What is chiral susceptibility probing? Hidenori Fukaya (Osaka U.)

S. Aoki, Y. Aoki, HF, S. Hashimoto, C. Rohrhofer, K. Suzuki [JLQCD collaboration], arXiv:2103.05954

For more details, see http://www-het.phys.sci.osaka-u.ac.jp/~hfukaya/slides/Fukaya-chiralsus-seminar.pdf

1. Introduction

QCD partition function

$$Z(m) = \int [dA] \det(D(A)+m)^{N_f} e^{-S_G(A)}$$

$$N_f = 2 \quad (m_u = m_d = m)$$

chiral condensate

$$-\langle \bar{q}q \rangle = \frac{1}{N_f V} \frac{\partial}{\partial m} \ln Z(m)$$

chiral susceptibility

$$\chi(m) = -\frac{\partial}{\partial m} \langle \bar{q}q \rangle (m)$$

probe for SU(2)$_L$xSU(2)$_R$

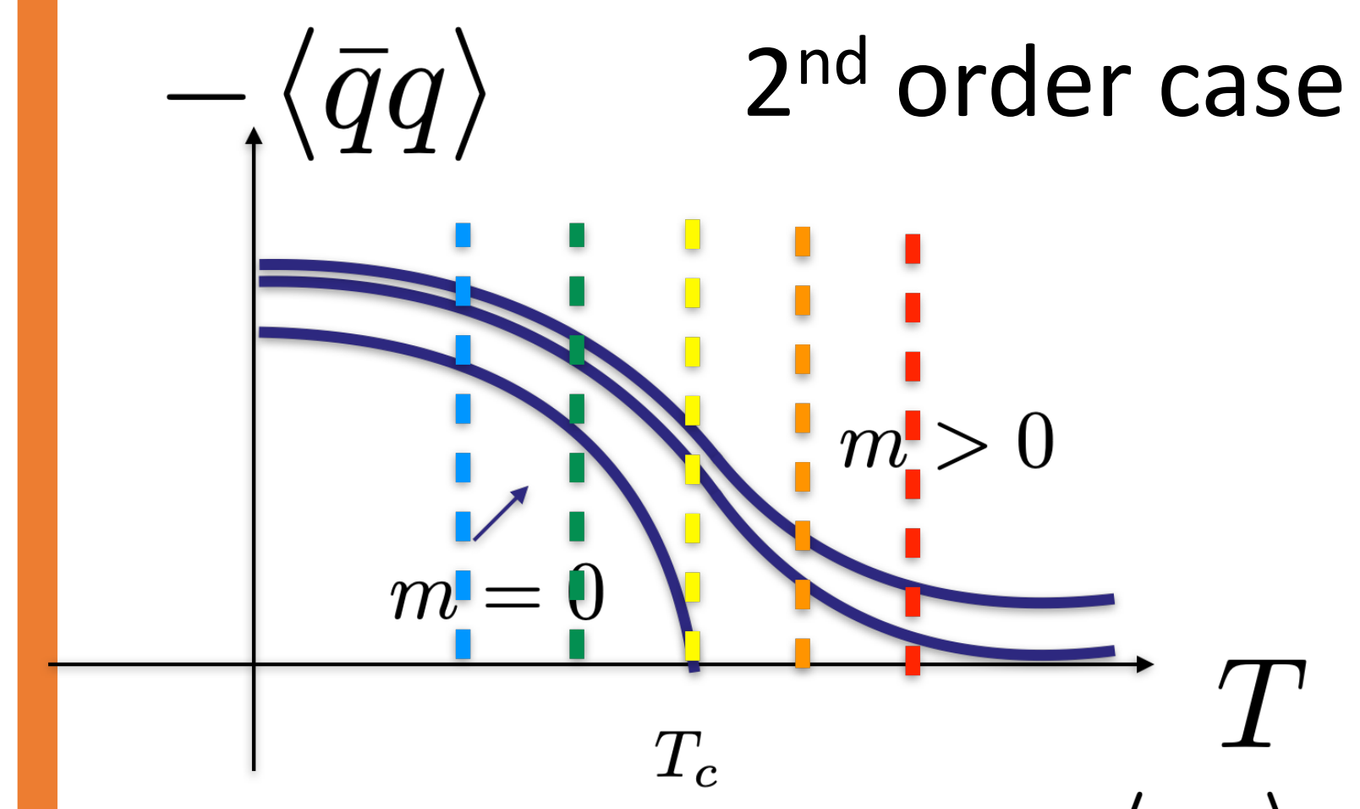
breaking/restoration (at m=0) :

T<Tc (~150MeV) , T>Tc,

$$\langle \bar{q}q \rangle \neq 0 \qquad \langle \bar{q}q \rangle = 0$$

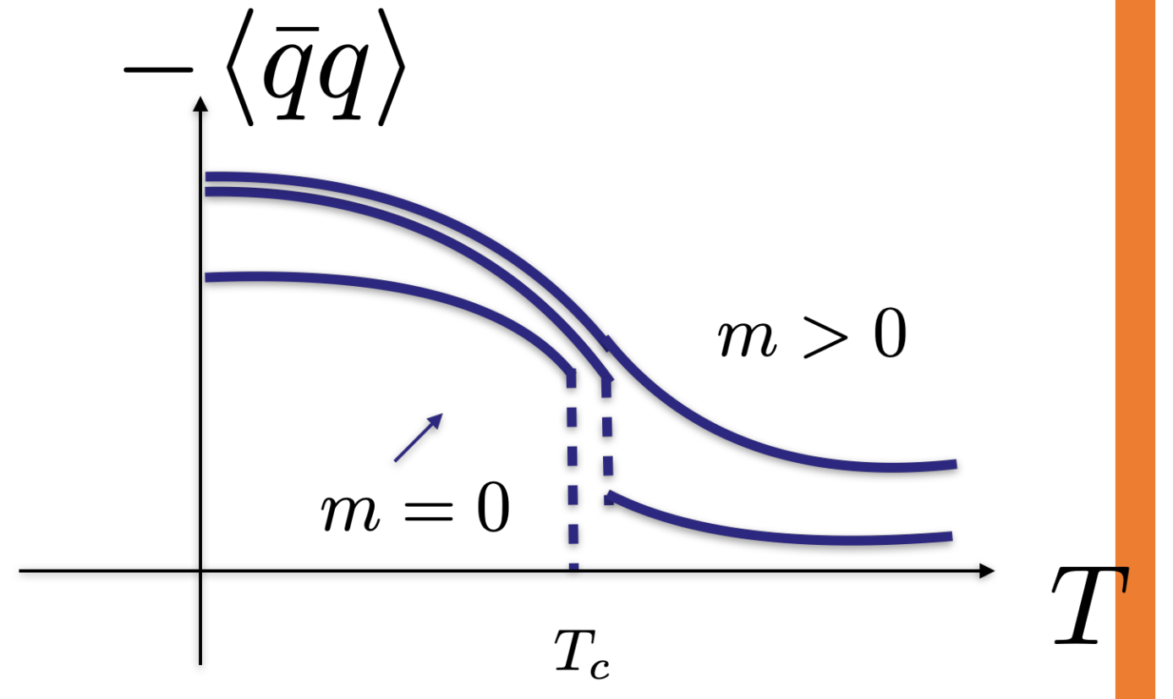
chiral phase transition

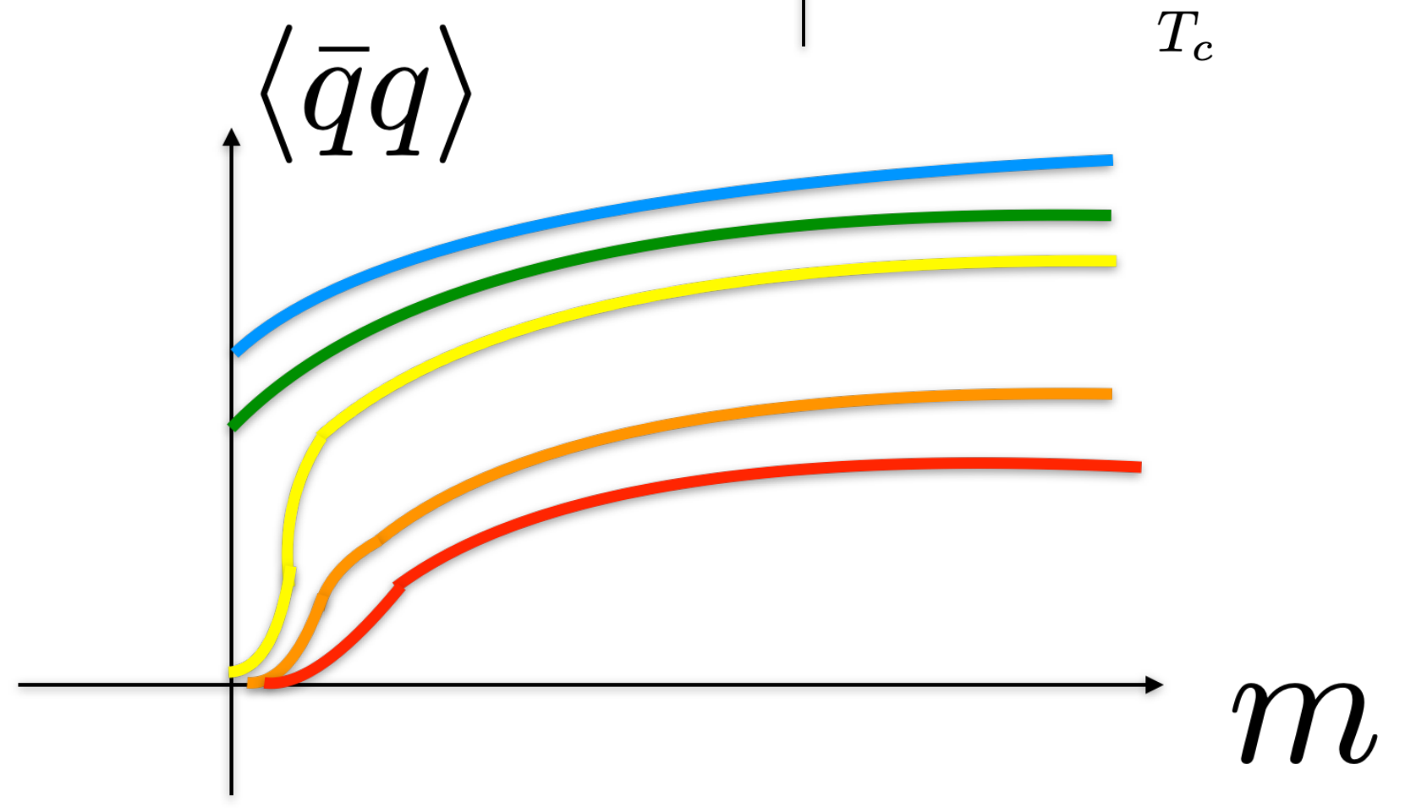
2. T and m dependence

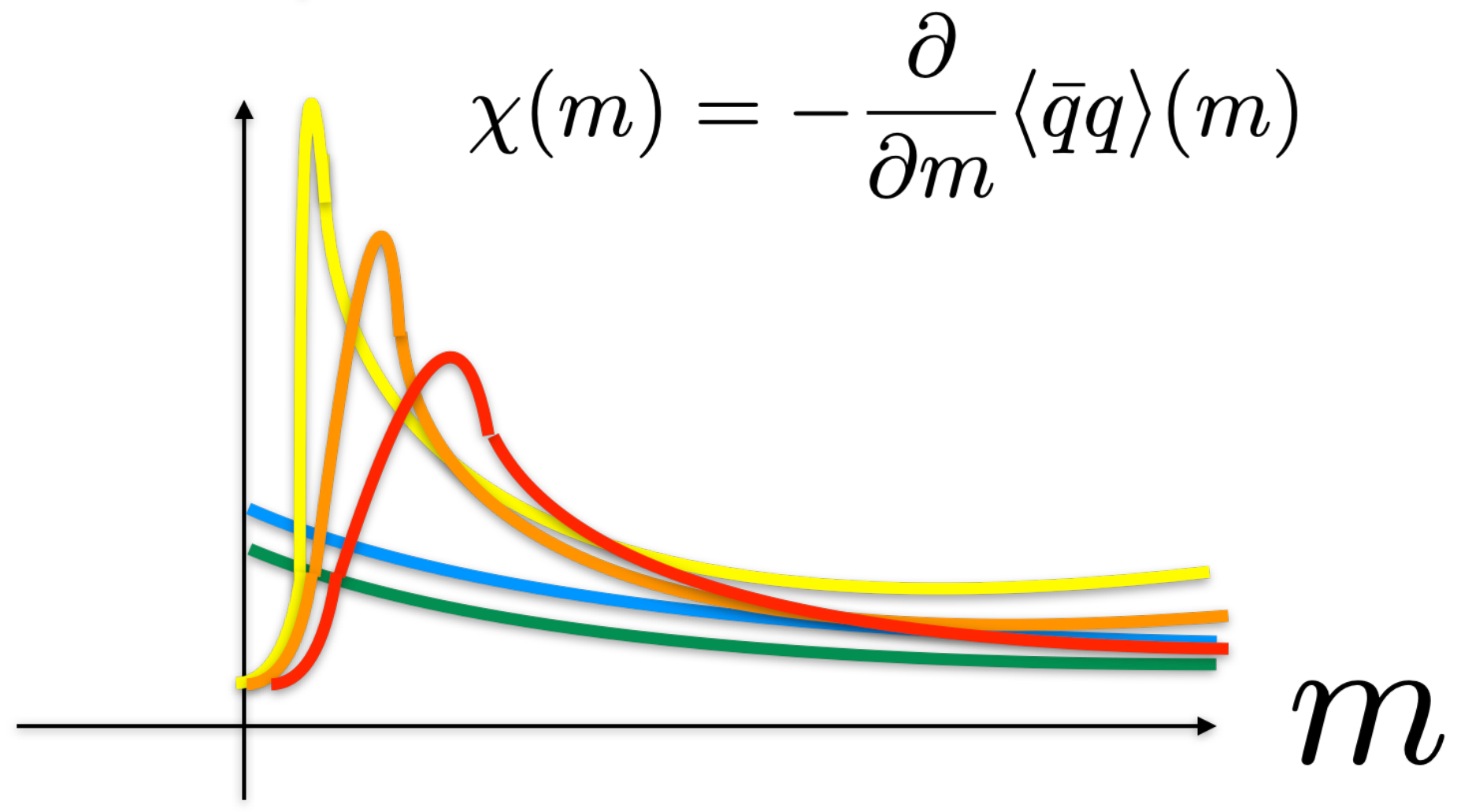


2nd order case

1st order case
(but will not
be so sharp
on finite lattices)







3. Question

condensate breaks both

$SU(2)_L \times SU(2)_R$ and $U(1)_A$

How much of

$$\chi(m) = -\frac{\partial}{\partial m} \langle \bar{q}q \rangle (m)$$

comes from U(1)$_A$ breaking?

Cf. Callan-Dashen-Gross 1978
suggested instanton effect
= U(1)$_A$ anomaly
= trigger of SU(2)xSU(2) breaking.
It may indicate that
instanton disappears
= U(1)A anomaly disappears
= SU(2)xSU(2) restoration.

Let us examine this in lattice QCD w/ chiral fermions.

4. Separating U(1)$_A$ part

$$\chi(m) = \chi^{\text{con.}}(m) + \chi^{\text{dis.}}(m)$$

$$\chi^{\text{con.}}(m) = \underbrace{-\Delta_{U(1)}(m) + \frac{\langle |Q(A)| \rangle}{m^2 V}}_{\text{U(1)A anomaly contribution}} \underbrace{- \frac{-\langle \bar{q}q \rangle_{\text{sub.}}(m)}{m}}_{\text{mixed}}$$

$$\chi^{\text{dis.}}(m) = \overbrace{\frac{N_f}{m^2} \chi_{\text{top.}}(m)} \underbrace{+\Delta^{(1)}_{SU(2)}(m) - \Delta^{(2)}_{SU(2)}(m)}_{\text{SU(2)xSU(2) breaking}}$$

$$\Delta_{U(1)}(m) \equiv \sum_x \langle P^a(x)P^a(0) - S^a(x)S^a(0) \rangle$$

$$\Delta^{(1)}_{SU(2)}(m) \equiv \sum_x \langle S^0(x)S^0(0) - P^a(x)P^a(0) \rangle$$

$$\Delta^{(2)}_{SU(2)}(m) \equiv \sum_x \langle S^a(x)S^a(0) - P^0(x)P^0(0) \rangle$$

$$\chi_{\text{top.}}(m) = \frac{\langle Q^2 \rangle}{V} \qquad Q(A) : \text{topological charge}$$

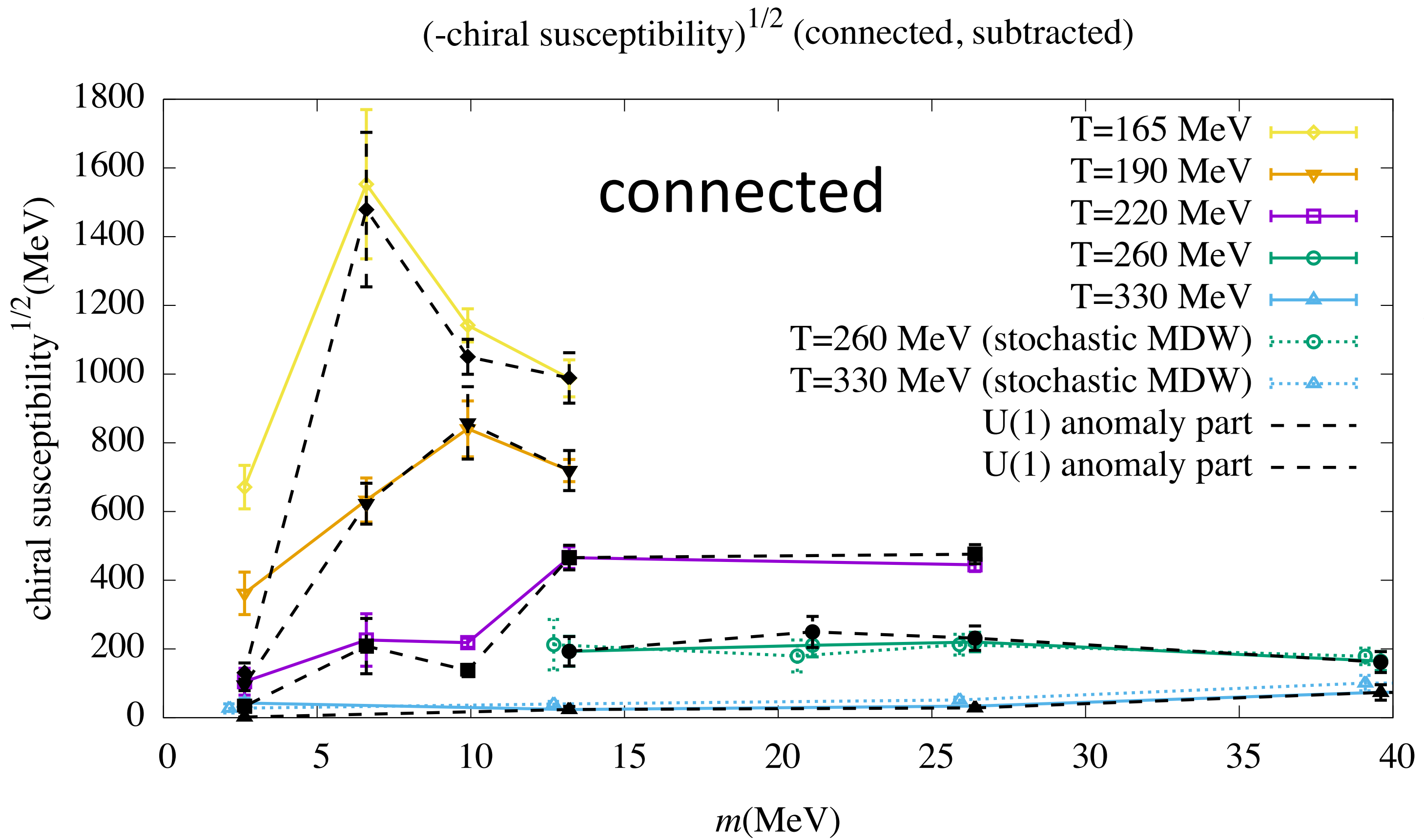
The formulas are known in continuum but true on a lattice only with overlap fermions.

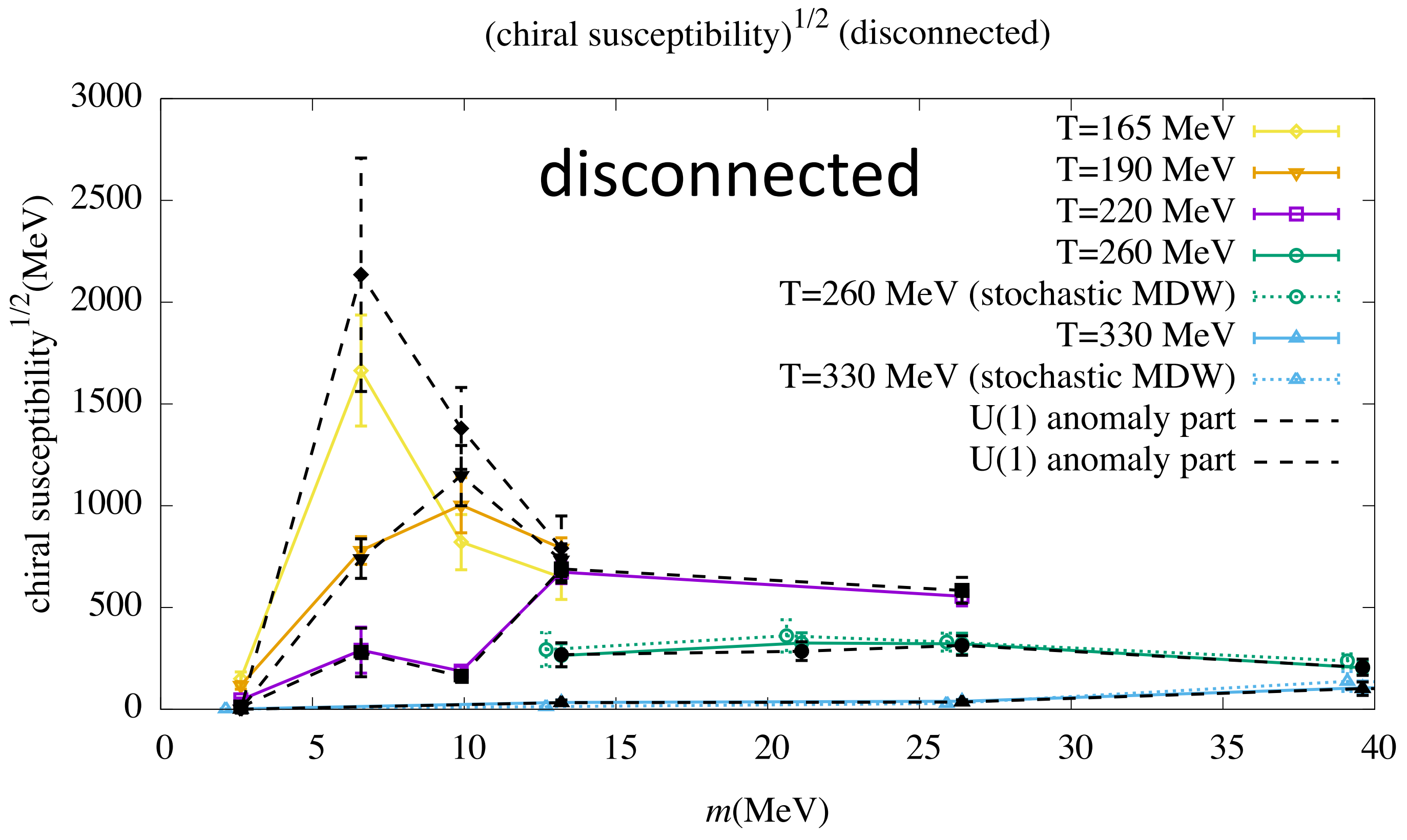
[LLNL/RBC 2014, Nicola et al. 2018,2020]

5. Lattice set-up

Nf=2 flavor QCD
T=190(~1.1Tc), 220, 260, 330 MeV.
(Lt=8,10,12,14)
1/a = 2.6 GeV (0.075fm)
L=24,32,40,48 [1.8-3.6fm] (at T=220MeV)
Mobius domain-wall fermion + reweighted overlap fermion
Quark mass from 3MeV
(< phys. pt. ~4MeV) to 30MeV.
Measurement is w/ spec. decomposition.

6. Numerical Results





~90% of signals comes from axial U(1) breaking effect.

Black : U(1)A breaking effect
Colored : total contribution
T=165MeV results are preliminary.

7. Summary

Chiral susceptibility is dominate by U(1) breaking at T>=165MeV.

Conn. part ~ U(1)$_A$ susceptibility
Discon. part ~ top. susceptibility

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