From zero flavors to twelve: the opening of the conformal window

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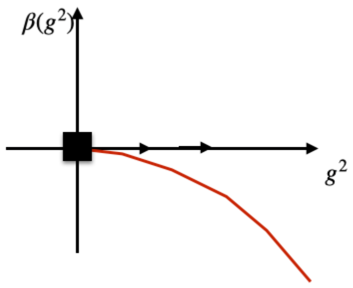
YITP Workshop
QCD phase diagram and lattice QCD

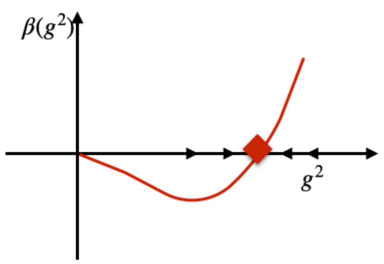
Oct 24 2021

The phase diagram as the function of N_f

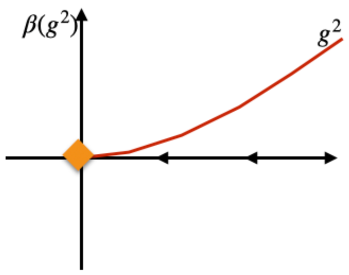
- $N_f = 0$ YM is confining (zero temperature)
- QCD with 2 light flavors is chirally broken, confining (zero temperature)



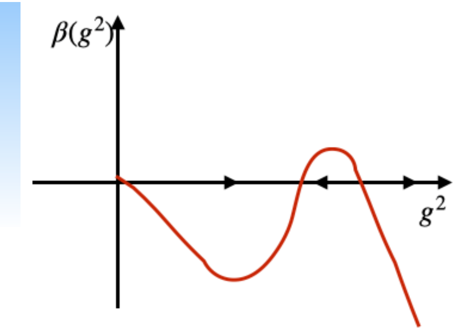
- As N_f increases a phase transition to a conformal phase occurs
  ($N_f^* \approx 8-10$ for SU(3) color, fundamental fermions)
  - the conformal phase is characterized by in infrared fixed point
  - there might be a UV fixed point with a new relevant operator (4-fermion?)



- At $N_f = 16.5$ (SU(3) color) asymptotic freedom is lost
  - the system could be “trivial”
  - or a UV-safe fixed point might emerge (again, new relevant operator)

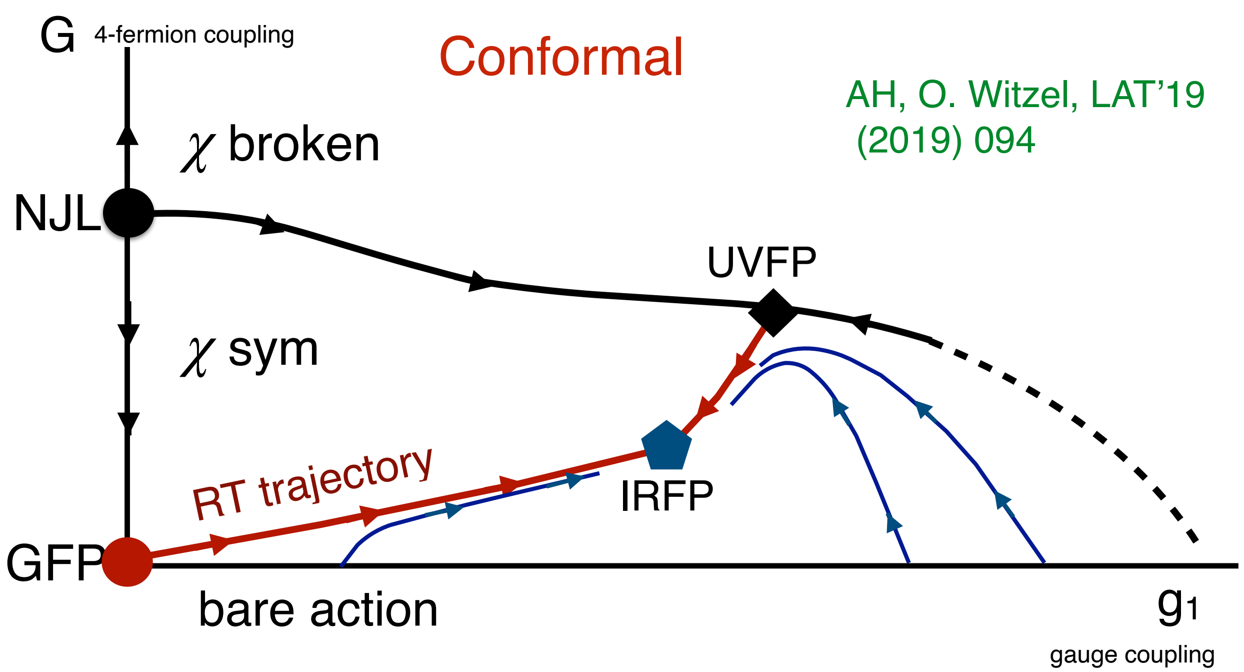


Phase structure of (near-)conformal systems

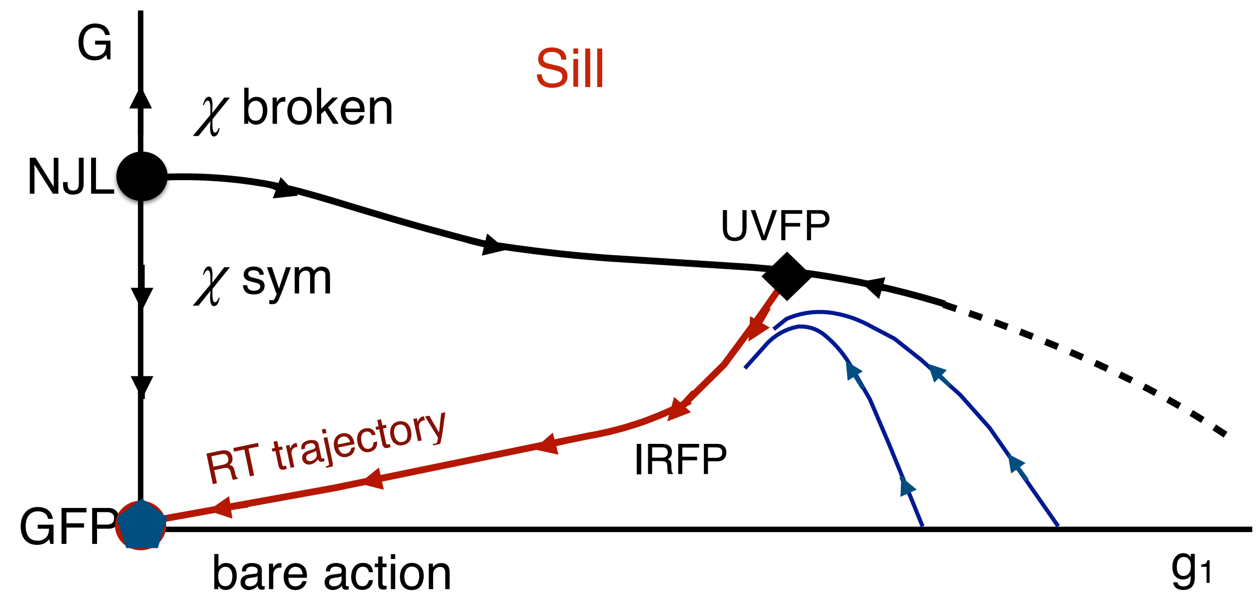


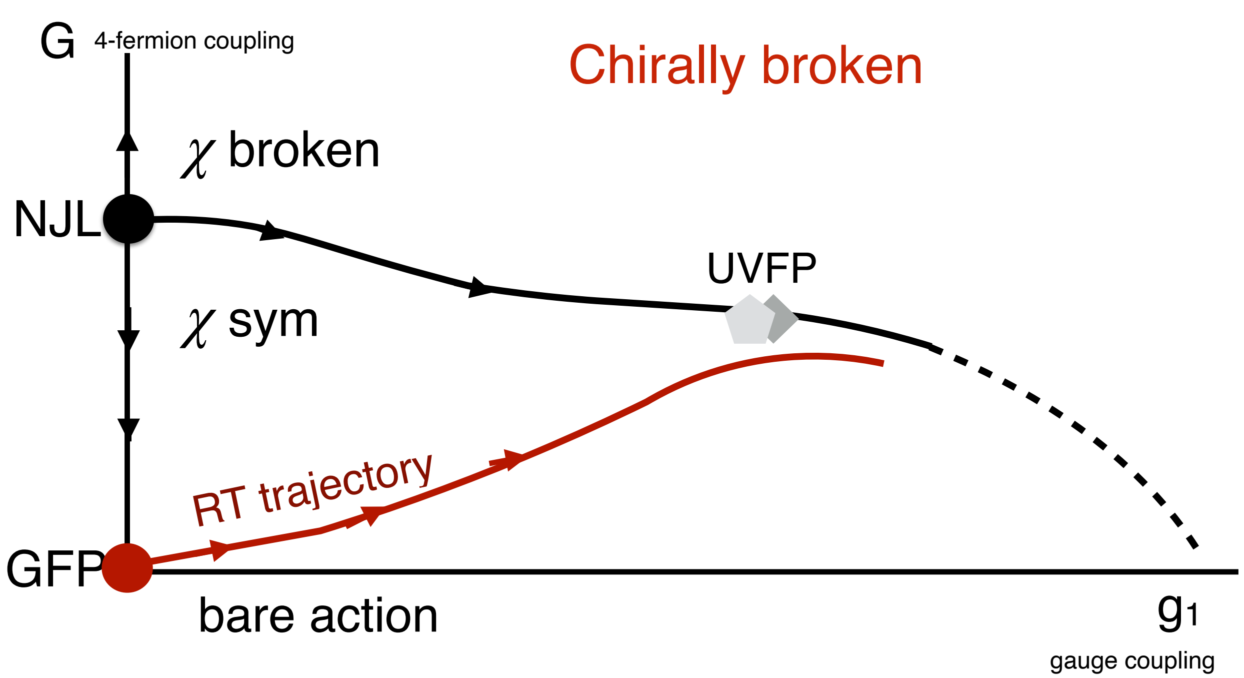
Conjectured phase diagram in the extended parameter space

Kaplan et al, *Phys.Rev.D* 80 (2009) 125005
Gorbenko et al, *JHEP* 10 (2018) 108



AH, O. Witzel, LAT'19 (2019) 094





Goal is to determine the RG flow along the renormalized trajectory:

- this is a 1-paramter function: RG β fn
- can be done without NJL action

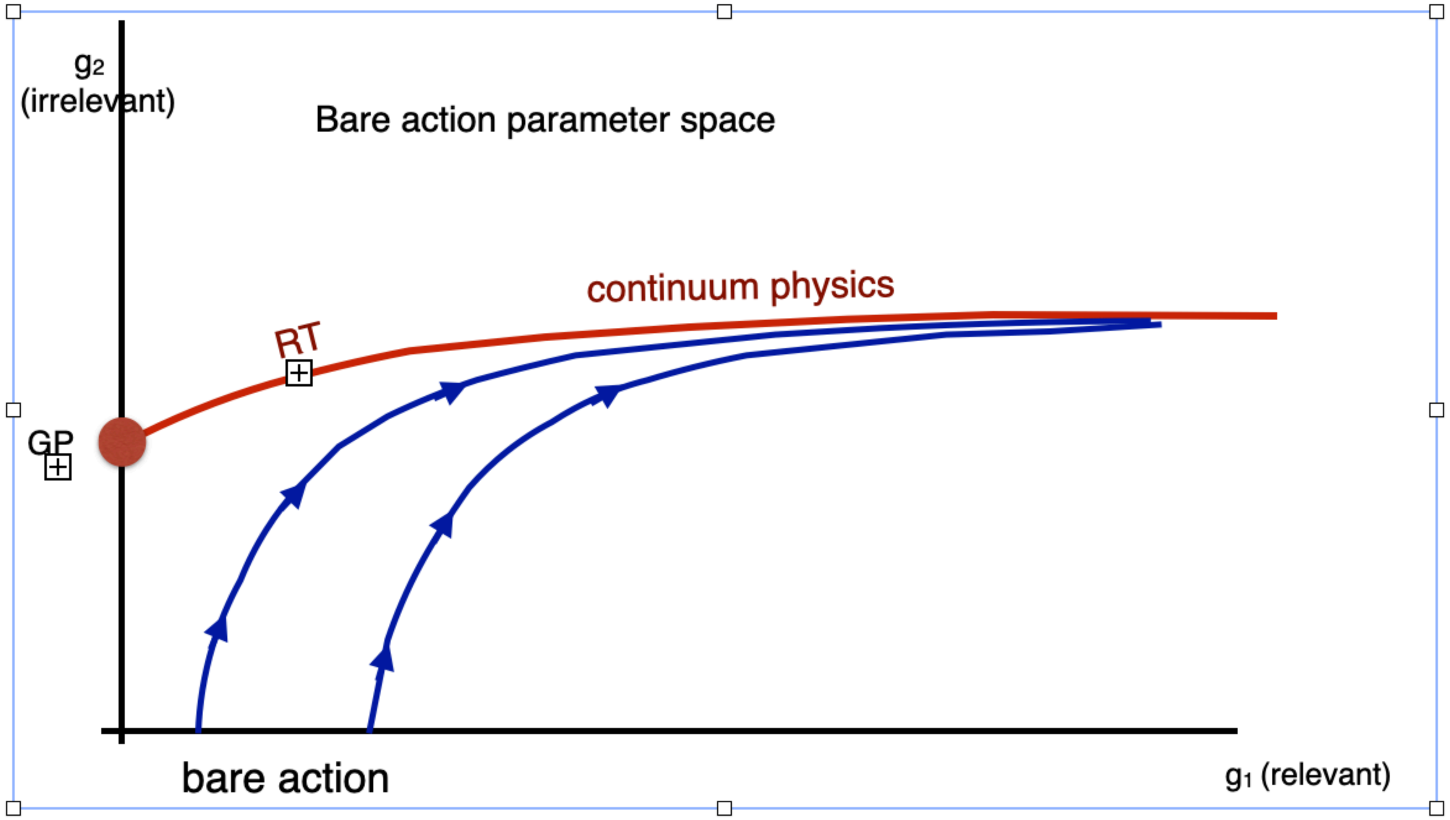
The continuous β function on the lattice

In Wilsonian RG language:

✦ RG transformation: determines the FP and its renormalized trajectory (RT) (continuum physics)

✦ Lattice action: starting point of RG flow
- along the RT RG flows from different bare couplings overlap, and describe continuum physics

✦ Operator: g^2_{GF} should correspond to scaling operator along the RT
- $g^2_{GF} \propto \mu^{-4}\langle E \rangle$ (no anomalous dimension)



gauge-fermion only

The continuous β function from gradient flow

GF is a smoothing transformation (Luscher JHEP 2010, 071)

It can be used to define a real-space Wilsonian RG transformation: $\mu \propto 1/\sqrt{8t}$,

$$\beta(g_{GF}) = -\,t\frac{dg^2_{GF}}{dt}$$

A. Carosso, AH, E. Neil, PRL 121,201601 (2018)

Sonoda, H., Suzuki, H. PTEP,023B05 (2021)

Lattice details:

The RG picture is valid

- in infinite volume
- in $am = 0$ chiral limit

The continuum limit is $t/a^2 \to \infty$

- the flows approach the RT
- the correct scaling operator is projected out
- Remaining cut-off effects are removed by $a^2/t \to 0$

AH, O. Witzel, *Phys.Rev.D* 101 (2020) 3

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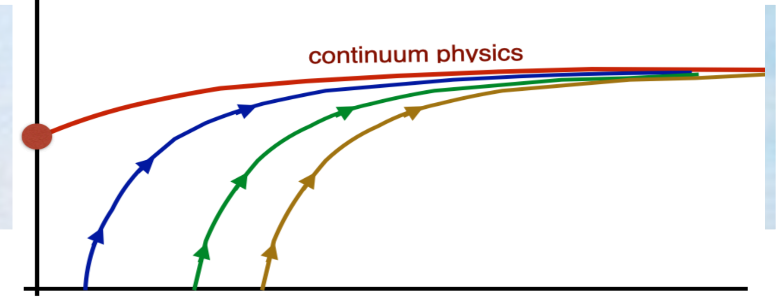
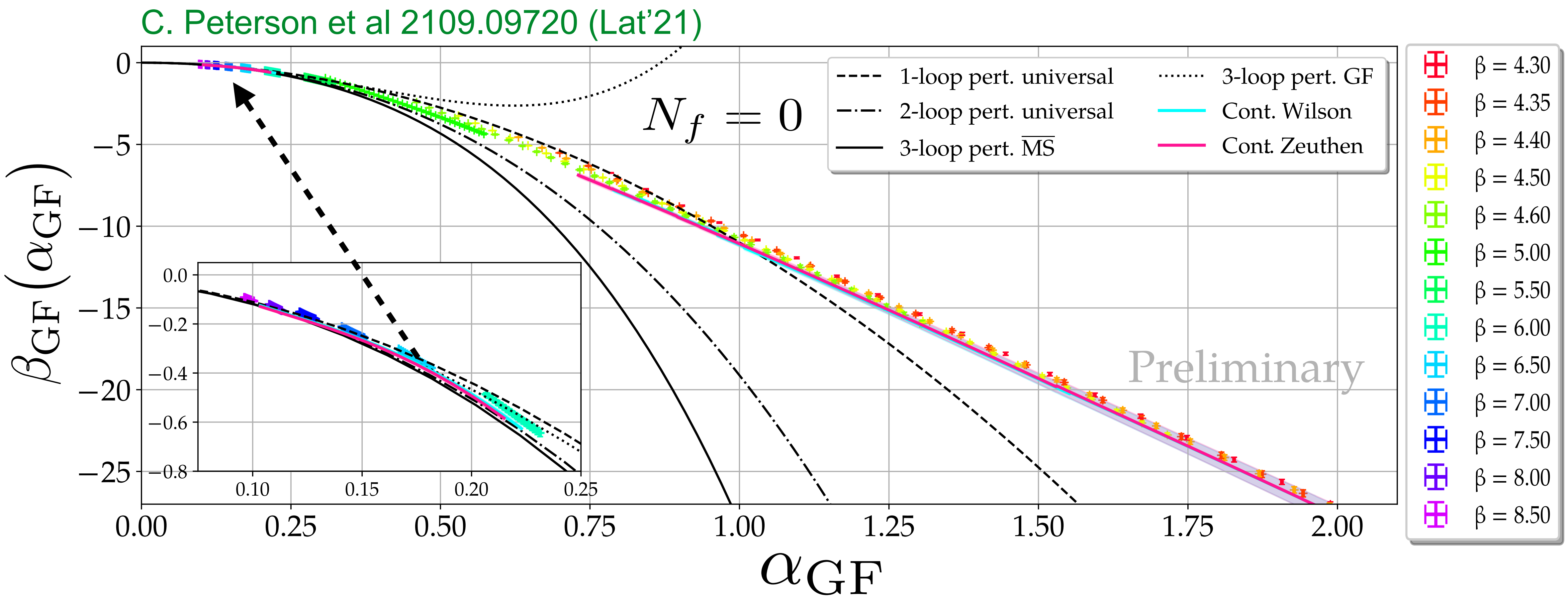
AH, O. Witzel, *Phys.Rev.D* 101 (2020) 3

- Step scaling function requires that the volume is the only scale; valid only in the deconfined regime
- Continuous β function requires infinite volume extrapolation but it is correct even in the confining regime

The continuous β function, $N_f = 0$

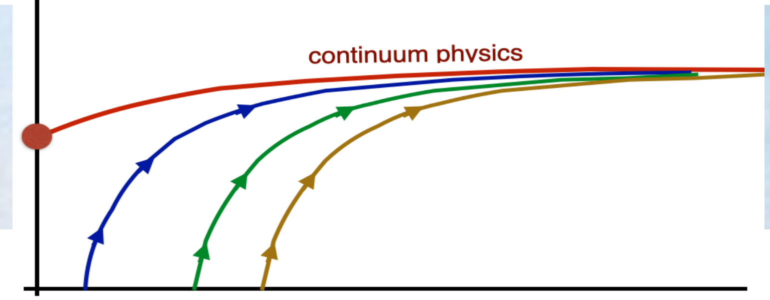




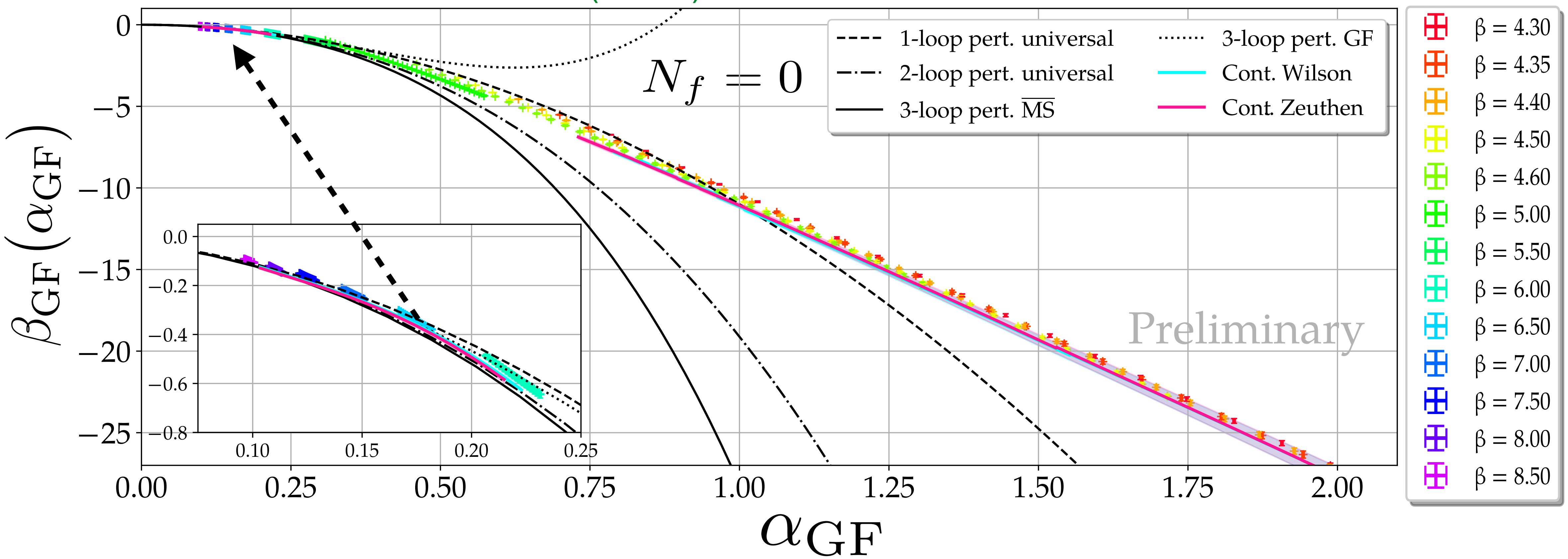
✦Different bare couplings overlap, form a unique curve :

- flow reached the RT
- cutoff effects are small

The continuous β function, $N_f = 0$



C. Peterson et al 2109.09720 (Lat'21)



Nakamura, Schierholz
2106.11369

✦In the confining regime $\beta(g^2) \propto g^2$: non-perturbative; Topology?

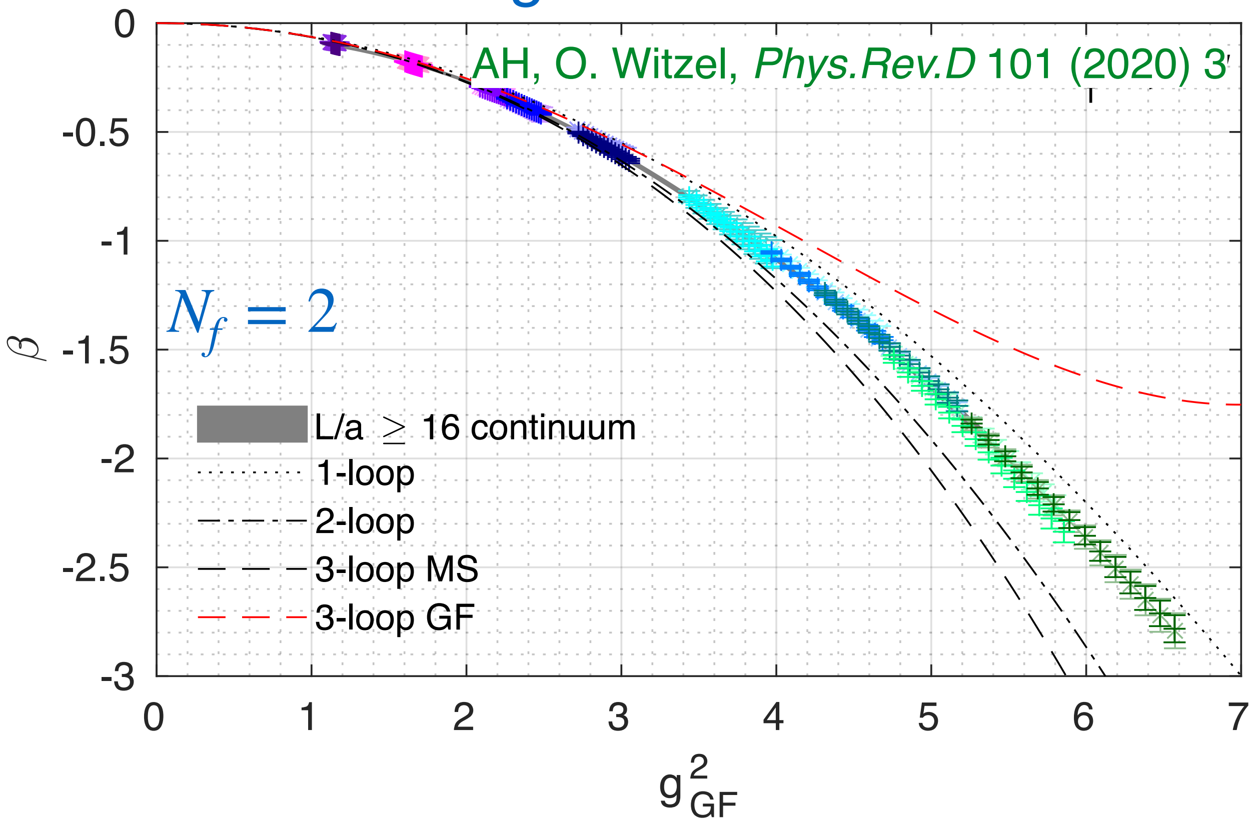
$N_T = N_I + N_A$ instantons → vacuum energy density: $\langle E \rangle = \langle E \rangle_{N_T=0} + N_T * S_I/V$

GF coupling: $g^2_{GF} = g^2_{GF,0} + ct^2 N_T/V$

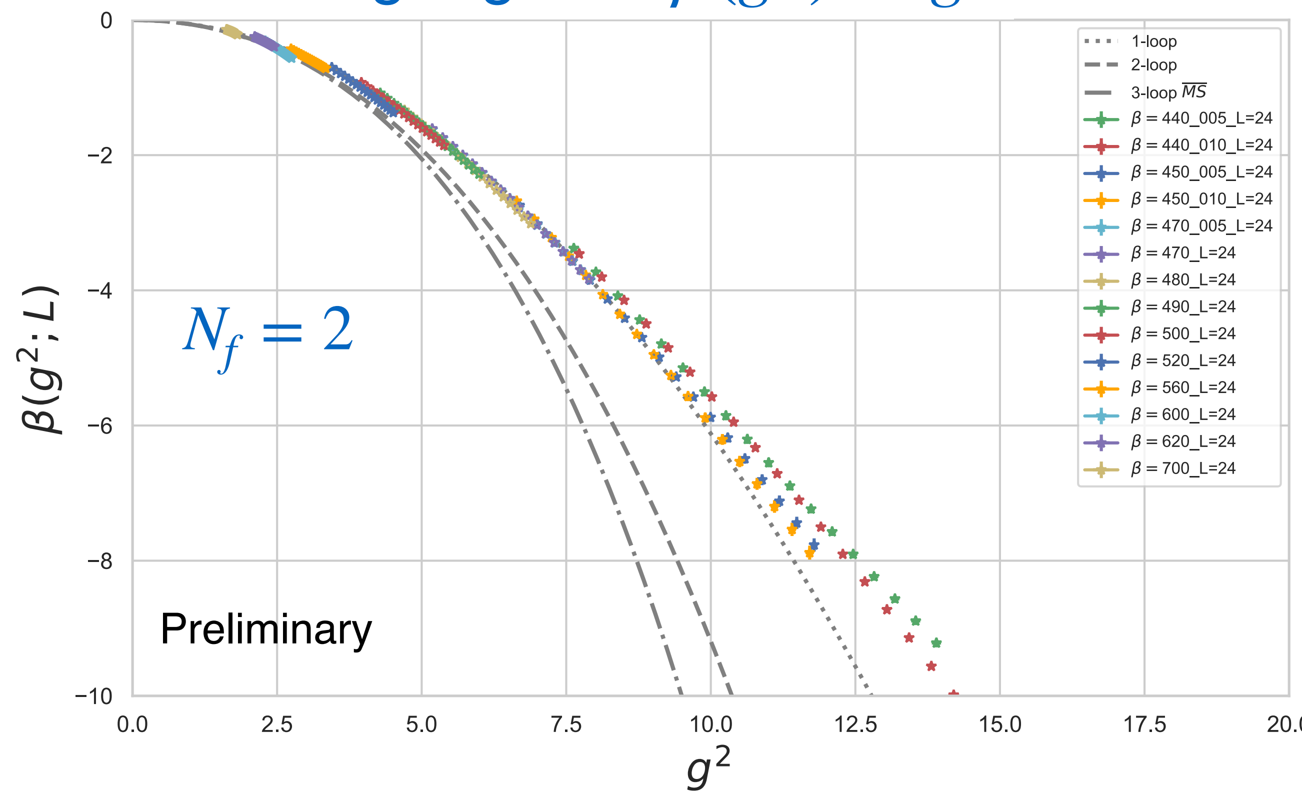
If $lim_{V\to\infty}\ N_T/V =$ finite, topology dominates and $\beta(g^2) \propto g^2$

The continuous β function, $N_f = 2$ DWF, $m_f = 0$

Deconfined regime



Confining regime : $\beta(g^2) \propto g^2$



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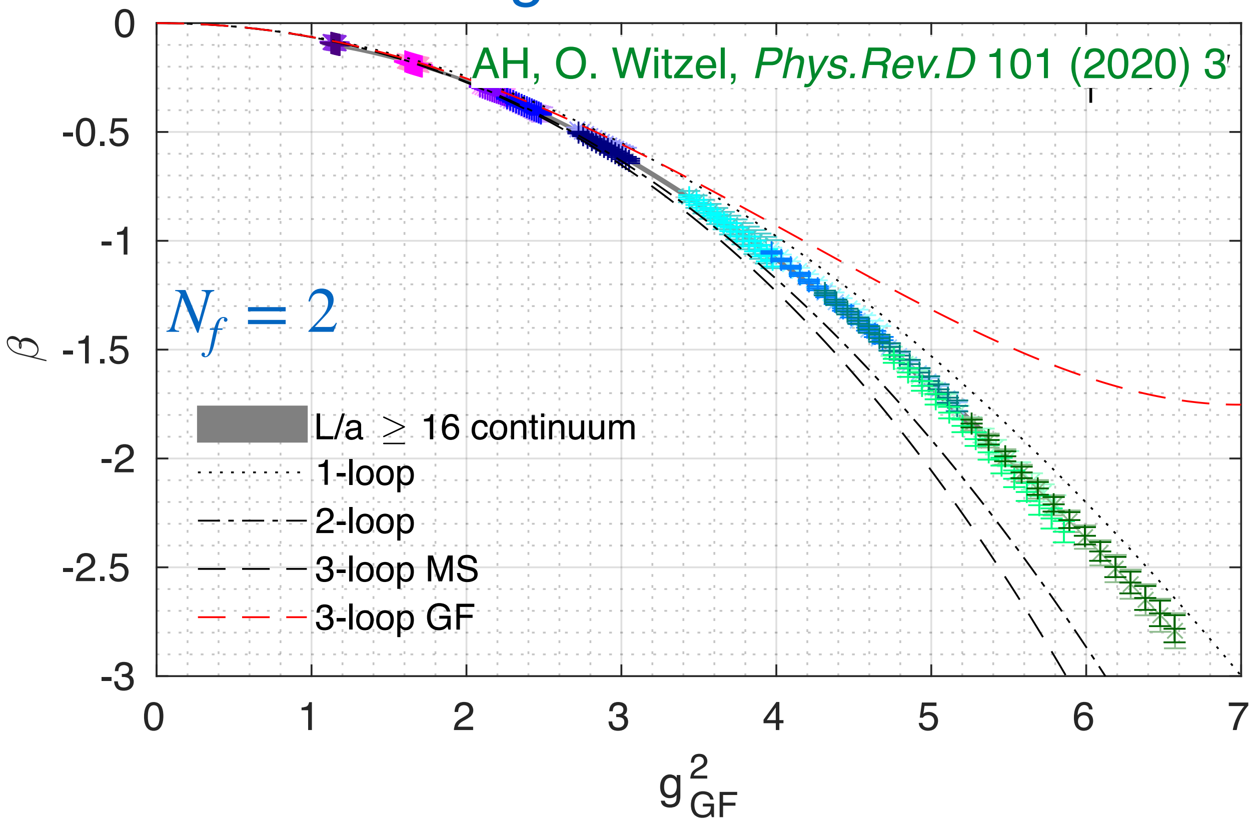

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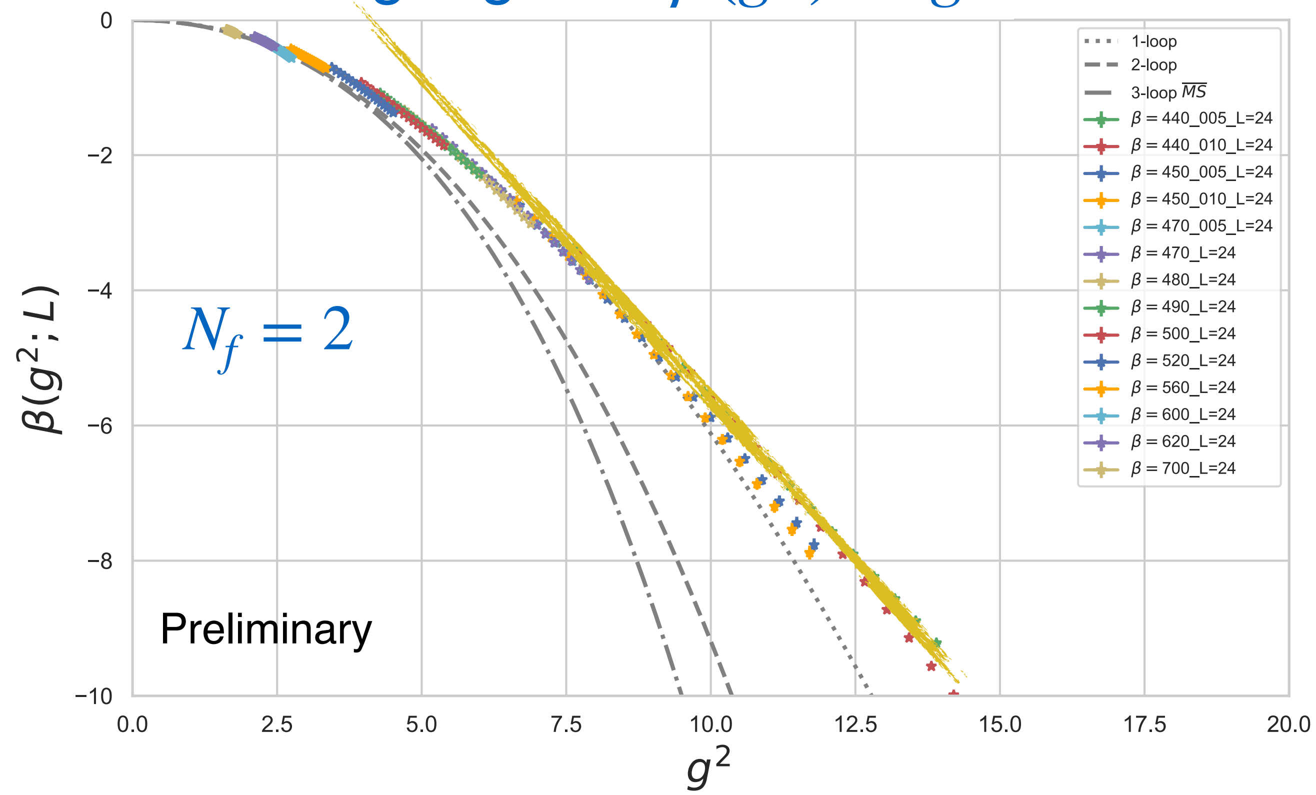
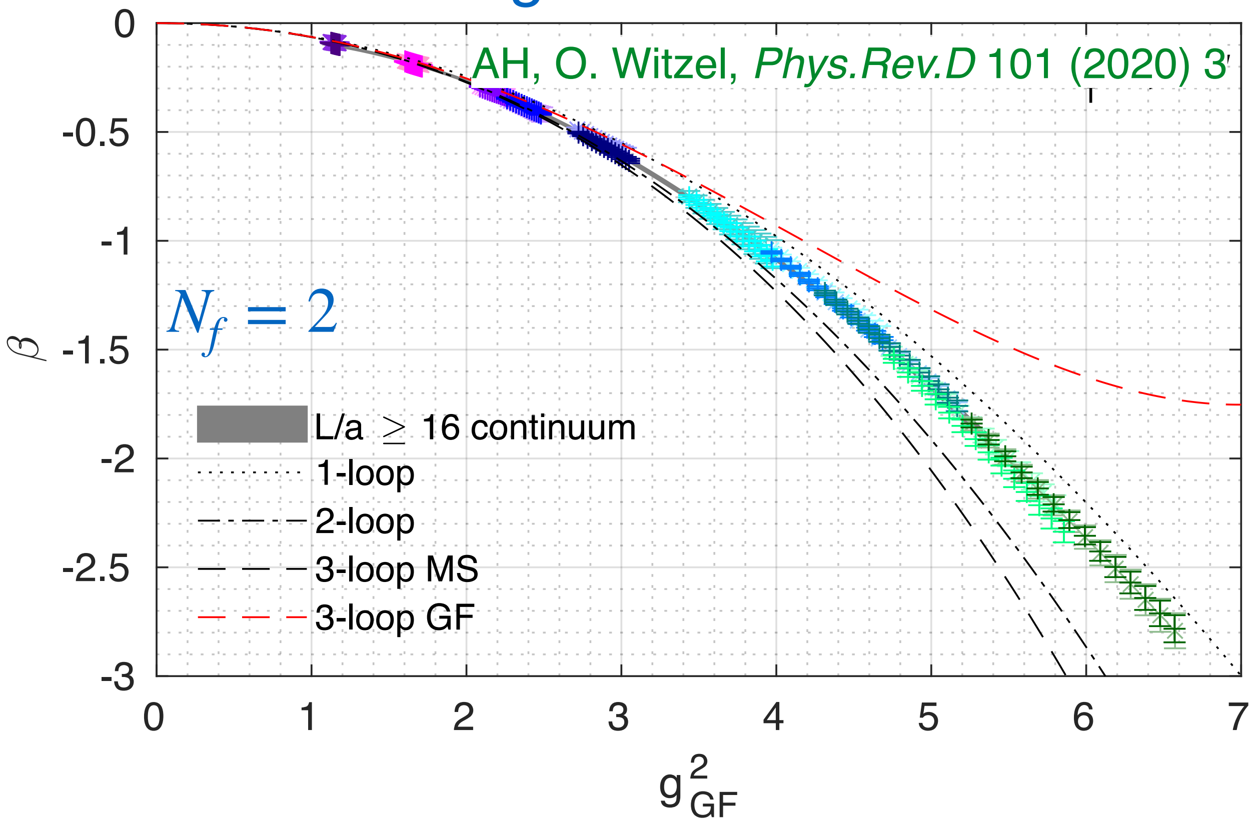


The continuous β function, $N_f = 2,\ 2+1$ DWF, $m_f = 0$

Deconfined regime



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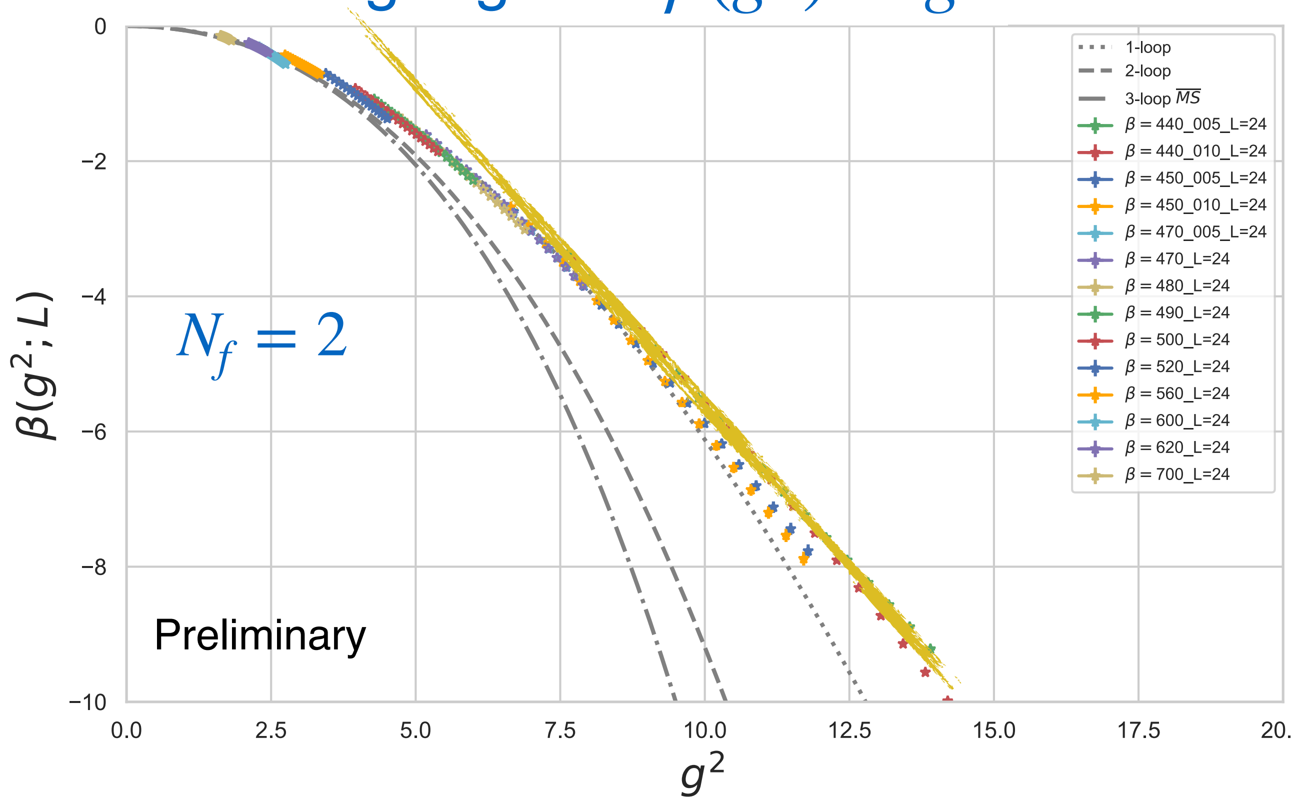
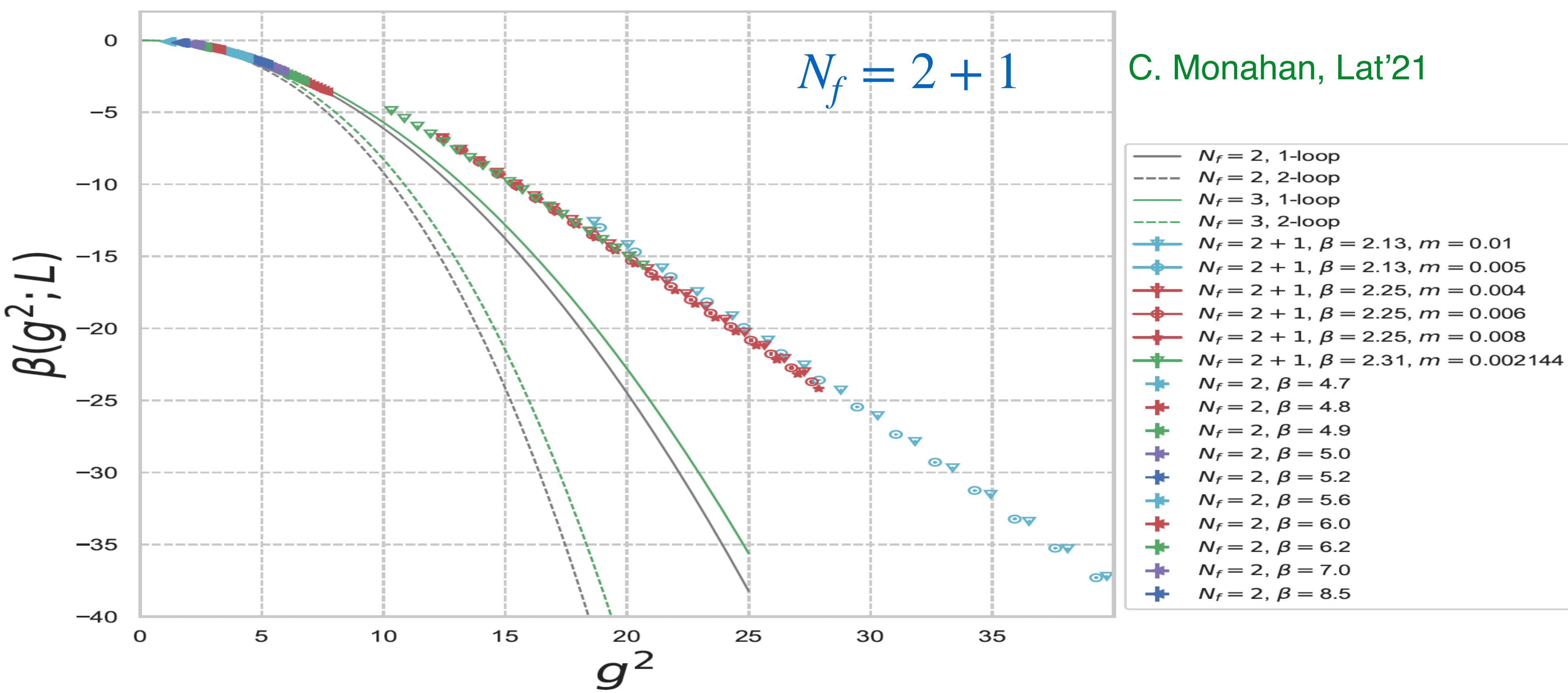




C. Monahan, Lat'21

Confining regime : $\beta(g^2) \propto g^2$ for $N_f = 0, 2, 2+1$

The continuous β function, $N_f = 4$ DWF, $m_f = 0$

AH, O. Witzel,in preparation

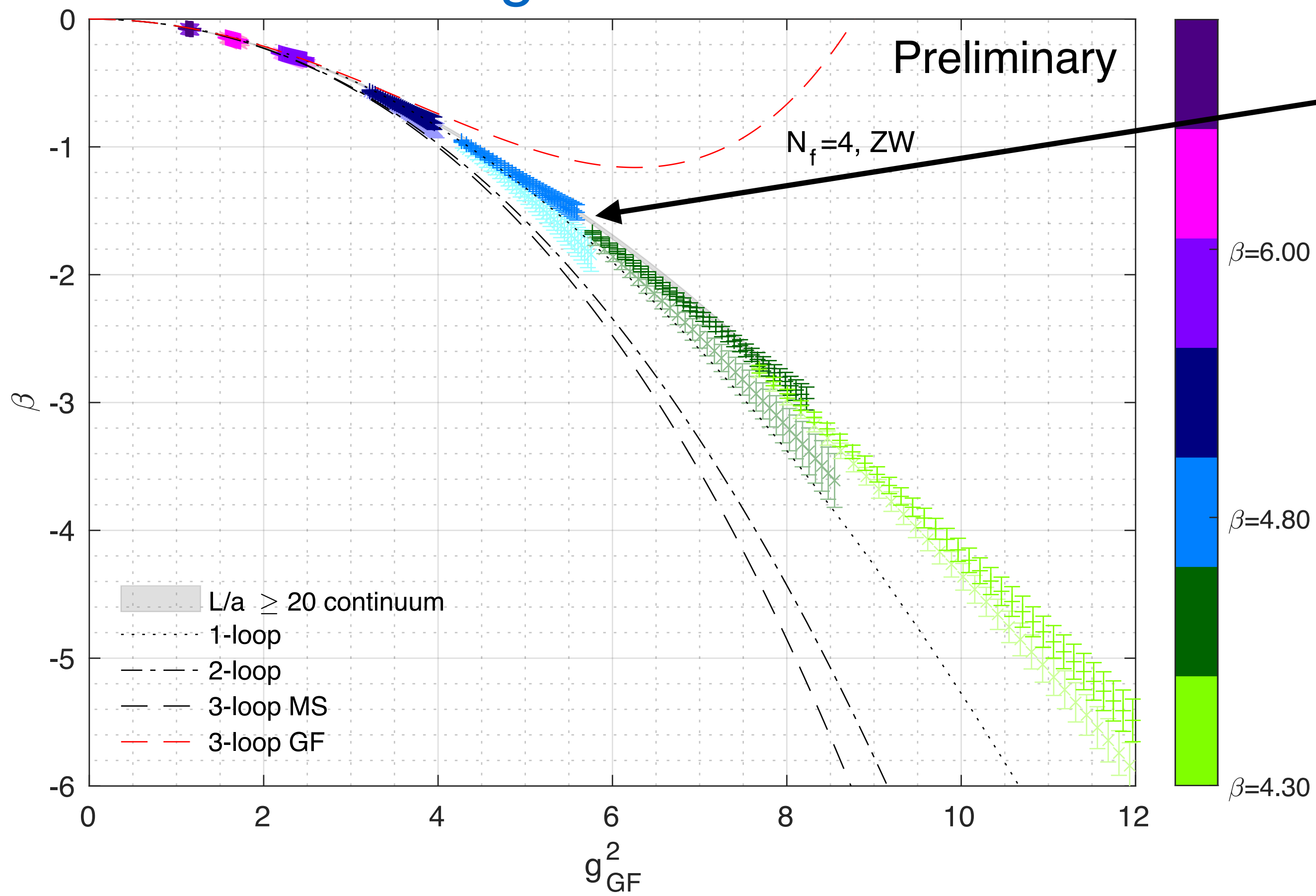
Simulations in the weak coupling chirally symmetric regime, 20^4, 24^4, 32^4 volumes

Raw data (Zeuthen flow, Wilson op)

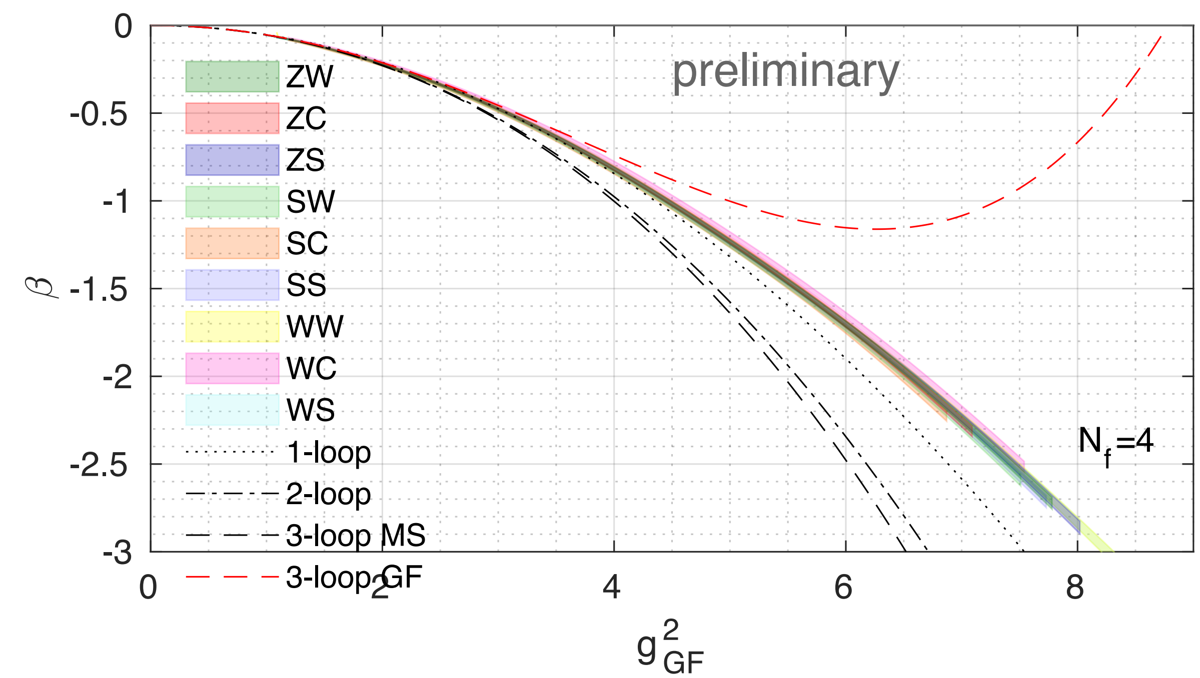
- shows significant overlap and minimal finite volume effects

➡ sits almost on top of continuum extrapolation

Deconfined regime



continuum limit



The continuous β function, $N_f = 6$ DWF, $m_f = 0$

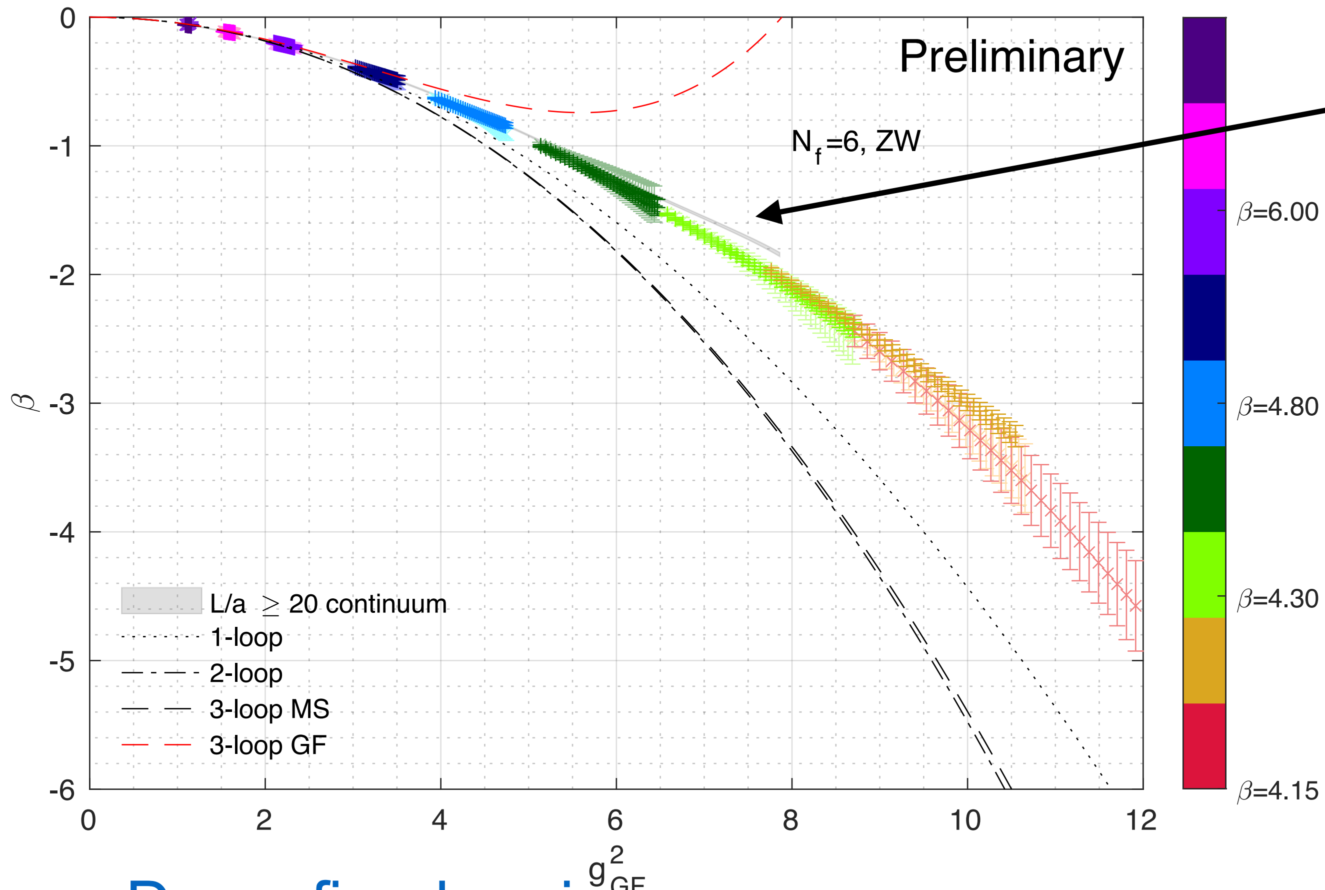
AH, O. Witzel, in preparation

Simulations in the weak coupling chirally symmetric regime, 20^4, 24^4, 32^4, 40^4 volumes

Raw data

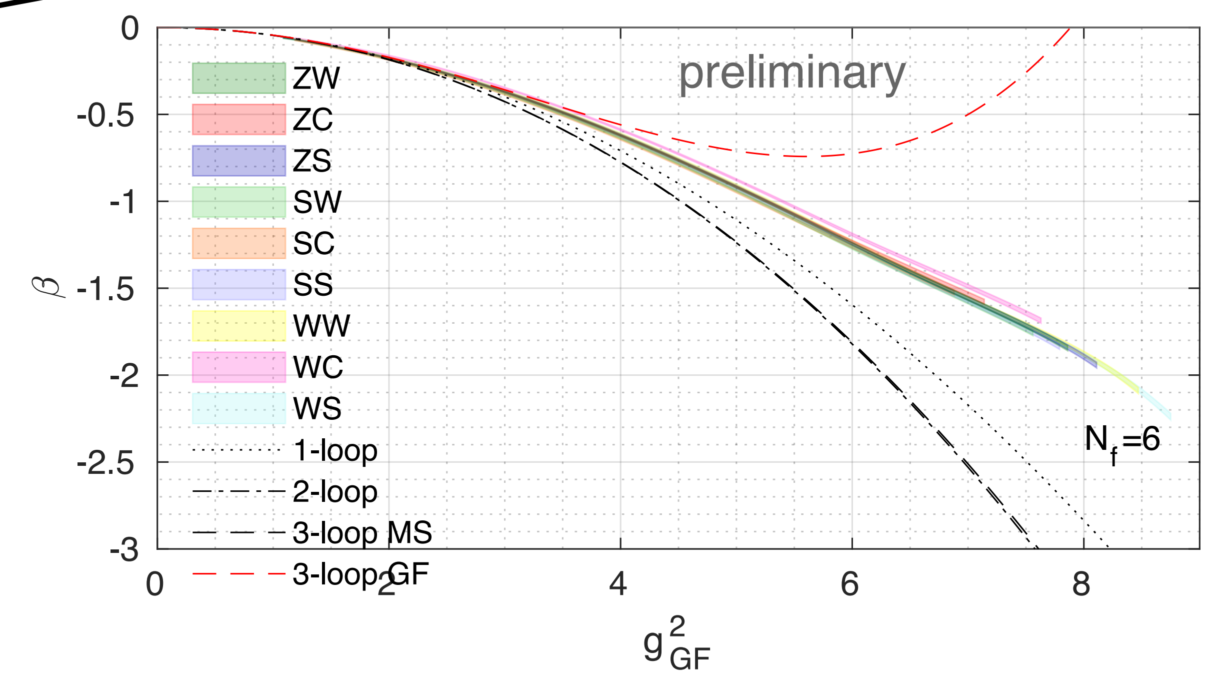
- shows significant overlap and minimal finite volume effects

➡ sits almost on top of continuum extrapolation



Deconfined regime

continuum limit



The continuous β function, $N_f = 8$ DWF, $m_f = 0$

AH, O. Witzel,in preparation

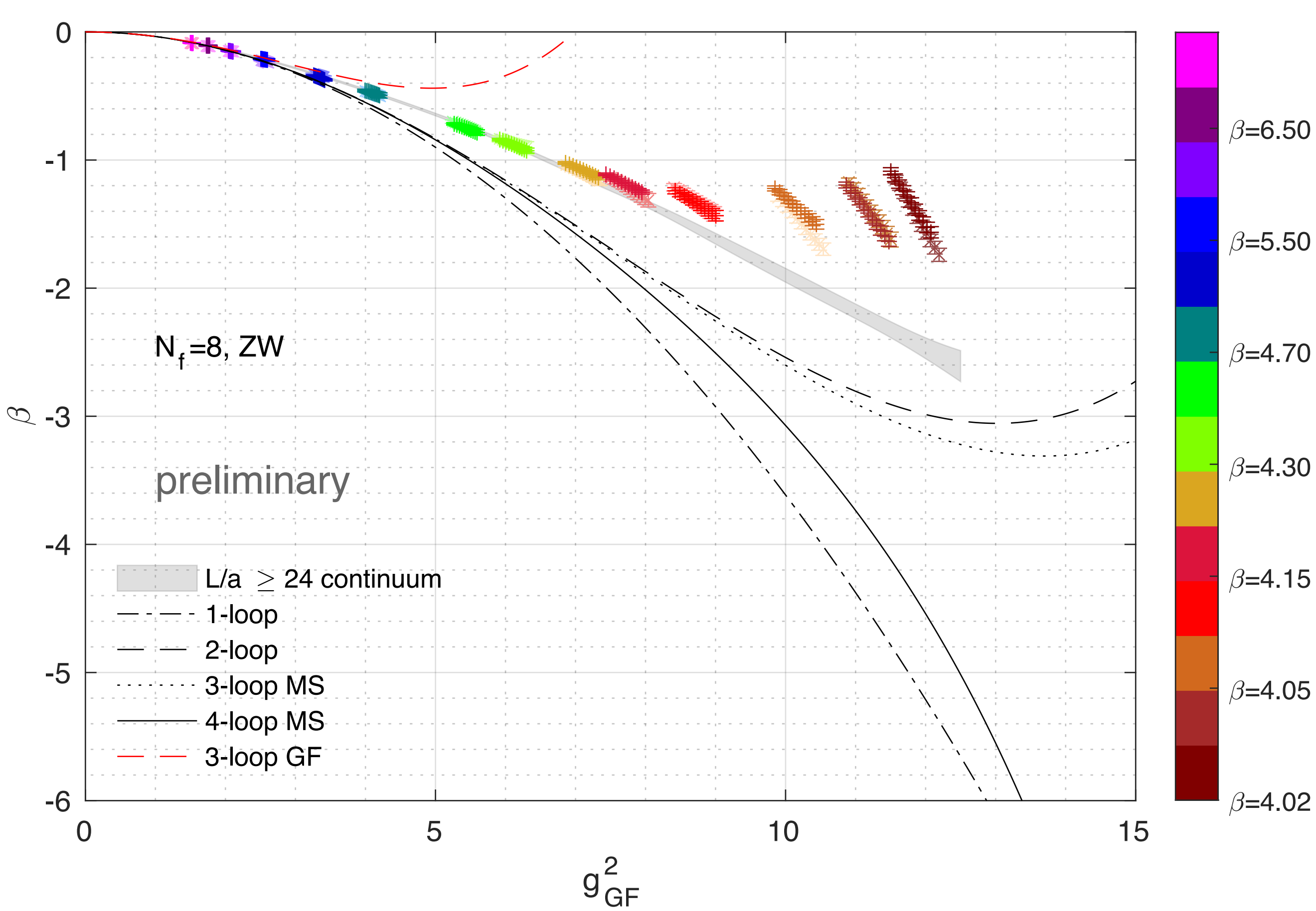
Simulations in the weak coupling chirally symmetric regime, 16^4, 24^4, 32^4, 48^4 volumes

Raw data

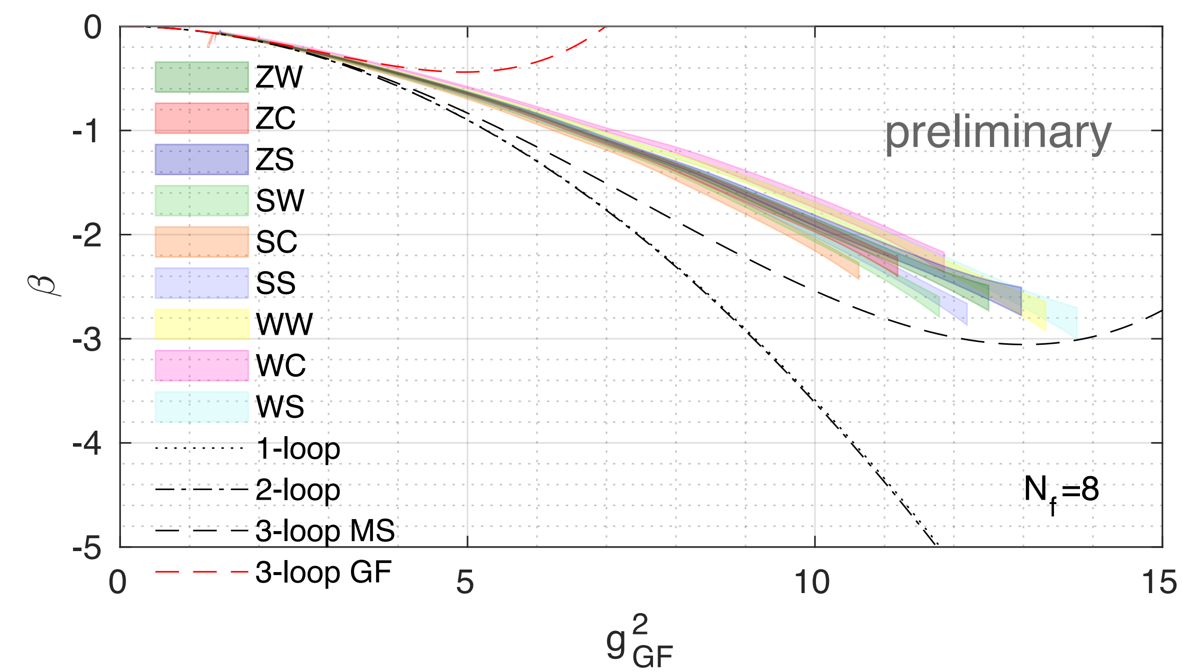
- shows significant overlap and minimal finite volume effects

➡ sits almost on top of continuum extrapolation up to $g^2_{GF} \approx 8$

at stronger coupling the perturbative guidance breaks down, no overlap, no unique curve traced out by raw data



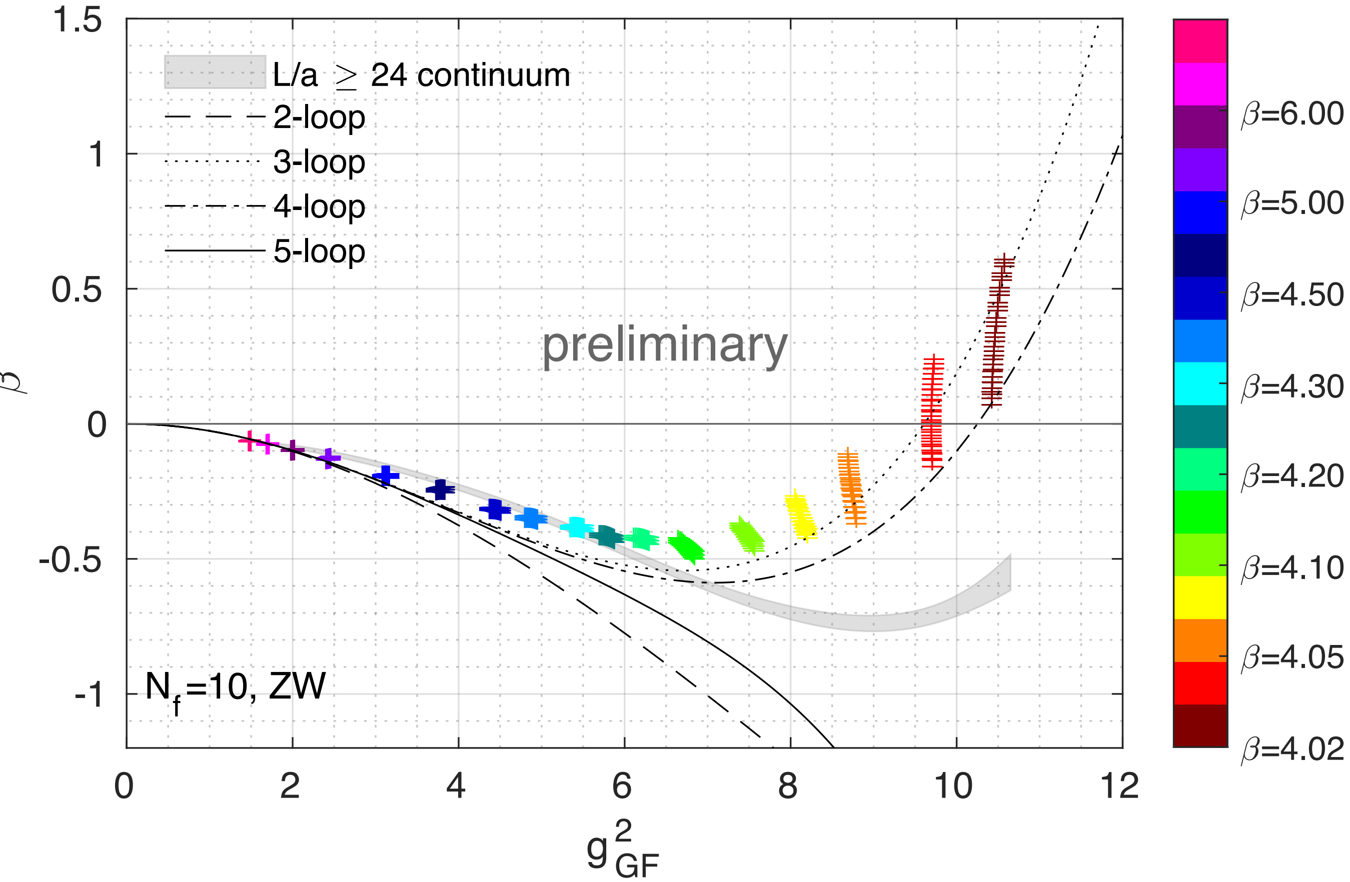
continuum limit

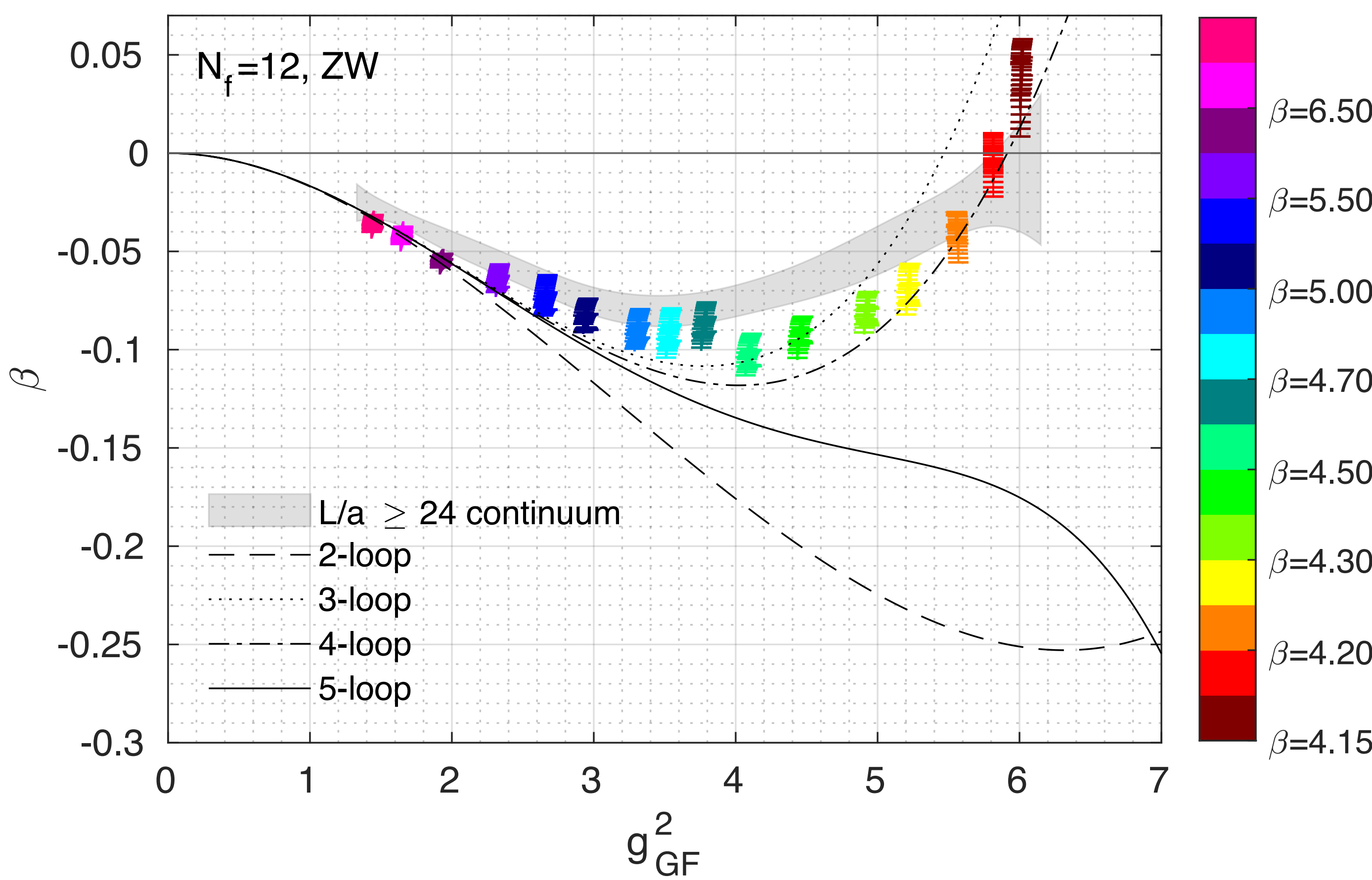


Is this a signal of nearby IRFP/UVFP ? (Somewhere at $N_f \gtrsim 8$)

The continuous β function, $N_f = 10$ and 12 DWF

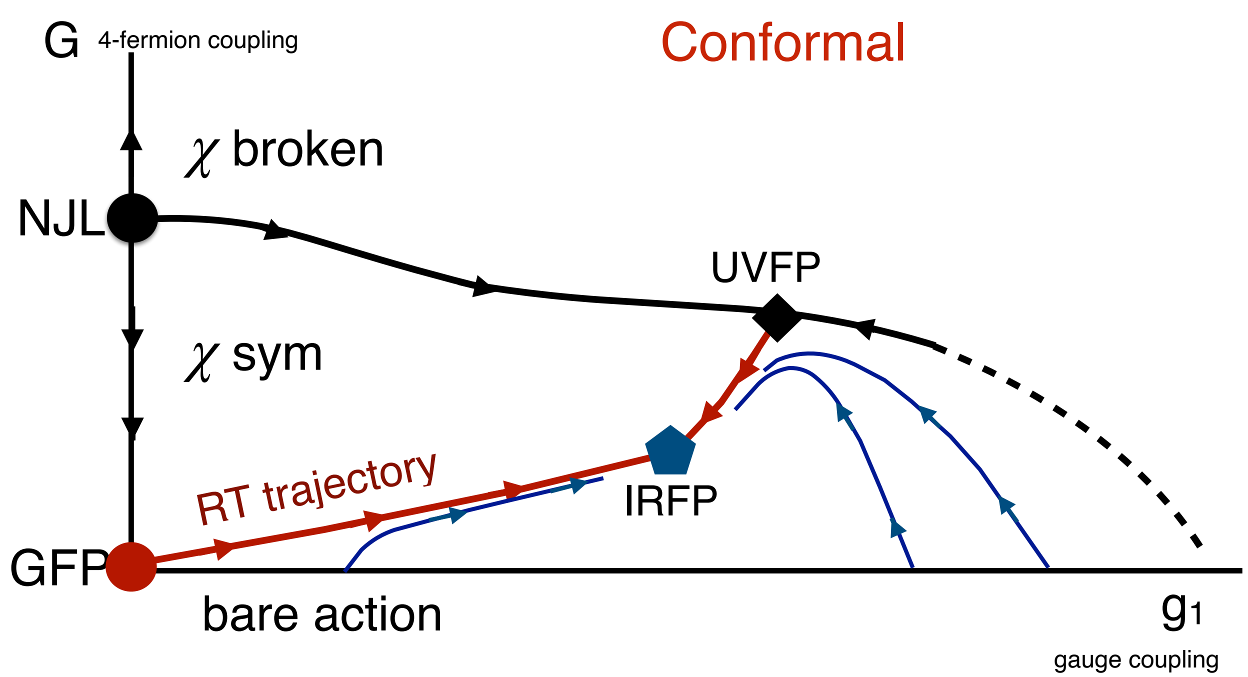
AH,O.Witzel, *Phys.Rev.D* 101 (2020) 11, 114508





- Very slow running
- At stronger couplings there is no overlap, no unique curve traced out by raw data
- Is this a signal of nearby IRFP/UVFP ?

The β function of (near-)conformal systems



Any calculation based on the perturbative Gaussian FP will cover only the regime between GFP and IRFP:

- $N_f = 0 - 6$ shows slow running but cutoff effects are easy to control
- $N_f \geq 8$ shows increasing non-perturbative effects
  - perturbatively justified extrapolation in $(a/L)^2$ or (a^2/t) might need to be replaced with $(a/L)^\alpha$, $(a^2/t)^{\alpha/2}$
  - new relevant operator at the UVFP means that a 1-parameter β function might not describe the RG flows

Most simulations are plagued by (bulk) 1st order transition that prevent investigations at strong coupling

To solve these issues will require a new approach

Improved action with Pauli-Villars bosons

AH,T.DeGrand PRD 49 (1994) 466
AH,B. Svetitsky, Y. Shamir, *Phys.Rev.D* 104 (2021) 7

Simulations in the strong coupling are difficult. Many flavor studies are particularly hard due to the induced effective gauge action from fermions

$$e^{-S_{ind}[U]} = \int \prod_{i=1}^{N_f} d\psi_i \, d\bar{\psi}_i \, e^{\left[-\sum_{i=1}^{N_f} \bar{\psi}_i (D+m_f)\psi_i\right]} = Det(D+m_f)^{N_f} = e^{N_f Tr \log(D+m_f)}$$

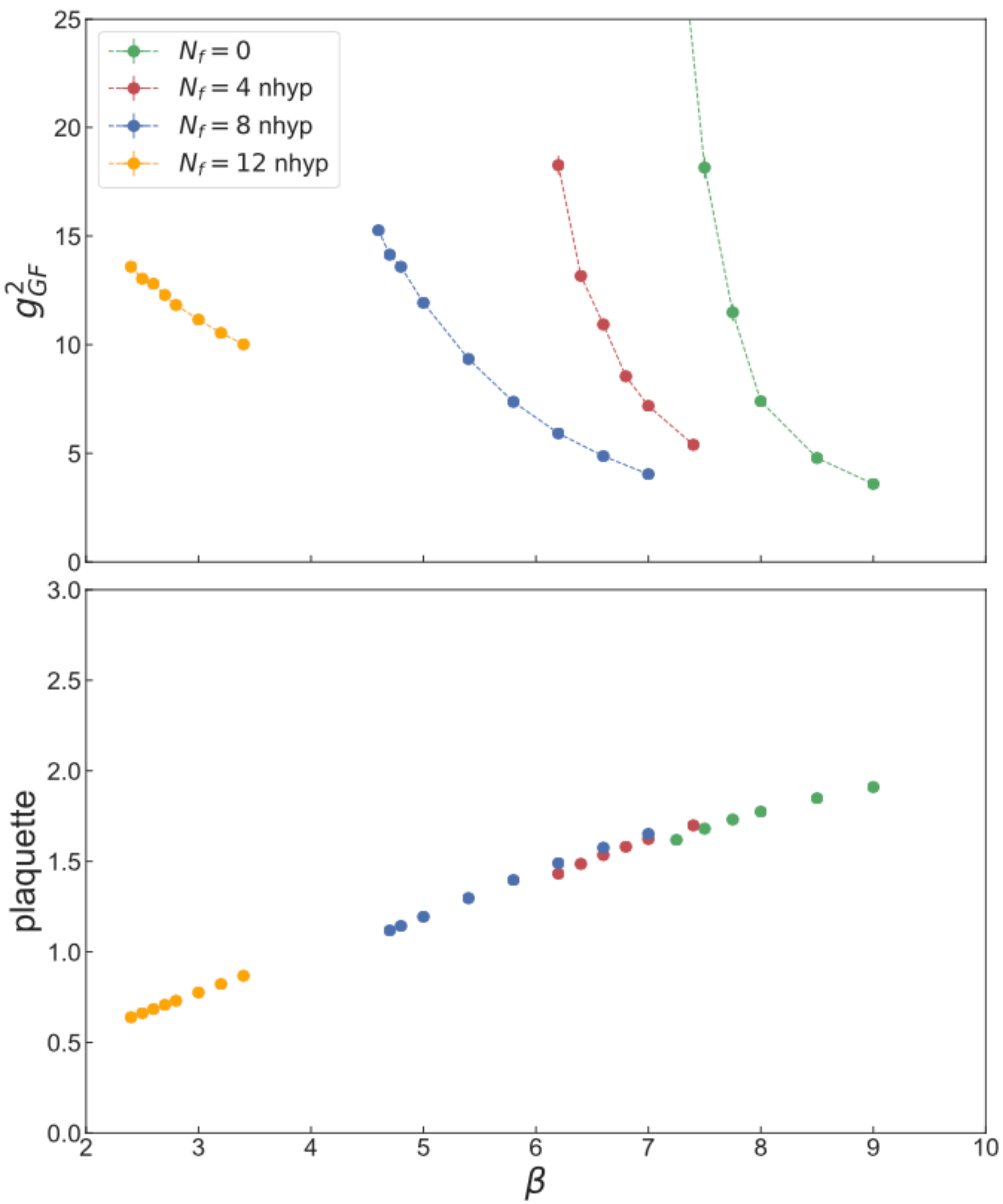
Expanding S_{ind} in $1/m_f$ gives a sum of gauge loops. For staggered fermions

$$S_{ind} = -N_f/4 \sum_{\ell} \frac{(-1)^{\ell/2}}{\ell (2am_f)^{\ell}} \sum_x \sum_{\mathscr{C}_\ell} \mathscr{E}_{\mathscr{C}_\ell} Tr U_{\mathscr{C}_\ell}$$

Leading term is plaquette: $\beta^{(p)}_{\rm ind} = \dfrac{6N_f}{4(2am_f)^4}$: $\beta_{eff} = \beta_g + \beta_{ind}$

The induced action of fermions

Compare simulations with $N_f = 0,\ 4, 8,$ and 12 fundamental (staggered) fermions



As N_f grows,

- identical renormalized coupling corresponds to smaller the bare coupling
- the plaquette much rougher

Simulations to be done at strong bare coupling

- lattice artifacts are large
- spurious bulk transitions limit the renormalized coupling range

Both $N_f = 8$ and 12 exhibit 1st order bulk transition that limits the accessible coupling range

The induced action of fermions

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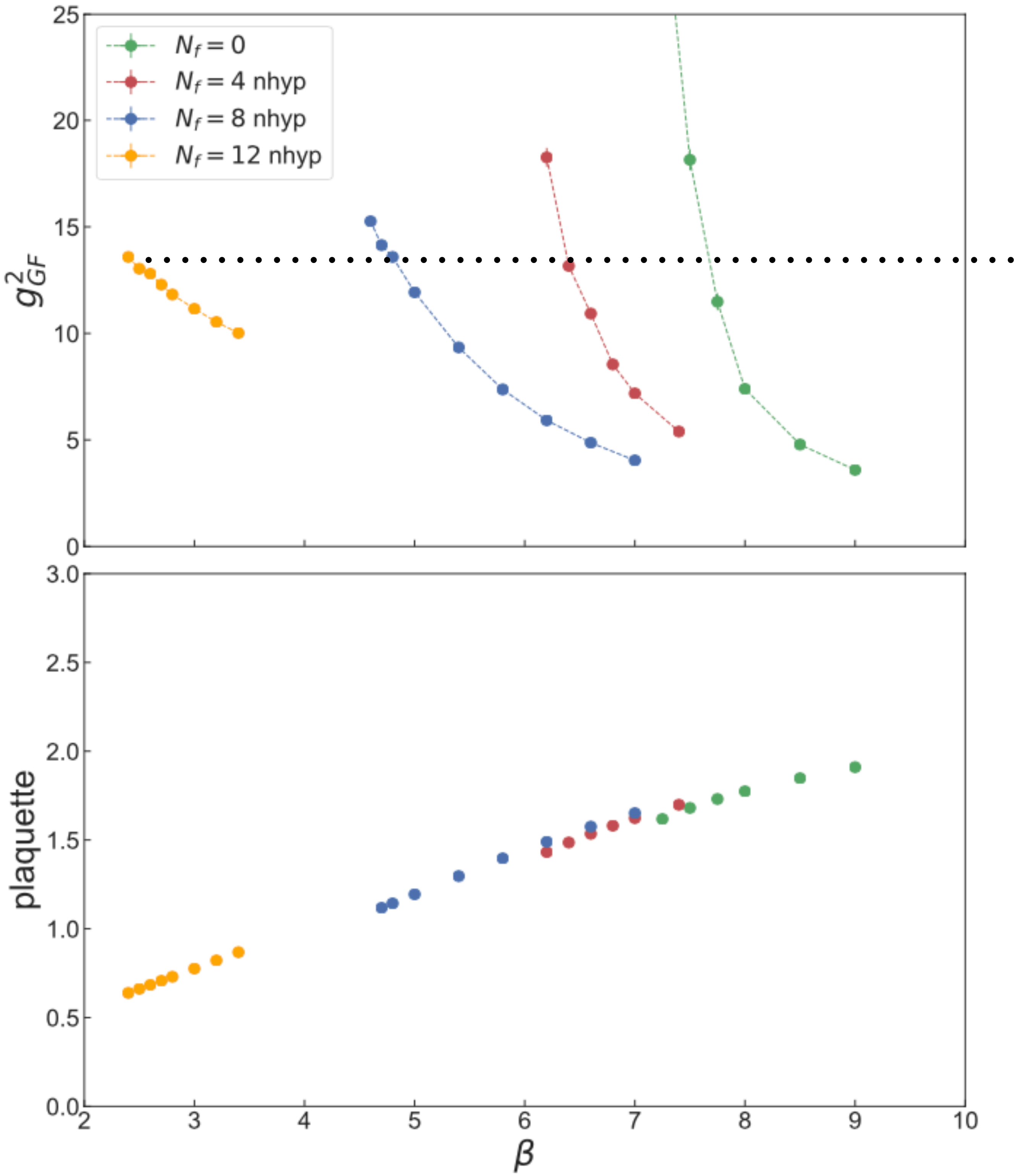

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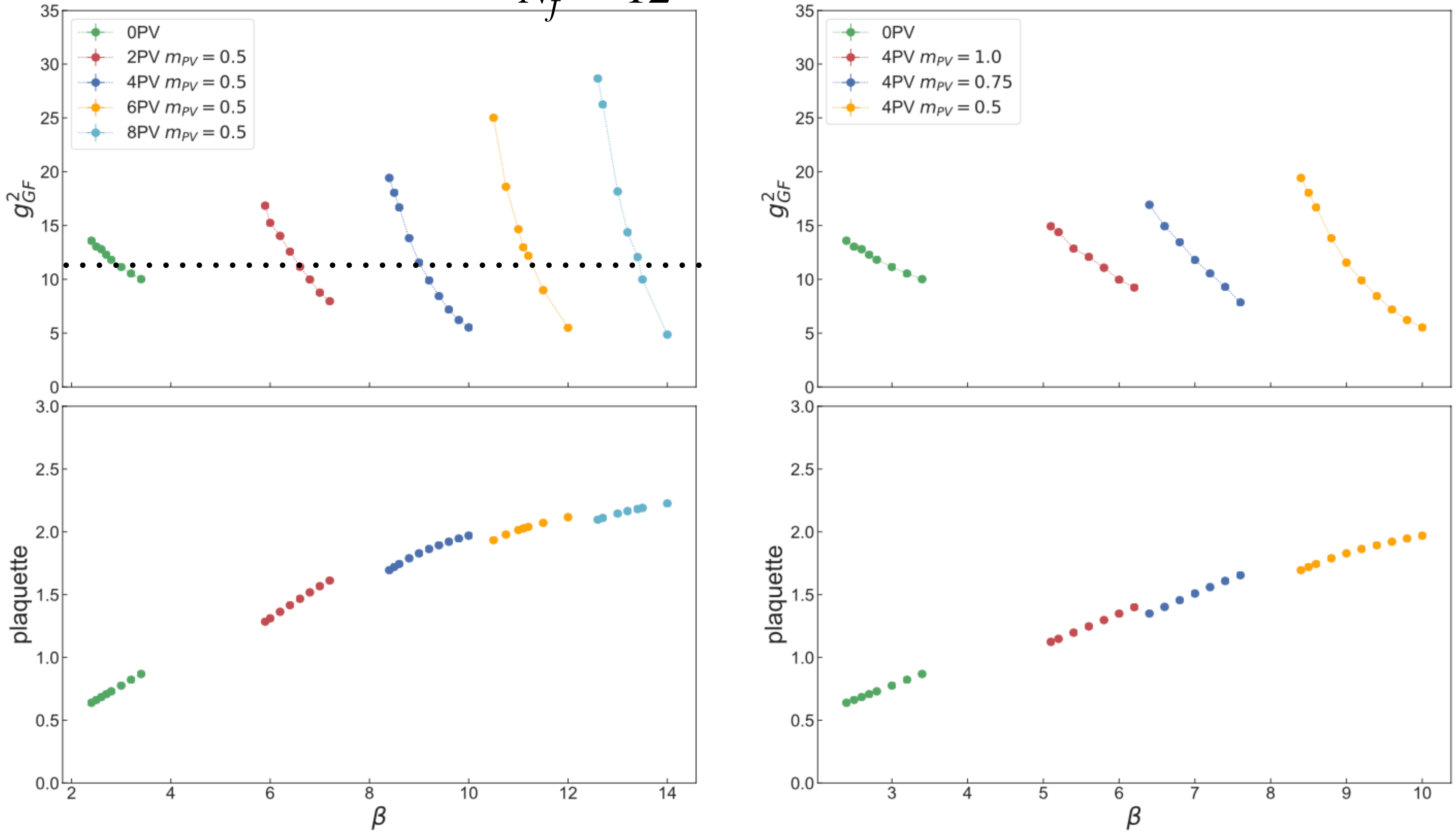
Improved action with Pauli-Villars bosons

Compensate the effect of fermions by adding heavy Pauli-Villars bosons:

- If $m_{PV} \gg m_f, 1/L, \Lambda_{QCD}$ the PV fields decouple and do not change the IR dynamics
- PV bosons induce a negative effective action
- any number of PV bosons can be added (even with different Dirac operator)

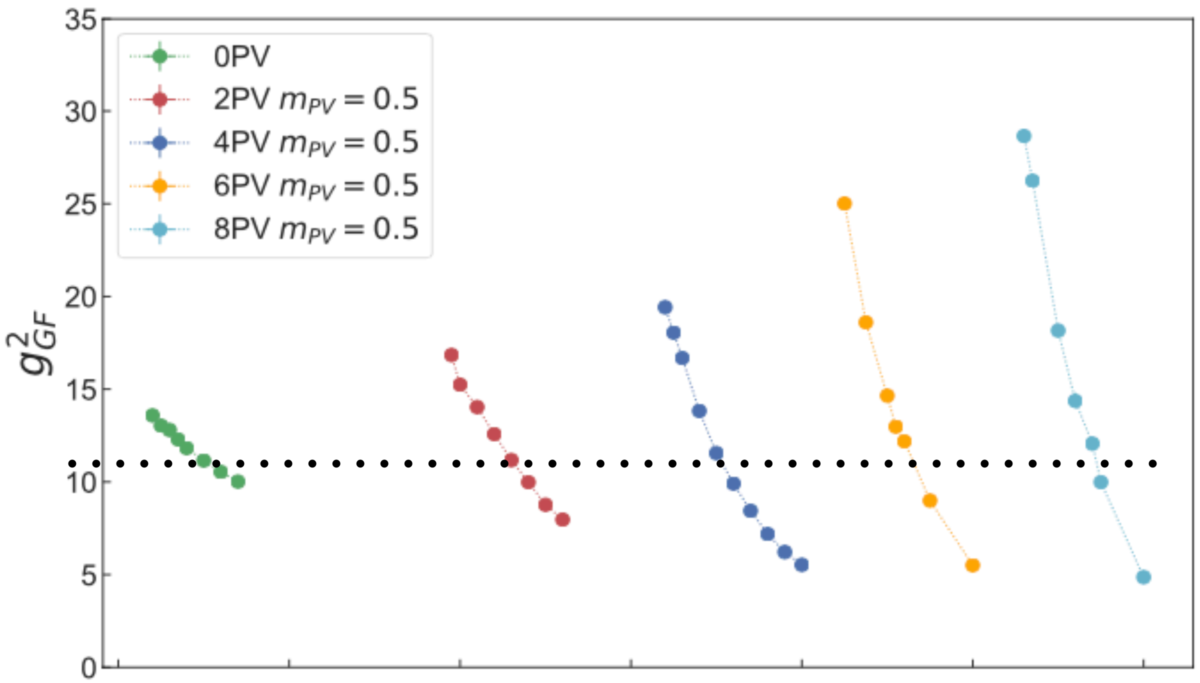
\- this is already done in DW simulations

$N_f = 12$



Improved action with Pauli-Villars bosons

Including PV boson are computationally inexpensive
They likely speed up the simulations while remove cutoff effects and open up the strong coupling parameter space

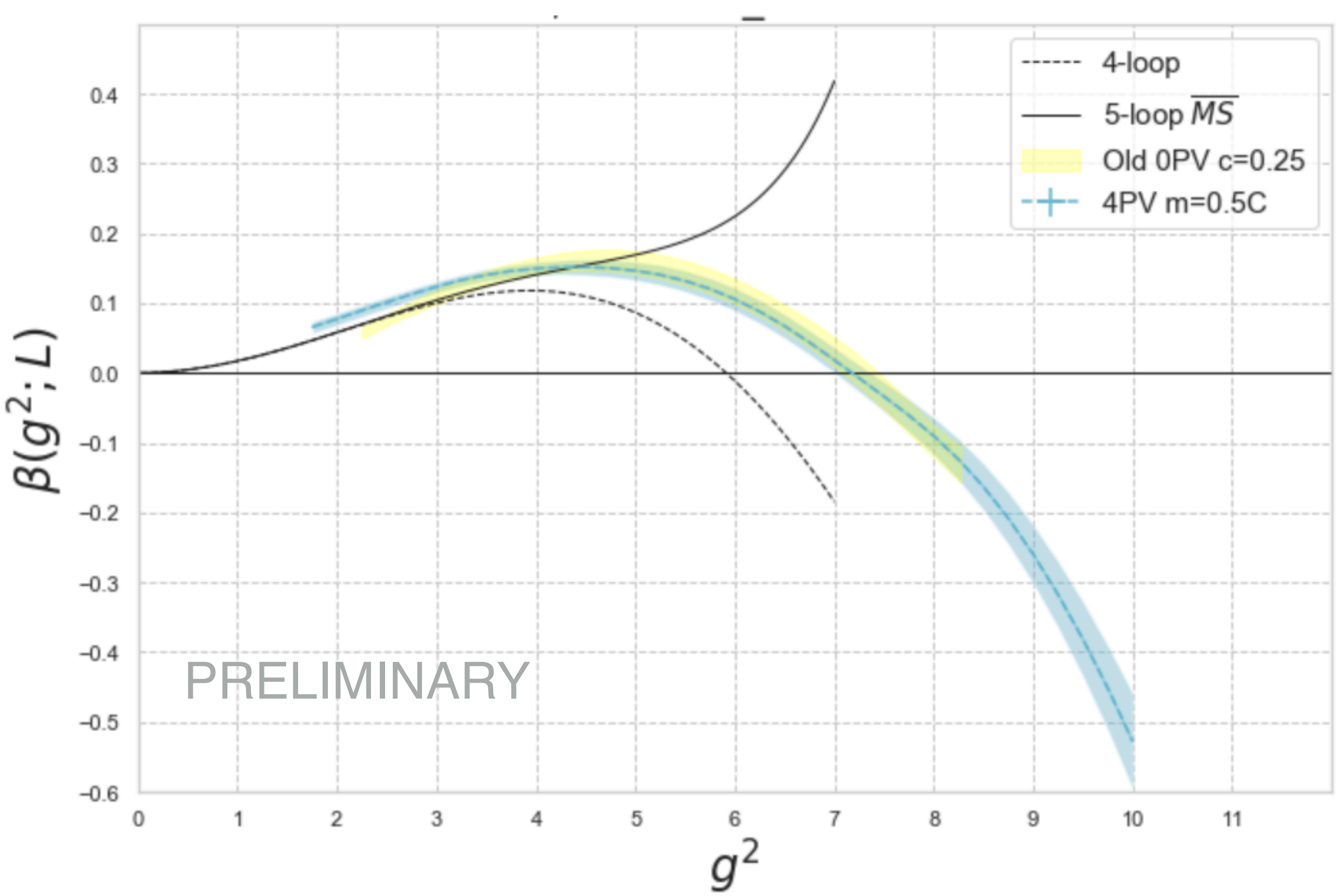


$N_f = 12$

$N_{\rm PV}$	β	$g^2_{\rm GF}$	$N_{\rm step}$	$N_{\rm CG}$	$\lvert\delta H\rvert$
0	2.8	10.87(6)	15	1190	0.45
2	6.8	10.89(9)	12	400	0.062
4	9.0	10.58(8)	12	365	0.033
6	11.2	10.87(30)	12	353	0.033
8	13.4	10.87(100)	12	359	0.034

$N_f = 12$ with/wo Pauli-Villars bosons

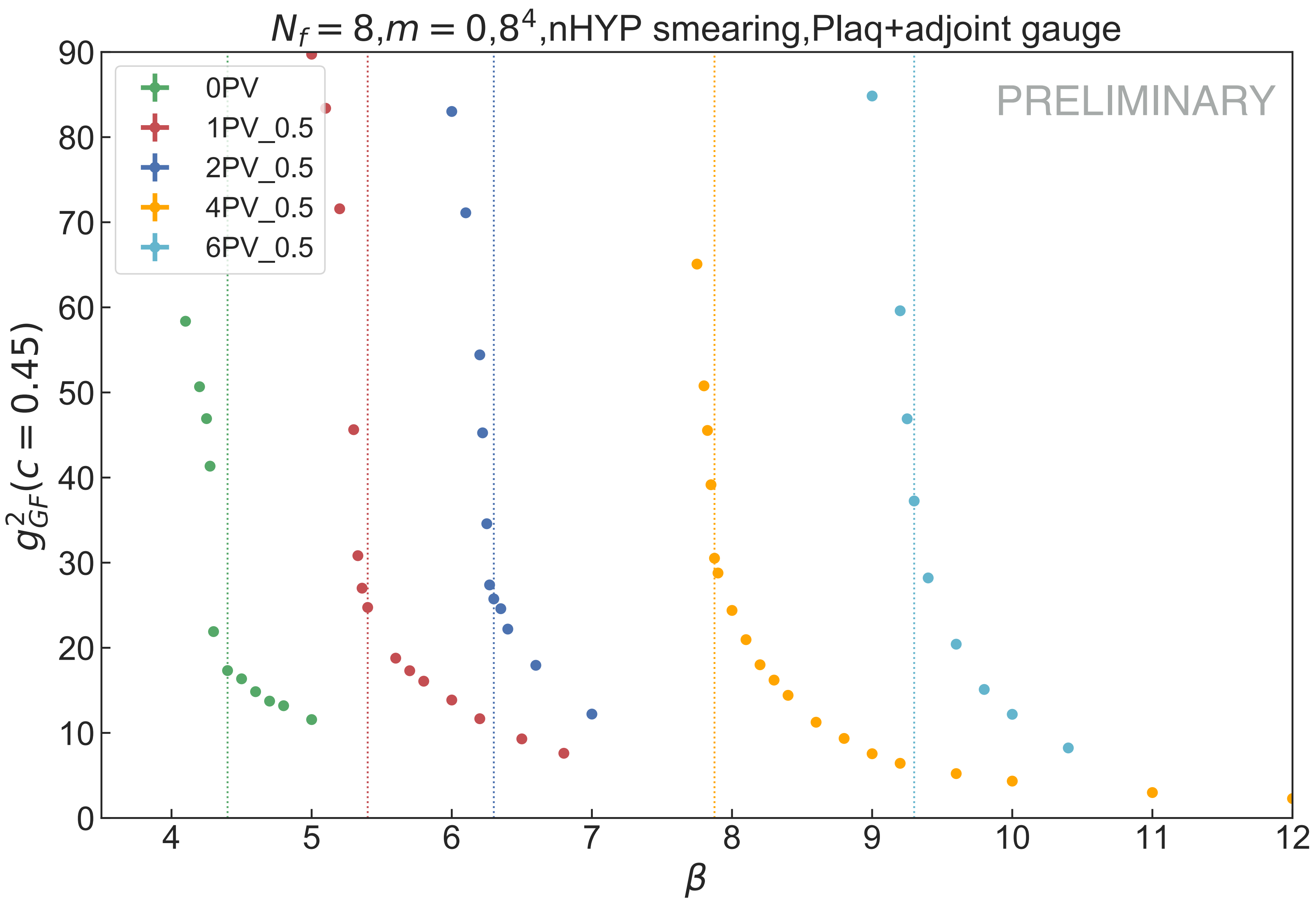
Predicted step scaling beta functions are identical (staggered)
PV action can explore much stronger couplings



Old:
AH, D. Schaich JHEP 02 (2018) 132

$N_f = 8$ with/wo Pauli-Villars bosons

Expected to be very close to the conformal window
Simulations without PV bosons are limited by a 1st order bulk transition to an
S4 phase where staggered shift symmetries are broken



S4 phase remains as PV bosons are added but the transition becomes smooth, appears continuous

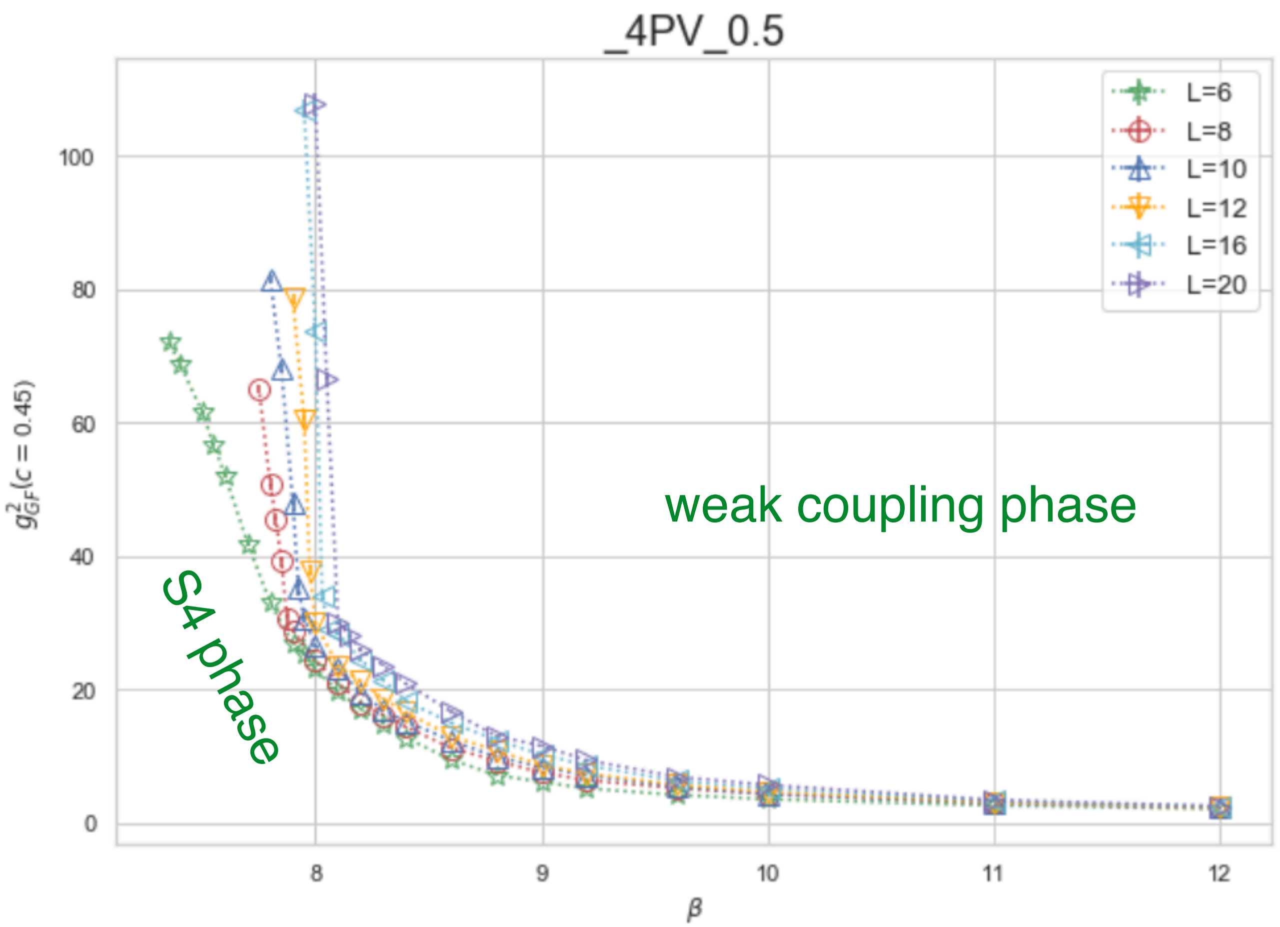
Could $N_f = 8$ be the boundary to the conformal window?

$N_f = 8$ phase structure with 4PV, $am_{PV} = 0.4$

Running coupling g^2_{GF} at flow time $\sqrt{8t} = 0.45L, L = 6,\ldots,20$
Significant finite volume dependence
—> finite size scaling

Topological susceptibility is rising even though simulations are at $m_f = 0$

Staggered fermions are Dirac-Kaehler, equivalent to Dirac only at $g_0^2 = 0$

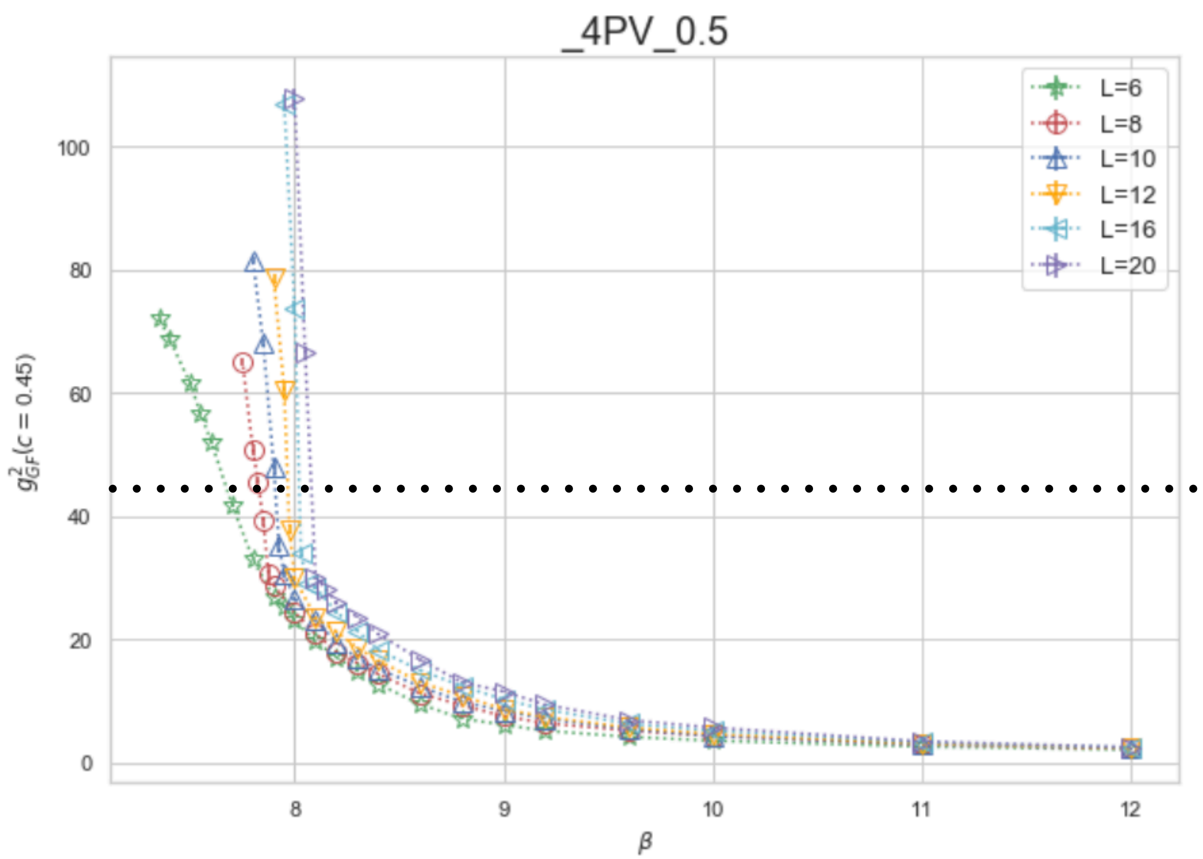


PRELIMINARY

$N_f = 8$:finite size scaling

PRELIMINARY

Running coupling at flow time $\sqrt{8t} = 0.45L$



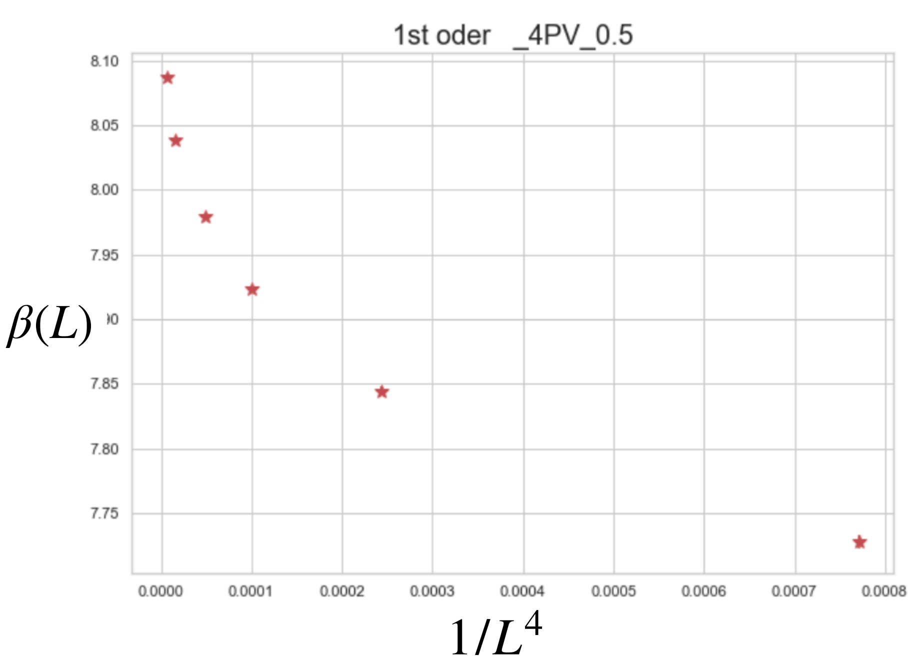
At fixed g^2_{GF} the coupling scale as

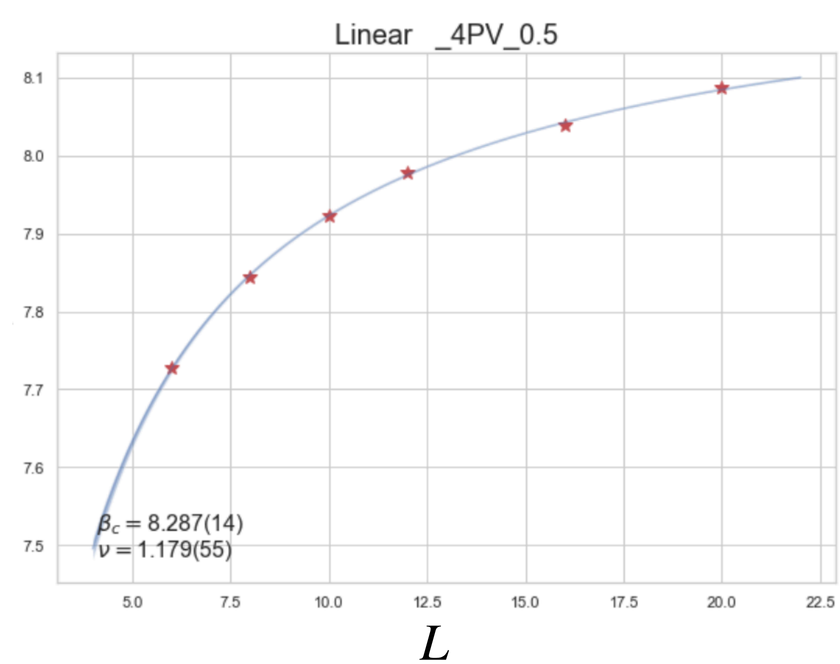
1st order : $\beta(L) - \beta^* \propto L^{-1/4}$

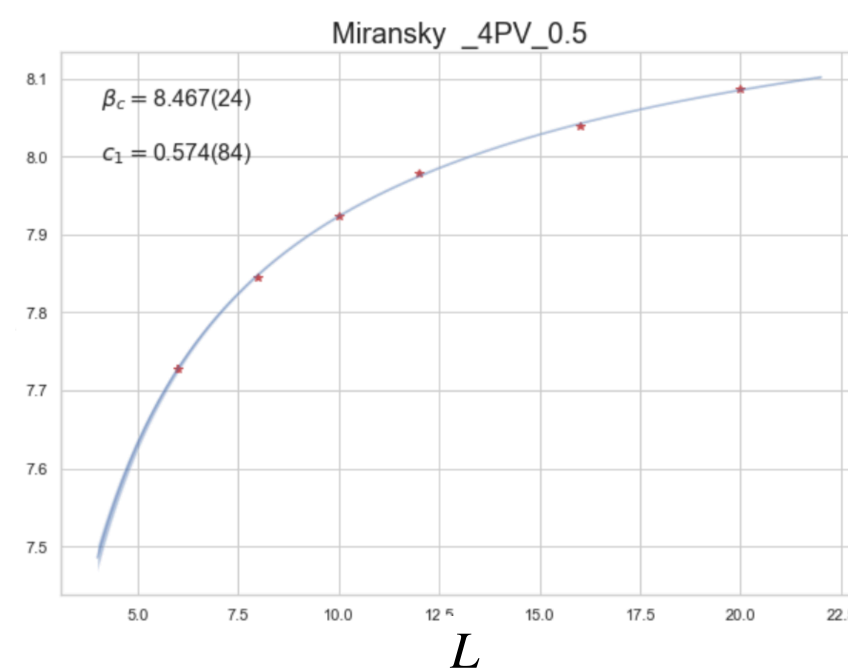
2nd order : $\beta(L) - \beta^* \propto L^{-1/\nu}$

Essential singularity (XY model/ Miransky) : $\beta(L) - \beta^* \propto 1/\log^2(cL)$

Data are not consistent with 1st order transition



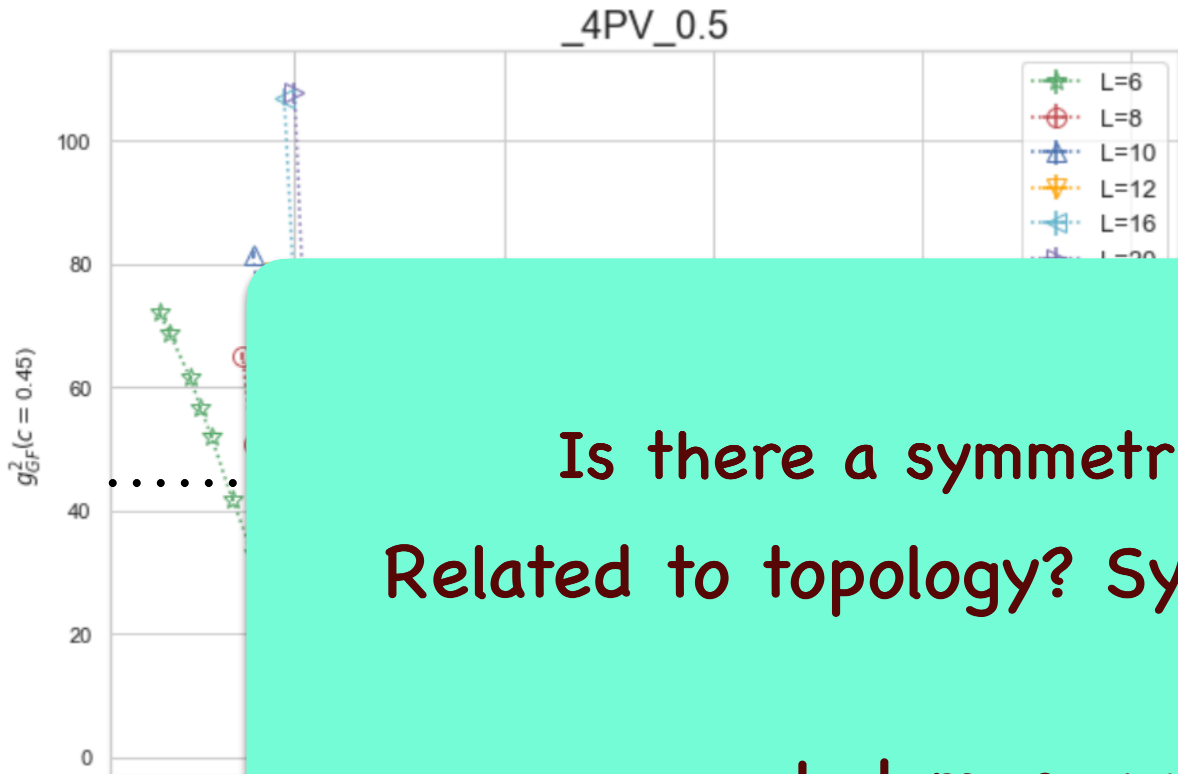




$N_f = 8$:finite size scaling

PRELIMINARY

Running coupling at flow time $\sqrt{8t} = 0.45L$



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el/

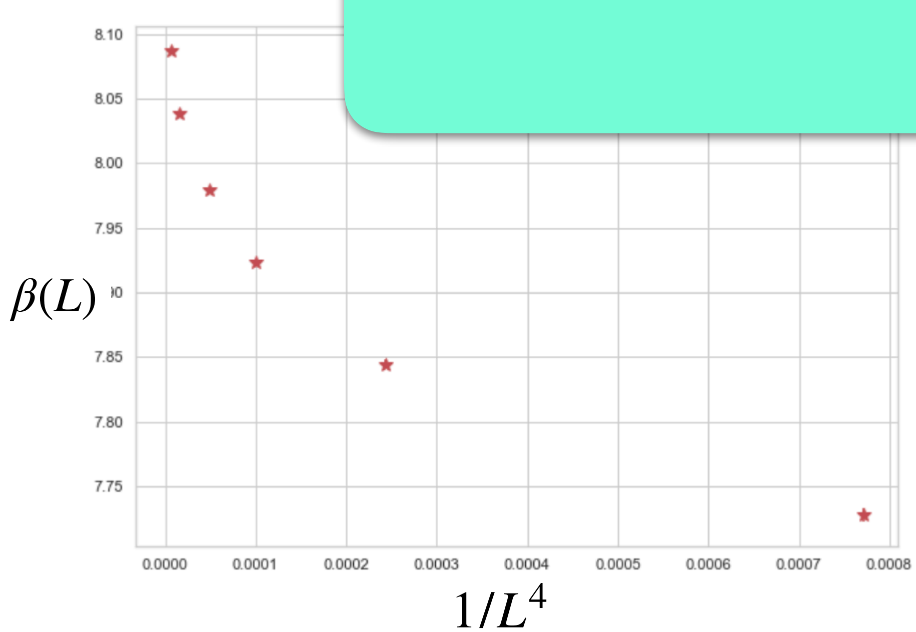
$g^2(cL)$

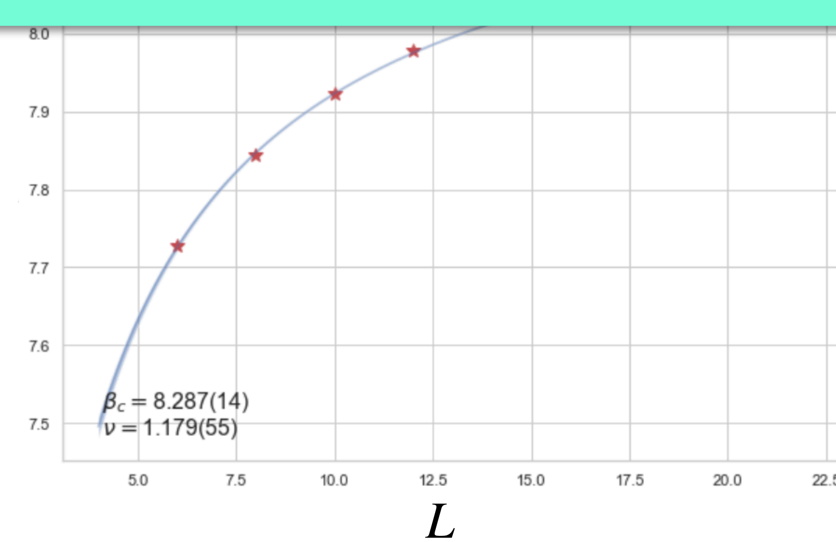
order

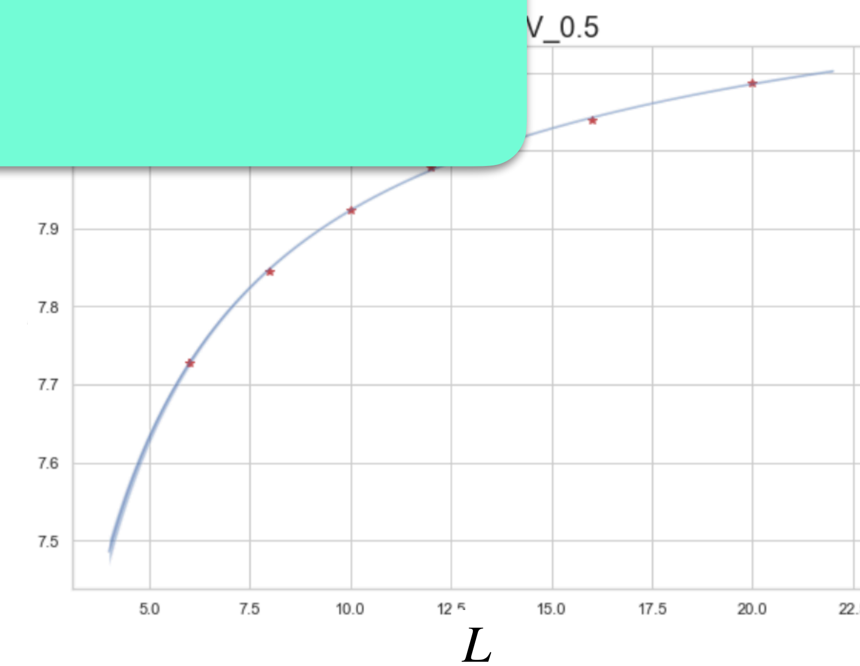
Is there a symmetry that selects $N_f = 8$?

Related to topology? Symmetric mass generation?

Lot more work is needed!







Summary and Outlook

The equivalence of Wilsonian RG and gradient flow allows a theoretically solid description of the strong coupling regime of lattice models

✦ This is particularly important in near-conformal / conformal systems where new fixed points, new relevant operators appear

The continuous β function:

✦In QCD-like chirally broken systems it is well controlled with minimal cutoff effects with improved action/flow/operator in QCD-like chirally broken systems

✦Near the conformal sill strong non-perturbative effects appear :

- extended actions (4-fermion term?) might be necessary
- larger volumes (brute-force approach) can extend the accessible range, but will not cover the IRFP or UVFP beyond

PV bosons reduce cutoff effects, open up the parameter space

✦ cleaner numerical results

✦ possibly revealing new dynamics

✦ is $N_f = 8$ special?

EXTRA SLIDES

The continuous β function, $N_f = 2$

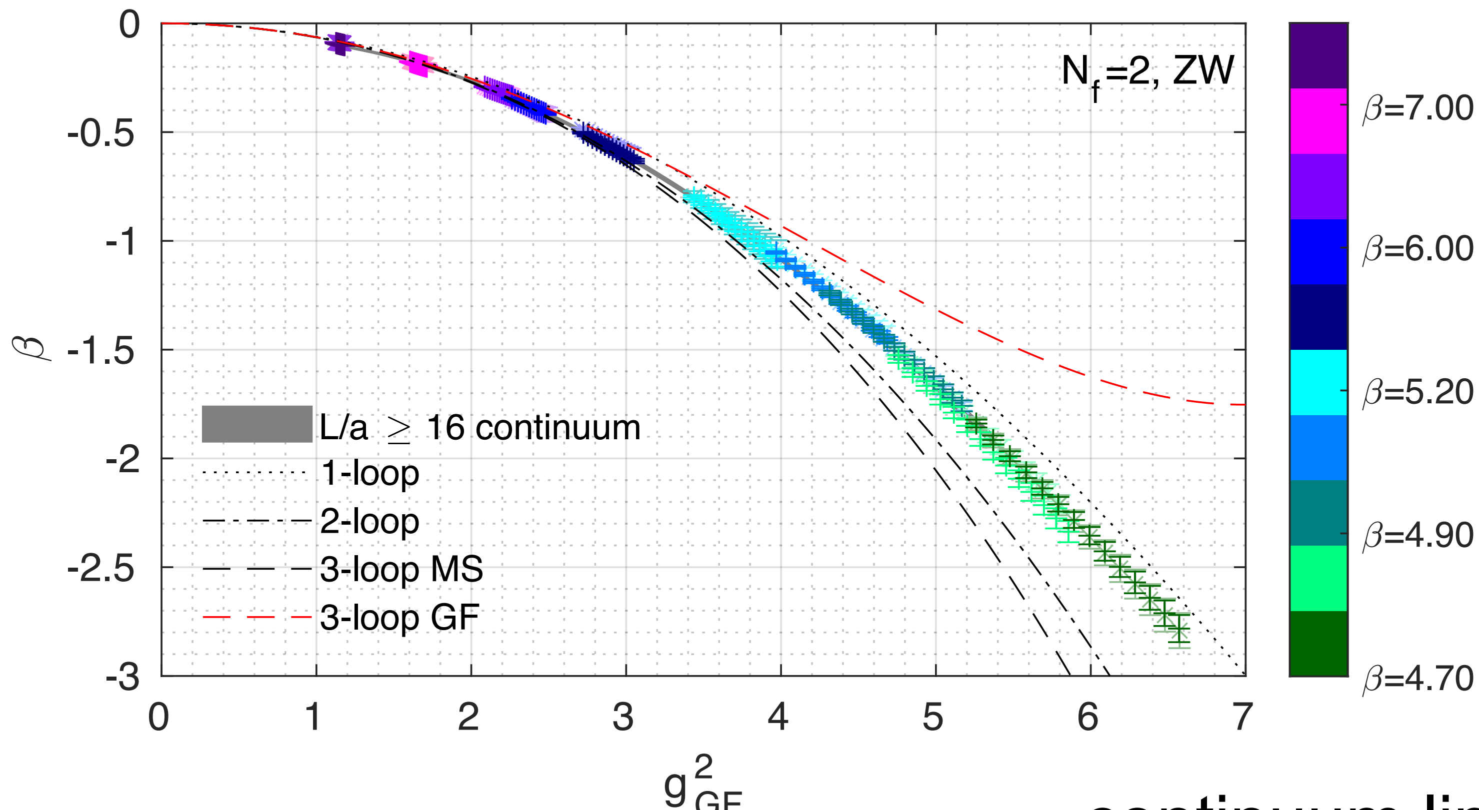
AH, O. Witzel, *Phys.Rev.D* 101 (2020) 3

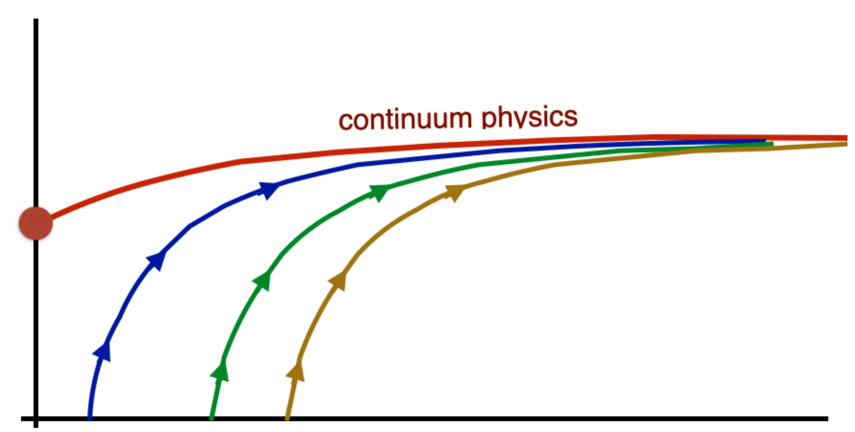
We use

- Symanzik gauge action, Mobius domain wall fermions (Grid)
- Zeuthen (Z), Wilson(W) and Symanzik(S) flows : optimize to pull the RT close
- Wilson plaquette(W), clover(C) and Symanzik(S) operators : combine to optimize for the scaling operator

ZW raw data

- shows minimal finite volume effects and significant overlap





continuum limit (below colored points)

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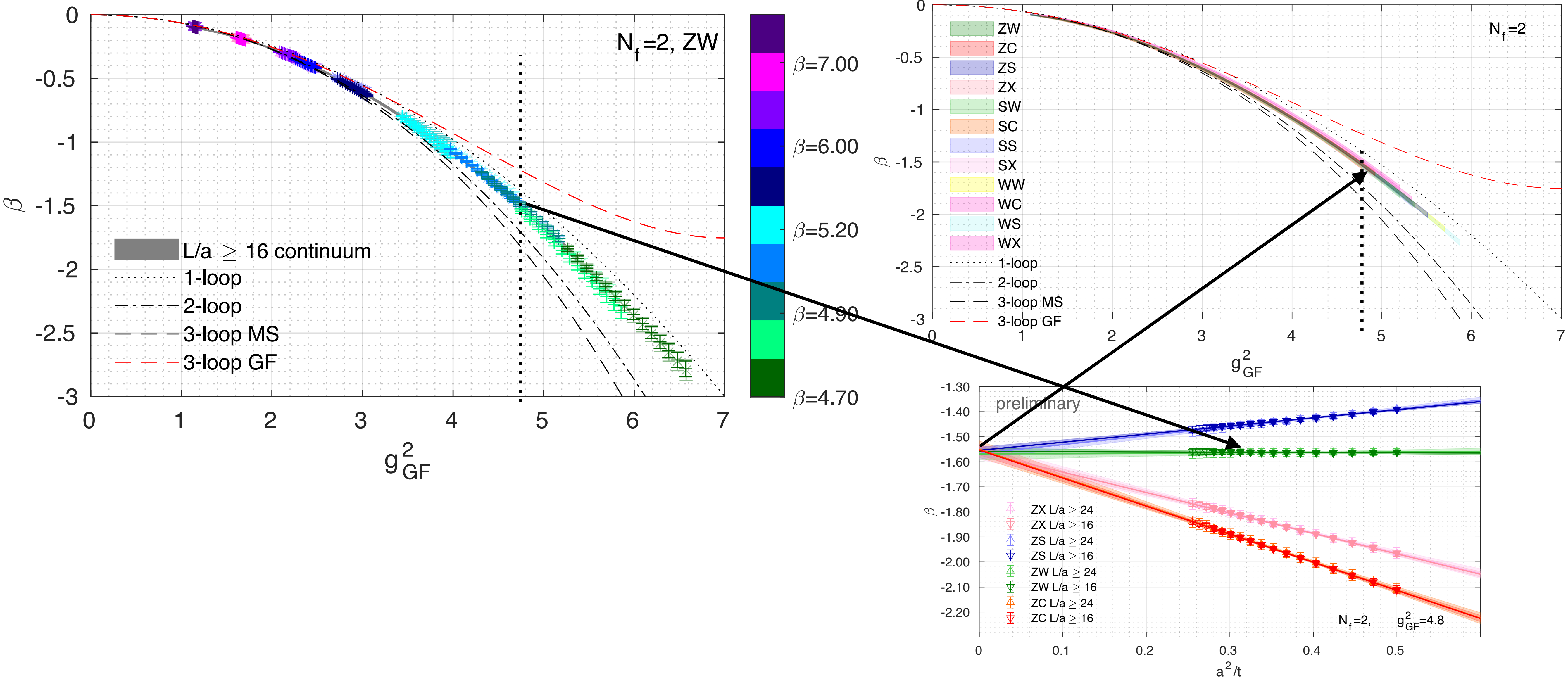
Analysis steps for continuum limit:

1. Infinite volume extrapolation ($1/L^4$ in the chirally symmetric regime)
2. Infinite flow time extrapolation (a^2/t) at fixed g^2_{GF}

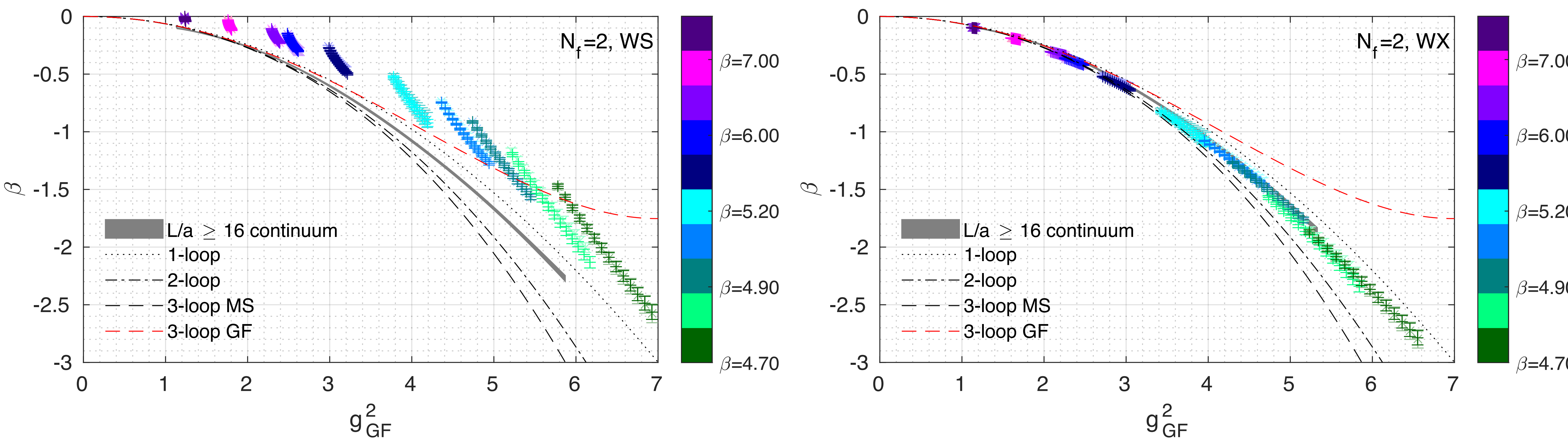
continuum limit



The continuous β function, $N_f = 2$

AH, O. Witzel, *Phys.Rev.D* 101 (2020) 3

- Other flow/operator combos predict the same continuum limit, sometimes with larger cut-off effects (WS)
- It is possible to "optimize" the operator for different flows and find "scaling operators" (e.g. $X = 0.25W + 0.75C$ is optimal for Wilson flow) (WX)



GF β function is closest to 1-loop; consistent with GF 3-loop up to $g^2_{GF} \approx 2.5$

Harlander et al, *JHEP* 06 (2016) 161

The continuous β function, $N_f = 0 - 8$

In all cases:

- The β functions runs slower than 1-loop PT
- minimal cutoff effects if Zeuthen flow+Wilson op (or tree level improved coupling)

In the confining regime apparently $\beta(g^2) \propto g^2$