Renormalization of field dependent couplings

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QCD phase diagram and lattice QCD 2021
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GF & A. Patkos, Phys. Rev. D**103**, 056015 (2021) [arXiv:2011.08387]
GF & A. Patkos [arXiv:2111.xxxxx]

Motivation

- What is a field dependent coupling?
- Classical action:

$$S[\phi] = \int \left[\frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2} m^2 \phi^2 + g\phi^4 + \eta\phi^6 + \ldots\right]$$

- Quantum action: (mom. space)

$$\Gamma[\phi] = \sum_n \int \Gamma^{(n)}(\{p_i\};\mu)\, \phi(p_1)\ldots\phi(p_n)$$

- Promoting $\Gamma^{(n)}(\{p_i\};\mu) \rightarrow \Gamma^{(n)}(\{p_i\};\mu,\underline{\underline{\phi}})$?!? $\Rightarrow$ senseless!
- But: internal symmetries & multicomponent ϕ^a, reorganize Γ!

  $\longrightarrow$ linear symmetries are inherited by Γ, therefore only invariant combinations appear: $I_1, I_2 \ldots I_N$

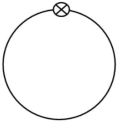
  $\longrightarrow$ if in the vacuum $I_1 \neq 0$ but $I_n = 0$ for all $n > 1$:

$$\Gamma[I_1, I_2, \ldots I_N] = \sum_{\{\alpha\}} \int \Gamma^{(\alpha)}(\{p_i\};\mu, I_1)\, I_2^{\alpha_2} I_3^{\alpha_3} \ldots I_N^{\alpha_N}$$

Functional Renormalization Group

- Flow equation of the effective action:

$$\partial_k \Gamma_k = \frac{1}{2}\int_{q,p} \mathrm{Tr}\left[\partial_k R_k(q,p)(\Gamma_{k,2}+R_k)^{-1}(p,q)\right] = \frac{1}{2}$$



- If one is interested in <u>zero momentum</u> couplings: (Local Potential Approximation - LPA)

$$\Gamma_k = \int \left[\frac{1}{2}(\partial_i \phi^a)^2 + V_k[I_1, I_2, \ldots]\right]$$

- Flow equation for the effective potential V_k:

$$\partial_k V_k[I_1, I_2, \ldots] = \frac{1}{2}\int_p \mathrm{Tr}\left[\partial_k R_k(p)\left(p^2 + V_{k,2} + R_k(p)\right)^{-1}\right]$$

- Extracting field dependent couplings via projection:

$$V_k[I_1, I_2, \ldots] = \sum_{\{\alpha\}} \underline{\underline{V_k^{(\alpha)}(I_1)}}\, I_2^{\alpha_2} I_3^{\alpha_3} \ldots I_N^{\alpha_N}$$

Application I: Yukawa coupling

- Three flavor quark-meson model: [M - mesons, ψ - quarks]

$$\mathcal{L} = \frac{1}{2}\mathrm{Tr}\,[\partial_i M^\dagger \partial_i M] + \bar{\psi}(\partial\!\!\!/ + g_Y M_5)\psi$$
$$+ \frac{1}{2} m^2\,\mathrm{Tr}\,[M^\dagger M] + g_1(\mathrm{Tr}\,[M^\dagger M])^2 + g_2\,\mathrm{Tr}\,(M^\dagger M M^\dagger M)$$

- Eff. potential (V) depends on invariants [e.g. $I_1 = \mathrm{Tr}\,(M^\dagger M)$]

$$V = \underline{\underline{U(I_1)}} + \underline{\underline{C(I_1)}}\,\mathrm{Tr}\,\Big[M^\dagger M - \frac{1}{3}\mathrm{Tr}\,(M^\dagger M)\Big]^2 + \ldots$$
$$= \underline{\underline{g_Y(I_1)}}\,\bar{\psi} M_5 \psi + \underline{\underline{g_{Y,2}(I_1)}}\,\bar{\psi} M_5 \Big[M_5^\dagger M_5 - \frac{1}{3}\mathrm{Tr}\,(M_5^\dagger M_5)\Big]\psi + \ldots$$

- Note that $g_Y \neq \delta^3 V / \delta\bar{\psi}\delta\psi\delta M_5$!
  $\longrightarrow$ naive calculation includes higher couplings (e.g. $g_{Y,2}$)
  $\longrightarrow$ one carefully needs to project out „contaminations”

Application I: Yukawa coupling

- Dressed Yukawa coupling as a function of the bare one (the UV scale was set to $\Lambda = 1\,\mathrm{GeV}$)

U''	$g_{Y,k=\Lambda}$	$g_{Y,k=0}$	Δg
10	5	6.0	16%
10	10	14.4	31%
10	15	22.7	34%
10	20	30.3	34%

U''	$g_{Y,k=\Lambda}$	$g_{Y,k=0}$	Δg
20	5	6.0	16%
20	10	14.1	29%
20	15	22.0	32%
20	20	29.2	32%

- Note: at zero field the Yukawa β-function is identically zero $\longrightarrow$ consistent with the one-loop calculation

Application II: $U_A(1)$ anomaly

- Three flavor meson model with anomaly: [M - mesons]

$$\mathcal{L} = \frac{1}{2}\mathrm{Tr}\,[\partial_i M^\dagger \partial_i M] + \underline{a\,(\det M^\dagger + \det M)} \\ + \frac{1}{2}m^2\,\mathrm{Tr}\,[M^\dagger M] + g_1(\,\mathrm{Tr}\,[M^\dagger M])^2 + g_2\,\mathrm{Tr}\,(M^\dagger M M^\dagger M)$$

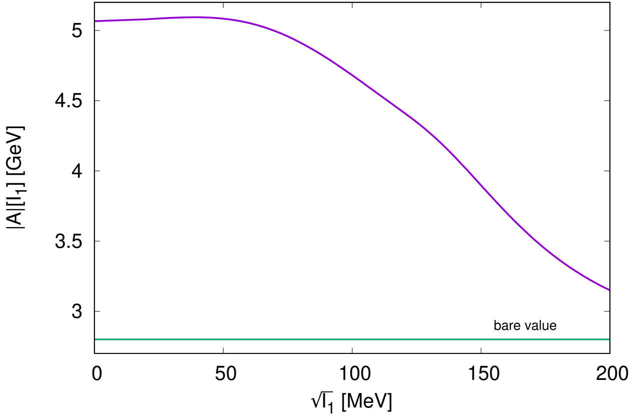
- Effective potential:

$$V[M] = \underline{\underline{U[I_1]}} + \underline{\underline{C[I_1]}}\,\mathrm{Tr}\left[M^\dagger M - \frac{1}{3}\mathrm{Tr}\,(M^\dagger M)\right]^2 + \ldots \\ + \underline{\underline{A[I_1]}}\,(\det M^\dagger + \det M) + B[I_1]\,(\det M^\dagger + \det M)^2 + \ldots$$

- Projection of the flow equation onto various operators provides individual flow equations for the field dependent couplings

Application II: $U_A(1)$ anomaly

- The $A(I_1)$ anomaly function is non-monotonous!
  $[I_1 = (v^2_{non-strange} + v^2_{strange})/2 \sim (100\,\mathrm{MeV}\,)^2$ in the vacuum]



⟶ when the condensate evaporates, the anomaly gets stronger, then weaker!

Summary

- Field dependent couplings can be defined via multicomponent variables + global symmetries
- Application I. Yukawa-coupling in the 3-flavor meson model
  $\longrightarrow$ field dependence is non-negligible even at $\mathcal{O}(1)$ initial (bare) values
- Application II. Determinant coupling of the $U_A(1)$ anomaly
  $\longrightarrow$ non-monotonous field dependence
  $\Rightarrow$ strenghtening, then weakening anomaly w.r.t. T, μ_B
- Is it really necessary to make couplings depend on the field?
  $\longrightarrow$ YES! A typical contribution in the flow equations looks

$$\int_p \frac{1}{p_R^2 + U_k'} = \int_p \frac{1}{p_R^2 + m_k^2 + g_{1,k} I_1 + \dots} = \int_p \frac{1}{p_R^2 + m_k^2} + \mathcal{O}(I_1)$$

- Symmetry breaking ($m^2 < 0$) blows up the integrals!
  $\longrightarrow$ resummation in I_1 is a necessity