Restoration of chiral symmetry in cold and dense Nambu－Jona-Lasinio model with tensor renormalization group

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Tensor renormalization group approach

Procedures

1) Write down the target function X defined on lattice as a tensor contraction (tensor network)

ex. Partition function, Path integral, …

2) Approximately perform the tensor contraction with the TRG

1) **TN representation for $\boldsymbol{X}$** : $(\#\text{ of tensors in TN}) = (\#\text{ of lattice sites})$

$$X \;\rightarrow\; \Sigma_{abcd\cdots} T_{aiw\cdots} T_{bjx\cdots} T_{cky\cdots} T_{dlz\cdots} \cdots$$

2) **TRG** : Block-spin trans. for T to reduce # of tensors in TN

$$\approx\; \Sigma_{a'b'c'd'\cdots} {T'}_{a'i'w'\cdots} {T'}_{b'j'x'\cdots} {T'}_{c'k'y'\cdots} {T'}_{d'l'z'\cdots} \cdots$$

Advantage of TRG approach

✔ TRG is a deterministic numerical method based on the idea of the real-space renormalization group

- **No sign problem**
- **The computational cost scales logarithmically w. r. t. the system size**
- **Direct evaluation of the Grassmann integrals**
- **Direct evaluation of the path integral**

✔ **Applicability to the higher-dimensional systems**

TRG is a kind of tensor-network method and the application of the TRG to the higher-dimensional systems has recently made remarkable progress

For applications of TN method to lattice field theory,
see Bañuls-Cichy, Rep. Prog. Phys. 83(2020)024401 (review)

Status of TRG in higher-dimensional systems (1/2)

D: bond dimension, L: linear system size

Algorithm	Cost	Applications to 3D	Applications to 4D
HOTRG Xie et al, PRB86(2012)045139	$D^{4d-1}\ln L$	Ising Xie+, Potts models Wang+, free Wilson fermion Sakai+, $\mathbb{Z}_2$ gauge theory Dittirich+, Kuramashi-Yoshimura	Ising model SA+, Staggered fermion w/strongly coupled U(N) Milde+
Anisotropic TRG (ATRG) Adachi-Okubo-Todo, PRB102(2020)054432	$D^{2d+1}\ln L$	Ising model Adachi+, SU(2) gauge Kuwahara-Tsuchiya, Real ϕ^4 theory SA+, Hubbard model SA-Kuramashi	Complex ϕ^4 theory SA+, NJL model SA+, Real ϕ^4 theory SA+
Triad RG Kadoh-Nakayama, arXiv:1912.02414	$D^{d+3}\ln L$	Ising model Kadoh-Nakayama, O(2) model Bloch+, $\mathbb{Z}_3$ (extended) clock model Bloch+	-

For the lower-dimensional applications,
see Meurice et al, arXiv:2010.06539 (review)

Status of TRG in higher-dimensional systems (2/2)

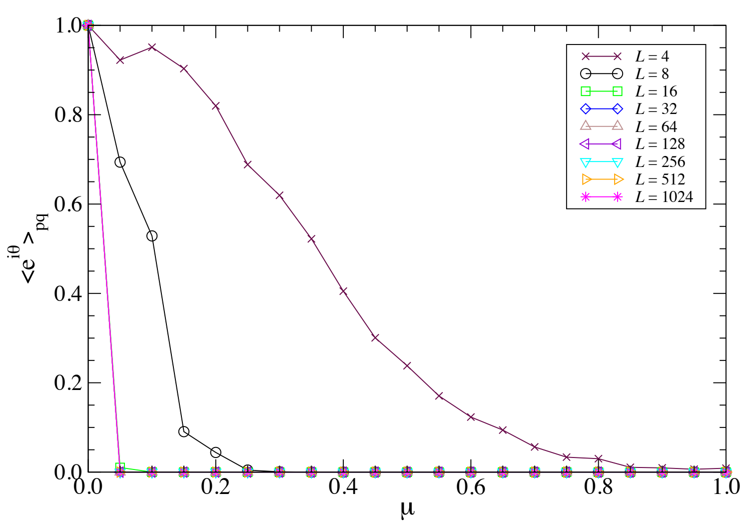
4D complex ϕ^4 theory at finite density

SA et al, JHEP09(2020)177

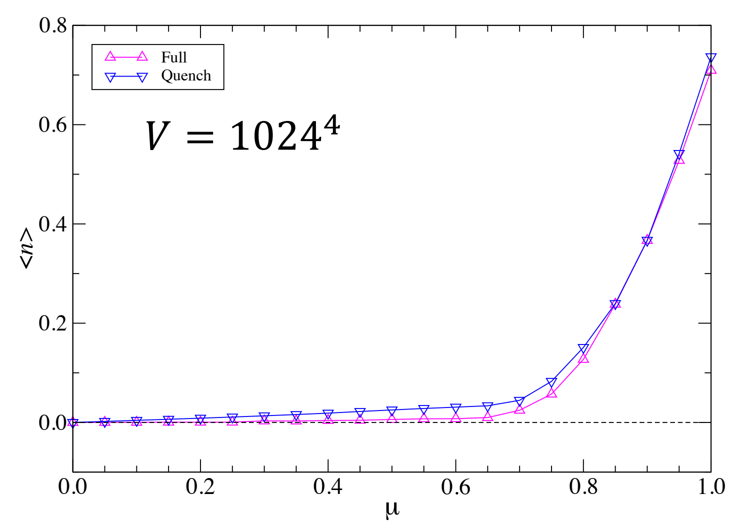
✔ Typical system w/ the sign problem

✔ Employing the Gauss quadrature rule, the TN rep for Z is approximately obtained

Average phase factor



Number density



TRG approach captures the Silver-Blaze phenomenon at finite density even in the regime with the vanishing APS

Expected phase diagram of the NJL model

✔ Effective theory of QCD

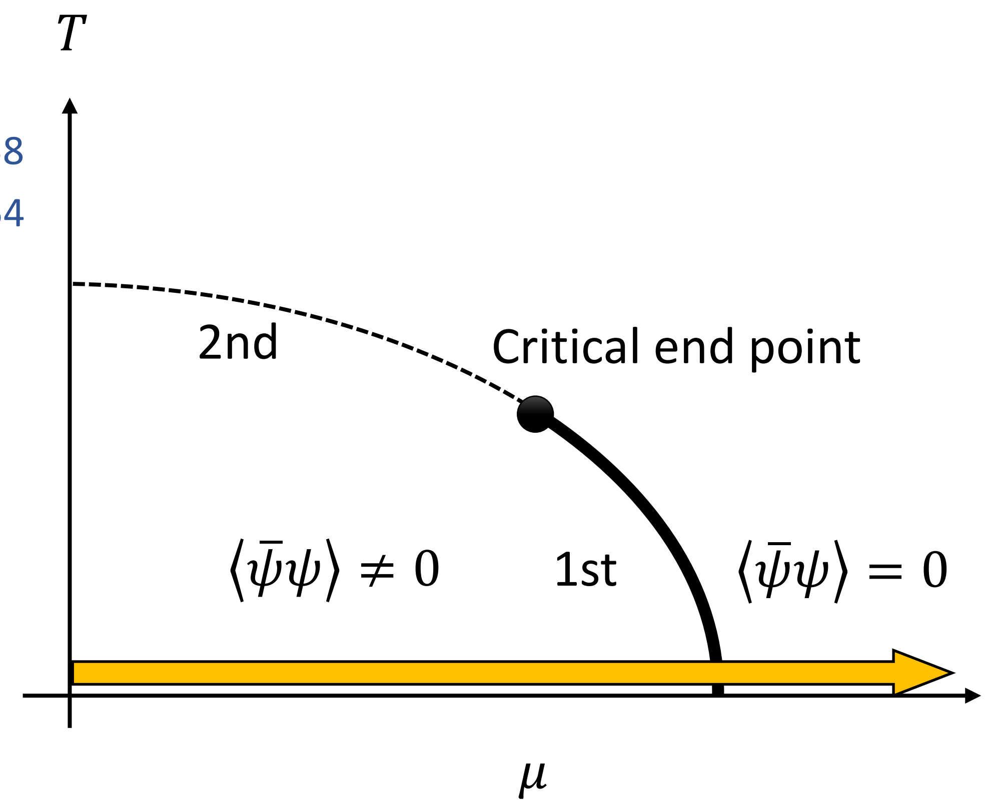
Nambu–Jona-Lasinio, PRD122(1961)345-358

Nambu–Jona-Lasinio, PRD124(1961)246-254

✔ **Chiral restoration is expected in cold & dense region**

Asakawa-Yazaki, NPA504(1989)668-684

✔ **Severe sign problem in cold & dense region**



We apply the Tensor Renormalization Group (TRG) approach to investigate the 1st order chiral phase transition in cold & dense region

NJL model at finite density

✔ w/ the Kogut-Susskind fermion

Single-component Grassmann variables w/o the Dirac structure

Staggered sign function $\eta_\nu(n) = (-1)^{n_1+\cdots+n_{\nu-1}}$ with $\eta_1(n) = 1$

✔ μ : chemical potential

$$S_{\mathrm{lat}} = \frac{1}{2} a^3 \Sigma_{n\in\Lambda} \Sigma_{\nu=1}^{4} \eta_\nu(n) \left[\mathrm{e}^{\mu a \delta_{\nu,4}} \bar{\chi}(n)\chi(n+\hat{\nu}) - \mathrm{e}^{-\mu a \delta_{\nu,4}} \bar{\chi}(n+\hat{\nu})\chi(n) \right]$$

$$+ m a^4 \Sigma_{n\in\Lambda} \bar{\chi}(n)\chi(n) - g_0 a^4 \Sigma_{n\in\Lambda} \Sigma_{\nu=1}^{4} \bar{\chi}(n)\chi(n)\bar{\chi}(n+\hat{\nu})\chi(n+\hat{\nu})$$

(This formulation follows Lee-Shrock, PRL59(1987)14)

✔ Continuous chiral symmetry for vanishing m :

$$\chi(n) \to \mathrm{e}^{\mathrm{i}\alpha\epsilon(n)}\chi(n), \qquad \bar{\chi}(n) \to \bar{\chi}(n)\mathrm{e}^{\mathrm{i}\alpha\epsilon(n)}$$

w/ $\alpha \in \mathbb{R}$ and $\epsilon(n) = (-1)^{n_1+n_2+n_3+n_4}$

Numerical Results

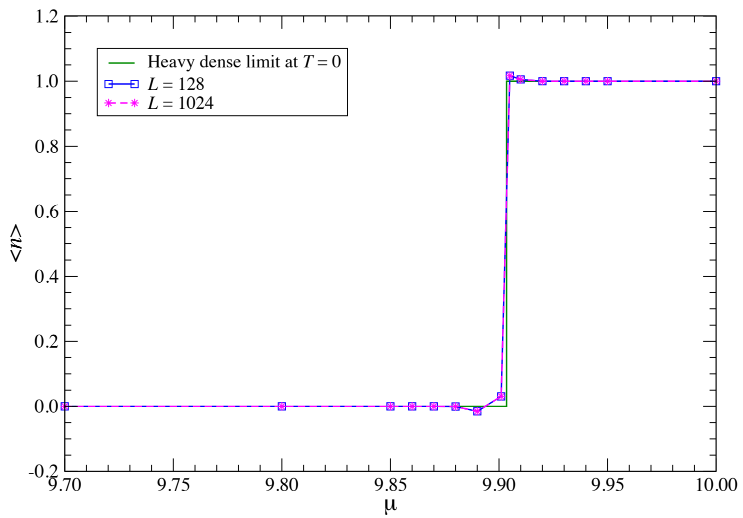
Heavy dense limit as a benchmark

with $m = 10^4, g_0 = 32, D = 30$

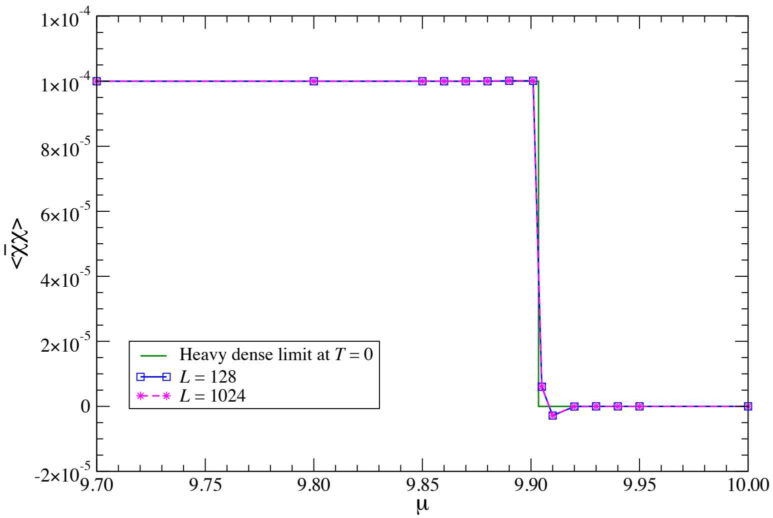
→ This limit allows us to compare numerical results w/ exact analytical ones

cf. Powlowski-Zielinski, PRD87(2013)094509

Number density



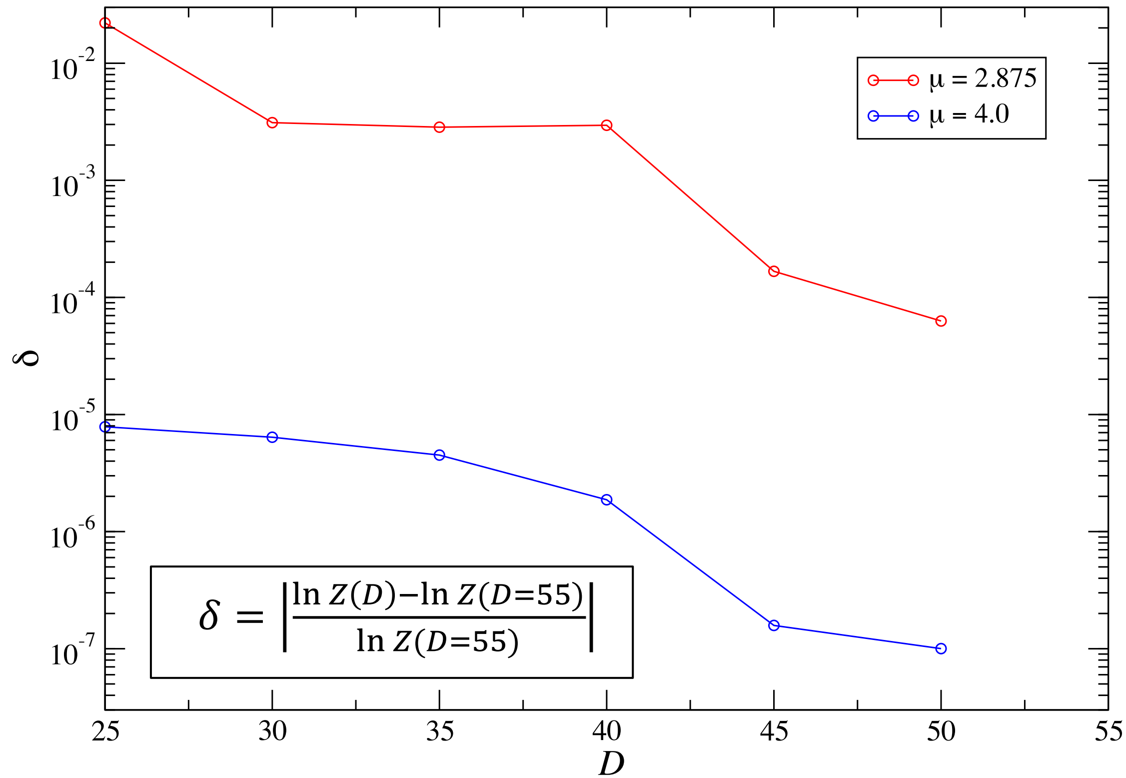
Fermion condensate



TRG calculation well reproduces the analytical results, including the location of $\mu_c = \ln(2m) = 9.903$

Converging behavior in bond dimension

with $m = 0.01, g_0 = 32, L = 1024$



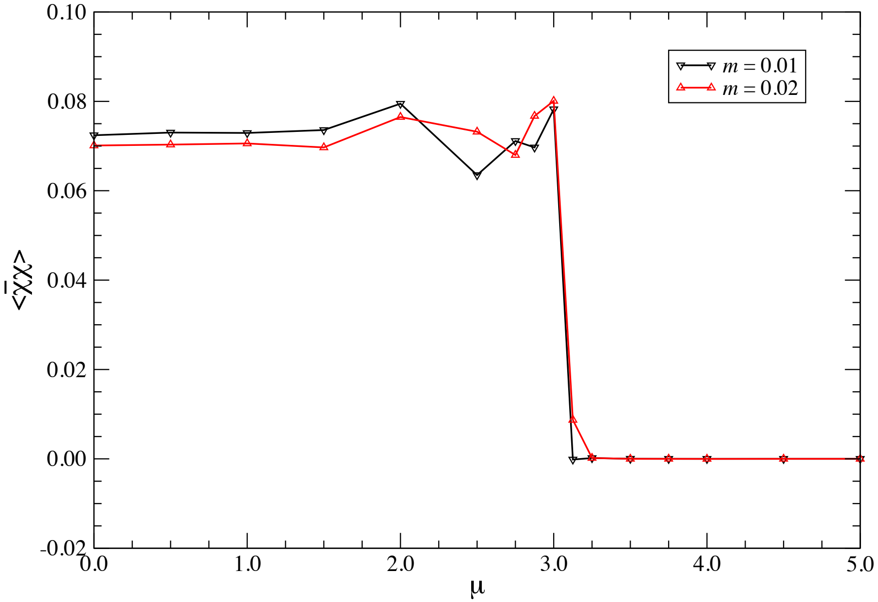
D: bond dimension (= the maximal size of tensors in TRG algorithm)

$\boldsymbol{\delta \lesssim 10^{-4}}$ **has been achieved up to** $\boldsymbol{D = 55}$ **at** $\boldsymbol{\mu \approx \mu_c}$

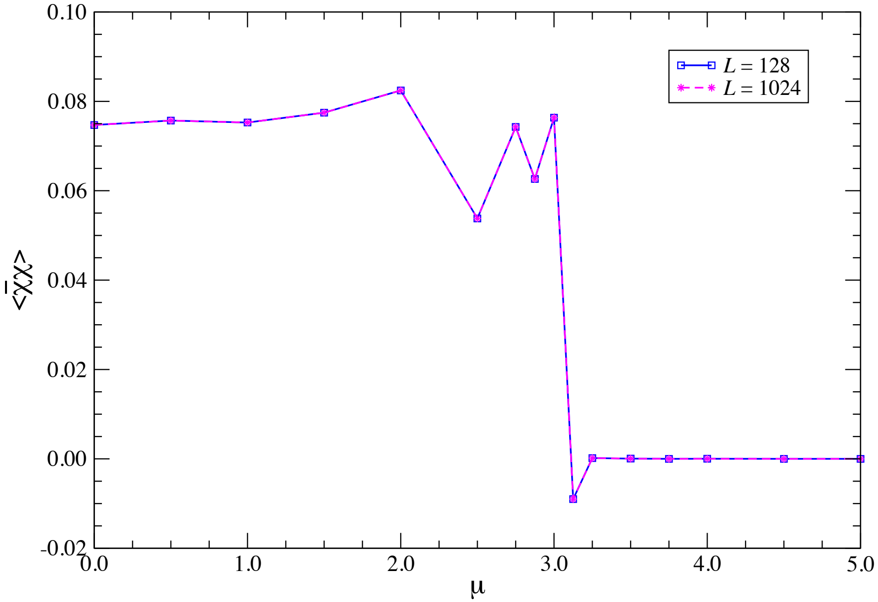
Chiral condensate

with $g_0 = 32, D = 55$

finite mass



chiral limit

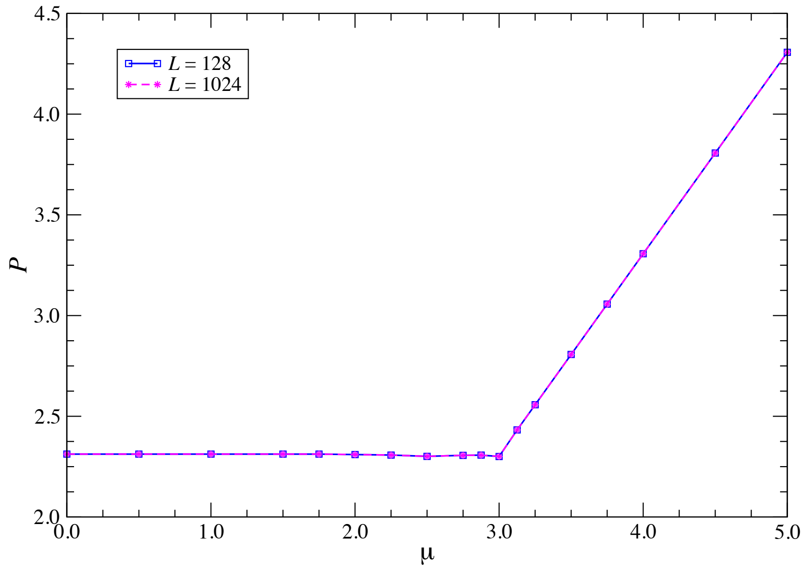


Chiral symmetry is restored in the region with $\mu \gtrsim 3.0$
A discontinuity at $\mu \approx 3.0$ indicates the 1st order transition

Ingredients of the equation of state

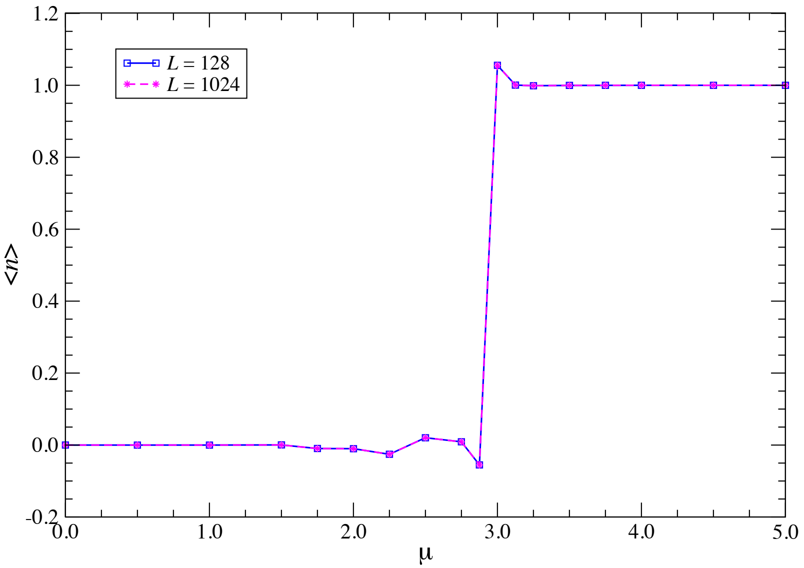
with $m = 0.01, g_0 = 32, D = 55$

Pressure
(~ Thermodynamic potential)



A sharp increase starting from $\mu \approx 3.0$

Number density



A jump from $\langle n \rangle = 0$ to $\langle n \rangle = 1$

Current numerical results clearly show that the chiral phase transition in cold & dense region is 1st order

Summary

- This study is the first application of TRG approach to the 4d fermionic system
- We have employed the TRG to study the lattice NJL model in cold & dense region ($\frac{\mu}{T} \gg 1$), there the faces a serious sign problem
- **We have observed the restoration of chiral symmetry in cold & dense region and current results (chiral condensate, pressure, number density) show the chiral phase transition is of first order in cold & dense regime**
- **Steady progress has been made toward numerical research on the QCD using the TRG approach**