h.c.}\right] + m\sum_{n=0}^{N-1}(-1)^n \chi_n^\dagger \chi_n$$

$$\left(w = \frac{1}{2a}, J = \frac{g^2 a}{2}\right)$$

Commutation relation:

$$[L_n, U_m] = U_m \delta_{nm}, \quad \{\chi_n, \chi_m^\dagger\} = \delta_{nm}$$

Gauss law:

$$L_n - L_{n-1} = q\left[\chi_n^\dagger \chi_n - \frac{1-(-1)^n}{2}\right]$$

Eliminate gauge d.o.f.

1. Take open b.c. & solve Gauss law:

$$L_n = L_{-1} + q\sum_{j=1}^{n}\left(\chi_j^\dagger \chi_j - \frac{1-(-1)^j}{2}\right) \qquad \text{w/ } L_{-1} = 0$$

2. Take the gauge $U_n = 1$

Then,

$$H = -\mathrm{i}w\sum_{n=1}^{N-1}\left[\chi_n^\dagger \chi_{n+1} - \mathrm{h.c.}\right] + m\sum_{n=1}^{N}(-1)^n \chi_n^\dagger \chi_n$$
$$+J\sum_{n=1}^{N}\left[\frac{\theta_0}{2\pi} + q\sum_{j=1}^{n}\left(\chi_j^\dagger \chi_j - \frac{1-(-1)^j}{2}\right)\right]^2.$$

This acts on finite dimensional Hilbert space

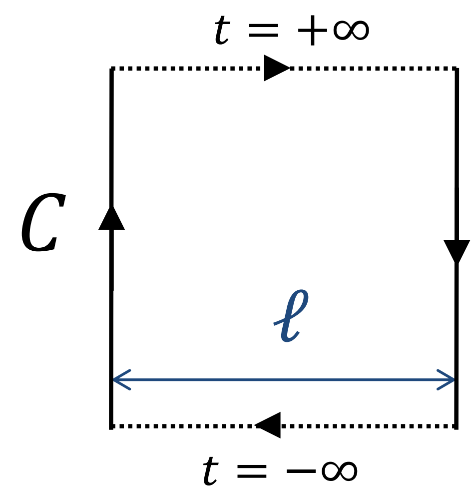
Insertion of the probe charges

① Introduce the probe charges $\pm q_p$:

$$e^{iq_p \int_C A}$$

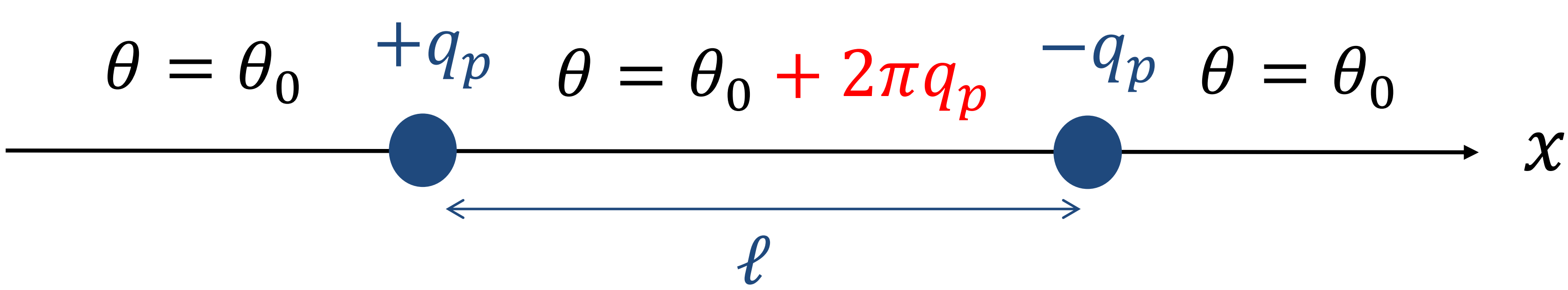
$$||$$

$$e^{iq_p \int_{S,\partial S=C} F}$$



local θ-term w/ $\theta = 2\pi q_p$!!

② Include it to the action & switch to Hamilton formalism



③ Compute the ground state energy (in the presence of the probes)

Going to spin system

$$\{\chi_n^\dagger, \chi_m\} = \delta_{mn}, \ \ \{\chi_n, \chi_m\} = 0$$

This is satisfied by the operator:

"Jordan-Wigner transformation"

[Jordan-Wigner'28]

$$\chi_n = \frac{X_n - \mathrm{i}Y_n}{2} \left(\prod_{i=1}^{n-1} -\mathrm{i}Z_i \right)$$

Now the system is purely a spin system:

$$H = -\mathrm{i}w \sum_{n=1}^{N-1} \left[\chi_n^\dagger \chi_{n+1} - \mathrm{h.c.} \right] + m \sum_{n=1}^{N} (-1)^n \chi_n^\dagger \chi_n + J \sum_{n=1}^{N} \left[\frac{\vartheta_n}{2\pi} + q \sum_{j=1}^{n} \left(\chi_j^\dagger \chi_j - \frac{1-(-1)^j}{2} \right) \right]^2$$

$$\Downarrow$$

$$H = J \sum_{n=0}^{N-2} \left[q \sum_{i=0}^{n} \frac{Z_i + (-1)^i}{2} + \frac{\vartheta_n}{2\pi} \right]^2 + \frac{w}{2} \sum_{n=0}^{N-2} \left[X_n X_{n+1} + Y_n Y_{n+1} \right] + \frac{m}{2} \sum_{n=0}^{N-1} (-1)^n Z_n$$

Qubit description of the Schwinger model !!

Comments on choices of setup

There were many choices of setup to come here…

- Formulation of continuum theory?
- Type of lattice fermion?
- Boundary condition?
- Impose Gauss law?
- How to map fermion to spin system?
- Even N or odd N?

Choice of continuum theory

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g\theta}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + igA_{\mu})\psi - m\bar{\psi}\psi$$

(used for the case w/ probes)

⇕ "chiral anomaly" [cf. Fujikawa'79]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + igA_{\mu})\psi - m\bar{\psi}e^{i\theta\gamma^5}\psi$$

(used for the case w/o probes)

- Equivalent for continuum theory w/o bdy.

—— (generically) inequivalent for theory on lattice or w/ bdy.

- The latter doesn't violate θ-periodicity even for open b.c.

Choice of boundary conditions

Gauss law: $L_n - L_{n-1} = q\left[\chi_n^\dagger \chi_n - \frac{1-(-1)^n}{2}\right]$

Open b.c.

- $L_n =$ (fermion op.)

$\longrightarrow$ $\dim(\mathcal{H}_{\mathrm{phys}}) < \infty$

- θ-periodicity is lost
- momentum not conserved

Periodic b.c.

- one of L_n's remains

$\longrightarrow$ $\dim(\mathcal{H}_{\mathrm{phys}}) = \infty$

additional truncation needed

- $^\exists\theta$-periodicity
- momentum conserved

Even N or odd N?

$\chi_0 \quad \chi_1 \quad \chi_2 \quad \chi_3 \quad \cdots \quad \chi_{N-2} \quad \chi_{N-1}$

Staggered fermion: $\dfrac{\chi_n}{a^{1/2}} \longleftrightarrow \psi(x) = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$ $\psi_u \rightarrow$ odd site, $\psi_d \rightarrow$ even site

- Usually even N is taken (p.b.c. allows only even N)
- Open b.c. allows both but parity is different: $\chi_n \rightarrow i(-1)^n \chi_{N-n-1}$

	n mod 2	$\bar{\psi}\psi \sim \sum_n (-1)^n \chi_n^\dagger \chi_n$	$\bar{\psi}\gamma^5\psi \sim \sum_n (-1)^n (\chi_n^\dagger \chi_{n+1} - \mathrm{h.c.})$
even N	changes	flipped	invariant
odd N	invariant	invariant	flipped

Odd N seems more like the continuum theory?

Contents

Adiabatic state preparation of vacuum

Step 1: Choose an initial Hamiltonian H_0 of a simple system whose ground state $|\mathrm{vac}_0\rangle$ is known and unique

Step 2:

Step 3:

Adiabatic state preparation of vacuum

Step 1: Choose an initial Hamiltonian H_0 of a simple system whose ground state $|\mathrm{vac}_0\rangle$ is known and unique

Step 2: Introduce adiabatic Hamiltonian $H_A(t)$ s.t.

- $H_A(0) = H_0,\ H_A(T) = H_{\mathrm{target}}$
- $\left|\frac{dH_A}{dt}\right| \ll 1$ for $T \gg 1$

Step 3:

Adiabatic state preparation of vacuum

Step 1: Choose an initial Hamiltonian H_0 of a simple system whose ground state $|\text{vac}_0\rangle$ is known and unique

Step 2: Introduce adiabatic Hamiltonian $H_A(t)$ s.t.

- $H_A(0) = H_0,\ H_A(T) = H_{\text{target}}$
- $\left|\frac{dH_A}{dt}\right| \ll 1$ for $T \gg 1$

Step 3: Use the adiabatic theorem

If $H_A(t)$ has a unique ground state w/ a finite gap for $\forall t$, then the ground state of H_{target} is obtained by

$$|\text{vac}\rangle = \lim_{T\to\infty} \mathcal{T} \exp\left(-i \int_0^T dt\, H_A(t)\right) |\text{vac}_0\rangle$$

Adiabatic state preparation in the presence of the probes

$$|\mathrm{vac}> = \lim_{T\to\infty} \mathcal{T} \exp\left(-i\int_0^T dt\ H_A(t)\right)|\mathrm{vac}_0>$$

$$\simeq U(T)U(T-\delta t)\cdots U(2\delta t)U(\delta t)|\mathrm{vac}_0>$$

$$\left(U(t) = e^{-iH_A(t)\delta t}\right)$$

Here we choose

$$\begin{cases} H_0 = H\Big|_{w\to 0,\, \vartheta_n \to 0,\, m\to m_0} \quad \Rightarrow \quad |\mathrm{vac}_0\rangle = |1010\cdots\rangle \\ H_A(t) = H\Big|_{w\to w(t), \vartheta_n\to\vartheta_n(t),\, m\to m(t)} \\ w(t) = \frac{t}{T}w,\, \vartheta_n(t) = \frac{t}{T}\vartheta_n,\ m(t) = \left(1-\frac{t}{T}\right)m_0 + \frac{t}{T}m \end{cases}$$

m_0 can be any positive number in principle
but it is practically chosen to have small systematic error

Comments on adiabatic state preparation

$$(\text{"systematic error"}) \sim \frac{1}{T\,(\text{gap})^2}$$

😄 Advantage:

- guaranteed to be correct for $T \gg 1$ & $\delta t \ll 1$ if $H_A(t)$ has a unique gapped vacuum
- can directly get excited states under some conditions

☹ Disadvantage:

- doesn't work for degenerate vacua
- costly — likely requires many gates

→ perhaps not so efficient in NISQ era

Tradeoff of symmetries in Suzuki-Trotter dec.

Suzuki-Trotter decomposition:

(more precisely, we actually use its improvement but I skip it)

$(M \in \mathbf{Z}, M \gg 1)$

$$e^{-iHt} = \left(e^{-iH\frac{t}{M}}\right)^M \simeq \left(e^{-iH_1\frac{t}{M}}e^{-iH_2\frac{t}{M}}\right)^M + \mathcal{O}(1/M)$$

$$\Rightarrow \quad H_{\text{eff}} = \frac{1}{-it}\log\left(e^{-iH_1\frac{t}{M}}e^{-iH_2\frac{t}{M}}\right)^M$$

Symmetries may be broken by decomposition

Tradeoff:

- Parity friendly (& translation if p.b.c.)

$$H = H_{XX} + H_{YY} + H_{ZZ} + H_Z \qquad \cancel{U(1)}$$

- $U(1)$ friendly

$$H = H_{XX+YY}^{(\text{even})} + H_{XX+YY}^{(\text{odd})} + H_{ZZ} + H_Z \qquad \cancel{P}$$

Contents

Expectation value of mass op. (chiral condensation)

$$\langle \bar{\psi}(x)\psi(x) \rangle = \langle \text{vac} | \bar{\psi}(x)\psi(x) | \text{vac} \rangle$$

Instead of the local op., we analyze the average over the space:

$$\frac{1}{2Na} \langle \text{vac} | \sum_{n=1}^{N} (-1)^n Z_n | \text{vac} \rangle$$

Once we get the vacuum, we can compute the VEV as

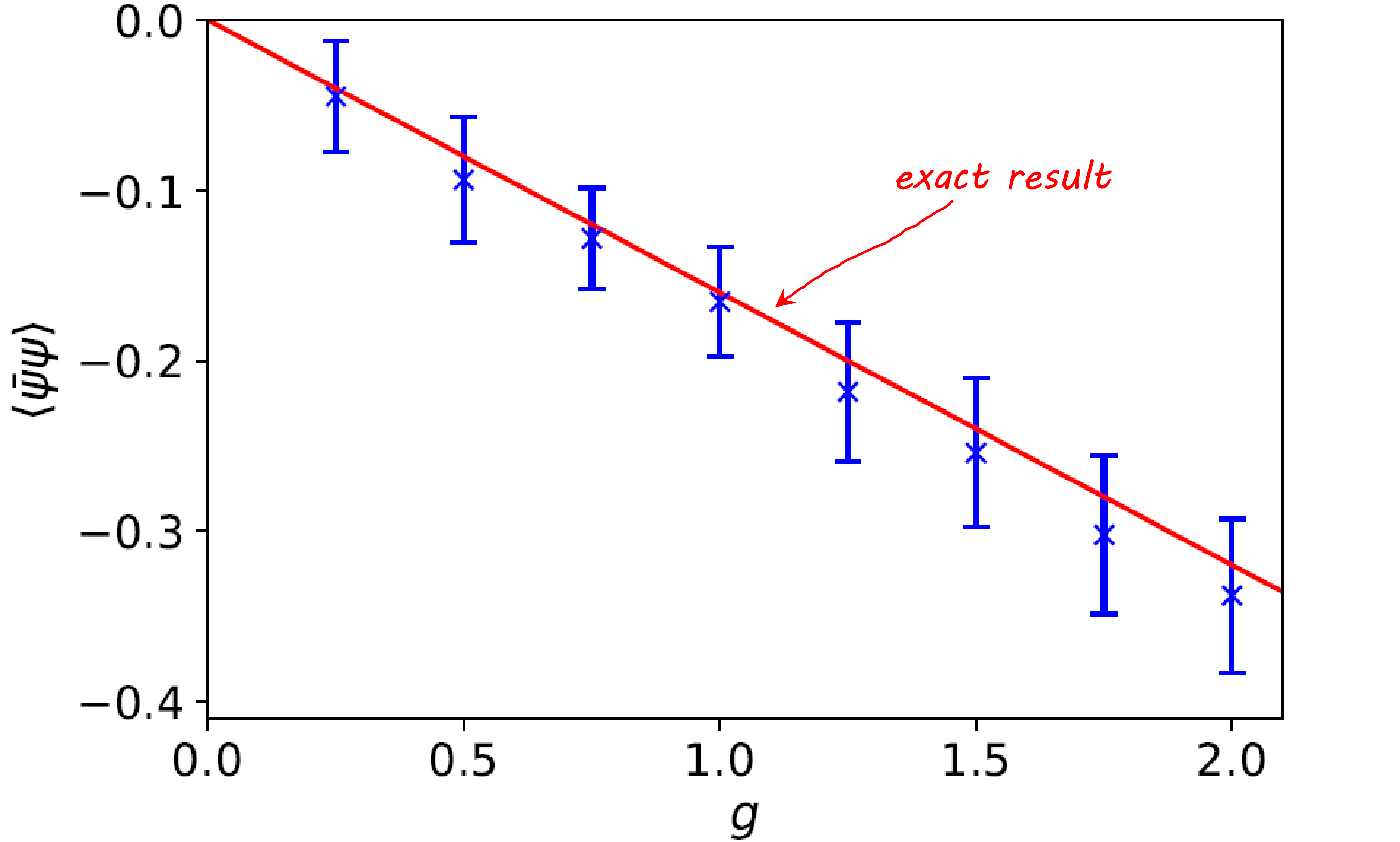
$$\frac{1}{2Na} \langle \text{vac} | \sum_{n=1}^{N} (-1)^n Z_n | \text{vac} \rangle = \frac{1}{2Na} \sum_{n=1}^{N} (-1)^n \sum_{i_1 \cdots i_N = 0,1} \langle \text{vac} | Z_n | i_1 \cdots i_N \rangle \langle i_1 \cdots i_N | \text{vac} \rangle$$

$$= \frac{1}{2Na} \sum_{n=1}^{N} \sum_{i_1 \cdots i_N = 0,1} (-1)^{n+i_n} | \langle i_1 \cdots i_N | \text{vac} \rangle |^2$$

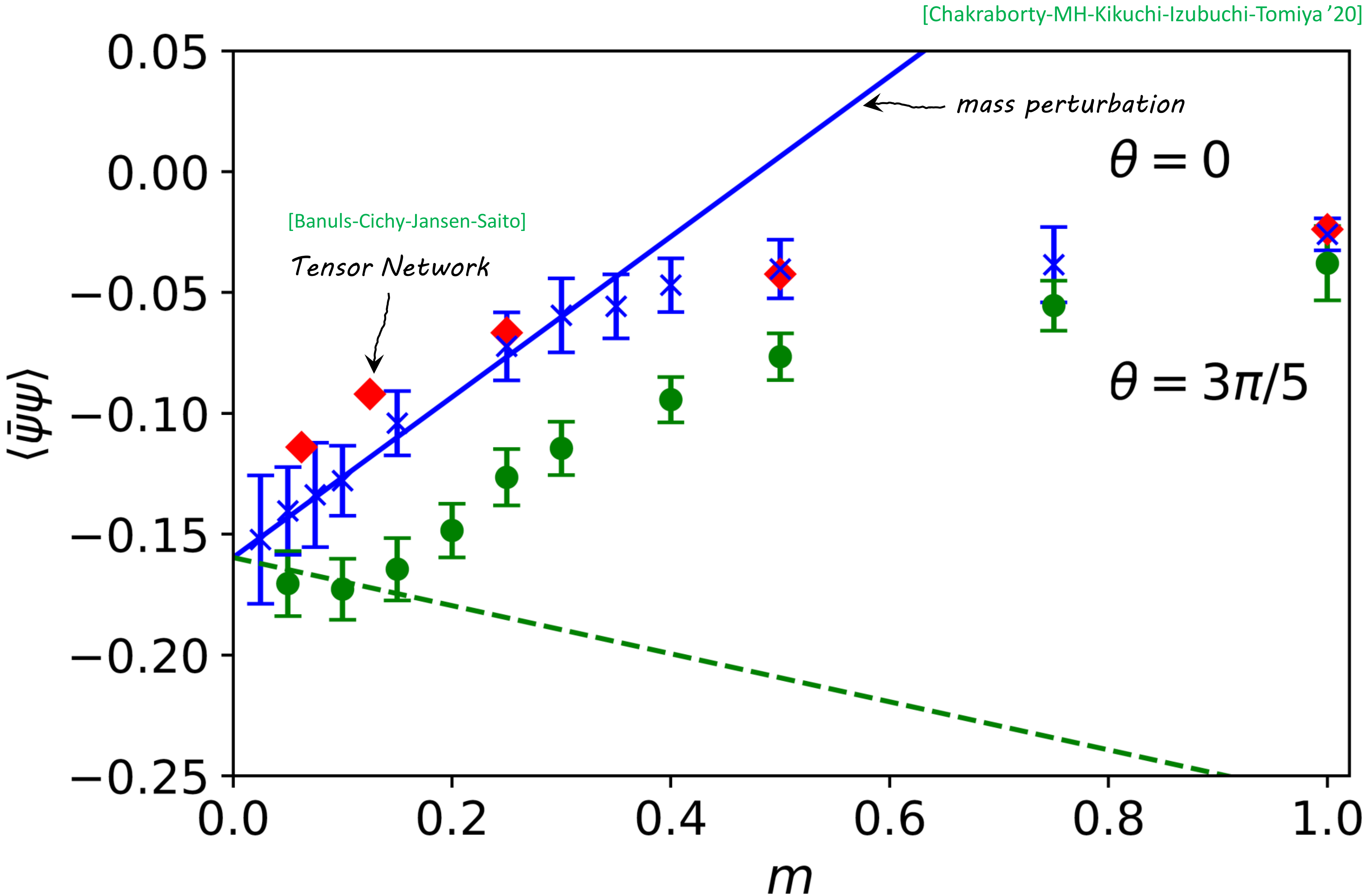
Chiral condens. for massless case (after continuum limit)

$T = 100, \delta t = 0.1, N_{\text{max}} = 16, 1M$ shots

[Chakraborty-MH-Kikuchi-Izubuchi-Tomiya '20]



Chiral condens. for massive case at g=1



Estimation of systematic errors

[Chakraborty-MH-Kikuchi-Izubuchi-Tomiya '20]

Approximation of vacuum:

$$|\text{vac} > \simeq U(T)U(T-\delta t)\cdots U(2\delta t)U(\delta t)|\text{vac}_0 > \equiv |\text{vac}_A\rangle$$

Approximation of VEV:

$$\langle\mathcal{O}\rangle \equiv \langle\text{vac}|\mathcal{O}|\text{vac}\rangle \simeq \langle\text{vac}_A|\mathcal{O}|\text{vac}_A\rangle$$

Introduce the quantity

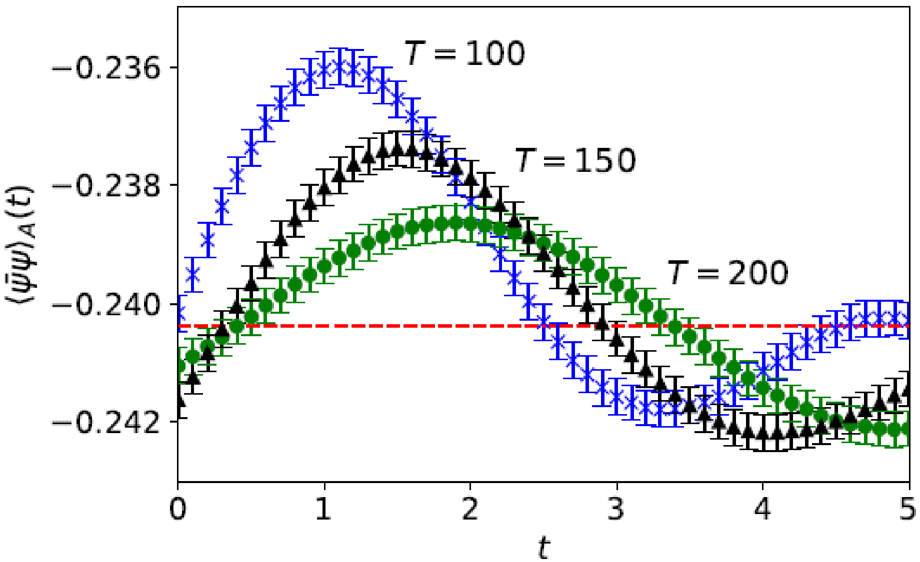
$$\langle\mathcal{O}\rangle_A(t) \equiv \langle\text{vac}_A|e^{i\hat{H}t}\mathcal{O}e^{-i\hat{H}t}|\text{vac}_A\rangle$$

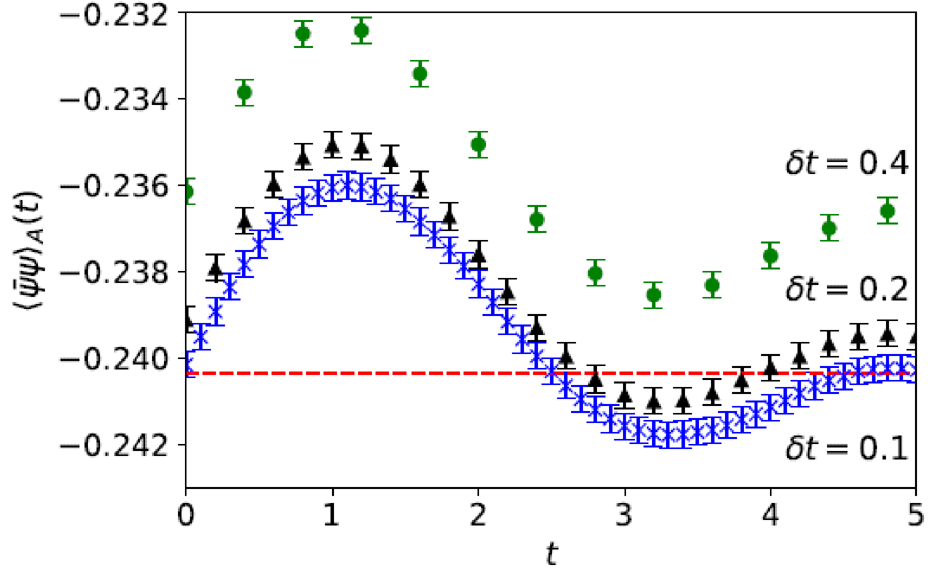
- independent of t if $|\text{vac}_A\rangle = |\text{vac}\rangle$
- dependent on t if $|\text{vac}_A\rangle \neq |\text{vac}\rangle$

This quantity describes intrinsic ambiguities in prediction

➡ Useful to estimate systematic errors

Estimation of systematic errors (Cont'd)





Oscillating around the correct value

Define central value & error as

$$\frac{1}{2}\left(\max\langle\mathcal{O}\rangle_A(t) + \min\langle\mathcal{O}\rangle_A(t)\right) \quad \& \quad \frac{1}{2}\left(\max\langle\mathcal{O}\rangle_A(t) - \min\langle\mathcal{O}\rangle_A(t)\right)$$

Contents

Expectations from previous analyzes (repeated)

Potential between probe charges $\pm q_p$ has been analytically computed

[Iso-Murayama '88, Gross-Klebanov-Matytsin-Smilga '95]

- massless case:

$\mu \equiv g/\sqrt{\pi}$

$$V(x) = \frac{q_p^2 g^2}{2\mu}(1 - e^{-q\mu x})$$

screening

- massive case: [cf. Misumi-Tanizaki-Unsal '19]

$\Sigma \equiv g e^{\gamma}/2\pi^{3/2}$

$$V(x) \sim mq\Sigma\left(\cos\left(\frac{\theta + 2\pi q_p}{q}\right) - \cos\left(\frac{\theta}{q}\right)\right) x \qquad (\mathrm{m} \ll g,\ |x| \gg 1/g)$$

$$\begin{cases} = \text{Const.} & \text{for } \mathrm{q_p/q} = \mathbf{Z} \quad \textit{screening} \\ \propto x & \text{for } \mathrm{q_p/q} \neq \mathbf{Z} \quad \textit{confinement?} \end{cases}$$

but sometimes negative slope!

Let's explore this aspect by quantum simulation!

FAQs on negative tension

Q1. It sounds that many pair creations are favored.
Is the theory unstable?

— No. Negative tension appears only for $q_p \neq q\boldsymbol{Z}$.

So, such unstable pair creations do not occur.

Q2. It sounds $E_{\mathrm{inside}} < E_{\mathrm{outside}} (= E_0 ?)$. Strange?

[cf. MH-Itou-Kikuchi-Tanizaki '21]

— $^\exists$ explanation from generalized (1-form) global sym.

The theory has something like superselection sector decomposed by Z_q 1-form symmetry. *"universe"*

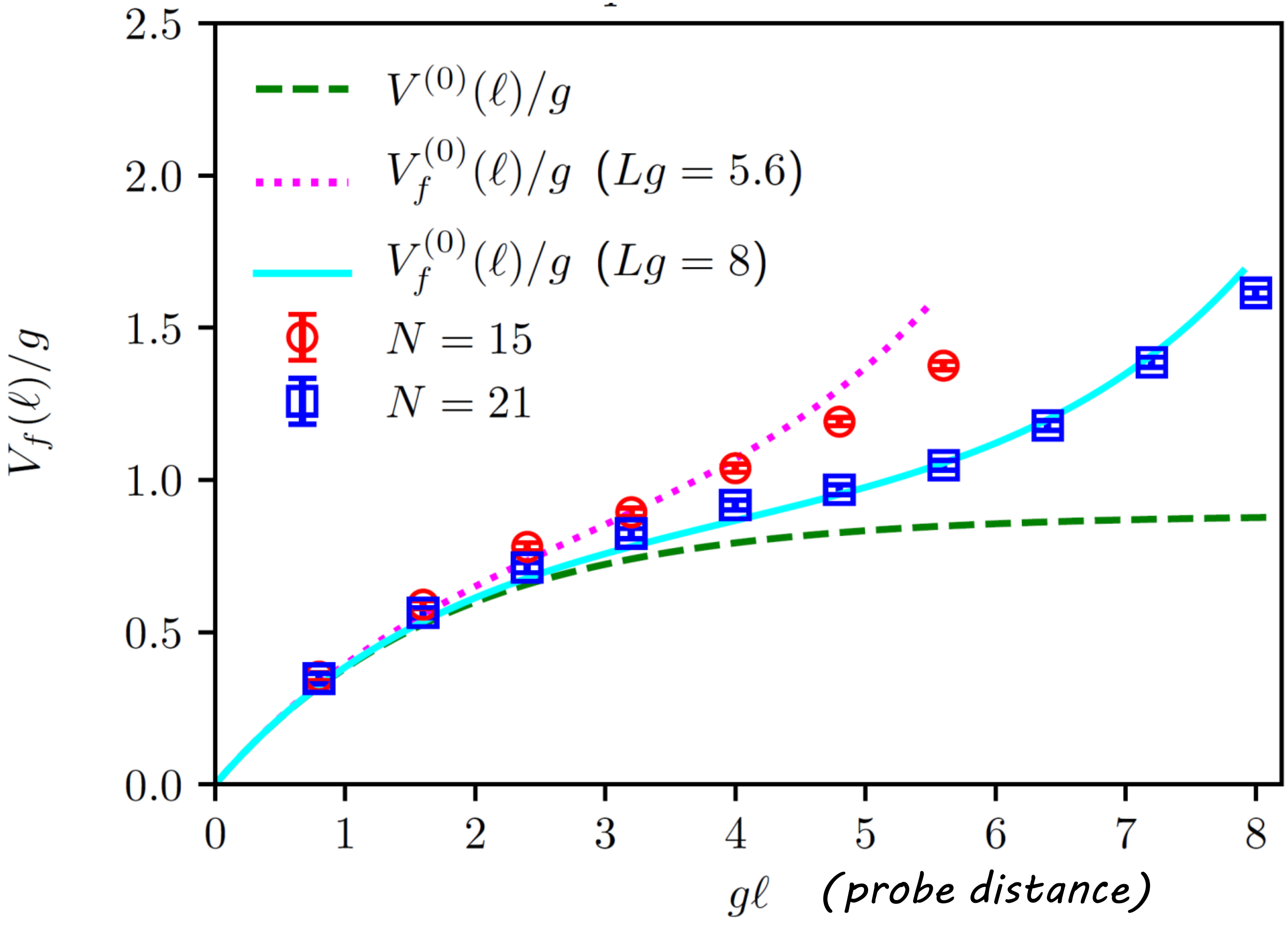
Inside & outside belong to different sectors.

Results for massless, $\theta_0 = 0$ & $q_p/q \in \mathbf{Z}$

[MH-Itou-Kikuchi-Nagano-Okuda '21]

Parameters: $g = 1, a = 0.4, N = 15$ & $21, T = 99, q_p/q = 1, m = 0$

Lines: analytical results in the continuum limit (finite & ∞ vols.)

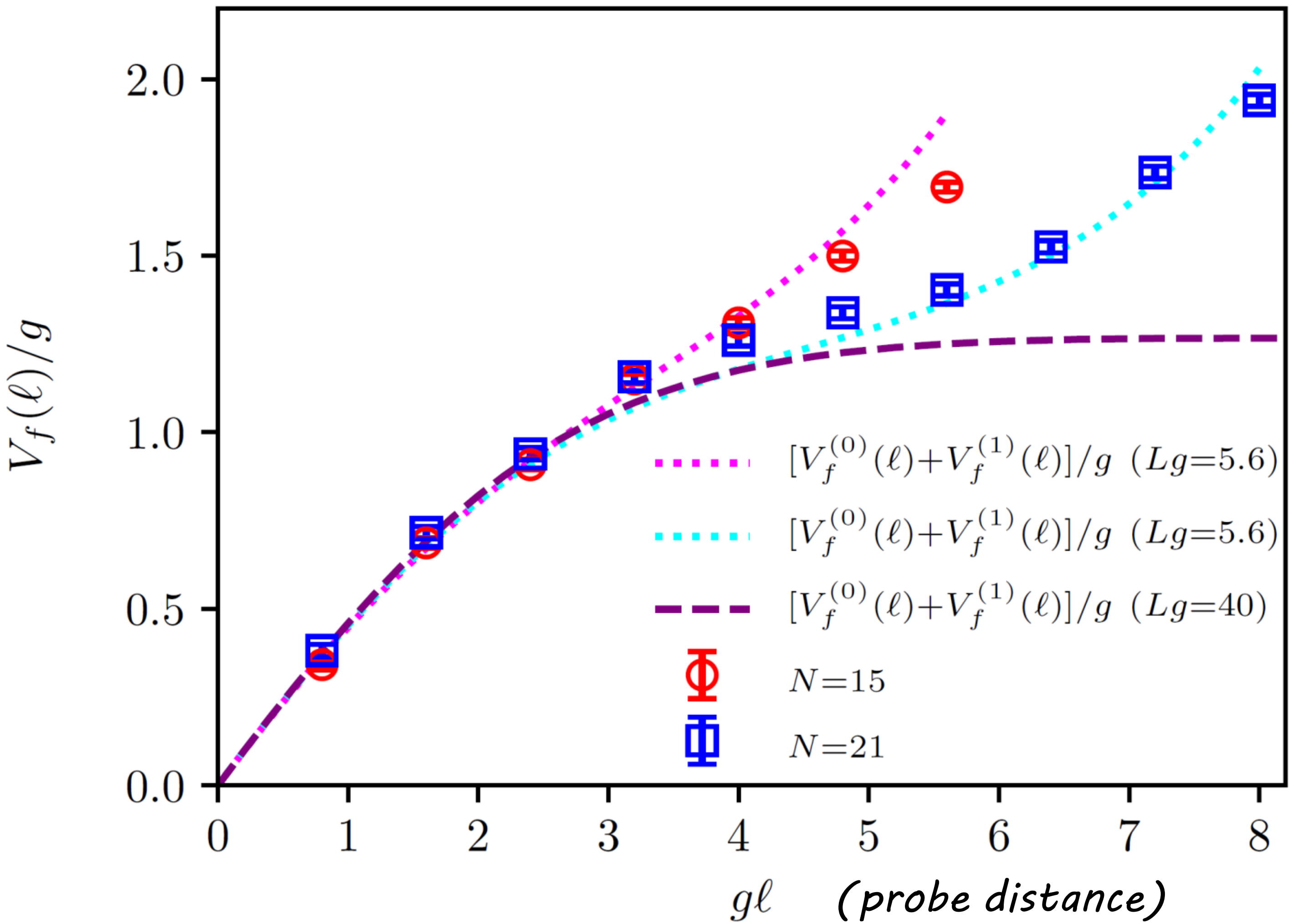


Results for massive, $\theta_0 = 0$ & $q_p/q \in \boldsymbol{Z}$

[MH-Itou-Kikuchi-Nagano-Okuda '21]

Parameters: $g = 1, a = 0.4, N = 15$ & $21, T = 99, q_p/q = 1, m = 0.2$

Lines: analytical results in the continuum limit (finite & ∞ vols.)



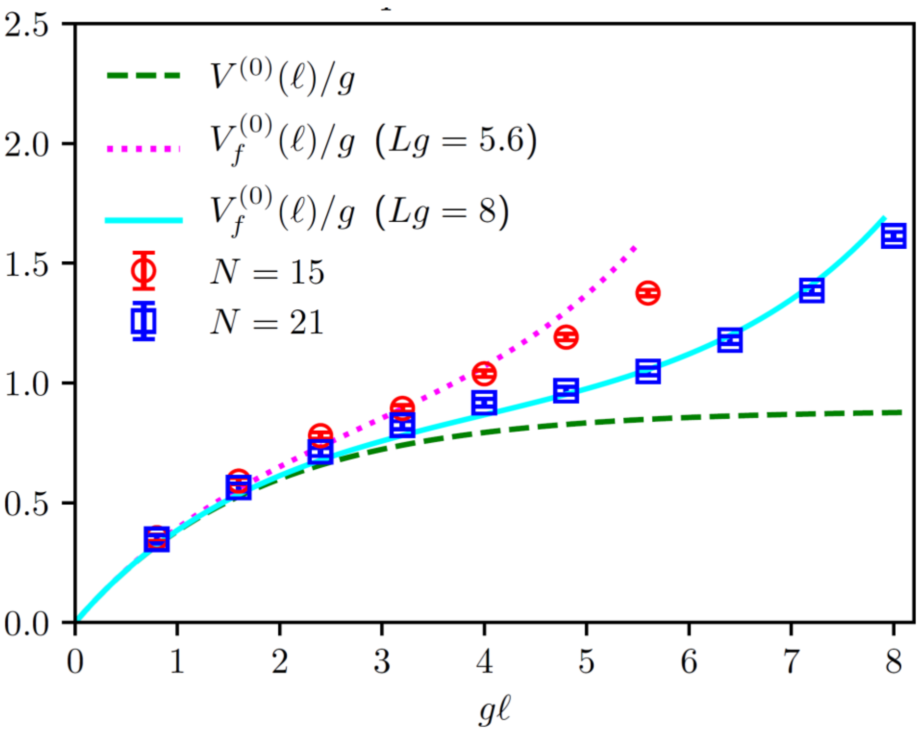
Massless vs massive for $\theta_0 = 0$ & $q_p/q \in \mathbf{Z}$

[MH-Itou-Kikuchi-Nagano-Okuda '21]

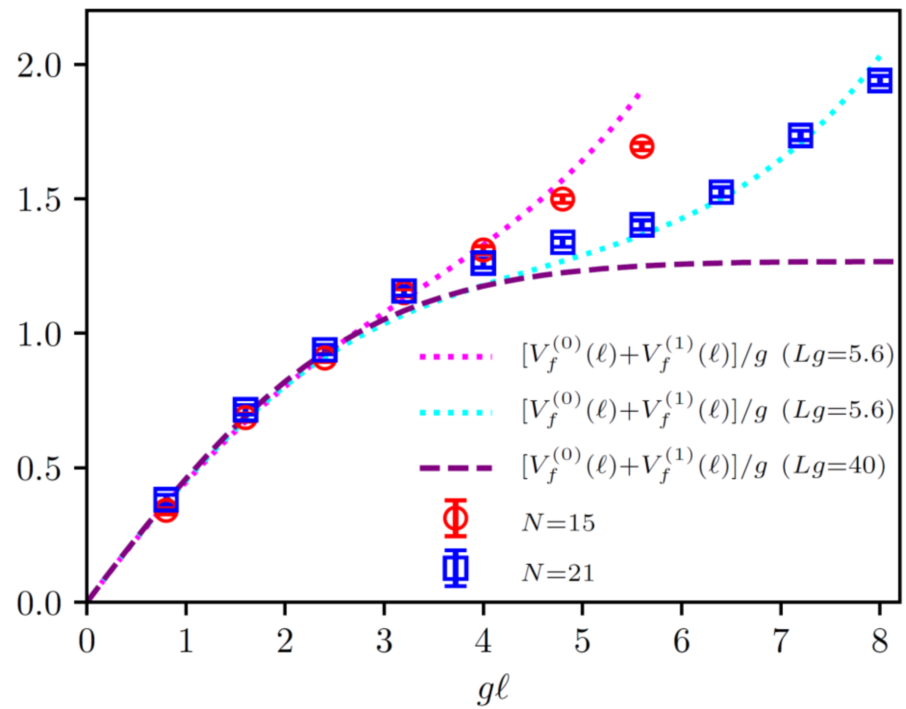
Parameters: $g = 1, a = 0.4, N = 15$ & $21, T = 99, q_p/q = 1$

Lines: analytical results in the continuum limit (finite & ∞ vols.)

$q_p = 1, m = 0$



$q_p = 1, m/g = 0.2$



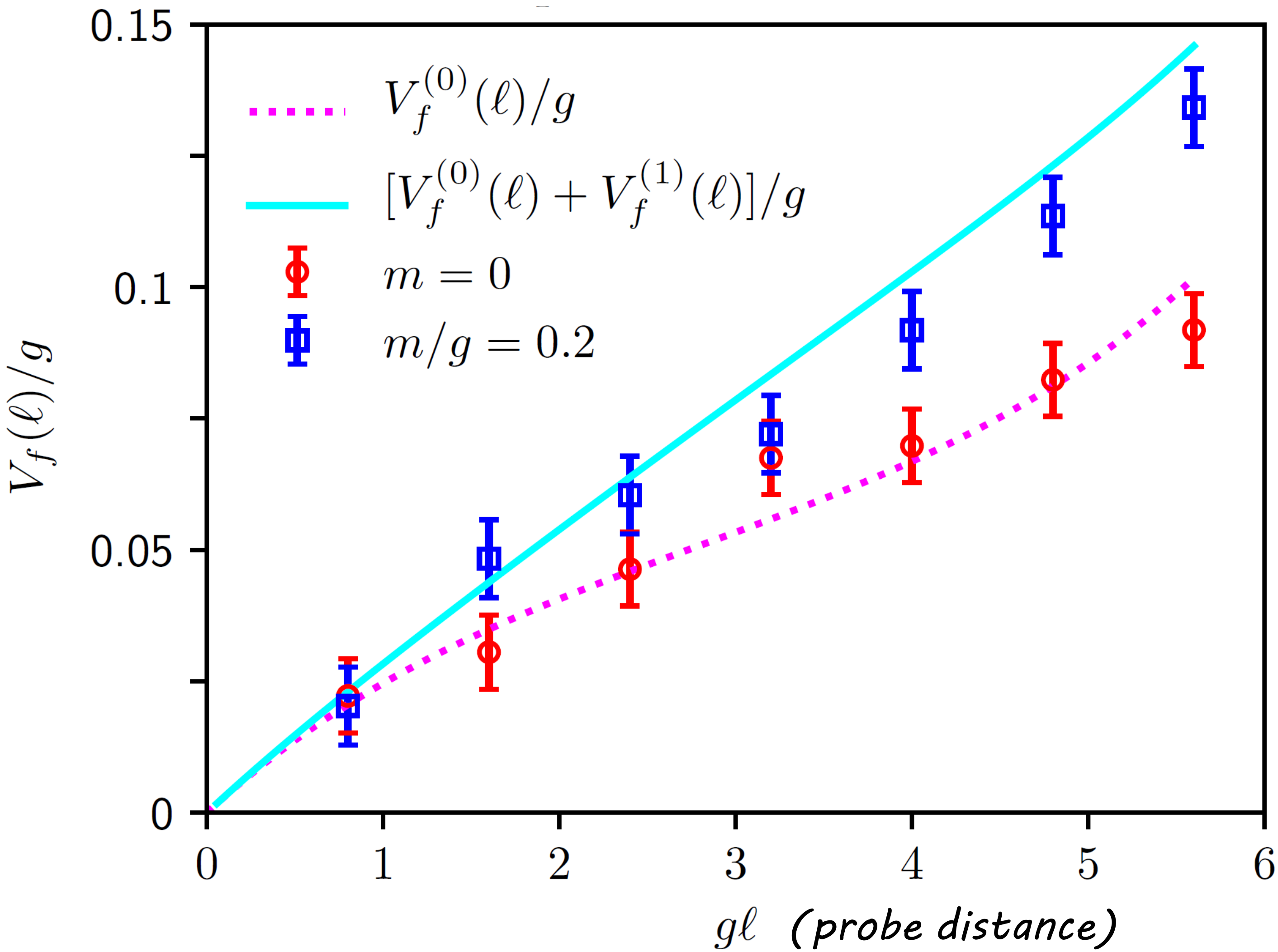
Consistent w/ expected screening behavior

Results for $\theta_0 = 0$ & $q_p/q \notin \mathbf{Z}$

[MH-Itou-Kikuchi-Nagano-Okuda '21]

Parameters: $g = 1, a = 0.4, N = 15, T = 99, q_p/q = 1/4, m = 0$ & 0.2

Lines: analytical results in the continuum limit (finite & ∞ vol.)



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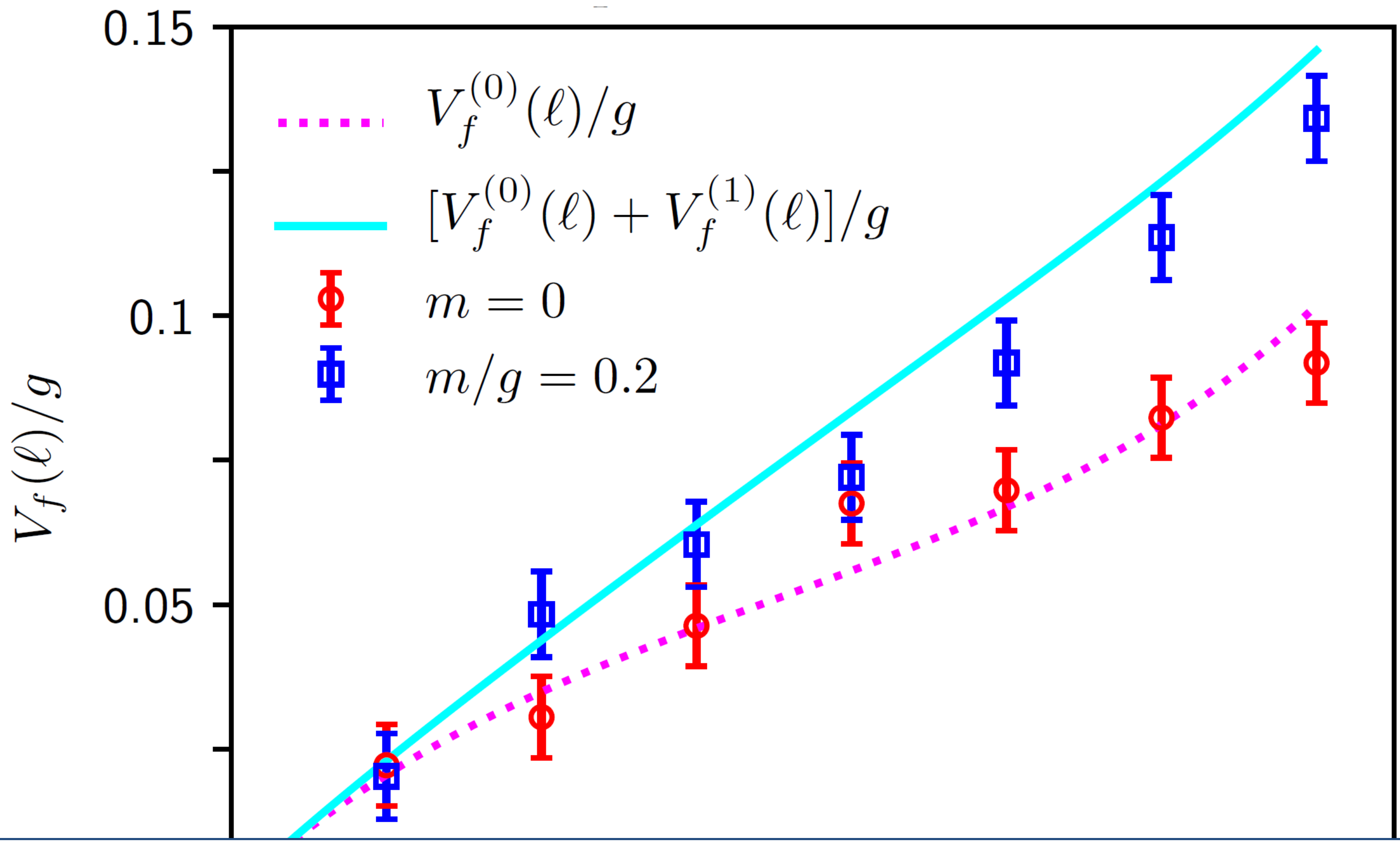


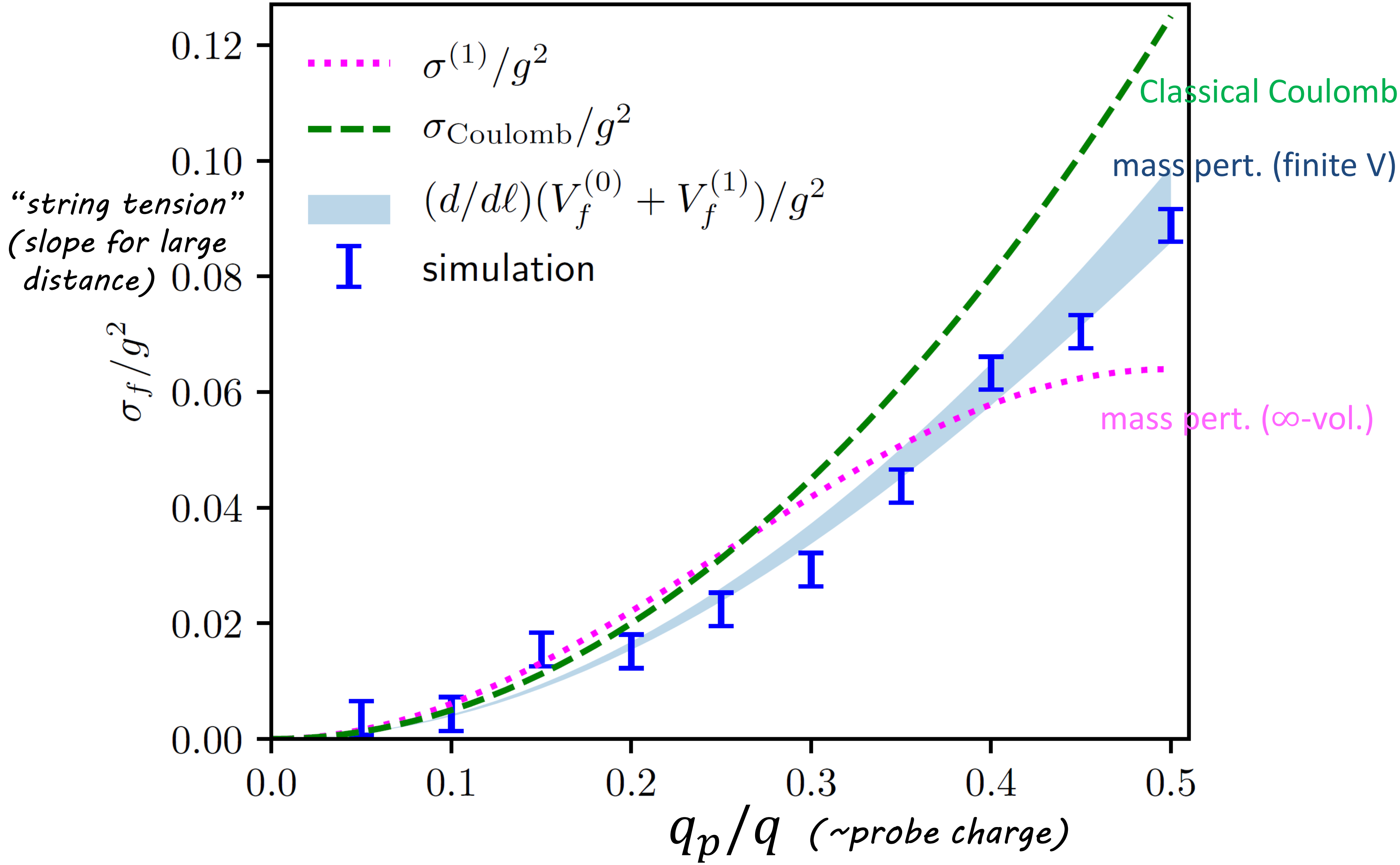
Consistent w/ expected confinement behavior

-> interesting to estimate string tension for various q_p?

“String tension” for $\theta_0 = 0$

Parameters: $g = 1, a = 0.4, N = 15, T = 99, m/g = 0.2$

[MH-Itou-Kikuchi-Nagano-Okuda ’21]



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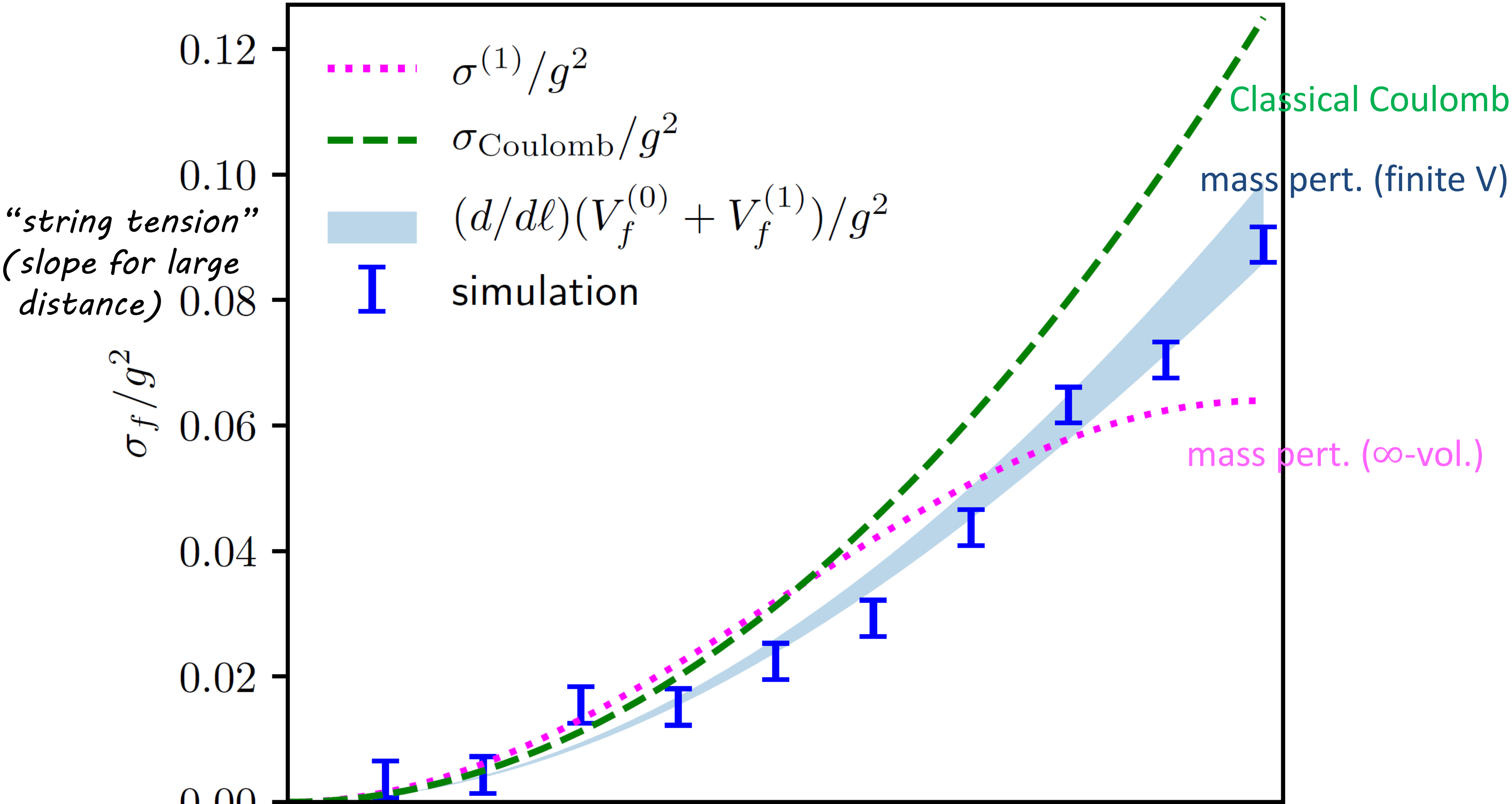
[MH-Itou-Kikuchi-Nagano-Okuda ’21]

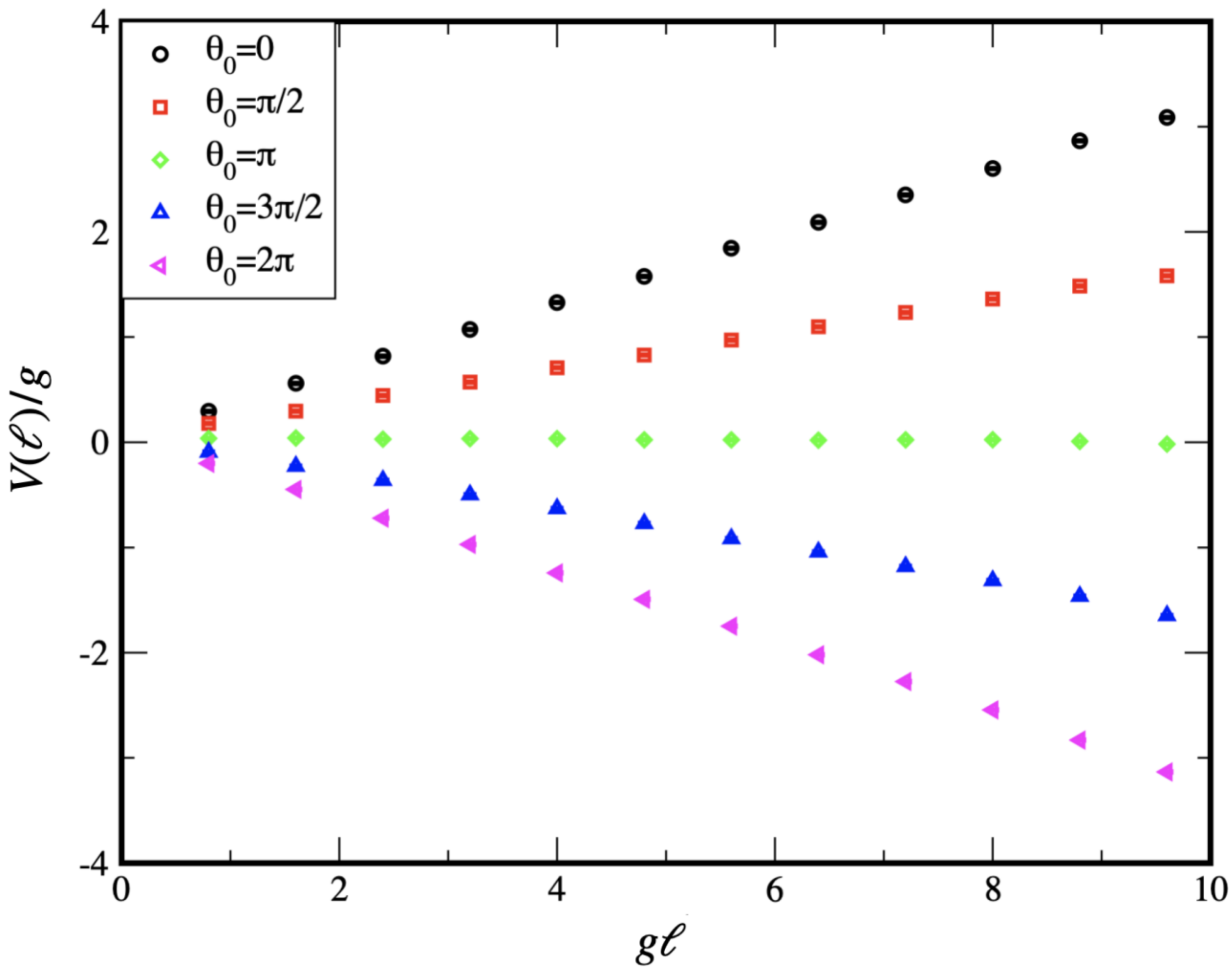


confinement by nontrivial dynamics!

Positive / negative string tension

[MH-Itou-Kikuchi-Tanizaki '21]

Parameters: $g = 1, a = 0.4, N = 25, T = 99, q_p/q = -1/3, m = 0.15$

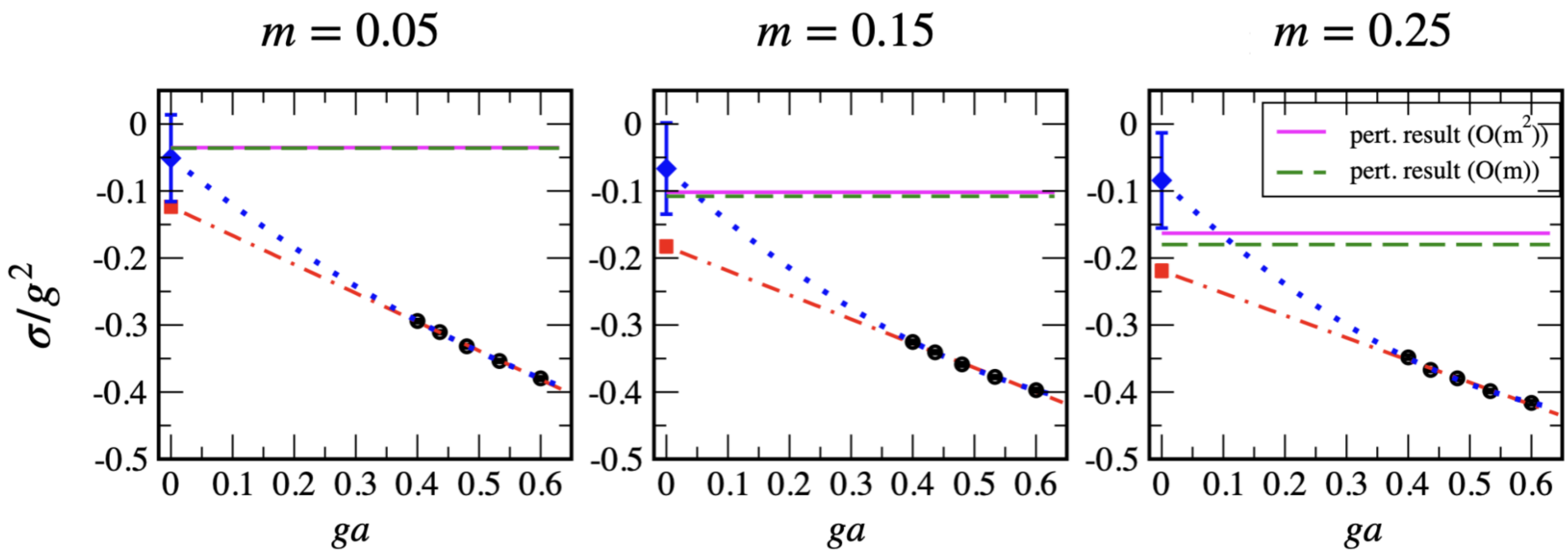


Sign(tension) changes as changing θ-angle!!

Continuum limit of string tension

[MH-Itou-Kikuchi-Tanizaki '21]

$g = 1,\ (\mathrm{Vol.}) = 9.6/g, T = 99,\ q_p/q = -1/3, m = 0.15, \theta_0 = 2\pi$

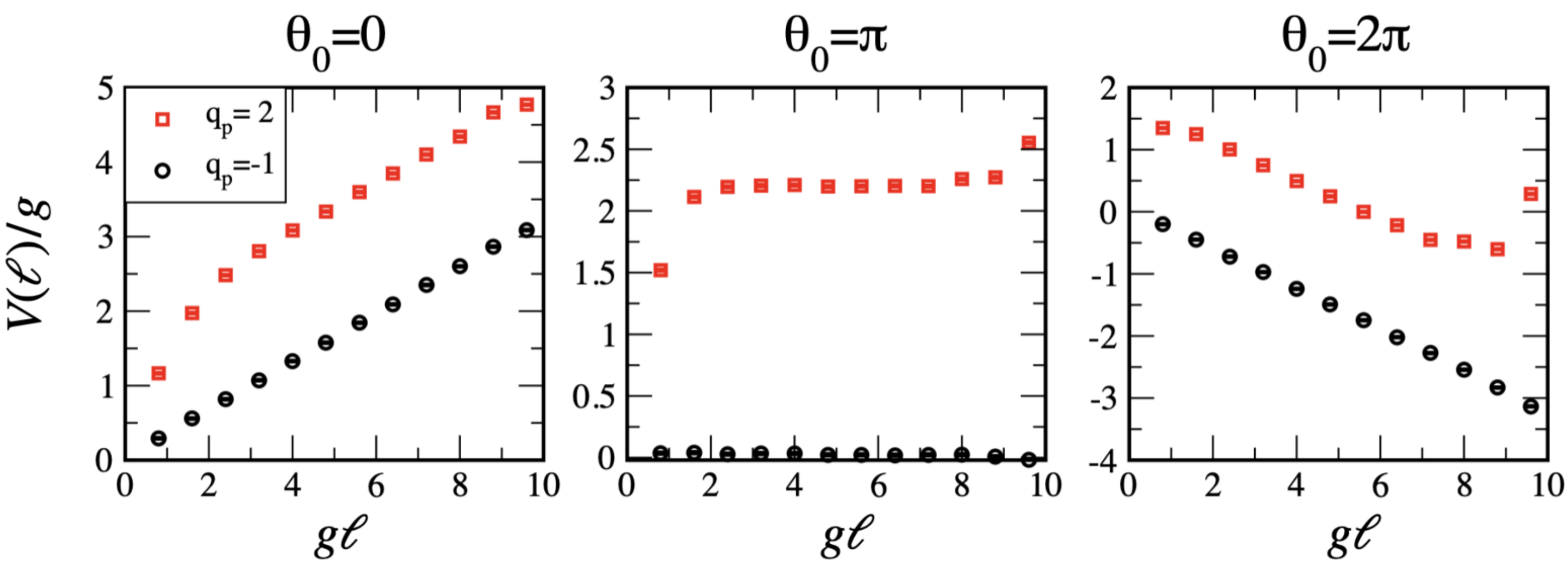


basically agrees with mass perturbation theory

Comparison of $q_p/q = -1/3$ & $q_p/q = 2/3$

[MH-Itou-Kikuchi-Tanizaki '21]

Parameters: $\mathrm{q} = 3, g = 1, a = 0.4, N = 25, T = 99, m = 0.15$

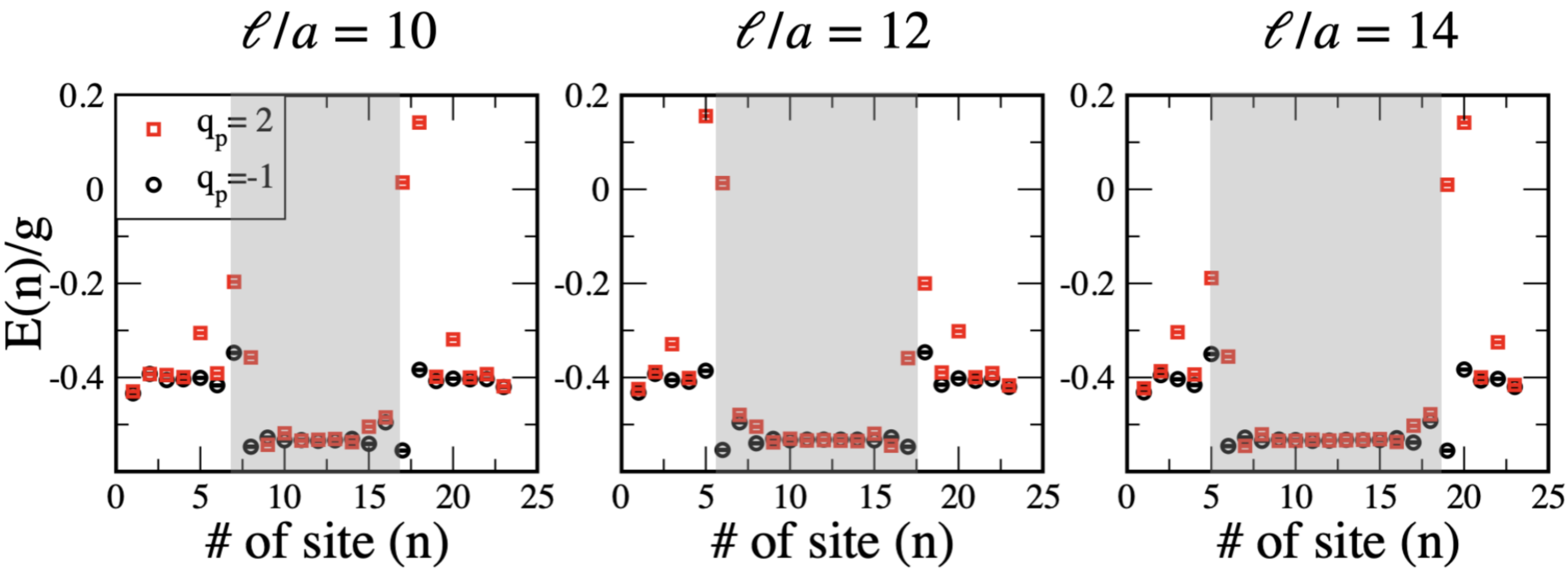


Similar slopes → (approximate) Z_3 symmetry

Energy density @ negative tension regime

[MH-Itou-Kikuchi-Tanizaki '21]

$$g = 1, a = 0.4, N = 25, T = 99,\ q_p/q = -1/3, m = 0.15, \theta_0 = 2\pi$$



Lower energy inside the probes!!

Summary & Outlook

Summary

- Quantum computation is suitable for Hamiltonian formalism which is free from sign problem

- Instead we have to deal with huge vector space. Quantum computers in future may do this job.

- We've constructed the vacuum of Schwinger model w/ the topological term by adiabatic state preparation

- found agreement in the chiral condensate with the exact result for $m = 0$ & mass perturbation theory for small m
  [Chakraborty-MH-Kikuchi-Izubuchi-Tomiya '20]

- explored the screening vs confinement problem & negative string tension behavior
  [MH-Itou-Kikuchi-Nagano-Okuda '21]
  [MH-Itou-Kikuchi-Tanizaki '21]

Outlook

- The problems in this talk involve only ground state
  → Tensor Network is better → DMRG w/ $N = \mathcal{O}(100)$

  [work in progress, MH-Itou-Kikuchi-Tanizaki]

- Searching critical point at $\theta = \pi$ [work in progress, Chakraborty-MH-Kikuchi-Izubuchi-Tomiya]

- Other ways to prepare vacuum (e.g. variational method, imaginary time evolution)

  [work in progress, MH-Kikuchi-Rendon]

- Real time simulation? [work in progress, Chakraborty-MH-Inotani-Itou-Kikuchi]

- Scattering amplitude?

- Other field theories (bosonic, higher dim., etc…)

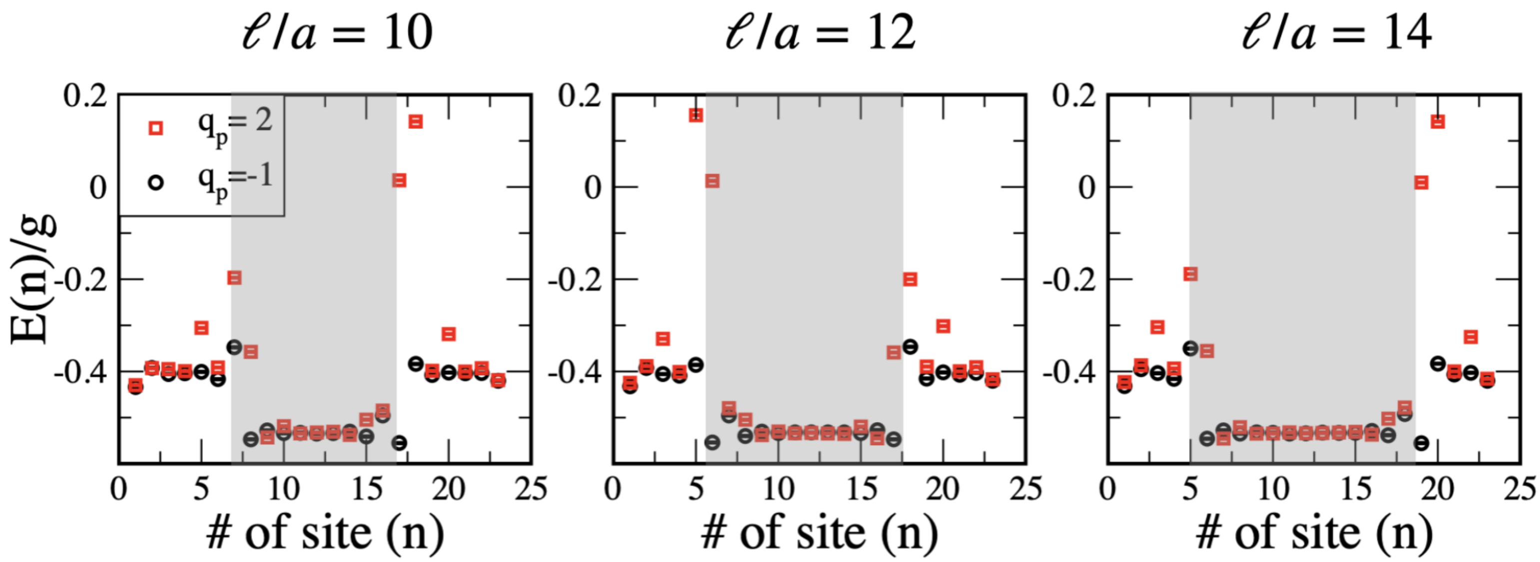
- Including quantum error correction/mitigation?

- Something not efficiently simulated by MC & TN

etc…

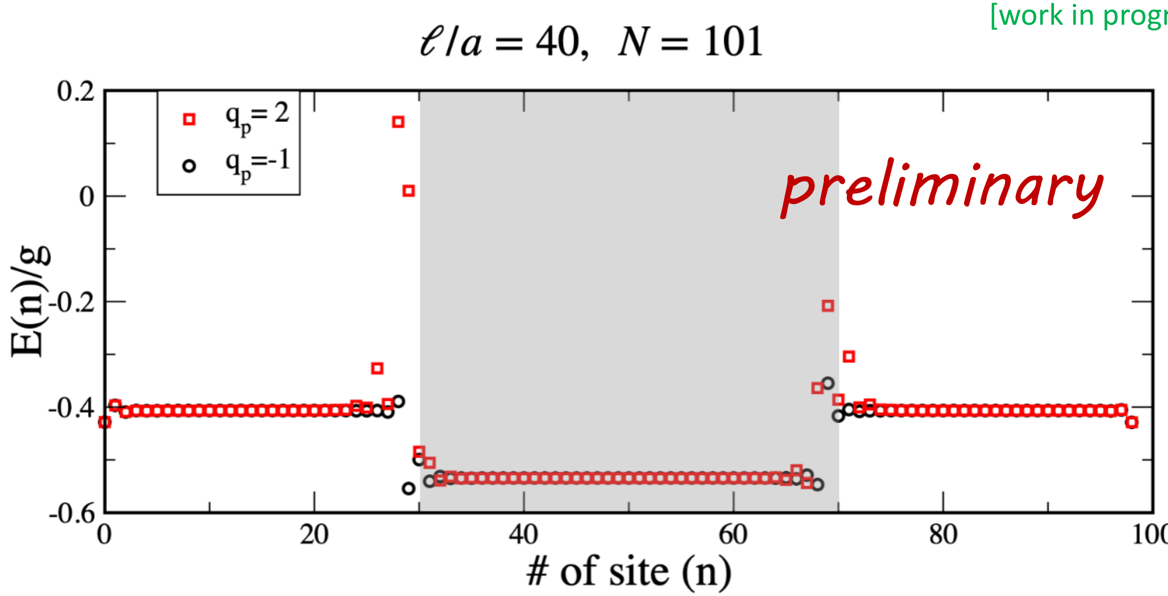
Adiabatic state preparation:

[MH-Itou-Kikuchi-Tanizaki ’21]



DMRG:

[work in progress: MH-Itou-Kikuchi-Tanizaki]



Thanks!

Appendix

Without probes

Massless case

For massless case,

θ is absorbed by chiral rotation ⇒ $\theta = 0$ w/o loss of generality

No sign problem

Nevertheless,

it's difficult in conventional approach because computation of fermion determinant becomes very heavy

$^\exists$ Exact result:

[Hetrick-Hosotani '88]

$$\langle\bar{\psi}(x)\psi(x)\rangle = -\frac{e^{\gamma}}{2\pi^{3/2}}g \simeq -0.160g$$

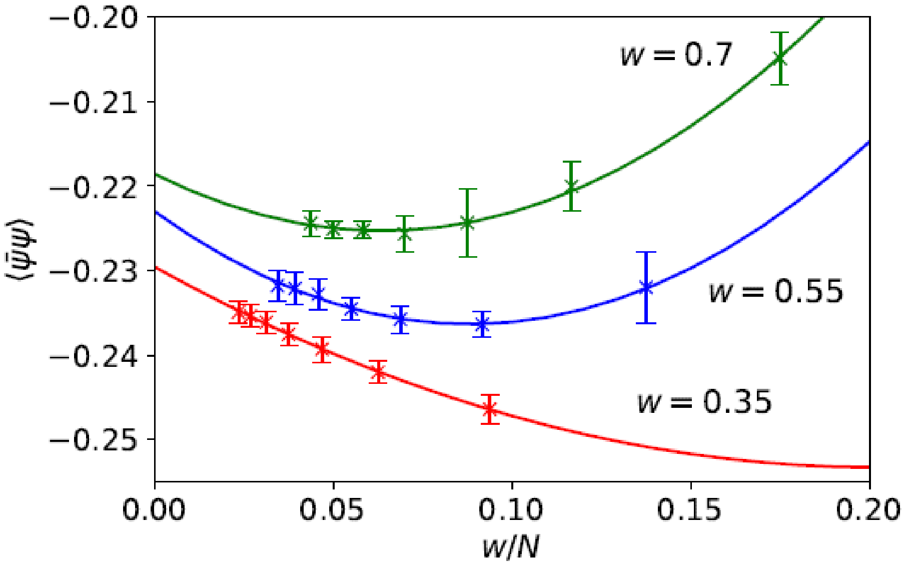
Can we reproduce it?

Thermodynamic & Continuum limit

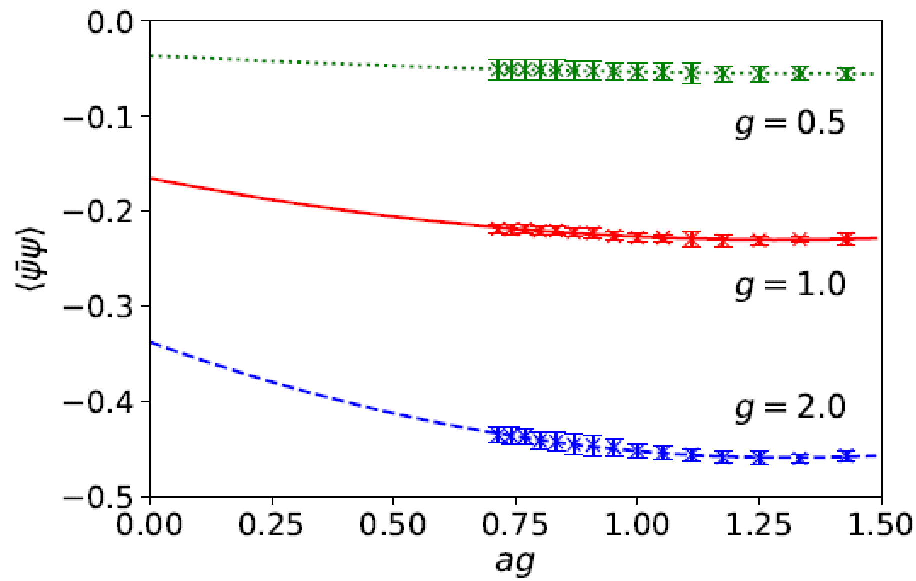
$g = 1, m = 0, N_{\text{max}} = 16, T = 100, \delta t = 0.1, 1M$ shots

#(measurements)

Thermodynamic limit (w/ fixed a)



Continuum limit (after $V \to \infty$)



Massive case

Result of mass perturbation theory: [Adam '98]

$$\langle \bar{\psi}(x)\psi(x) \rangle \simeq -0.160g + 0.322m\cos\theta + \mathcal{O}(m^2)$$

However,

∃ subtlety in comparison: this quantity is UV divergent

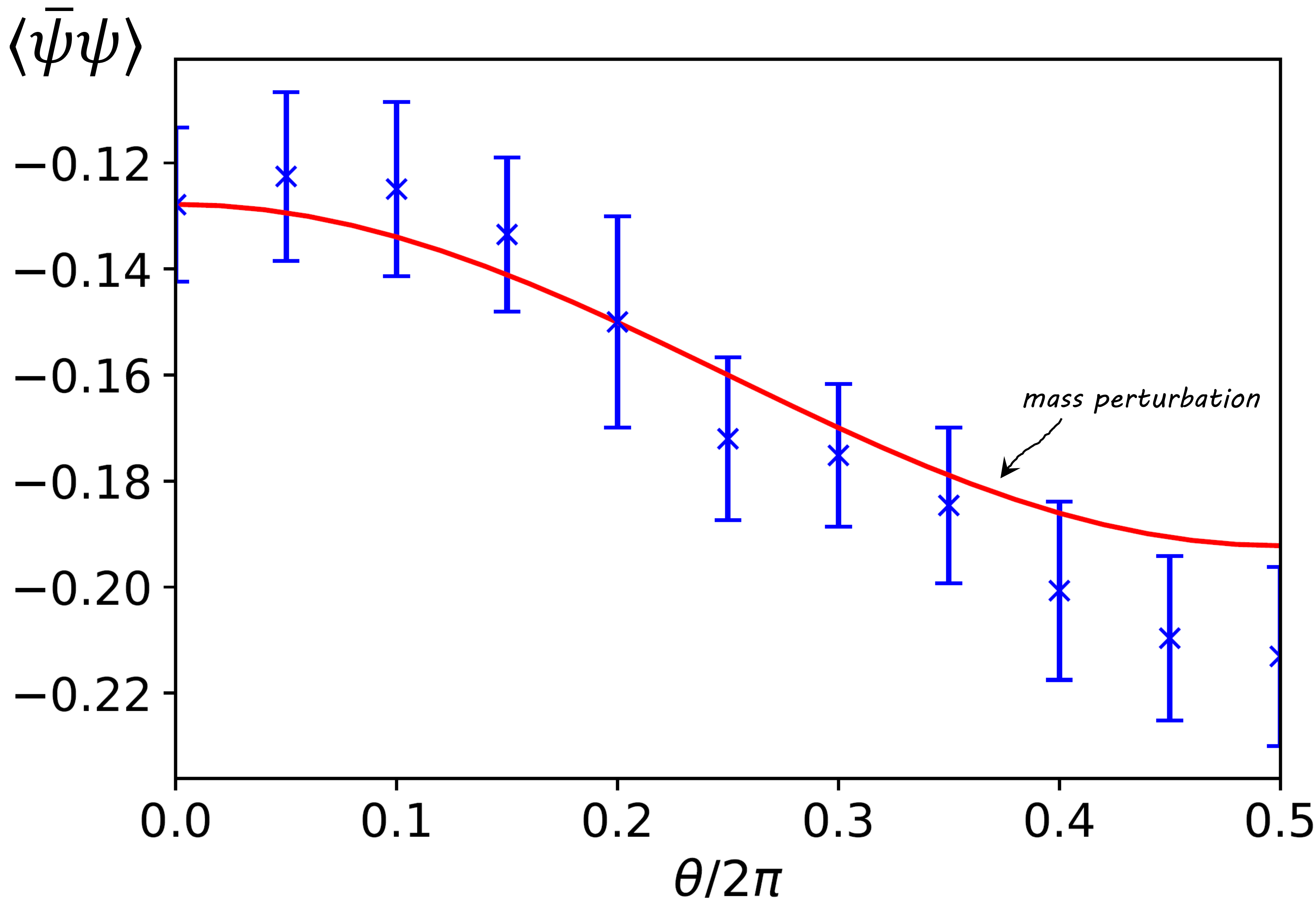
$(\sim m \log \Lambda)$

➡ Use a regularization scheme to have the same finite part

Here we subtract free theory result before taking continuum limit:

$$\lim_{a\to 0}\left[\langle\bar{\psi}\psi\rangle - \langle\bar{\psi}\psi\rangle_{\text{free}}\right]$$

θ dependence at $m = 0.1$ & $g = 1$



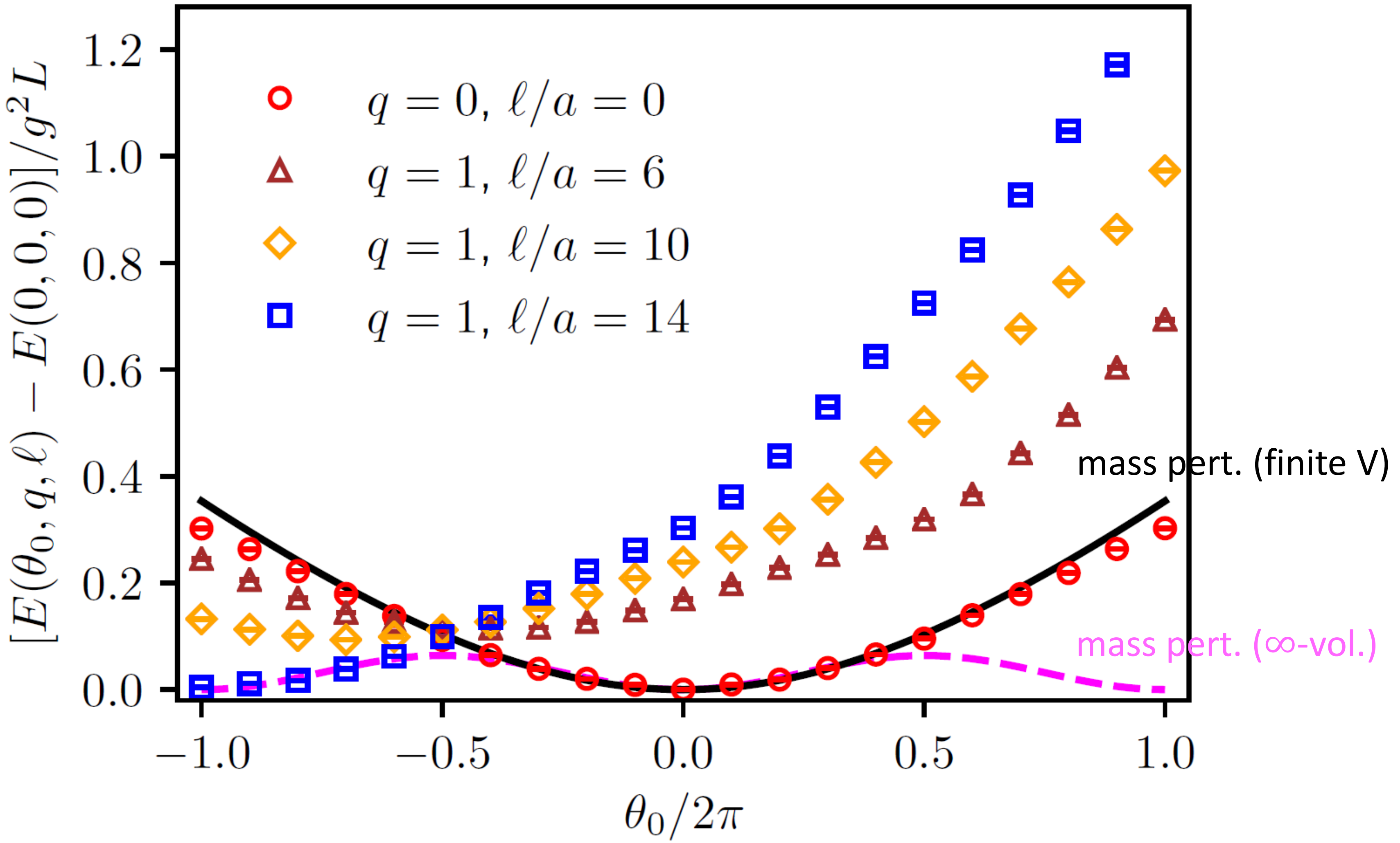
With probes

Results for $\theta_0 \neq 0$

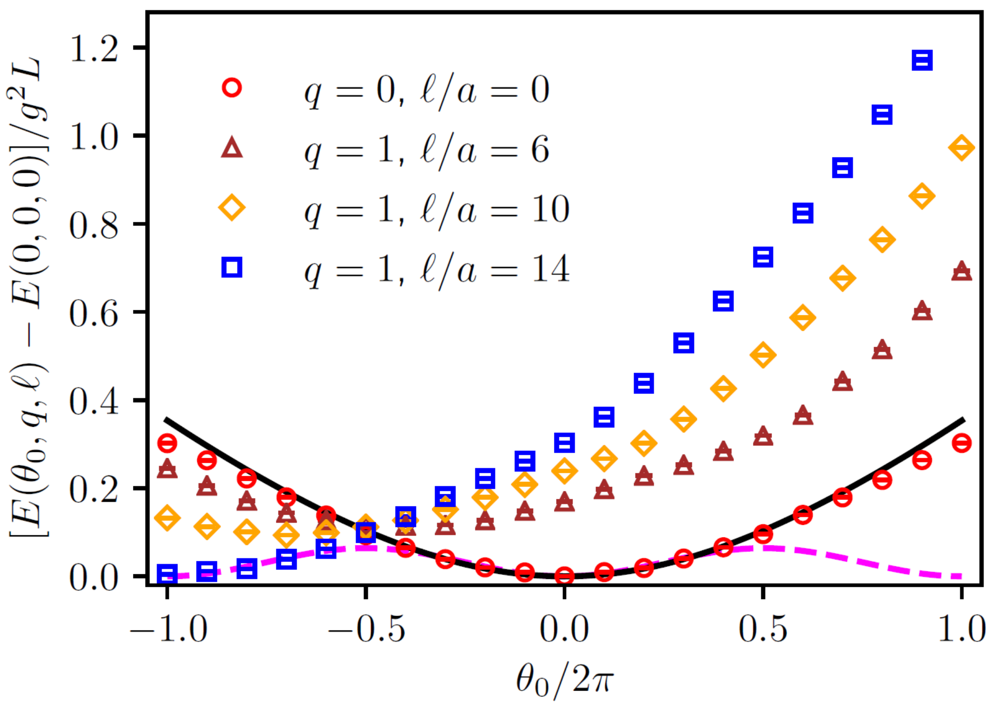
[MH-Itou-Kikuchi-Nagano-Okuda '21]

(difficult to explore by the conventional Monte Carlo approach)

Parameters: $g = 1, a = 0.4, N = 15, T = 99, q_p/q = 1, m/g = 0.2$



Comment on theta angle periodicity



Absence of the periodicity: $\theta_0 \sim \theta_0 + 2\pi$?

This is expected because we're taking open b.c.

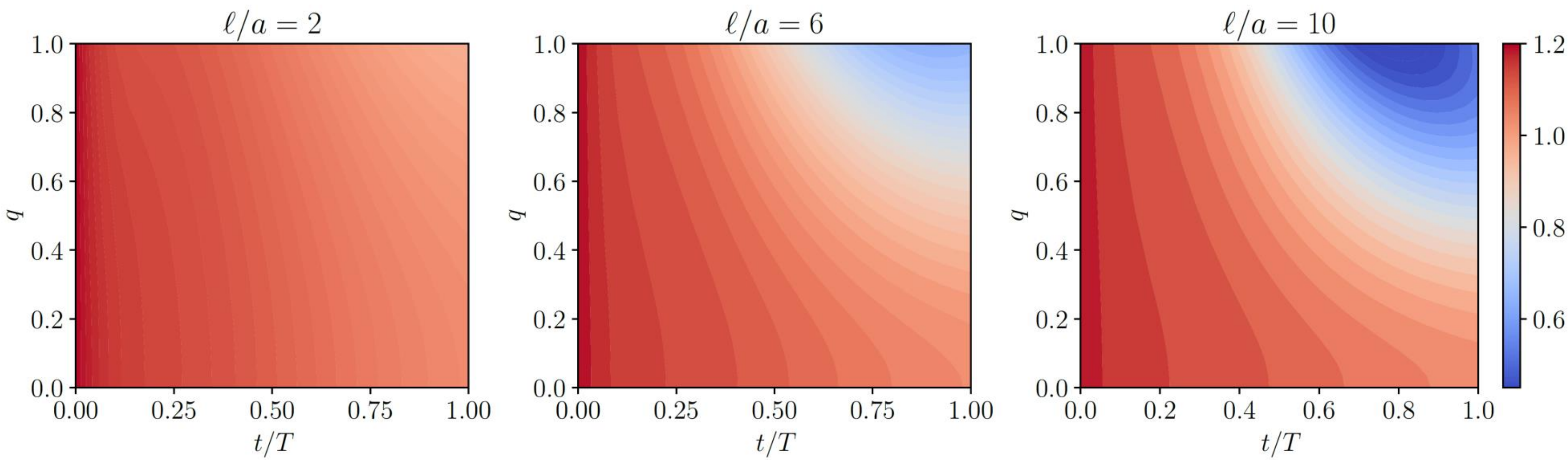
To get the periodicity back, we need to take ∞-vol. limit

[detailed study: work in progress, MH-Itou-Kikuchi-Tanizaki]

Comment: density plots of energy gap

(known as “Tuna slice plot” inside the collaboration) [MH-Itou-Kikuchi-Nagano-Okuda ’21]

Parameters: $g = 1, a = 0.4, N = 15, q_p/q = 1, m/g = 0.15$

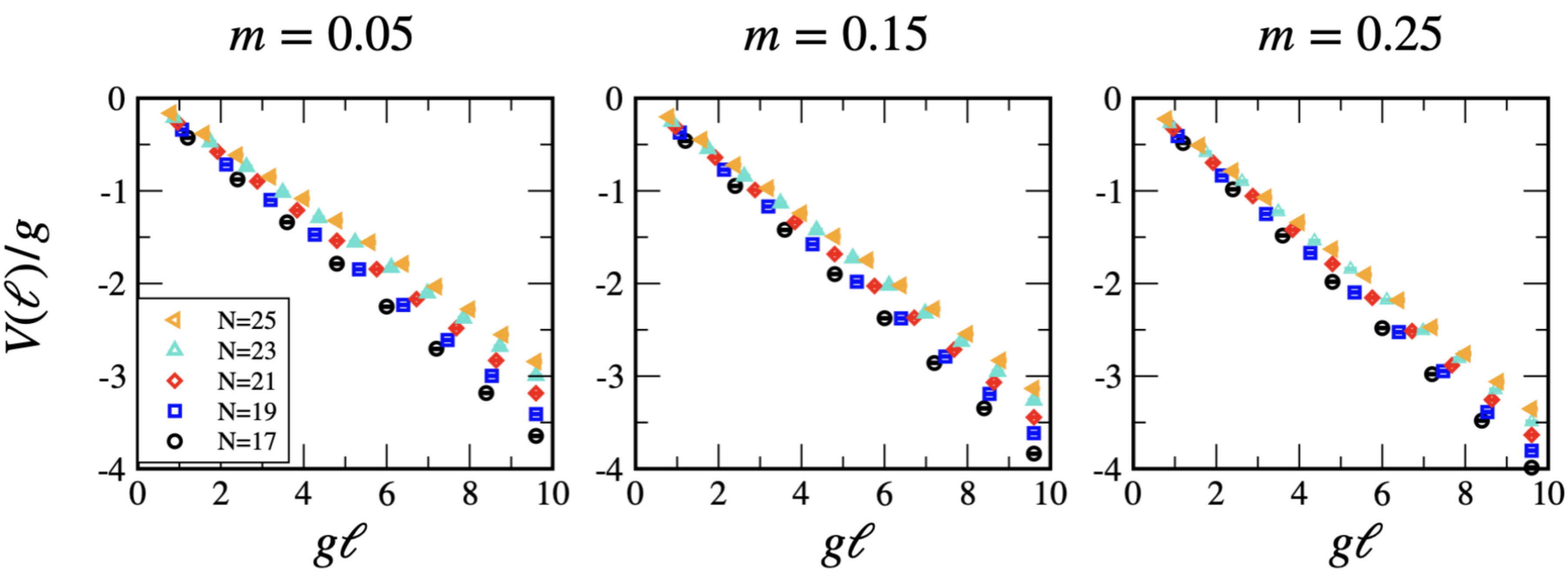


smaller gap for larger ℓ

⇒ larger systematic error for larger ℓ

N-dependence of V w/ fixed physical volume

[MH-Itou-Kikuchi-Tanizaki ’21]



Adiabatic scheduling

[MH-Itou-Kikuchi-Tanizaki '21]

$$N = 17, ga = 0.40, m = 0.20, q_p = 2, \theta_0 = 2\pi,$$