The phase diagram of QCD at low temperature with the complex Langevin method

F. Attanasio[a], B. Jäger[b] and F.P.G. Ziegler[b,c]
[a]Institute for Theoretical Physics, Heidelberg University
[b]CP3-Origins and D-IAS, Department of Mathematics and Computer Science, University of Southern Denmark
[c]School of Physics and Astronomy, The University of Edinburgh

Motivation

Physics challenge

- QCD phase diagram at finite chemical potential μ and low temperatures T largely unkown
- subject to experimental efforts at LHC, RHIC, FAIR & NICA
- QCD equation of state relevant for understanding neutron stars.

Sign Problem as a Computational Challenge

- $\mu > 0$ renders the Euclidean action complex making sampling with standard phase reweighting exponentially hard. This worsens as T is decreased and as μ is increased.
- can be circumvented by the **complex Langevin (CL) method** [1]

Lattice QCD at finite density

Partition function

$$Z = \int_{\mathrm{SU}(3)^{4\Omega}} \mathrm{d}U \exp(-S_G[U]) \det D(U,\mu)\,,$$

Wilson Dirac operator

$$D_{x,y} = (4+m)\delta_{x,y} + \frac{1}{2}\sum_\nu \Gamma_\nu\, e^{\mu\delta_{0,\nu}}\, U_{x,\nu}\delta_{x+\hat\nu,y} + \Gamma_{-\nu}\, e^{-\mu\delta_{0,\nu}}\, U^\dagger_{x-\hat\nu,\nu}\delta_{x-\hat\nu,y}$$

CL simulation

- Since $\det(D(U,\mu)) \in \mathbb{C}$ perform holomorphic extension of the integration manifold $SU(3)$ to the complexified gauge group $SL(3,\mathbb{C})$, in particular for the gauge field $U^\dagger_{x,\mu} \to U^{-1}_{x,\mu}$.
- Perform Euler-Maruyama update scheme

$$U^{n+1}_{x,\nu} = \exp[-it^a(-D_{x,\nu,a}S[U] + \eta_{x,\nu,a})]U^n_{x,\nu}$$

$$\text{where } \langle\eta_{x,\nu,a}\rangle = 0\,, \quad \langle\eta_{x,\nu,a}\eta_{y,\rho,b}\rangle = 2\delta_{x,y}\delta_{\nu,\rho}\delta_{a,b},\, a = 0,\dots,N_c^2-1\,.$$

Lattice setup

- $N_f = 2$ mass-degenerate quarks
- $N_t \in \{4,..,32\}$, $L/a = 24$, $a \approx 0.08$ fm
- Wilson plaquette action, Wilson-Dirac fermions (tree-level)
- $\beta = 5.8$, $\kappa = 0.1544$, see [2]
- $m_\pi \approx 480$ MeV
- $T \in [100, 800]$ MeV, $\mu \in [0, 6500]$ MeV

Stabilizing the CL simulation

Since $SL(3,\mathbb{C})$ is **non-compact** stabilizing methods need to be applied during the CL simulation to avoid run-away trajectories and to ensure correct results:

- **Adaptive step size**
- **gauge cooling** [3] $\to$ minimize unitarity norm

$$F[U] = \frac{1}{\Omega N_c}\sum_{x,\nu} \mathrm{tr}[U^\dagger_{x,\nu}U_{x,\nu} + (U^\dagger_{x,\nu}U_{x,\nu})^{-1} - 2\mathbb{1}]$$

- **Dynamic Stabilization** [4] $\to$ extension of the drift term

$$K_{x,\nu,a} \to K_{x,\nu,a} + i\alpha_{DS}M_{x,a}\,,$$

$$\text{where } M_{x,a} = ib_{x,a}\left(\sum_c b_{x,c}b_{x,c}\right)^3, \quad b_{x,a} = \mathrm{tr}\left(\lambda^a \sum_\nu U_{x,\nu}U^\dagger_{x,\nu}\right)$$

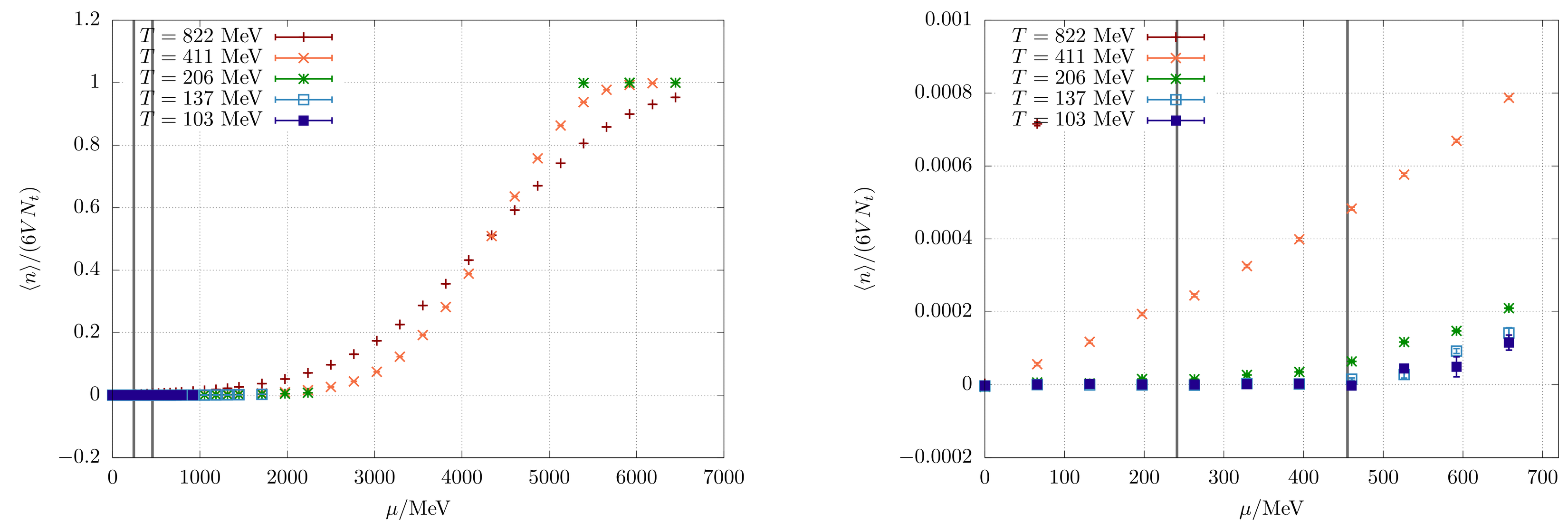
Observables

Polyakov loop and density

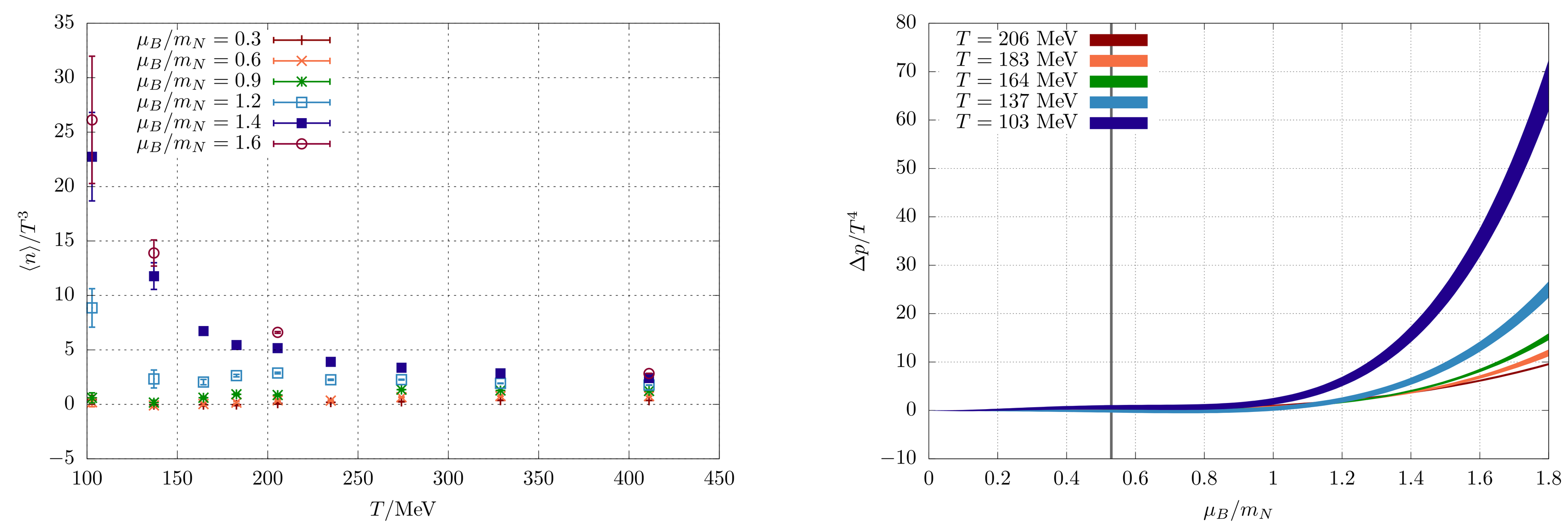
$$P(\vec{x}) = \mathrm{tr}\left(\prod_{x_0=0}^{N_\tau-1} U_0(\vec{x},x_0)\right)\,, \quad \langle n\rangle = \frac{1}{\Omega}\frac{\partial\log(Z)}{\partial\mu}$$

Results

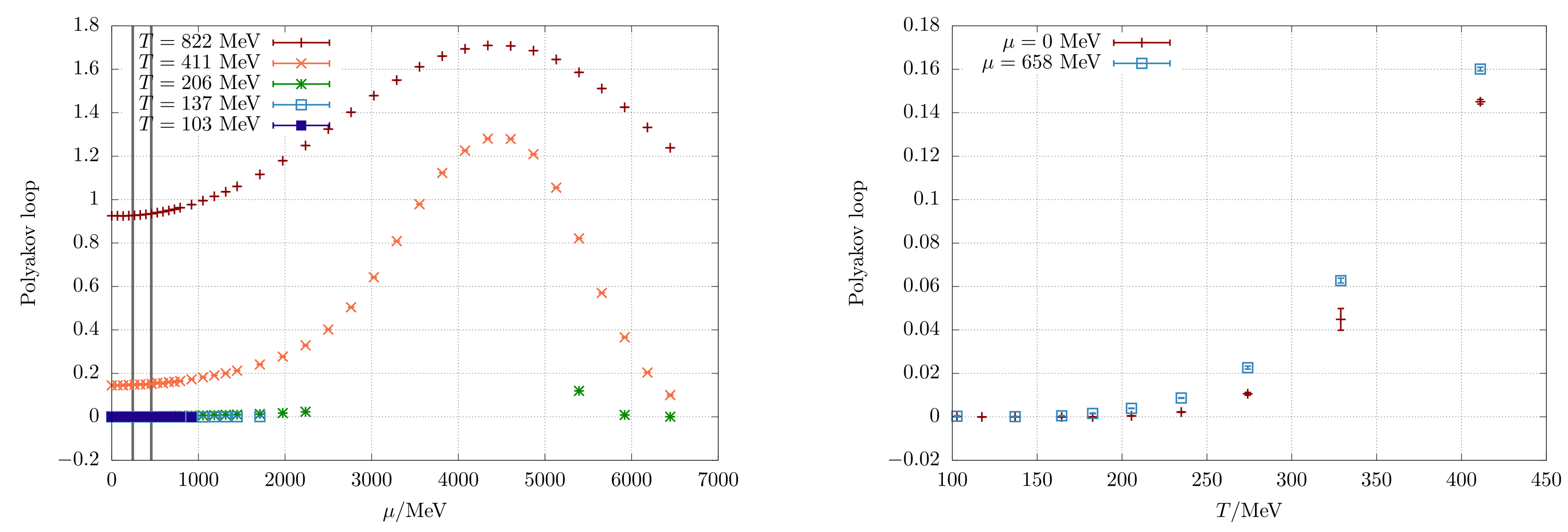
Density and Silver blaze phenomenon



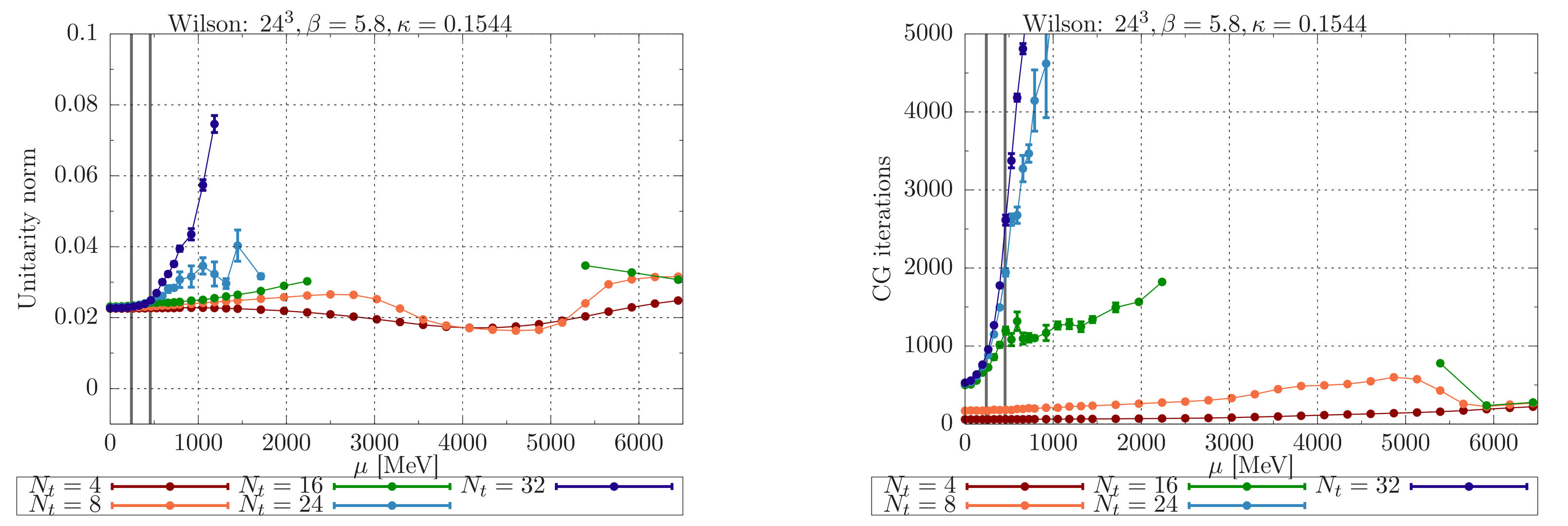
Density and pressure



Polyakov loop



Unitarity norm and solver performance



Perspectives

- further investigate CL systematics [5,6], see talk by B. Jäger
- finite size scalings and phase transitions

[1] Aarts and Stamatescu, JHEP **09** (2008) 018
[2] Del Debbio, Giusti, Lüscher, Petronzio and Tantalo, JHEP 02 (2006) 011
[3] Seiler, Sexty and Stamatescu, Phys.Lett.B 723 (2013) 213-216
[4] Attanasio, Jäger, Eur.Phys.J.C 79 (2019) 1, 16
[5] Scherzer, Seiler, Sexty and Stamatescu, Phys.Rev.D 101 (2020) 1, 014501
[6] Nagata, Nishimura and Shimasaki, Phys.Rev.D 94 (2016) 11, 114515

pyfelipe@thphys.uni-heidelberg.de, felix.ziegler@ed.ac.uk