Global Symmetry and Partial Deconfinement

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Based on MH-Holden-Knaggs-O'Bannon, 2021 (hep-th, to appear) + multiple papers since 2016

Outline

- Partial deconfinement at infinite N
- Partial deconfinement at finite N
- Chiral symmetry vs Partial deconfinement
- CP symmetry vs Partial deconfinement
- Conclusion & speculation

Confinement/deconfinement transition at large N

- Confinement ~ individual colors are not visible

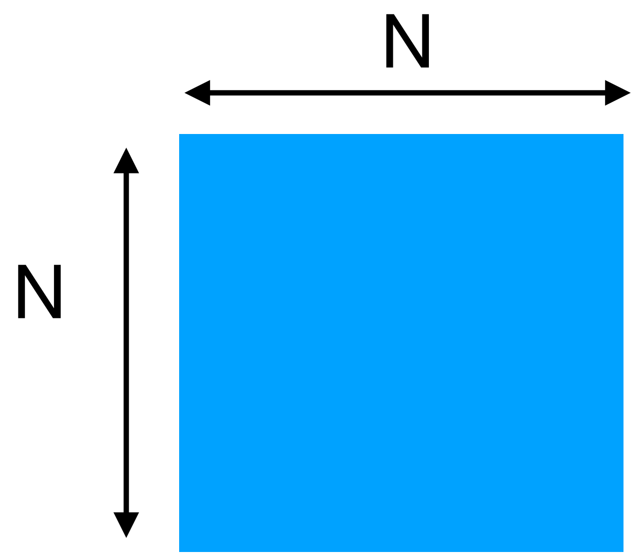
$$S \sim E \sim N^0$$

- Deconfinement ~ N^2 d.o.f. unlocked

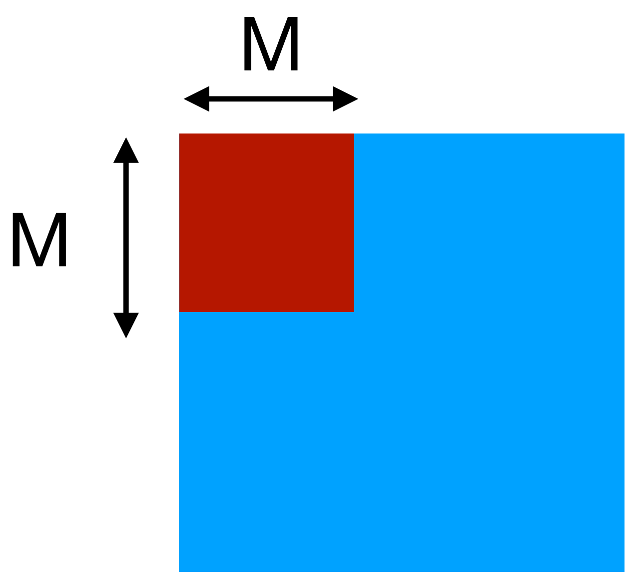
$$S \sim E \sim N^2$$

- What happens in between these two phases?

$$N^0 \ll E \ll N^2$$



Completely Confined

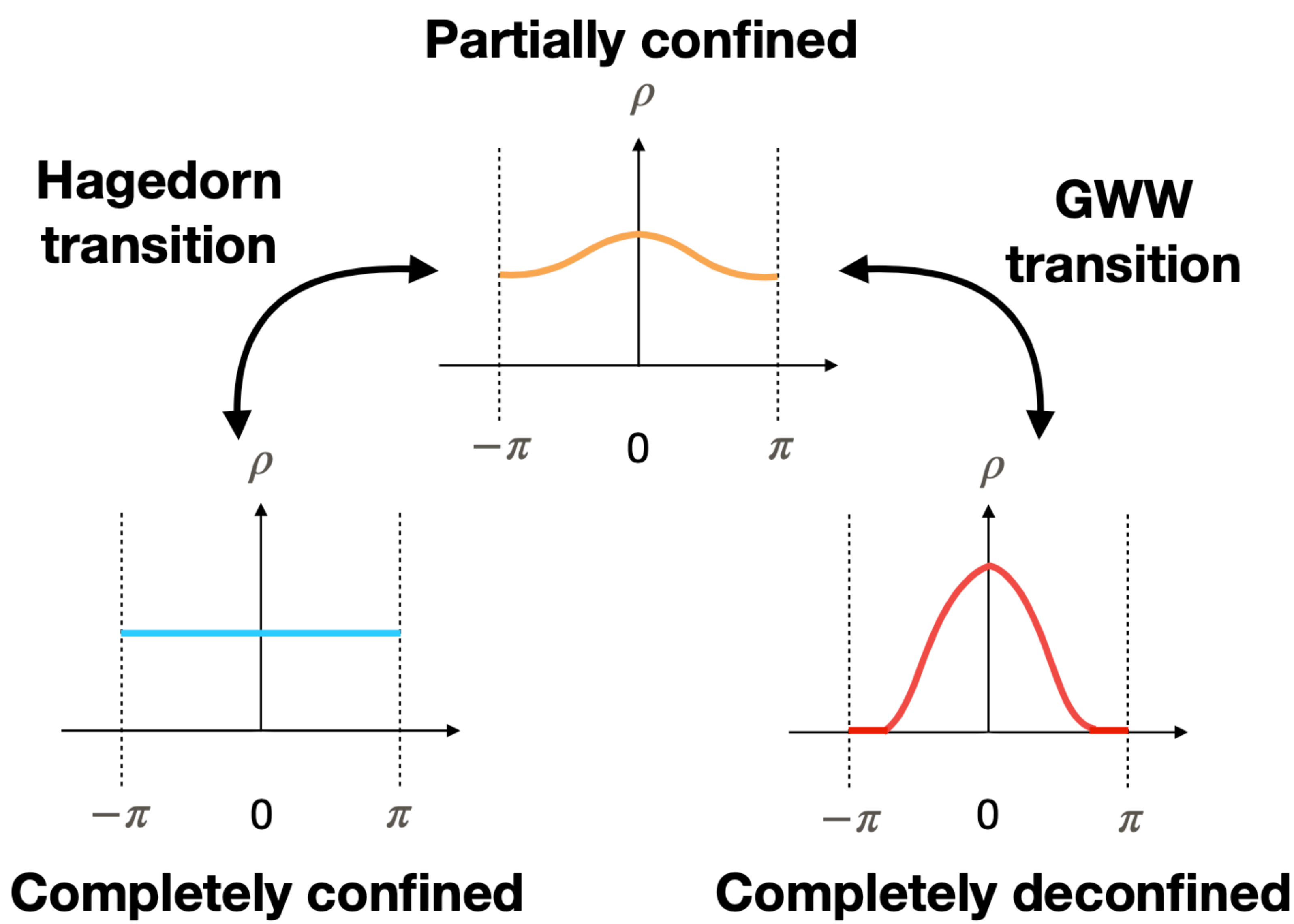


Partially Confined
(= Partially Deconfined)

MH-Maltz, 2016 (JHEP) **First proposal, for 4d SYM (as dual of small BH)**
MH-Ishiki-Watanabe, 2018 (JHEP) **Qualitative arguments for generic theories**
MH-Jevicki-Peng-Wintergerst, 2019 (JHEP) **Explicit demonstration for weakly-coupled theories**
MH-Shimada-Wintergerst, 2020 (JHEP)
Underlying mechanism (~BEC in color space)

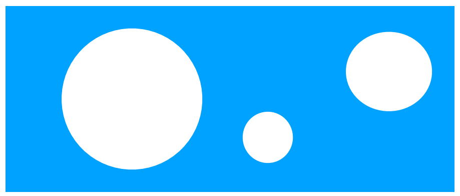
Completely Deconfined

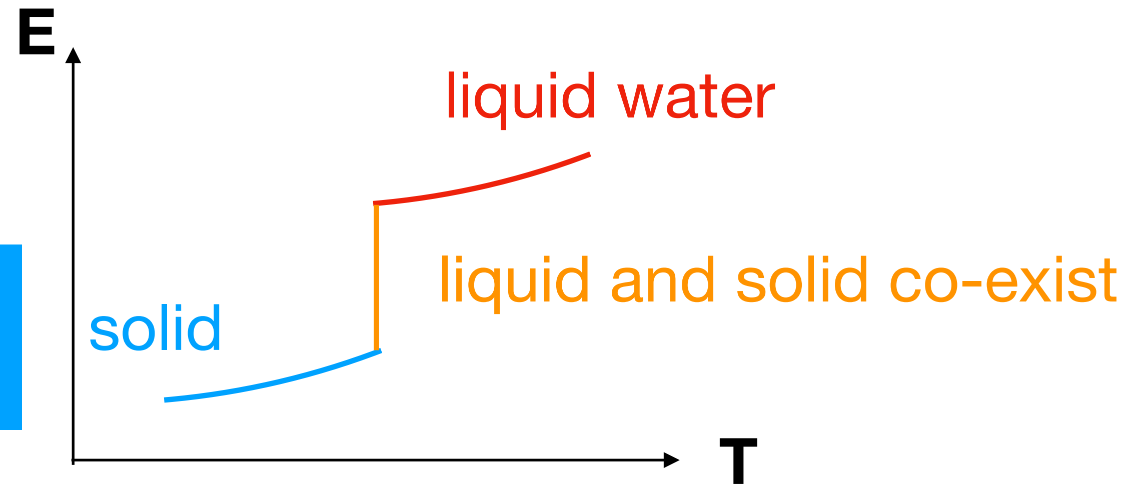
Polyakov Loop $P = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$

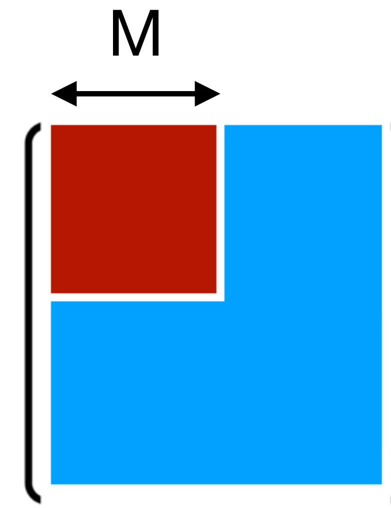


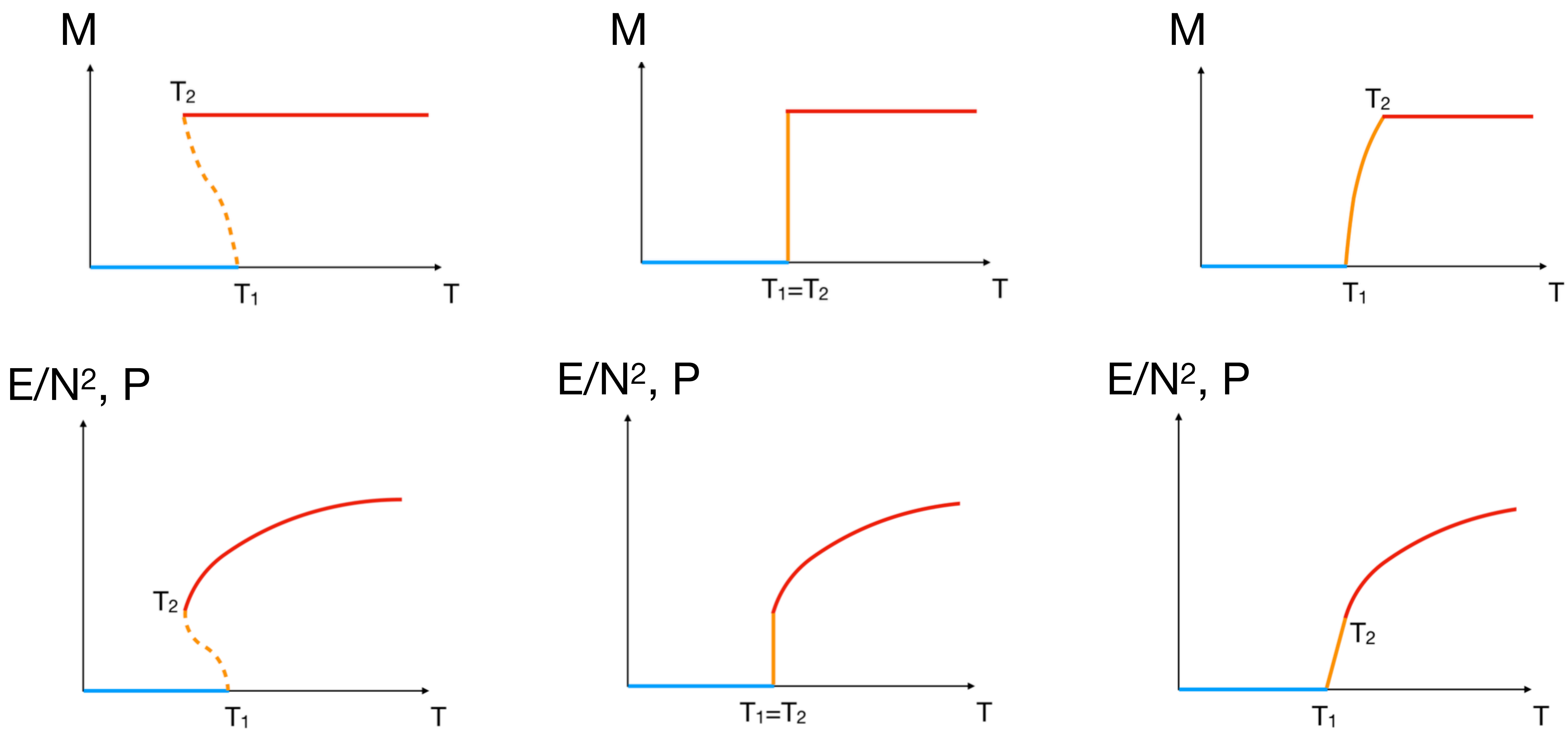
MH-Ishiki-Watanabe, 2018 (JHEP)
MH-Shimada-Wintergerst, 2020 (JHEP)

cf) water/ice









All-to-all interaction → Nontrivial T-dependence

Gauge singlet constraint in Hilbert space

- "Gauge-singlet constraint appears when A_0, is integrated out."

$$G = \prod_{\vec{x}} [\mathrm{SU}(N)]_{\vec{x}}$$

Gauge singlet constraint in Hilbert space

- "Gauge-singlet constraint appears when A_0, is integrated out."

Path integral with A_0 ⟷ **textbook argument** **(Feynman)**

$$Z(T) = \frac{1}{\mathrm{vol}(G)} \int_G dg \mathrm{Tr}_{\mathcal{H}_{\mathrm{ext}}}(\hat{g} e^{-\hat{H}/T})$$

projection to singlet

(~Polyakov loop; gauge field A_0)

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Gauge singlet constraint in Hilbert space

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(~Polyakov loop; gauge field A_0)

Equivalent

$$Z(T) = \mathrm{Tr}_{\mathcal{H}_{\mathrm{inv}}}(e^{-\hat{H}/T})$$

$$G = \prod_{\vec{x}} [\mathrm{SU}(N)]_{\vec{x}}$$

$$Z(T) = \frac{1}{\mathrm{vol}(G)} \int_G dg \mathrm{Tr}_{\mathcal{H}_{\rm ext}} (\hat{g} e^{-\hat{H}/T})$$

G-symmetrization →

$$Z(T) = \mathrm{Tr}_{\mathcal{H}_{\rm inv}} (e^{-\hat{H}/T})$$

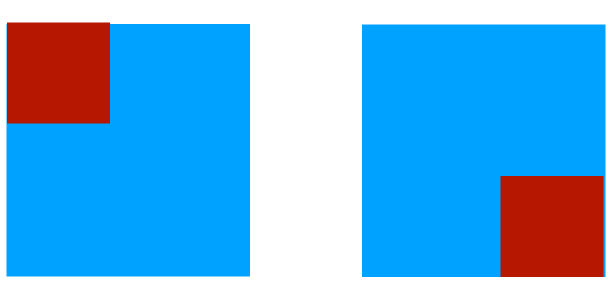
projection to singlet

(~Polyakov loop; gauge field A_0)

$$|\Phi\rangle \quad \rightarrow \quad \int_G dg (\hat{g}|\Phi\rangle)$$

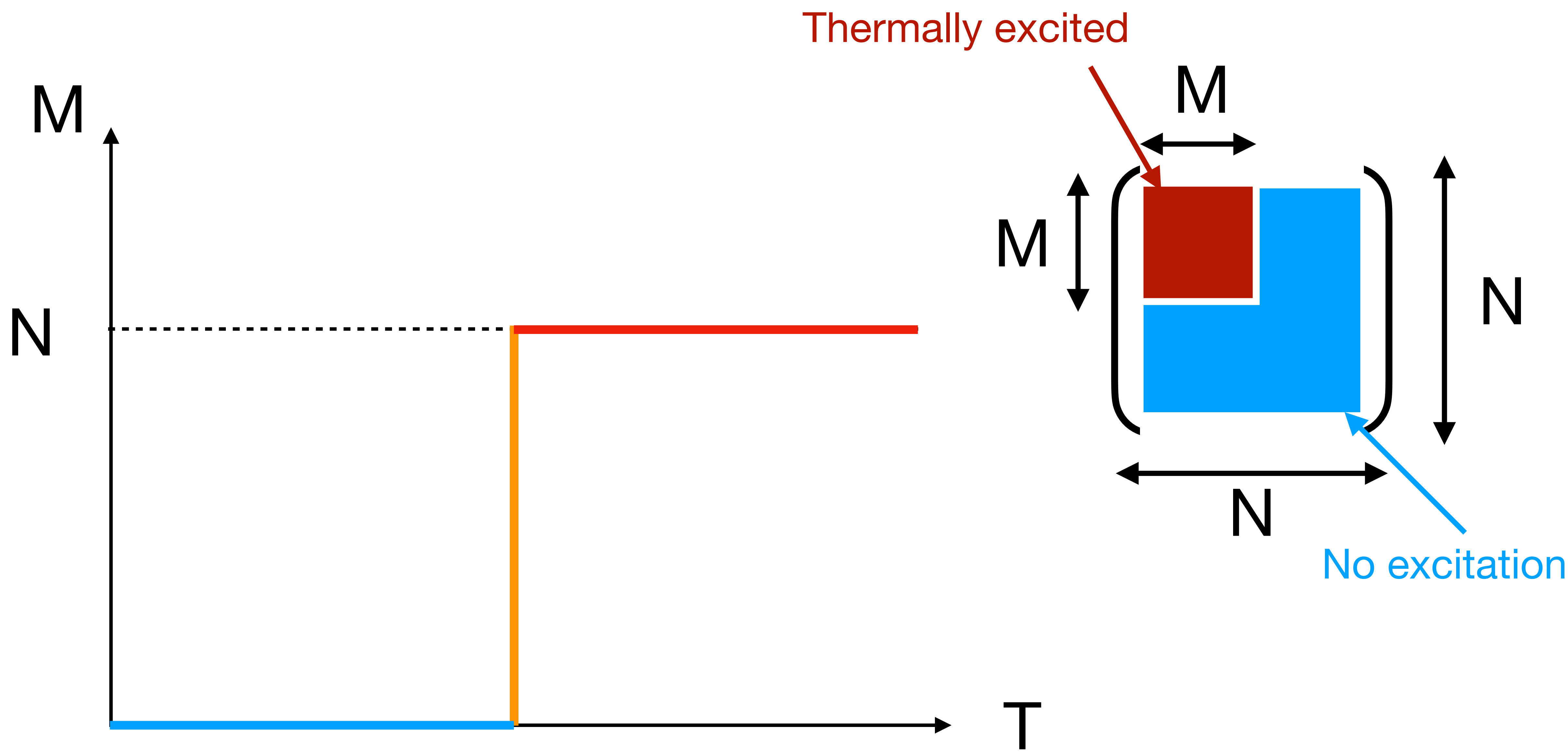
$$|\Phi\rangle \sim \hat{g}|\Phi\rangle$$

states related by a gauge transformation should be identified



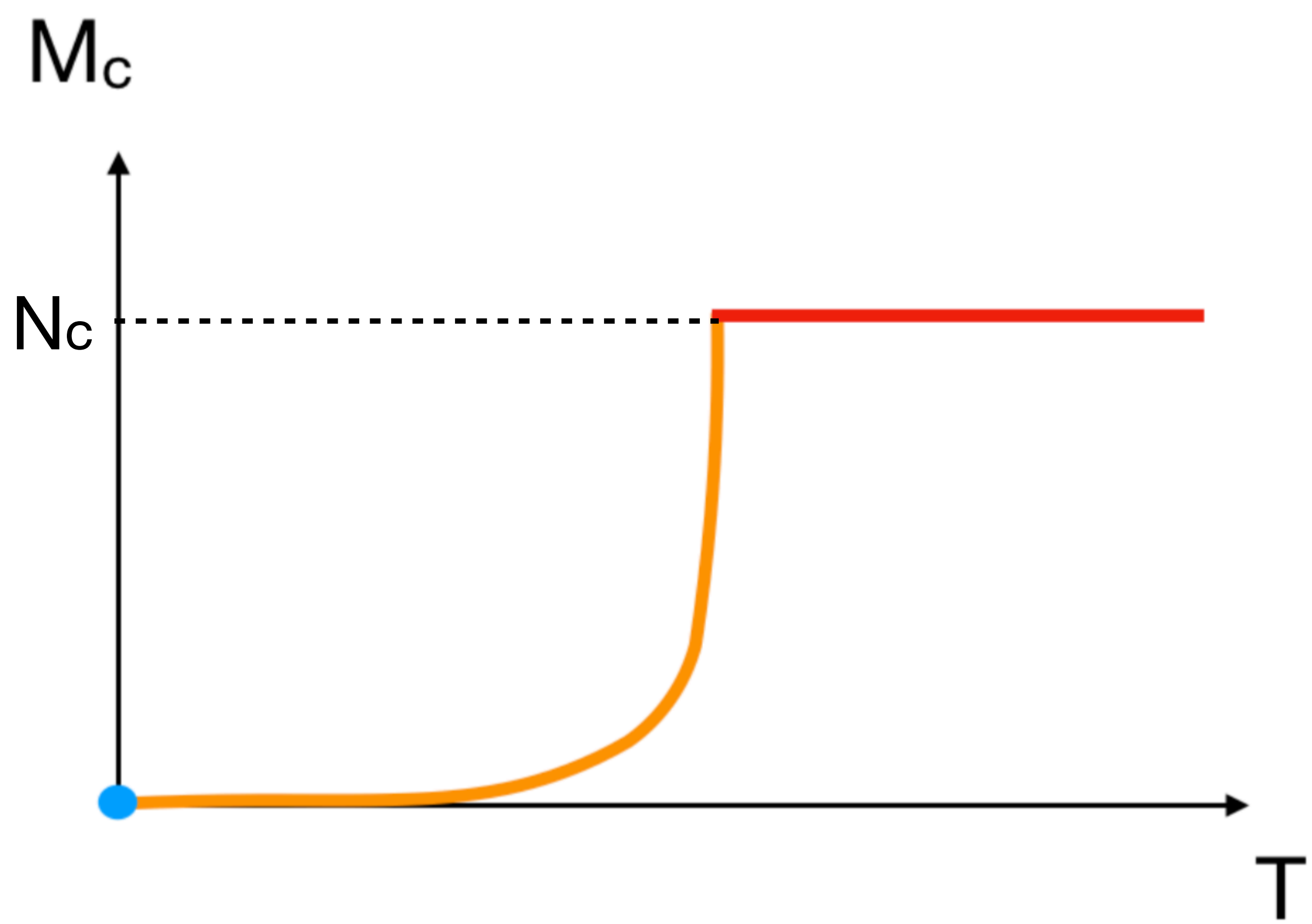
Any embedding of SU(M) to SU(N) is fine.

Weakly-coupled pure YM on S^3

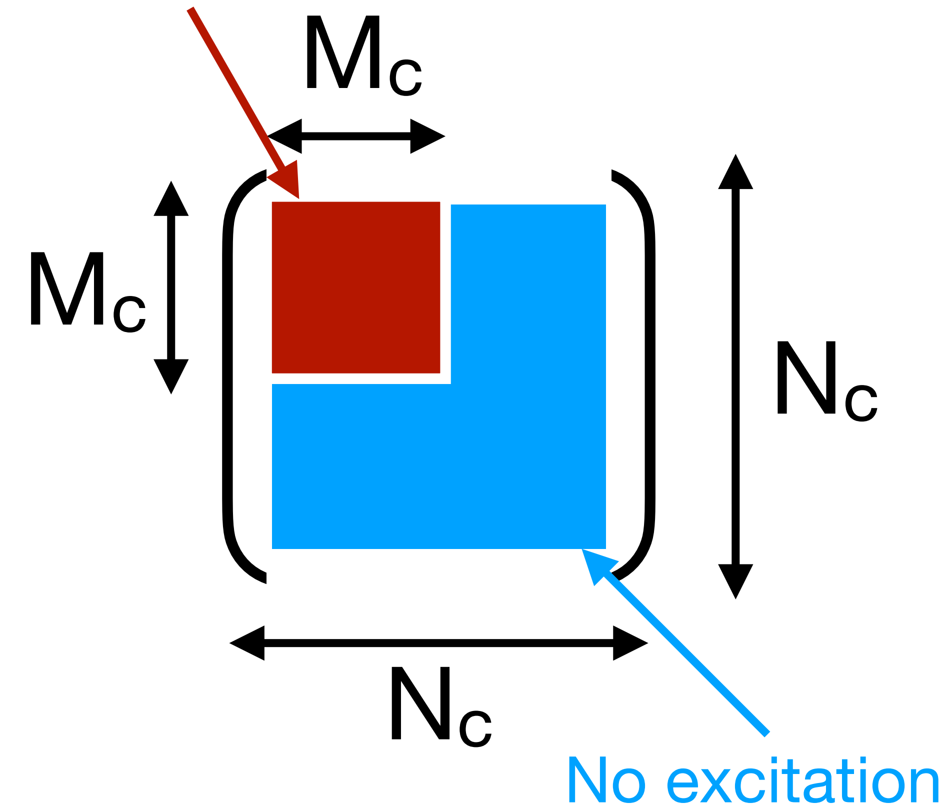


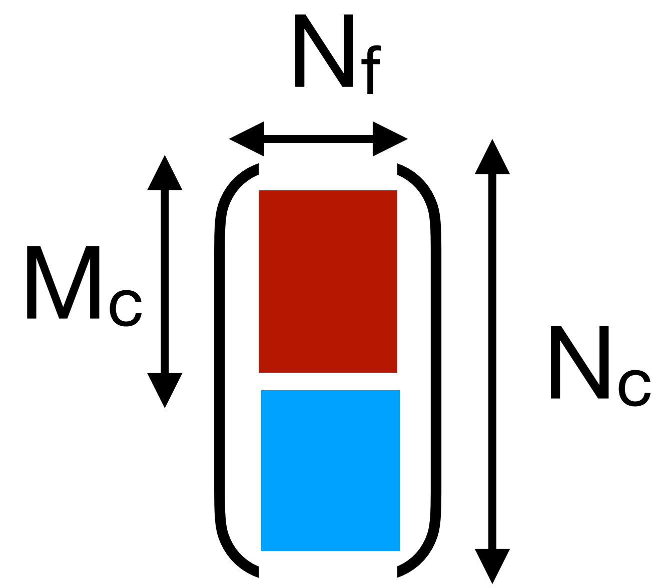
MH-Jevicki-Peng-Wintergerst, 1909.09118
(re-interpretation of Sundborg 1998, Aharony et al 2003)

Weakly-coupled QCD on S^3



MH-Robinson, 1911.06223
(re-interpretation of Schnitzer's calculation in 2004)



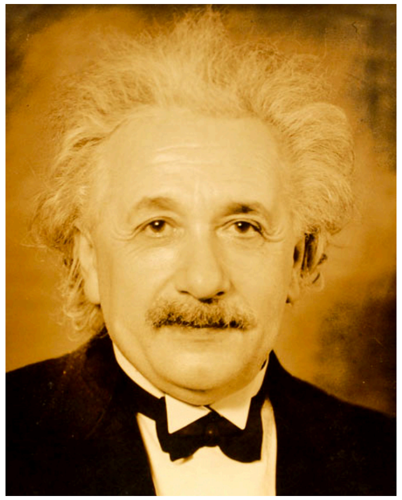


Free, but N_f/M_c changes
→ nontrivial T-dependence

Underlying Mechanism

BEC in color space

Bose



Einstein

Summation over singlet states $Z(T) = \mathrm{Tr}_{\mathcal{H}_{\mathrm{inv}}}(e^{-\hat{H}/T})$

Summation over all states & projection to singlet states

$$Z(T) = \frac{1}{\mathrm{vol}(G)} \int_G dg \mathrm{Tr}_{\mathcal{H}_{\mathrm{ext}}}(\hat{g} e^{-\hat{H}/T})$$

$G = \mathrm{SU}(N)$ + adjoint fields → Yang-Mills

$G = \mathrm{S}_N$ + fundamental fields → N indistinguishable bosons

N bosons in 3d harmonic trap

$$\hat{H} = \sum_{i=1}^{N} \left(\frac{\hat{\vec{p}}_i^2}{2m} + \frac{m\omega^2}{2} \hat{\vec{x}}_i^2 \right)$$

$$\hat{\vec{x}}_i = (\hat{x}_i, \hat{y}_i, \hat{z}_i)$$

$$\hat{\vec{p}}_i = (\hat{p}_{x,i}, \hat{p}_{y,i}, \hat{p}_{z,i})$$

Fock states $|\vec{n}_1, \vec{n}_2, \cdots, \vec{n}_N\rangle \equiv \prod_{i=1}^{3} \frac{\hat{a}_{i1}^{\dagger n_{i1}}}{\sqrt{n_{i1}!}} \frac{\hat{a}_{i2}^{\dagger n_{i2}}}{\sqrt{n_{i2}!}} \cdots \frac{\hat{a}_{iN}^{\dagger n_{iN}}}{\sqrt{n_{iN}!}} |0\rangle$

$$\begin{aligned} Z(T) &= \frac{1}{N!} \sum_{\sigma \in \mathrm{S}_N} \sum_{\vec{n}_1, \cdots, \vec{n}_N} \langle \vec{n}_1, \cdots, \vec{n}_N | \hat{\sigma} e^{-\hat{H}/T} | \vec{n}_1, \cdots, \vec{n}_N \rangle \\ &= \frac{1}{N!} \sum_{\vec{n}_1, \cdots, \vec{n}_N} e^{-(E_{\vec{n}_1} + \cdots + E_{\vec{n}_N})/T} \underline{\left(\sum_{\sigma \in \mathrm{S}_N} \langle \vec{n}_1, \cdots, \vec{n}_N | \vec{n}_{\sigma(1)}, \cdots, \vec{n}_{\sigma(N)} \rangle \right)} \end{aligned}$$

measures the amount of redundancy

$$Z(T) = \frac{1}{N!} \sum_{\sigma \in \mathrm{S}_N} \sum_{\vec{n}_1, \cdots, \vec{n}_N} \langle \vec{n}_1, \cdots, \vec{n}_N | \hat{\sigma} e^{-\hat{H}/T} | \vec{n}_1, \cdots, \vec{n}_N \rangle$$

$$= \frac{1}{N!} \sum_{\vec{n}_1, \cdots, \vec{n}_N} e^{-(E_{\vec{n}_1} + \cdots + E_{\vec{n}_N})/T} \left(\sum_{\sigma \in \mathrm{S}_N} \langle \vec{n}_1, \cdots, \vec{n}_N | \vec{n}_{\sigma(1)}, \cdots, \vec{n}_{\sigma(N)} \rangle \right)$$

$|\vec{0}, \vec{0}, \cdots, \vec{0}\rangle$ $\quad N!$

$|\vec{n}_1, \cdots, \vec{n}_N\rangle$ $\quad 1$

(all of them are different)

$|\vec{n}_1, \cdots, \vec{n}_M, \vec{0}, \cdots, \vec{0}\rangle$ $\quad (N-M)!$

This enhancement triggers BEC.

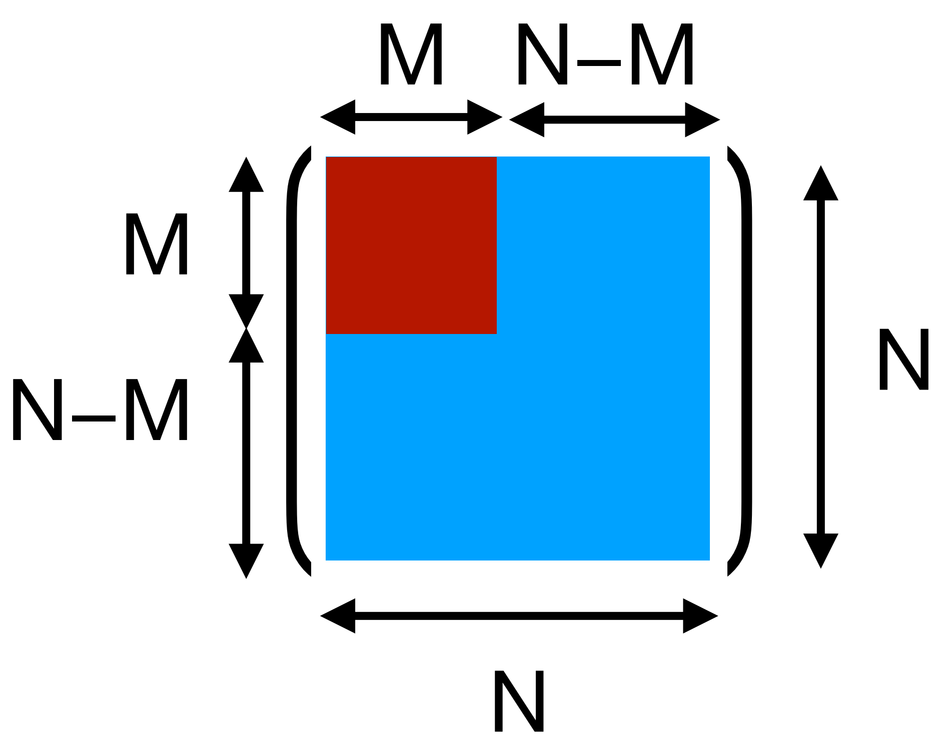
Einstein, 1924

Partially-BEC state

$$|\vec{n}_1, \cdots, \vec{n}_M, \vec{0}, \cdots, \vec{0}\rangle \quad (N-M)!$$

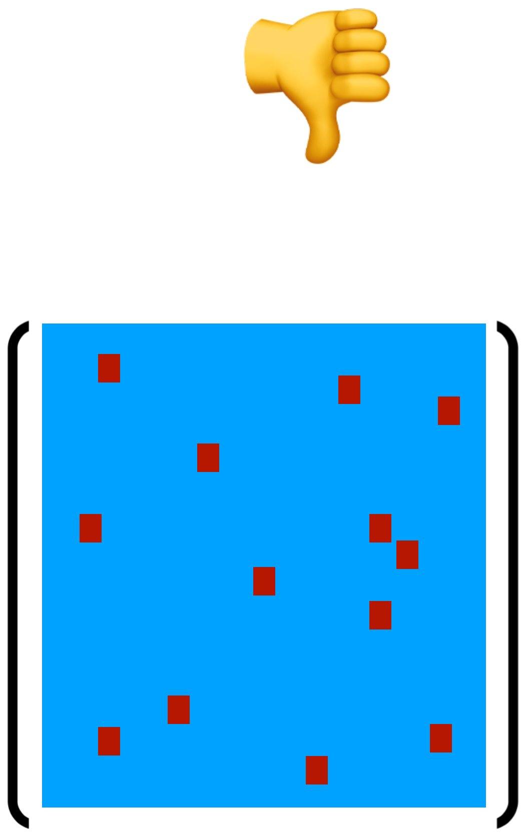
Partially-confined state

(MH-Maltz, 2016; Berenstein, 2018;
MH-Ishiki-Watanabe, 2018;
MH-Jevicki-Peng-Wintergerst, 2019;
Watanabe et al, 2020)

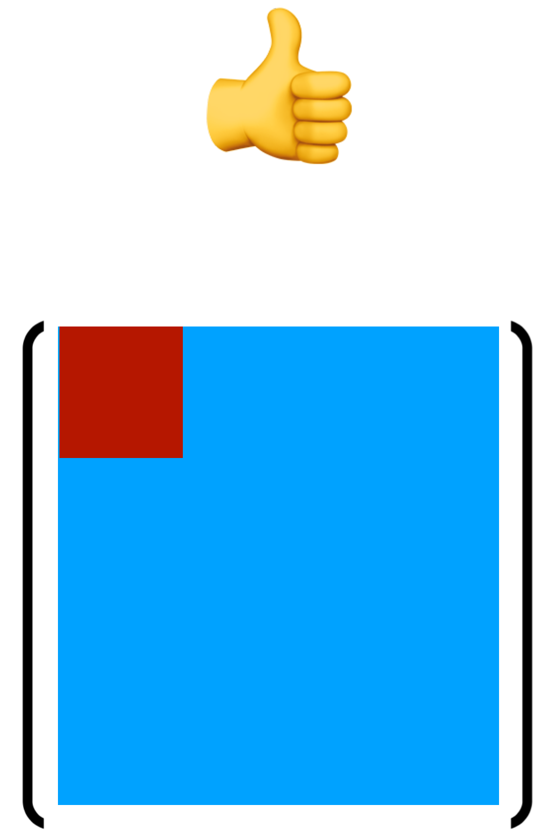


$$\mathrm{vol}(\mathrm{SU}(N-M)) \sim e^{(N-M)^2}$$

deconfined sector = extended bound state of strings and D-brane
= black hole



no symmetry



Larger enhancement factor (volume of SU(N-M))

This symmetry argument does not assume weak coupling.

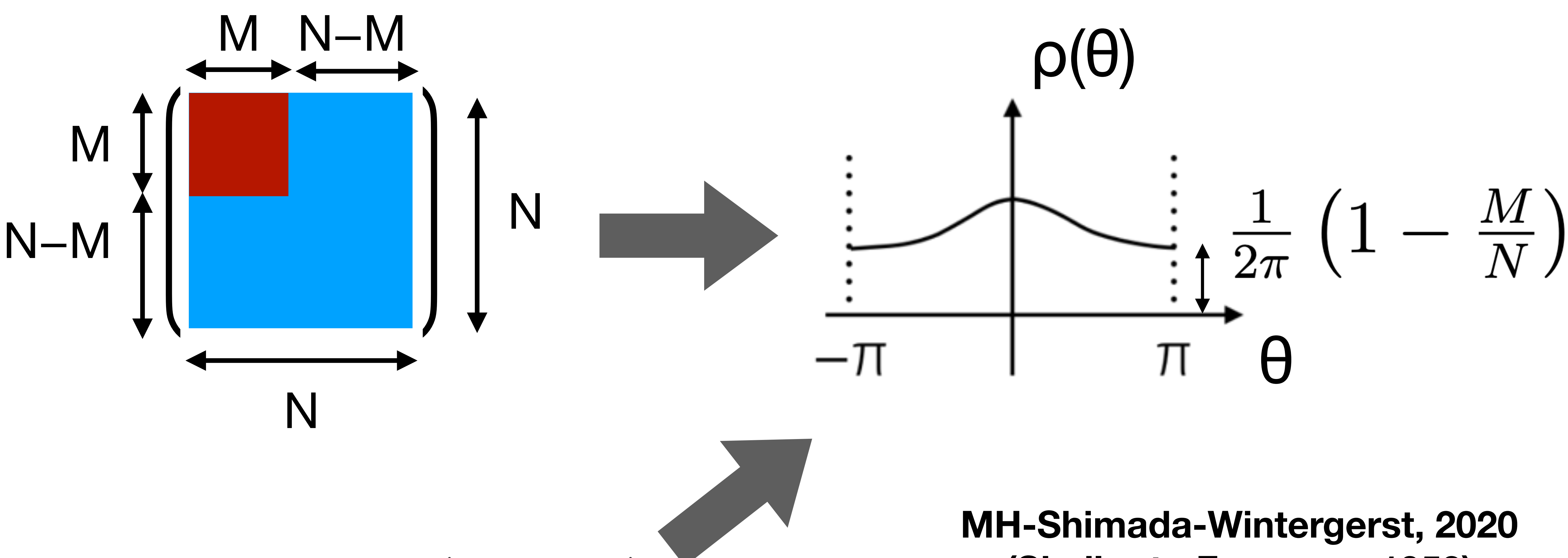
Explicit construction for SYM and matrix model: MH, 2102.08982 [hep-th]

$$Z(T) = \frac{1}{\mathrm{vol}G}\int_G dg\,\mathrm{Tr}_{\mathcal{H}_{\mathrm{ext}}}\left(\hat{g}e^{-\hat{H}/T}\right)$$

$$\sim \frac{1}{\mathrm{vol}G}e^{-E_{\mathrm{typical}}/T}\int_G dg\langle\mathrm{typical}|\hat{g}|\mathrm{typical}\rangle$$

Polyakov loop

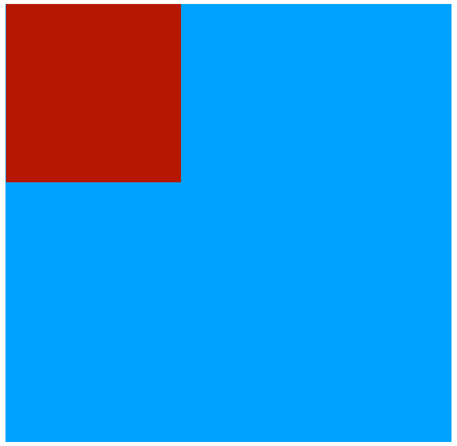
Typical $\hat{g}$'s which leave $|\mathrm{typical}\rangle$ unchanged dominate the phase distribution



$$|\vec{n}_1,\cdots,\vec{n}_M,\vec{0},\cdots,\vec{0}\rangle$$

MH-Shimada-Wintergerst, 2020
(Similar to Feynman, 1953)

- Partial deconfinement at infinite N
- Partial deconfinement at finite N
- Chiral symmetry vs Partial deconfinement
- CP symmetry vs Partial deconfinement
- Conclusion & speculation



We have to specify the embedding of SU(M) as boundary condition
→multiple superselection sectors

- Global part of gauge symmetry breaks spontaneously.
- It is convenient to fix the local part, like usual Higgsing.
- Gauge fixing of the local part makes physics more easily understandable.

$$SU(N) \to SU(M) \times SU(N-M)$$

'gauge symmetry breaking' like Higgs mechanism.

MH-Jevicki-Peng-Wintergerst, 2019

Works for finite N as well. No need for global symmetry.
→ It works for real-world QCD.
But the consequence is not immediately clear.

- Partial deconfinement at infinite N
- Partial deconfinement at finite N
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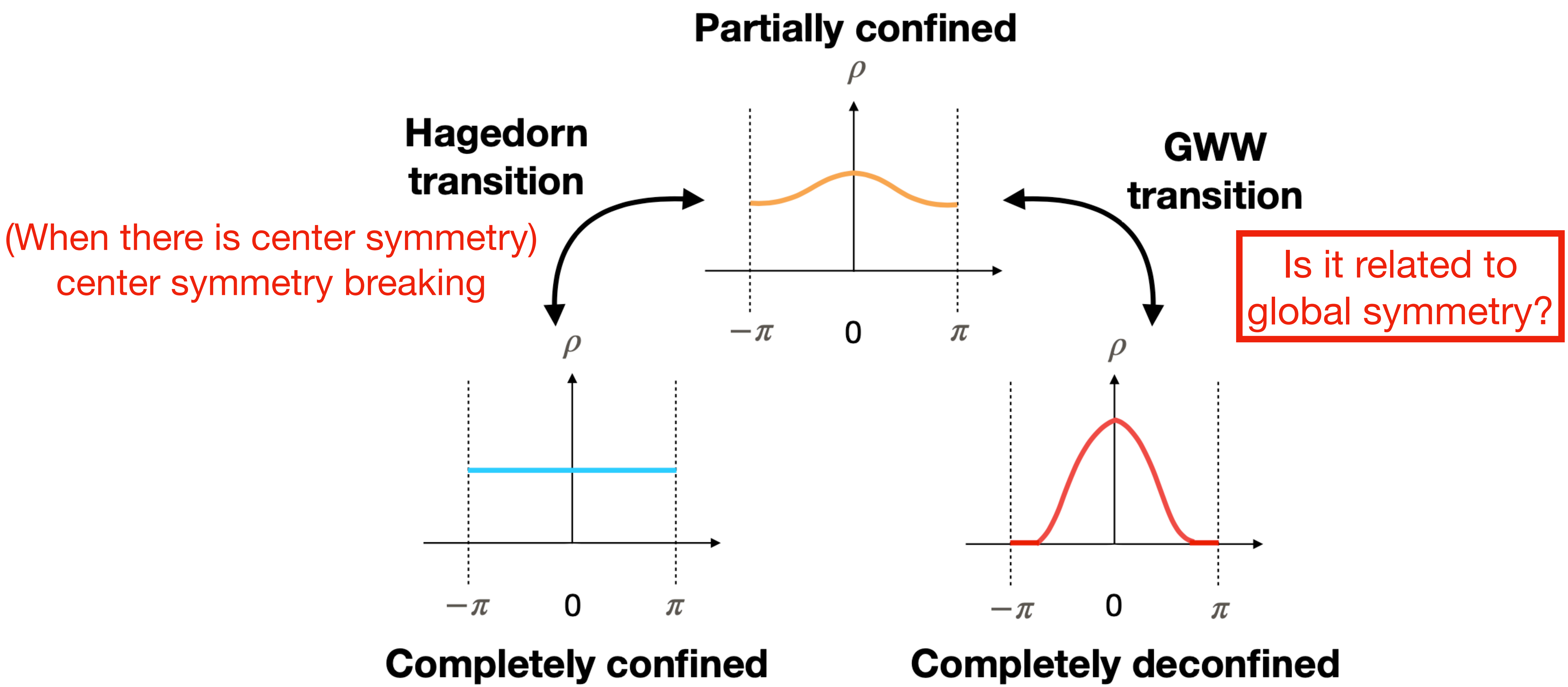
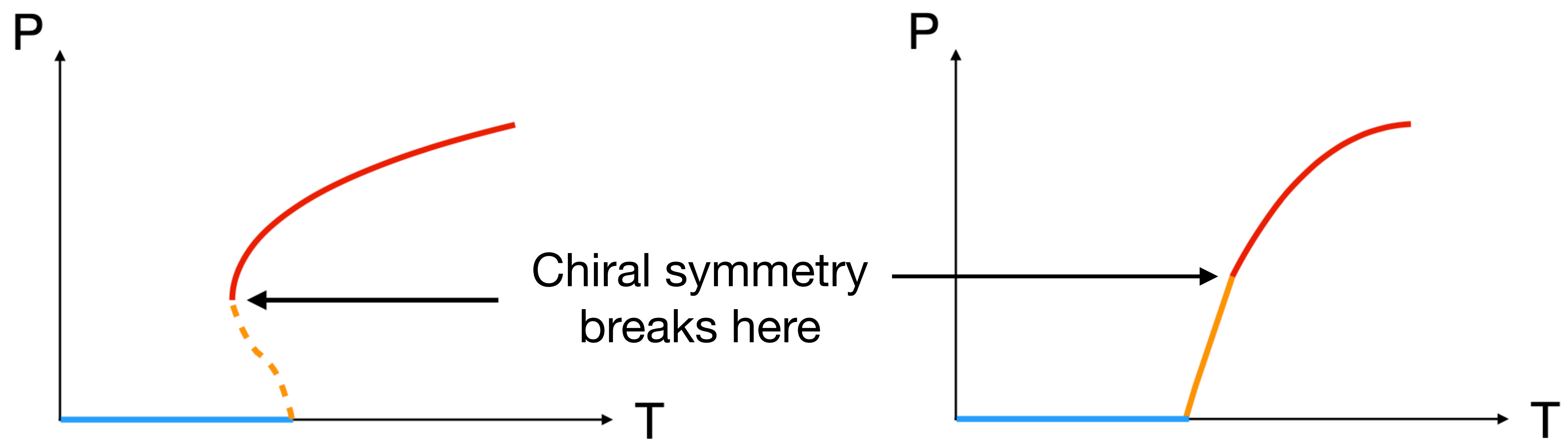
Polyakov Loop $P = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$



Our hypothesis

MH-Robinson, 1911.06223

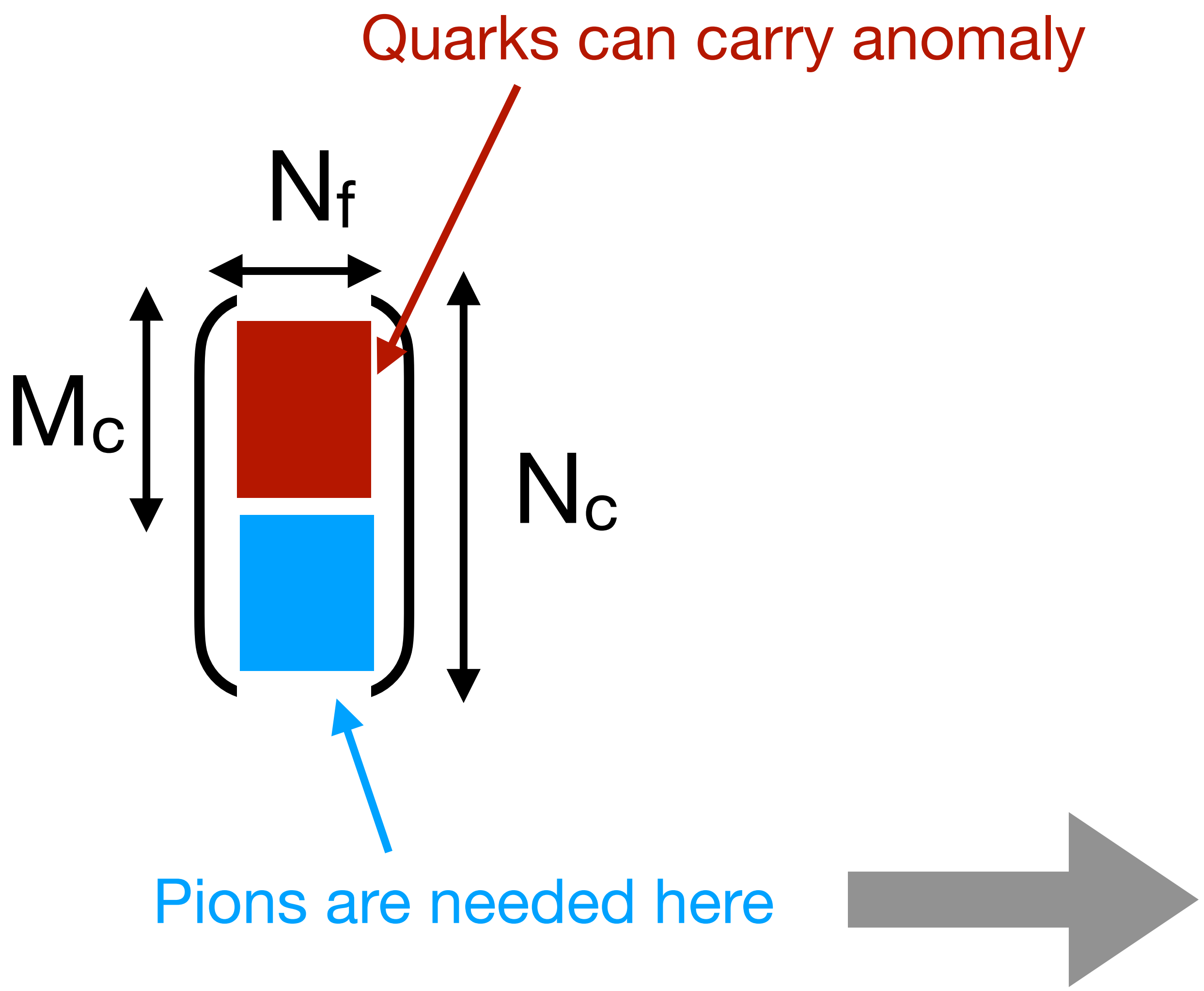


MH-Robinson, 1911.06223

Because:

- 't Hooft anomaly matching
  → chiral symmetry has to be broken in confined phase

- Anomaly matching is required at any energy scale

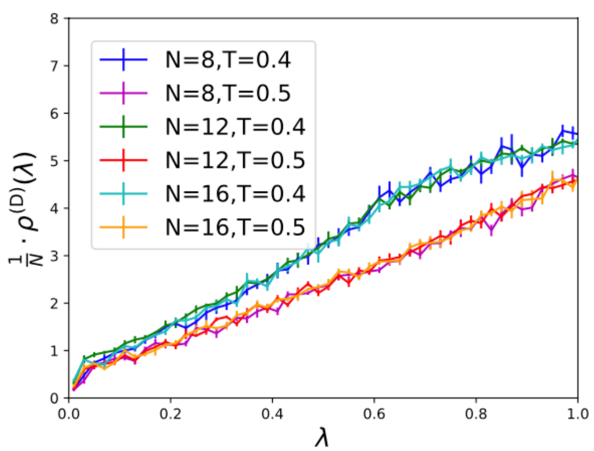


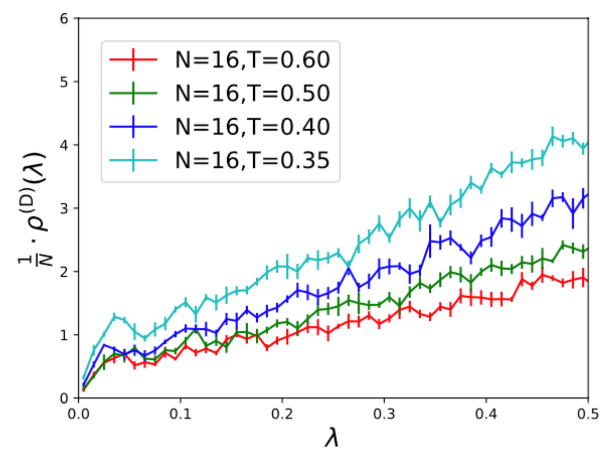
Chiral symmetry
should be broken

Chiral symmetry (1)

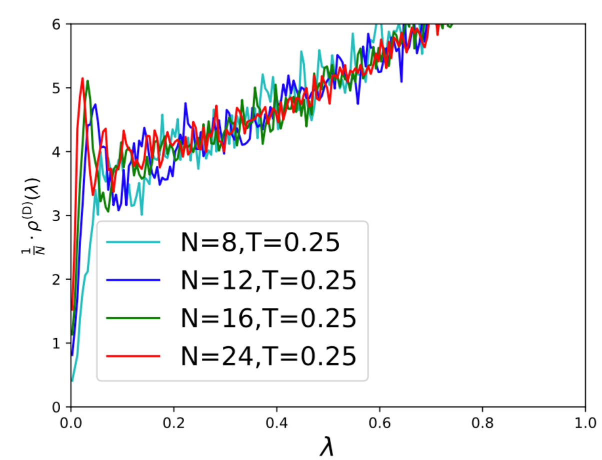
- Strong-coupling lattice YM + probe fermion (naive fermion with doublers)
- Eguchi Kawai equivalence → single-site model can be used
- Banks-Casher relation: chiral condensate ∝ $\rho_{\mathrm{Dirac}}(0)$

Completely-deconfined
(high T)





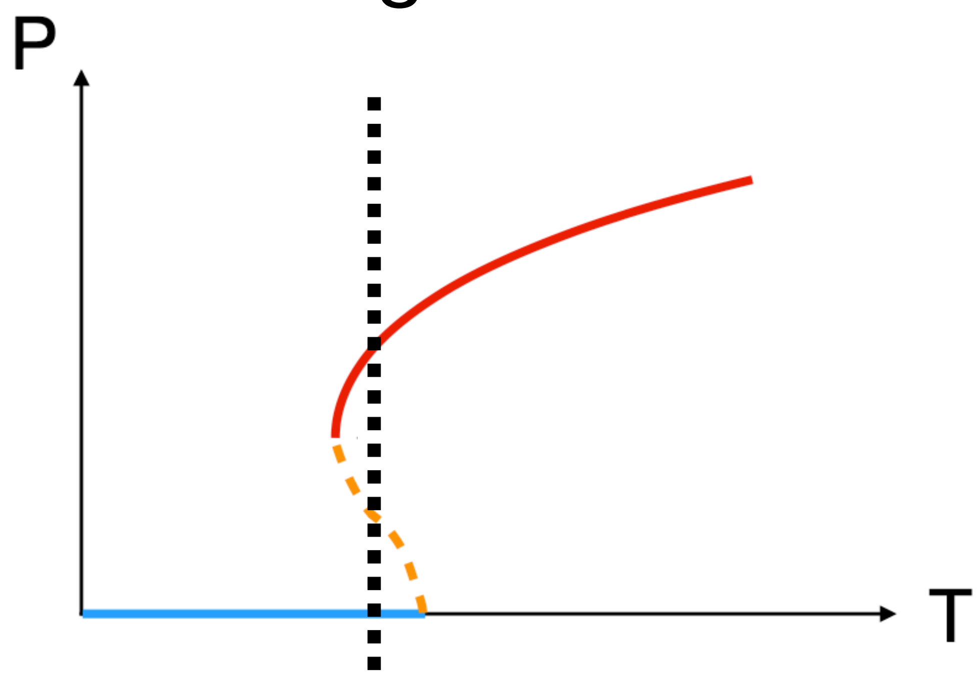
Completely-confined
(low T)



MH-Holden-Knaggs-O'Bannon, 2021 (hep-th, to appear)

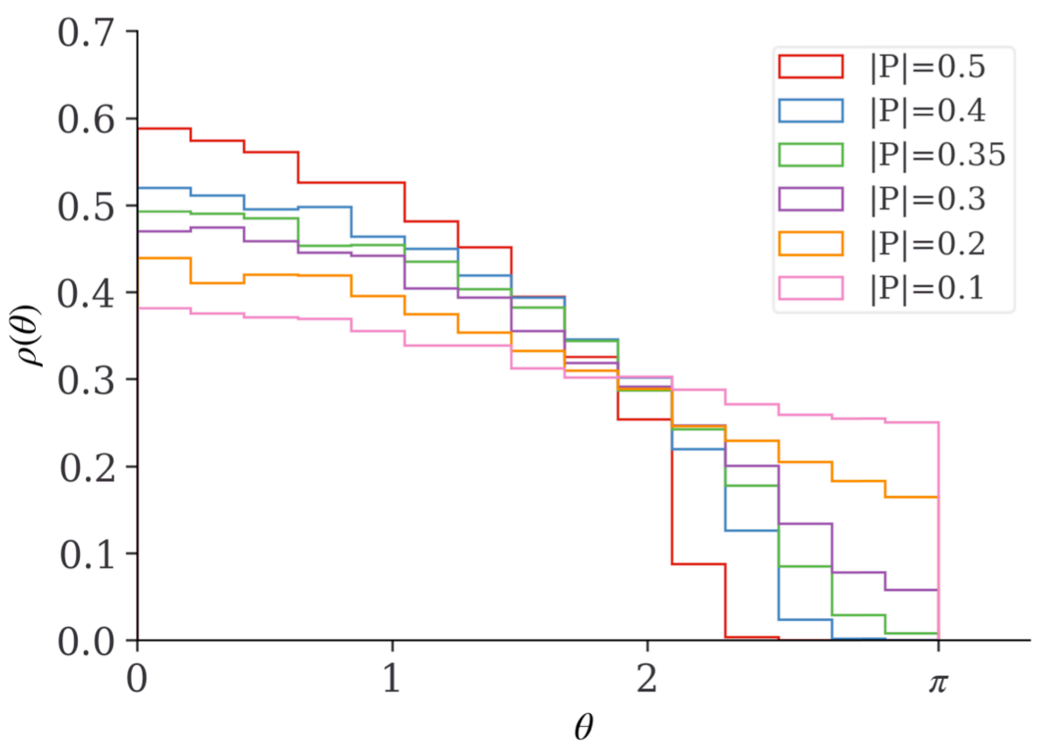
Chiral symmetry (2)

Vary P
along this line



T=0.29

Polyakov line phase
distribution
@ critical temperature

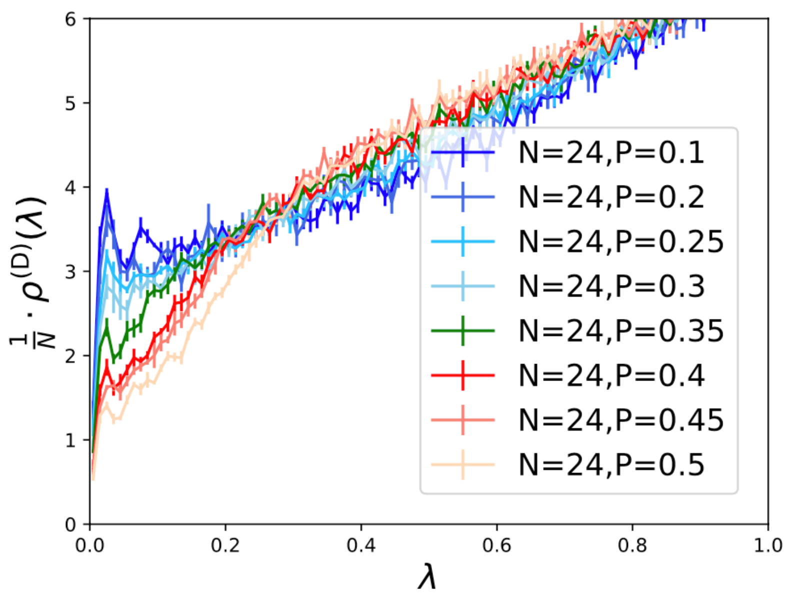


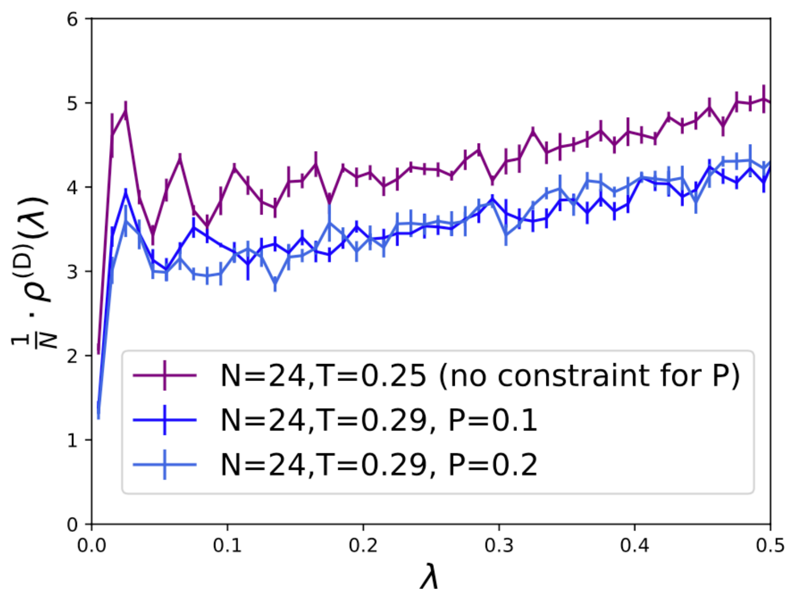
GWW transition
P ~ 0.35

MH-Holden-Knaggs-O'Bannon, 2021 (hep-th, to appear)

Chiral symmetry (3)

Dirac eigenvalue
distribution
@ critical temperature





GWW transition
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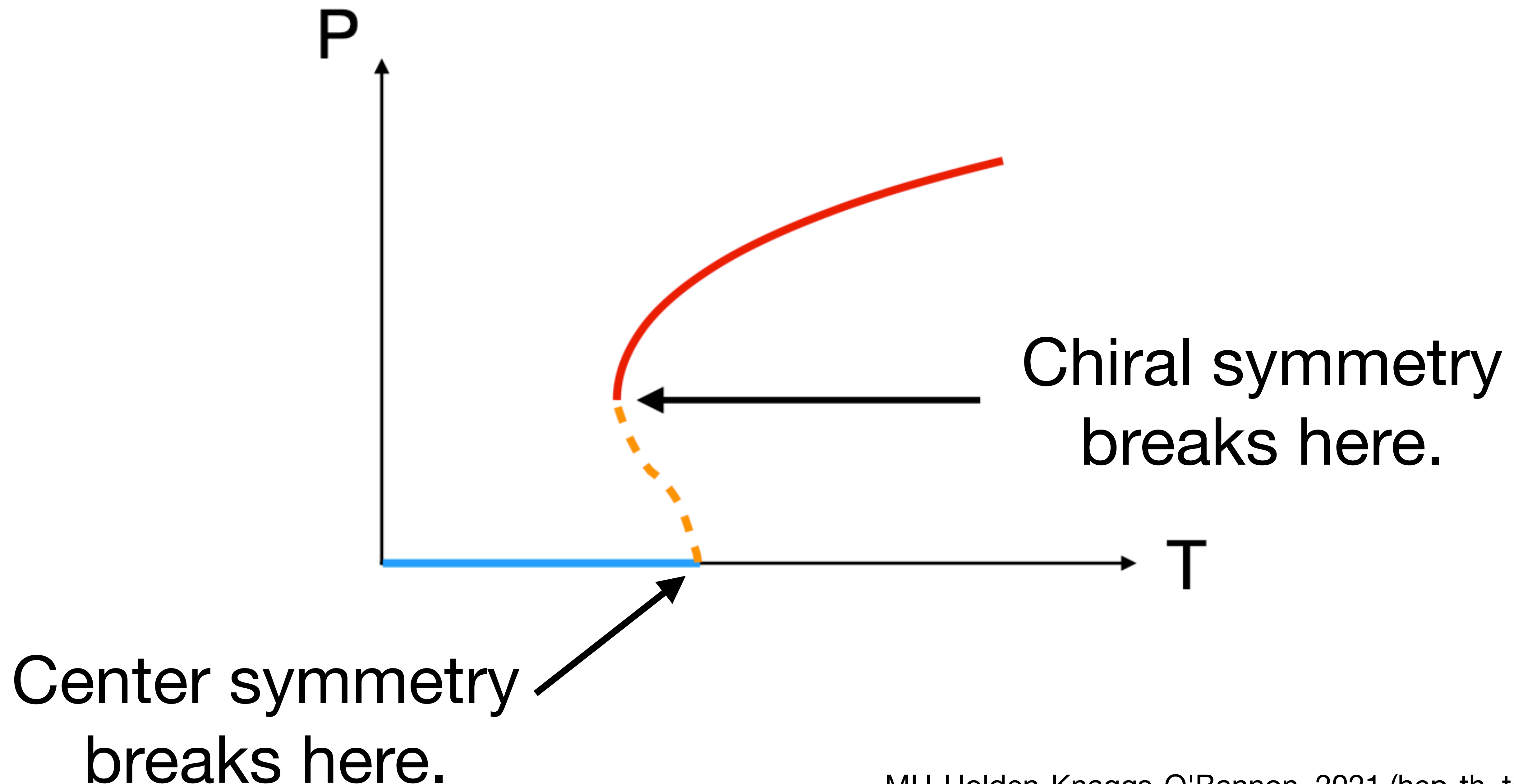
MH-Holden-Knaggs-O'Bannon, 2021 (hep-th, to appear)

Chiral symmetry (3)



MH-Holden-Knaggs-O'Bannon, 2021 (hep-th, to appear)

- Partial deconfinement at infinite N
- Partial deconfinement at finite N
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- CP symmetry vs Partial deconfinement
- Conclusion & speculation

CP symmetry (1)

4d $\mathcal{N}$=1SYM on $R^3 \times S^1$

$$\int d^4x \left(\frac{1}{4g^2}\mathrm{tr}(FF) - \frac{i\theta}{8\pi^2}\mathrm{tr}(F\tilde{F}) + \frac{2i}{g^2}\mathrm{tr}(\bar{\lambda}\bar{\sigma}^{\mu}D_{\mu}\lambda) \right)$$

- pbc for bosons and fermions
- Add soft mass for gluino

→ analytically controllable at very small S^1 radius

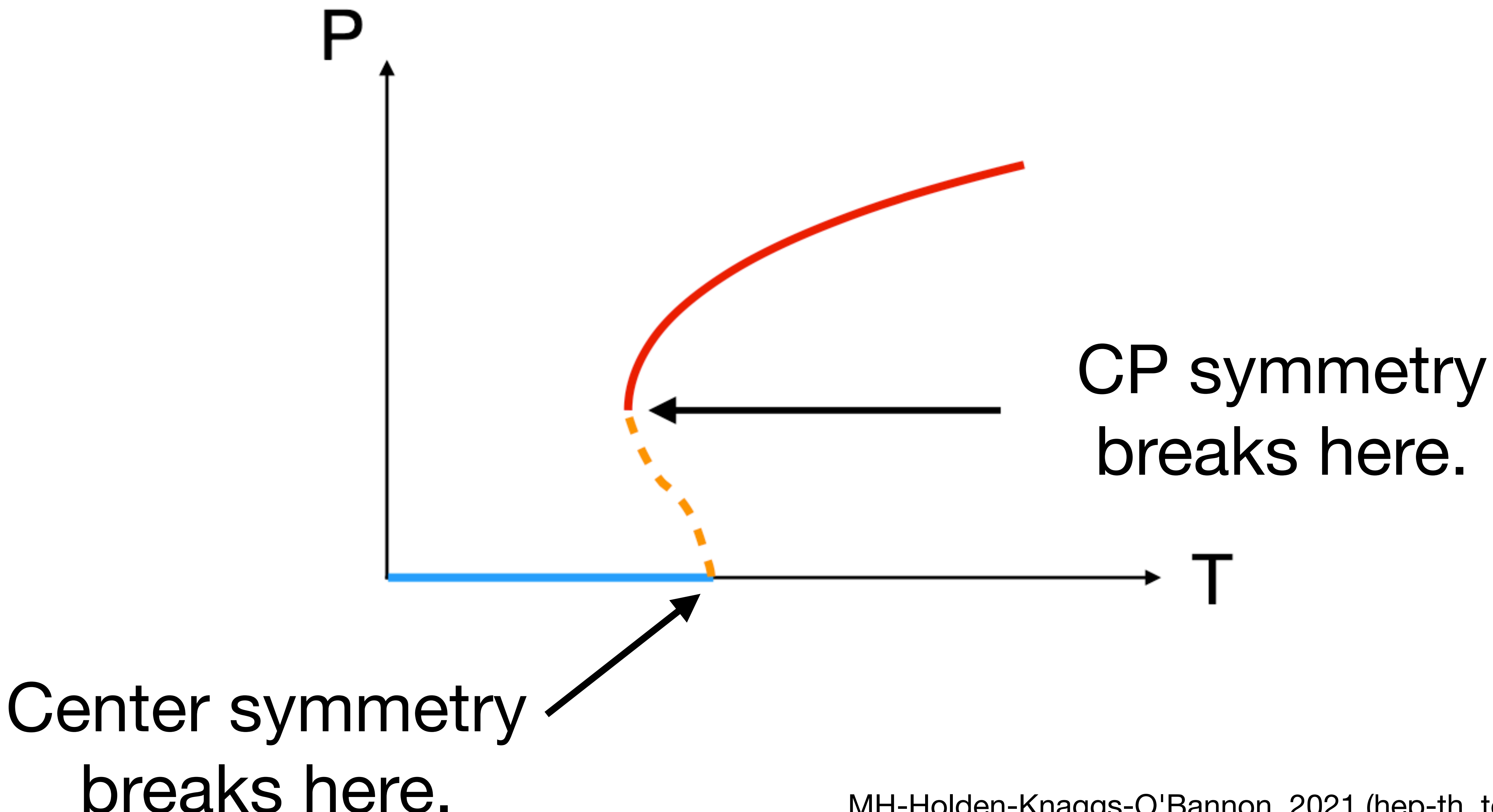
Poppitz, Schaefer, Unsal 2012

- Spontaneous CP breaking at θ = π

Gaiotto, Kapustin, Komargodski, Seiberg 2017
Chen, Fukushima, Nishimura, Tanizaki 2020

CP symmetry (2)

We will see:



MH-Holden-Knaggs-O'Bannon, 2021 (hep-th, to appear)

CP symmetry (3)

outline of the derivation

- Effective action in terms of Polyakov loop phases is known

Poppitz, Schaefer, Unsal 2012

minimum/maximum of effective potential = minimum/maximum of free energy

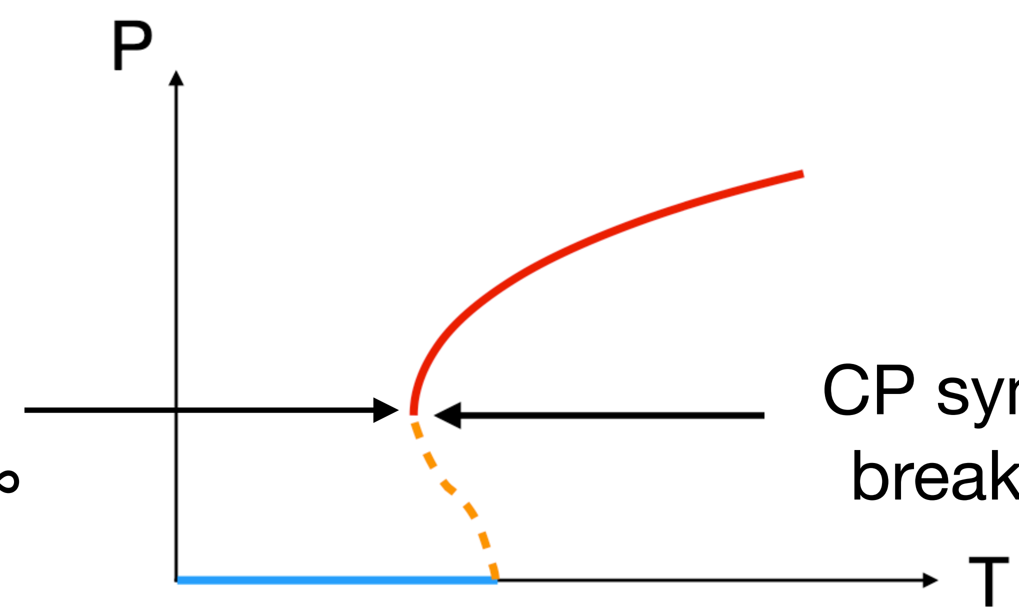
- Completely deconfined phase and completely confined phase were studied before

Chen, Fukushima, Nishimura, Tanizaki 2020

- Partially deconfined phase can be studied numerically

MH-Holden-Knaggs-O'Bannon, 2021 (hep-th, to appear)

CP symmetry (4)



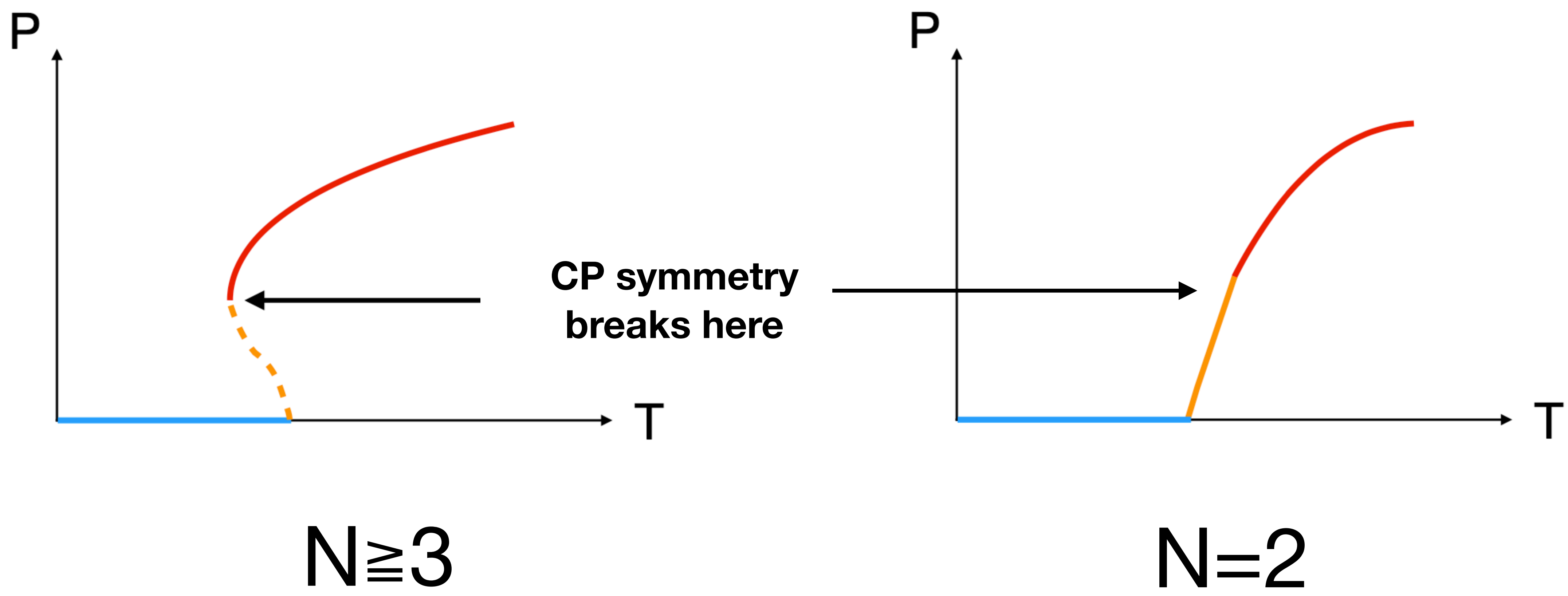
N≧3

(explicit calculation was done for N=30, 50. 70, 100 and ∞)

Chen, Fukushima, Nishimura, Tanizaki 2020 + MH-Holden-Knaggs-O'Bannon, 2021

CP symmetry (5)

Our results are consistent with:



A natural guess obtained by combining
Chen, Fukushima, Nishimura, Tanizaki 2020,
and MH-Holden-Knaggs-O'Bannon, 2021

- Partial deconfinement at <u>infinite</u> N
- Partial deconfinement at <u>finite</u> N
- Chiral symmetry vs Partial deconfinement
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- Conclusion & speculation

- Global symmetry breaks at the onset of confinement -- at least in the examples studied so far.
- How about in real-world QCD? I believe it's OK, but more efforts are required.
- Linear confinement in the confined sector? In progress, stay tuned.
- Application to superstring theory is even more fun! Close connection to the emergent geometry.