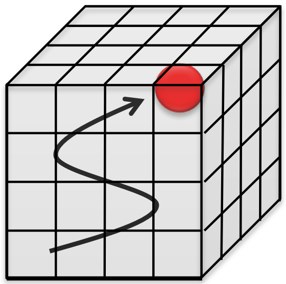
Quantum simulation for lattice gauge theory

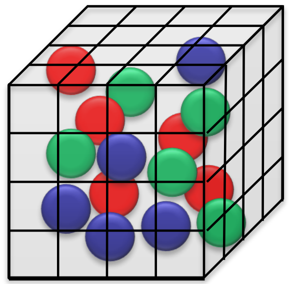
Arata Yamamoto (University of Tokyo)

Introduction

open problems in lattice gauge theory

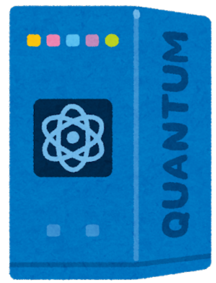


non-equilibrium



nonzero density

quantum computer

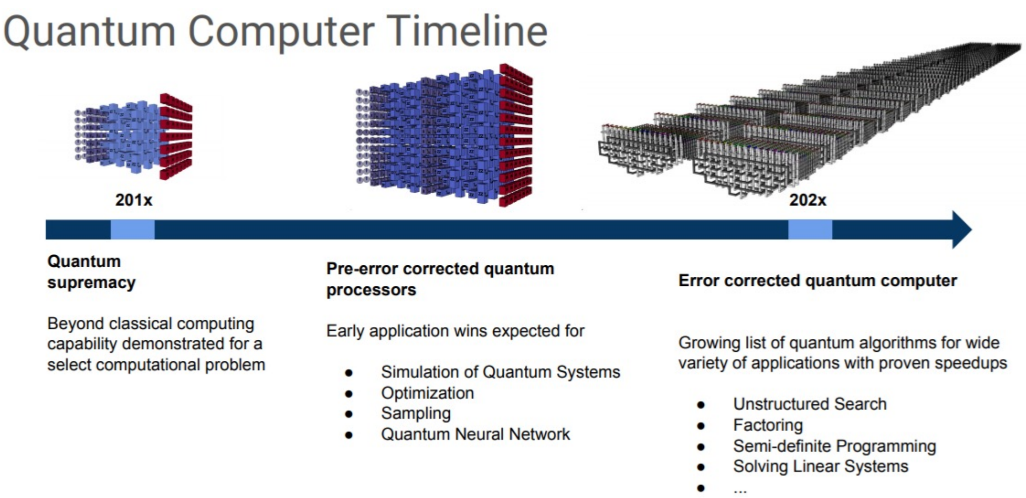


Introduction

noisy intermediate-scale quantum (NISQ)

noisy = device error

intermediate-scale = limited resource



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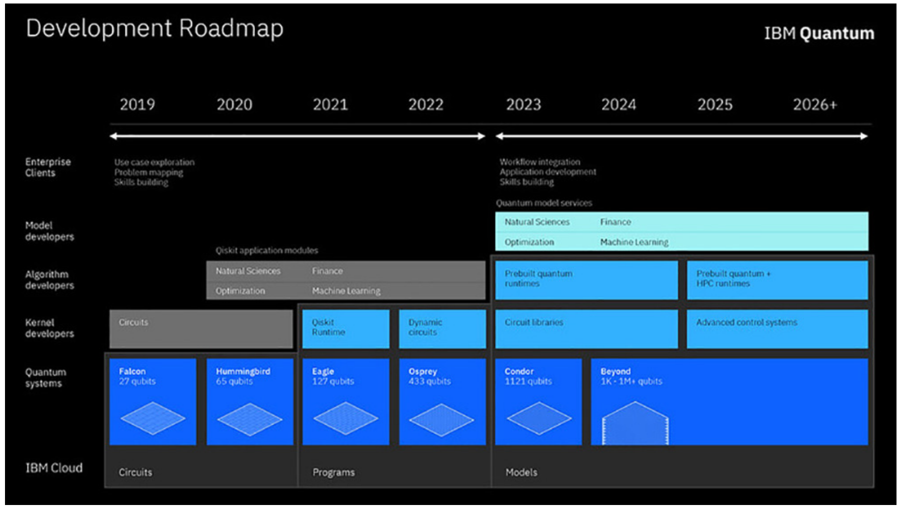

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Introduction

Hamiltonian formalism

$$E = \langle U | H | U \rangle$$

Hamiltonian operator — quantum state

Lagrangian formalism

$$\mathcal{Z} = \int dU \, e^{-S}$$

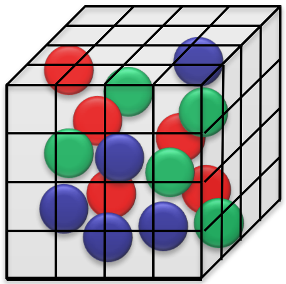
c-number — classical action

Which do you prefer?

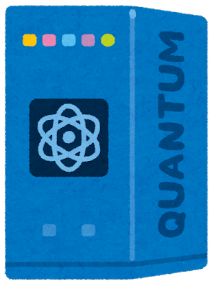
1. Hamiltonian formalism PRD 104, 014506 (2021) [arXiv:2104.10669]

2. Lagrangian formalism arXiv:2201.12556

1. Hamiltonian formalism

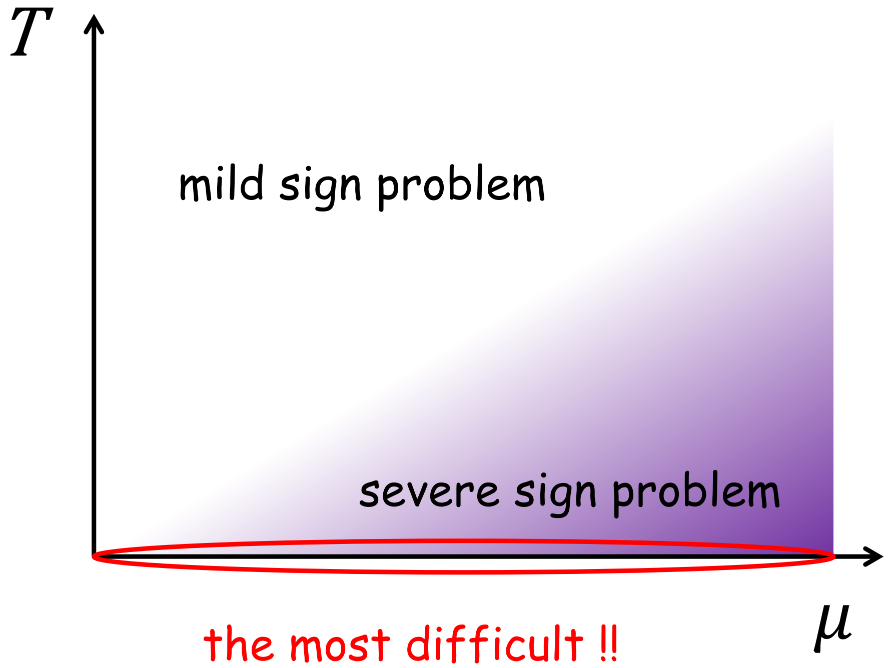


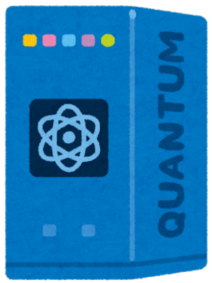
nonzero density



quantum computer

1. Hamiltonian formalism





quantum computer

1. Hamiltonian formalism

$$[H, Q] = 0 \quad \longrightarrow \quad Q|\Psi(q)\rangle = q|\Psi(q)\rangle$$

zero temperature = ground state $|\Psi(q)\rangle$

nonzero density = fermion number $q \neq 0$

1. Hamiltonian formalism

ground state of **full Hamiltonian** ↓

ground state of **solvable Hamiltonian** ↓

$$|\Psi(q)\rangle = \prod_{l=1,\cdots,L} U(l)\,|\Psi_0(q)\rangle$$

↑

quantum-gate operation s.t. $[U(l), Q] = 0$

1. Hamiltonian formalism

quantum adiabatic algorithm

Farhi *et al.* (2000)

- ✓ adiabatic theorem
- ✓ $U(l) = \exp\left[-i\delta t\left(H_0 + \frac{l}{L}H_1\right)\right]$
- ✓ exact in $L \to \infty$

quantum variational algorithm

Peruzzo *et al.* (2014)

- ✓ hybrid variational method
- ✓ $U(l) = \exp\left[-i\left(\alpha H_0 + \beta\frac{l}{L}H_1\right)\right]$

  variational parameters (α, β)
- ✓ L can be small

1. Hamiltonian formalism

benchmark test

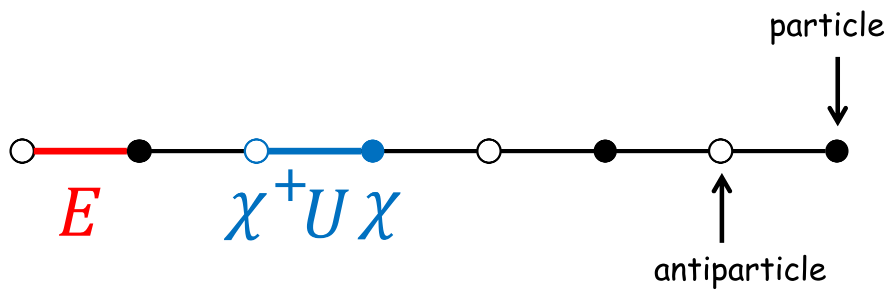
- ✓ noiseless simulator (classical computer to emulate quantum computer)
- ✓ lattice Schwinger model
- ✓ 8-site lattice

1. Hamiltonian formalism

lattice Schwinger model

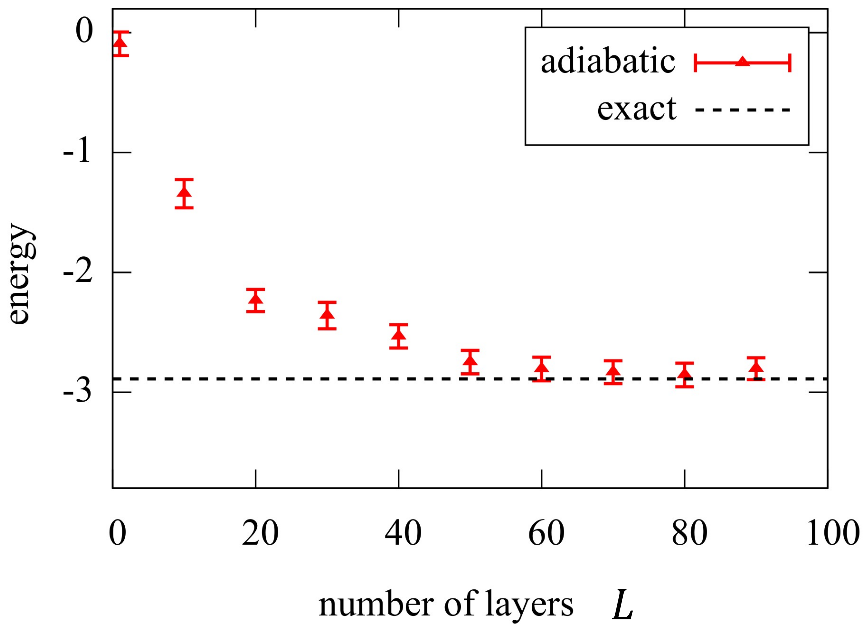
$$H = \sum_{\text{link}} E^2 - i \sum_{\text{site}} (\chi^+ U \chi - \chi^+ U^+ \chi)$$

$$Q = \chi^+ \chi - \frac{1}{2}[1 - (-1)^n]$$

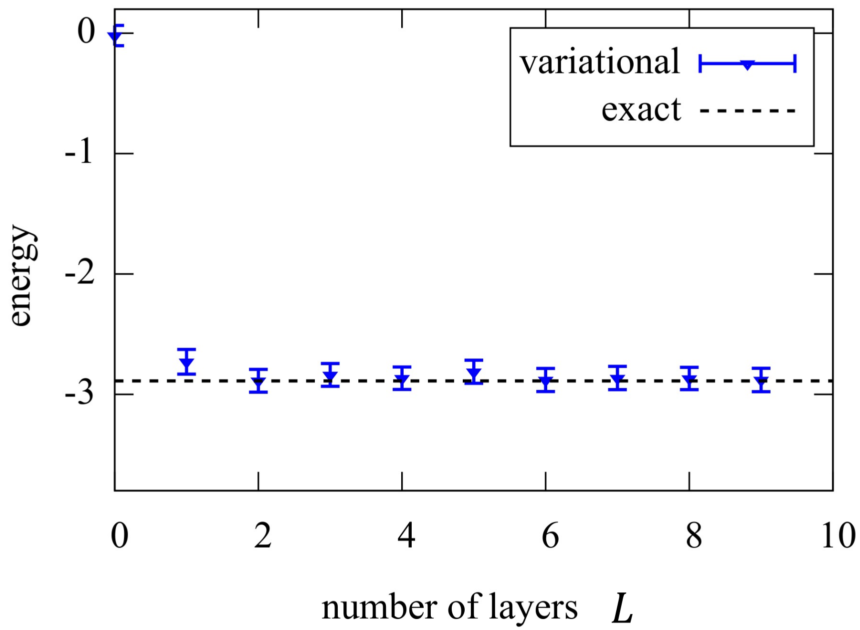


1. Hamiltonian formalism

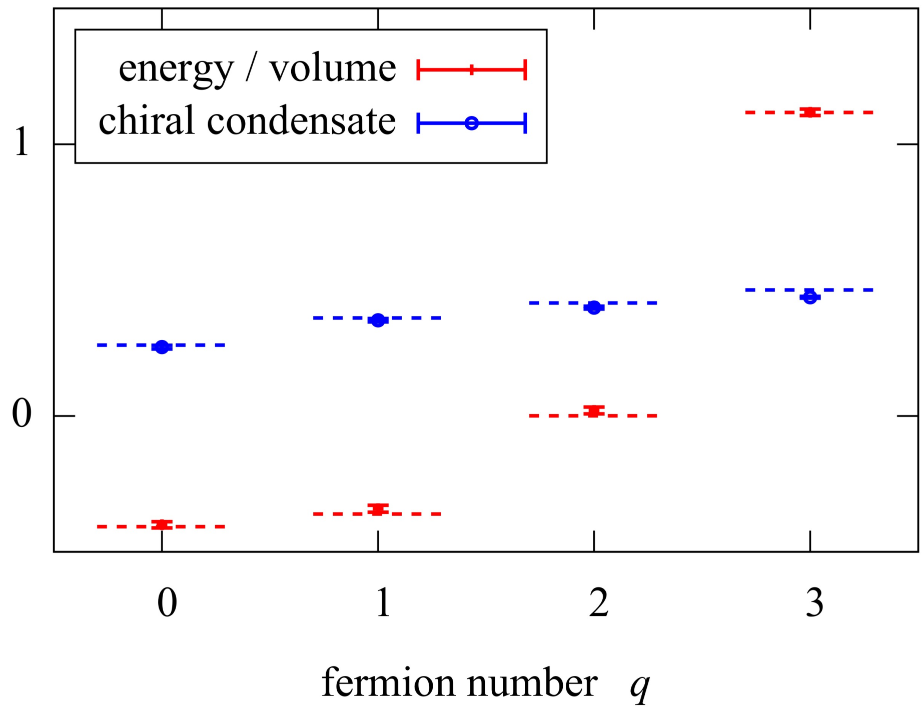
adiabatic algorithm



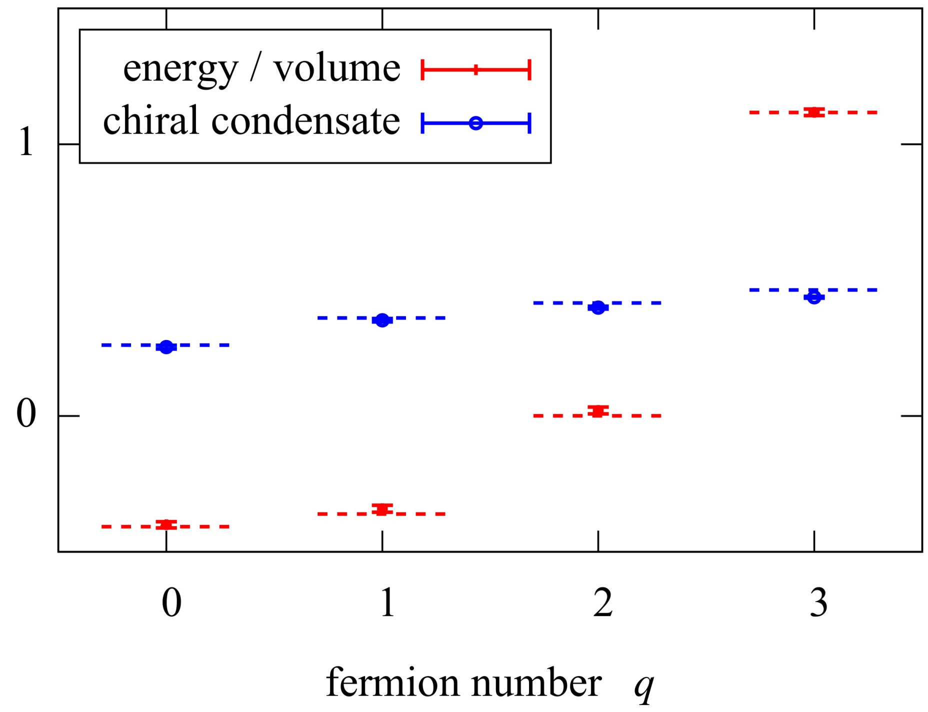
variational algorithm



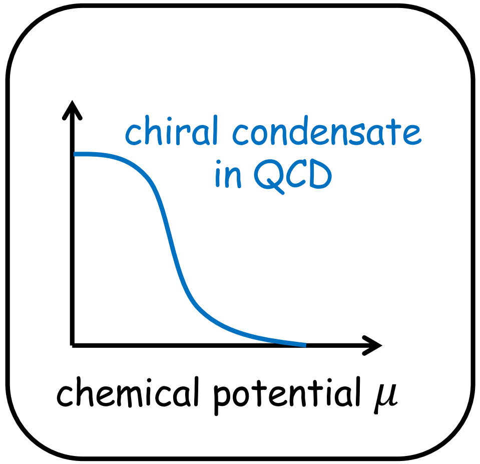
1. Hamiltonian formalism



1. Hamiltonian formalism



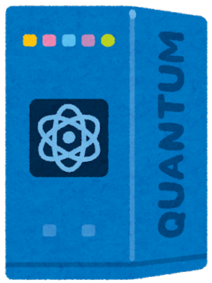
⇔



2. Lagrangian formalism

$$\mathcal{Z} = \int DU\, e^{-S}$$

how to encode?



Euclidean path integral

quantum computer

2. Lagrangian formalism

quantum sampling algorithm

- ✓ designed for classical statistics $\mathcal{Z} = \sum e^{-E/T}$
- ✓ classical random number is replaced by quantum fluctuation
- ✓ several algorithms

2. Lagrangian formalism

quantum sampling algorithm Wild *et al.* (2021)

matrix representation of classical action

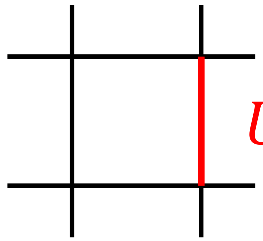
$$H = N\left(I - e^{-\mathcal{S}/2} M e^{\mathcal{S}/2}\right)$$

matrix representation of Markov chain

ground state ⟷ classical ensemble

2. Lagrangian formalism

Z_2 lattice gauge theory

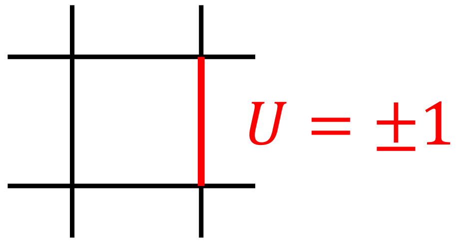


$$S = -\beta \sum_{\mathrm{plaq}} UUUU$$

$$\{U_1, U_2, \cdots, U_N\}$$

2. Lagrangian formalism

Z_2 lattice gauge theory



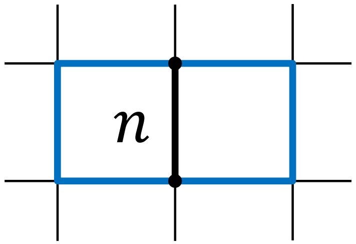
$$S = -\beta \sum_{\text{plaq}} UUUU \quad \xrightarrow{\text{matrix representation}} \quad \mathcal{S} = -\beta \sum_{\text{plaq}} ZZZZ$$

$$\{U_1, U_2, \cdots, U_N\} \qquad |U_1\rangle|U_2\rangle \cdots |U_N\rangle$$

2. Lagrangian formalism

$$H = \sum_n \frac{1}{2}\left(I - \tanh(\beta C_n)\, Z_n - \frac{1}{\cosh(\beta C_n)}\, X_n\right)$$

$$C_n = \sum_{\text{staple}} ZZZ$$



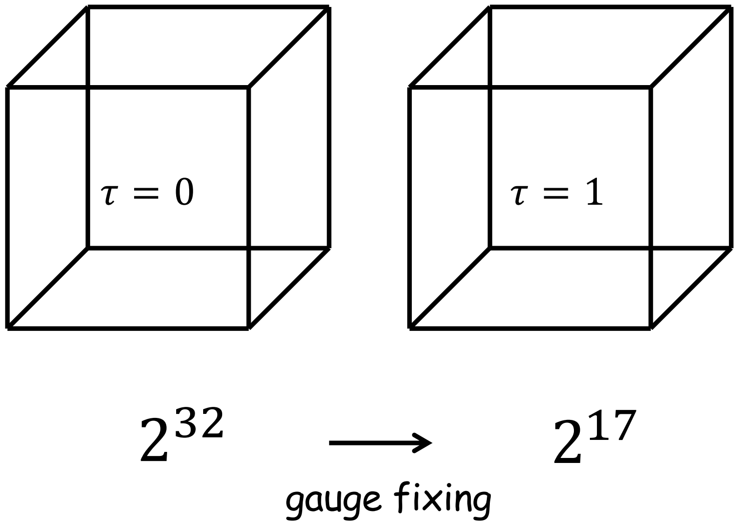
$$|\Psi\rangle = \frac{1}{\sqrt{\mathcal{Z}}} \sum e^{-\mathcal{S}/2} |U_1\rangle |U_2\rangle \cdots |U_N\rangle$$

gauge configuration !!

2. Lagrangian formalism

benchmark test

- ✓ noiseless simulator
- ✓ Z_2 pure gauge theory
- ✓ 2^4-site lattice

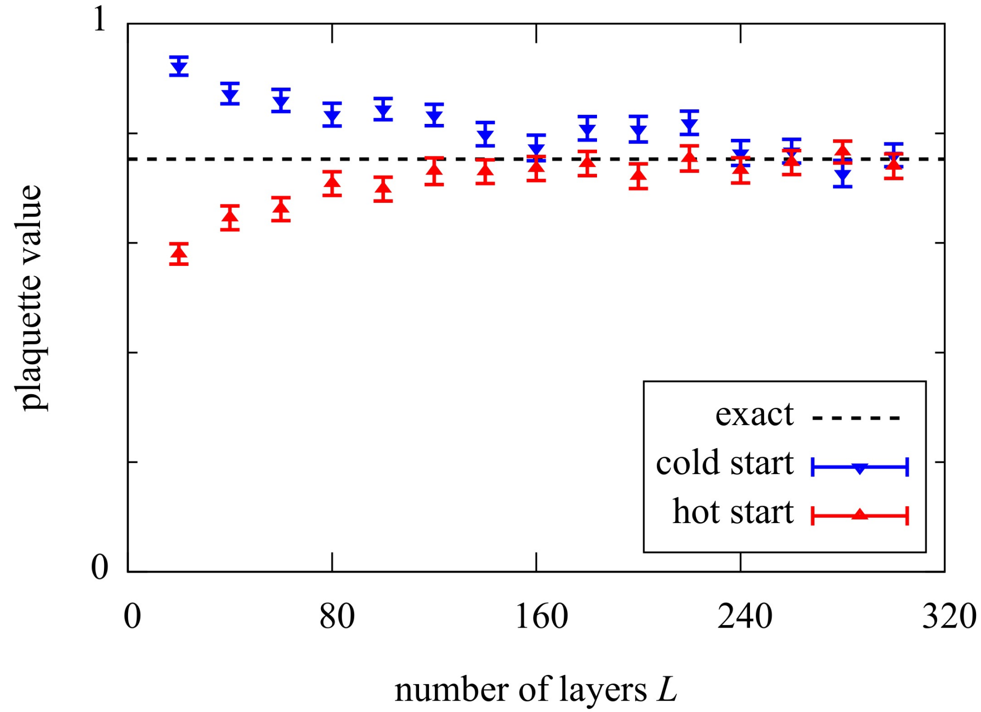


2. Lagrangian formalism

$$|\Psi(\beta)\rangle = \prod_{l=1,\cdots,L} U(l)\,|\Psi_0\rangle$$

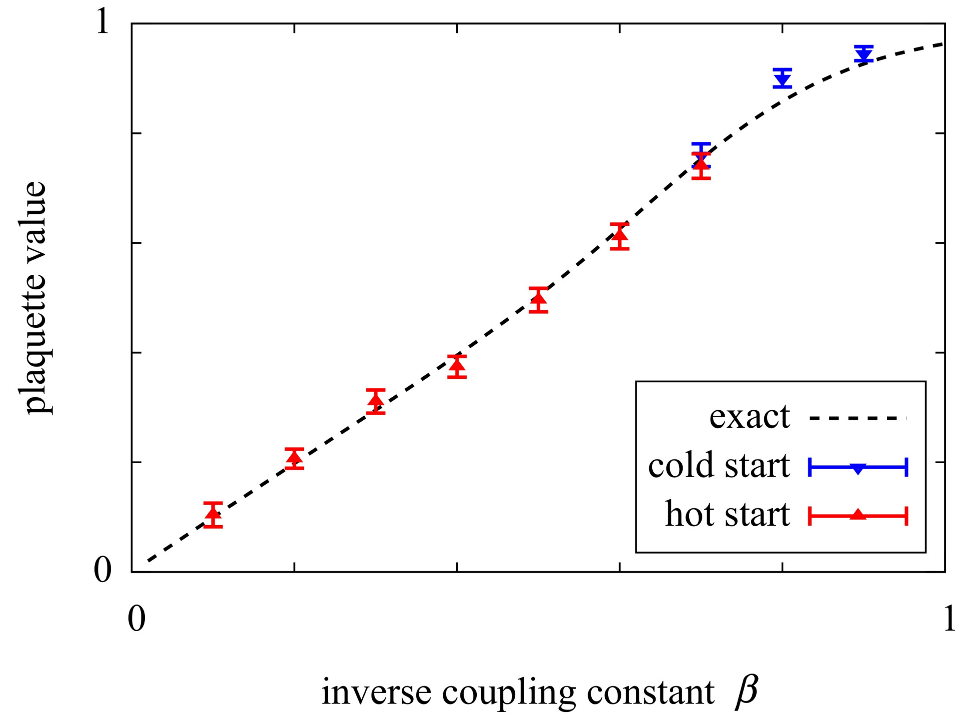
cold start $\quad |\Psi_0\rangle = |\Psi(\beta=\infty)\rangle$

hot start $\quad |\Psi_0\rangle = |\Psi(\beta=0)\rangle$



2. Lagrangian formalism

$$|\Psi(\beta)\rangle = \prod_{l=1,\cdots,L} U(l)\,|\Psi_0\rangle$$



cold start $|\Psi_0\rangle = |\Psi(\beta = \infty)\rangle$

hot start $|\Psi_0\rangle = |\Psi(\beta = 0)\rangle$

2. Lagrangian formalism

Compared with **Hamiltonian formalism**...

past experience, Lorentz invariance, classical storage

Compared with **classical simulation**...

quadratic speedup Wild *et al.* (2021)

fermion is more important

Summary

1. **Hamiltonian formalism**

    ✓ nonzero fermion density

    ✓ benchmark of the Schwinger model

2. **Lagrangian formalism**

    ✓ quantum sampling

    ✓ benchmark of Z_2 pure gauge theory

useful someday in the future...