The QCD phase diagram with complex Langevin simulations

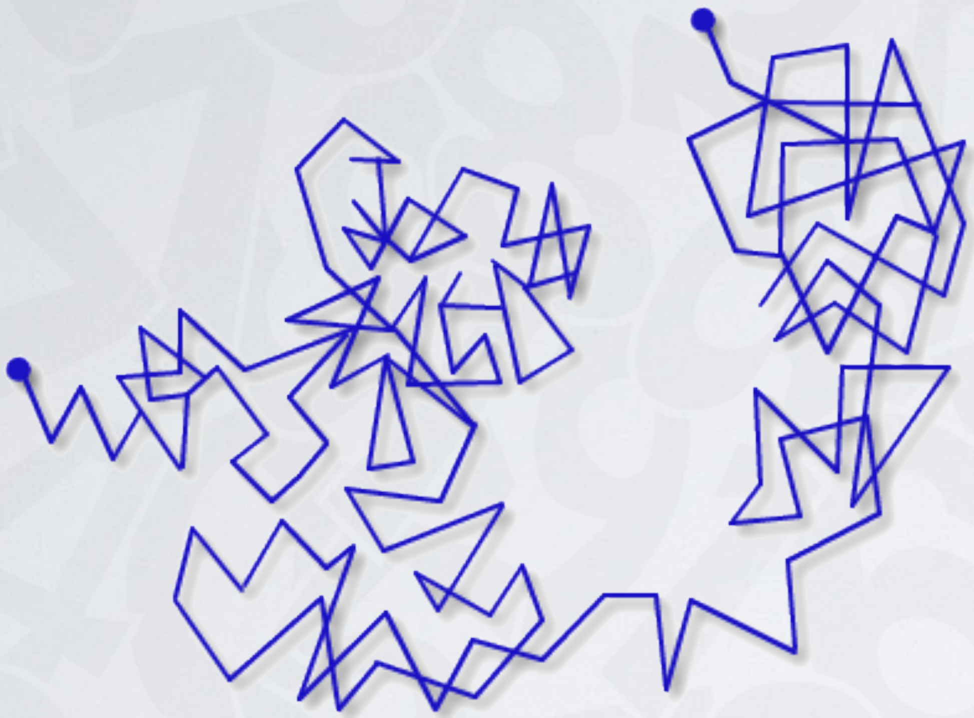
Felipe Attanasio, Benjamin Jäger & Felix Ziegler

UNIVERSITY OF SOUTHERN DENMARK

CP³ Origins
Cosmology & Particle Physics

D·IAS

Complex Langevin



- Complexify degrees of freedom

$$x \to z = x + \mathrm{i}\, y$$

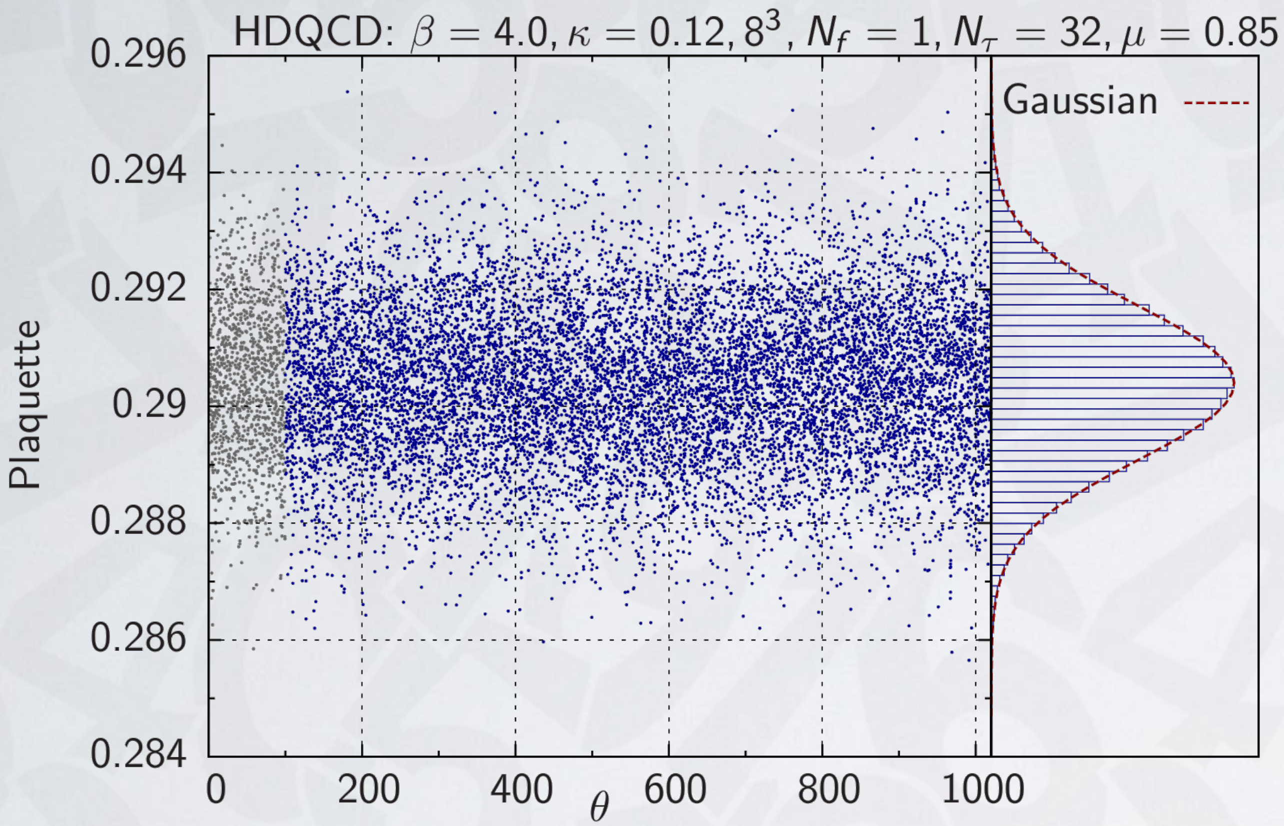
- Stochastic Quantization:
  Langevin Eq:

$$\frac{\partial z}{\partial \theta} = \frac{\partial S}{\partial z} + \eta(\theta)$$

- Sign problem can be circumvented, even if it is severe :)
- However, convergence only when:
  - Action and observables are holomorphic
  - Extension into the non-SU(3) manifold is compact

proof in
0912.3360
1606.07627

Complex Langevin



- Gauge theories (QCD)

$$\mathrm{SU}(3) \to \mathrm{SL}(3, \mathbb{C})$$

- Non-compact gauge group

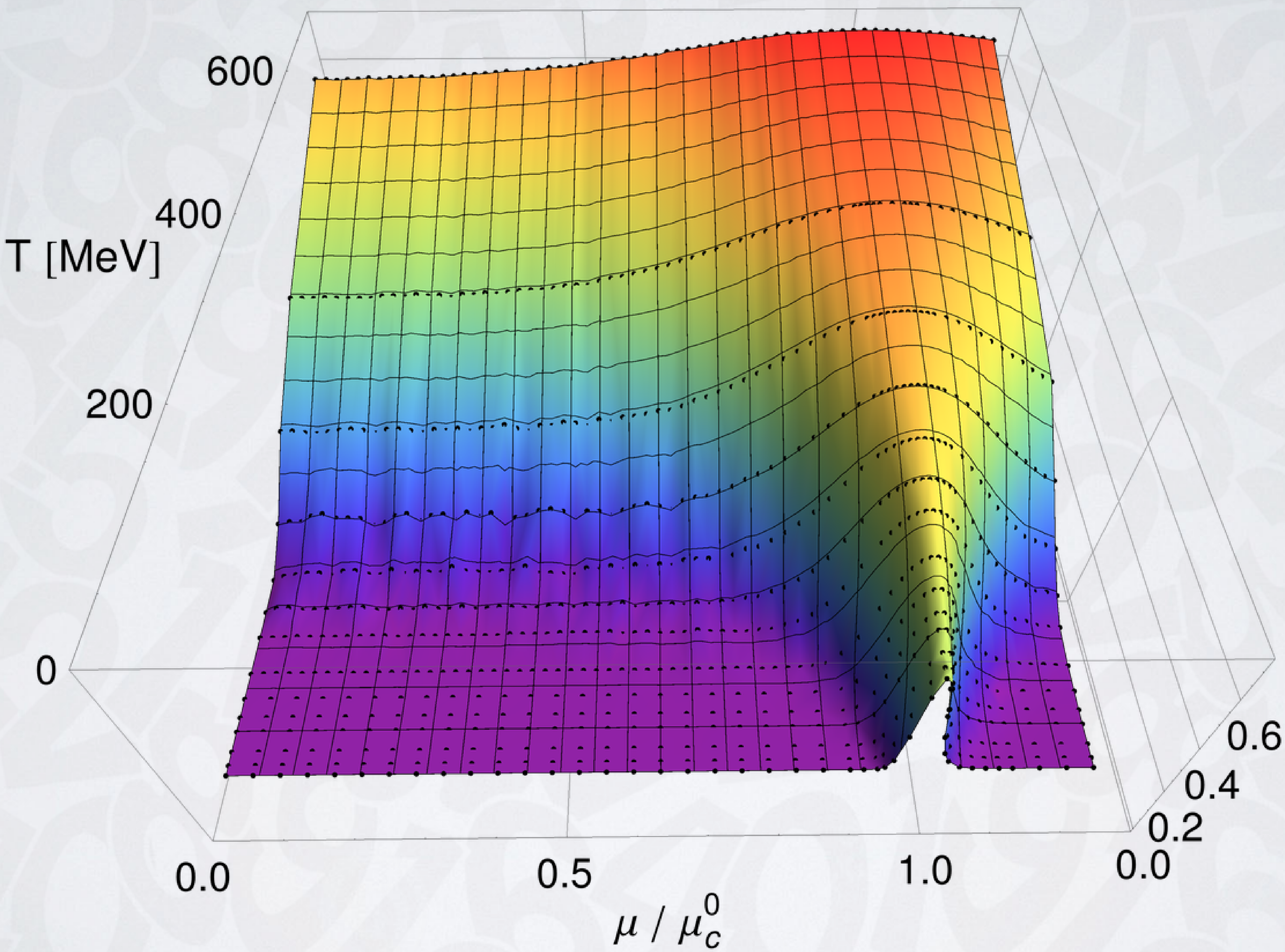
$$U_{x,\mu} = \exp\left[\mathrm{i}\, a\, \lambda_c \left(A^c_{x,\mu} + \mathrm{i} B^c_{x,\mu}\right)\right]$$

- Update scheme (First order discretisation)

$$U_{x,\mu}(\theta + \epsilon) = \exp\left[\mathrm{i}\, a\, \lambda_c \left(-\epsilon\, D^c_{x,\mu} S + \sqrt{\epsilon}\, \eta^c_{x,\mu}\right)\right] U_{x,\mu}(\theta)$$

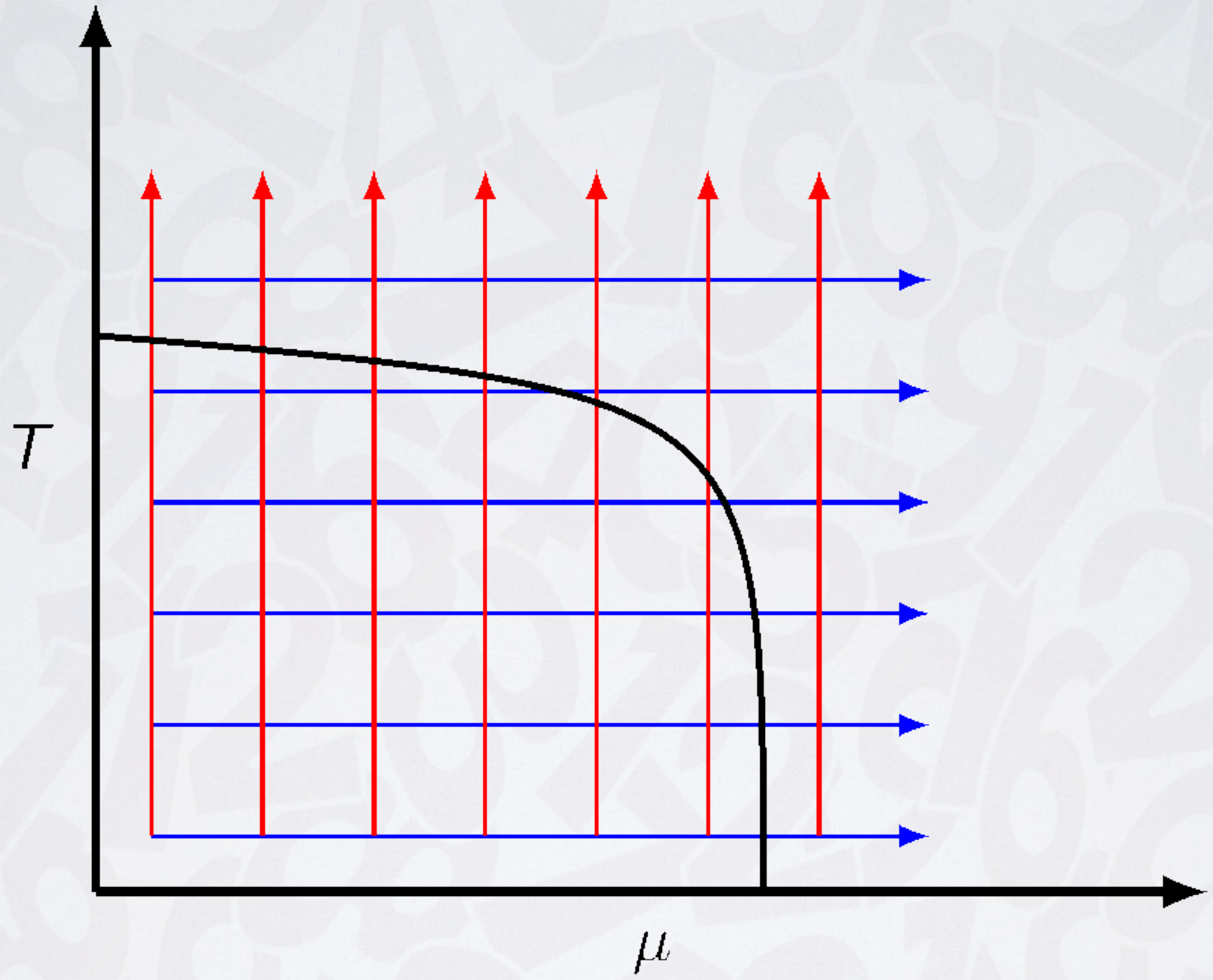
- Accept-reject step not possible, but extrapolation $\epsilon \to 0$

Previous Work (HDQCD)



based on
1606.05561
1808.04400
2006.00476

Strategy



Lattice Setup

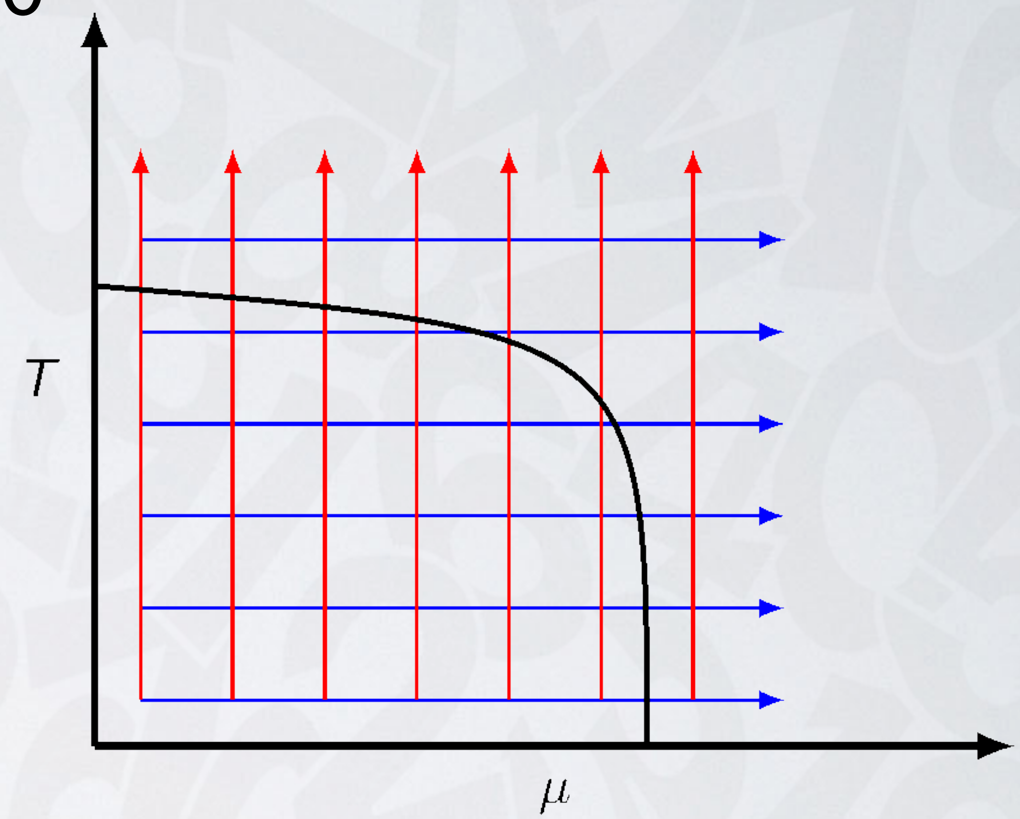
- **Lattice setup**
  - Wilson plaquette action $\beta = 5.8$
  - Two-flavour dynamical fermions Wilson Fermions $(c_{sw} = 0)$
  - Pion mass $\kappa = 0.1544 \leftrightarrow m_\pi \sim 500\ MeV$
  - Volume $V = 24^3 \leftrightarrow m_\pi L = 3.5$
  - Lattice spacing $a \sim 0.06\ fm$

parameters similar to hep-lat:0512021

- **Phase diagram scan**
  - Temperature $N_t = 4 - 128 \leftrightarrow T \sim 25 - 800\ MeV$
  - Chemical potential $a\mu = 0 - 2 \leftrightarrow \mu \sim 0 - 6500\ MeV$
  - Gauge Cooling, Adaptive Stepsize & Dynamic Stabilisation

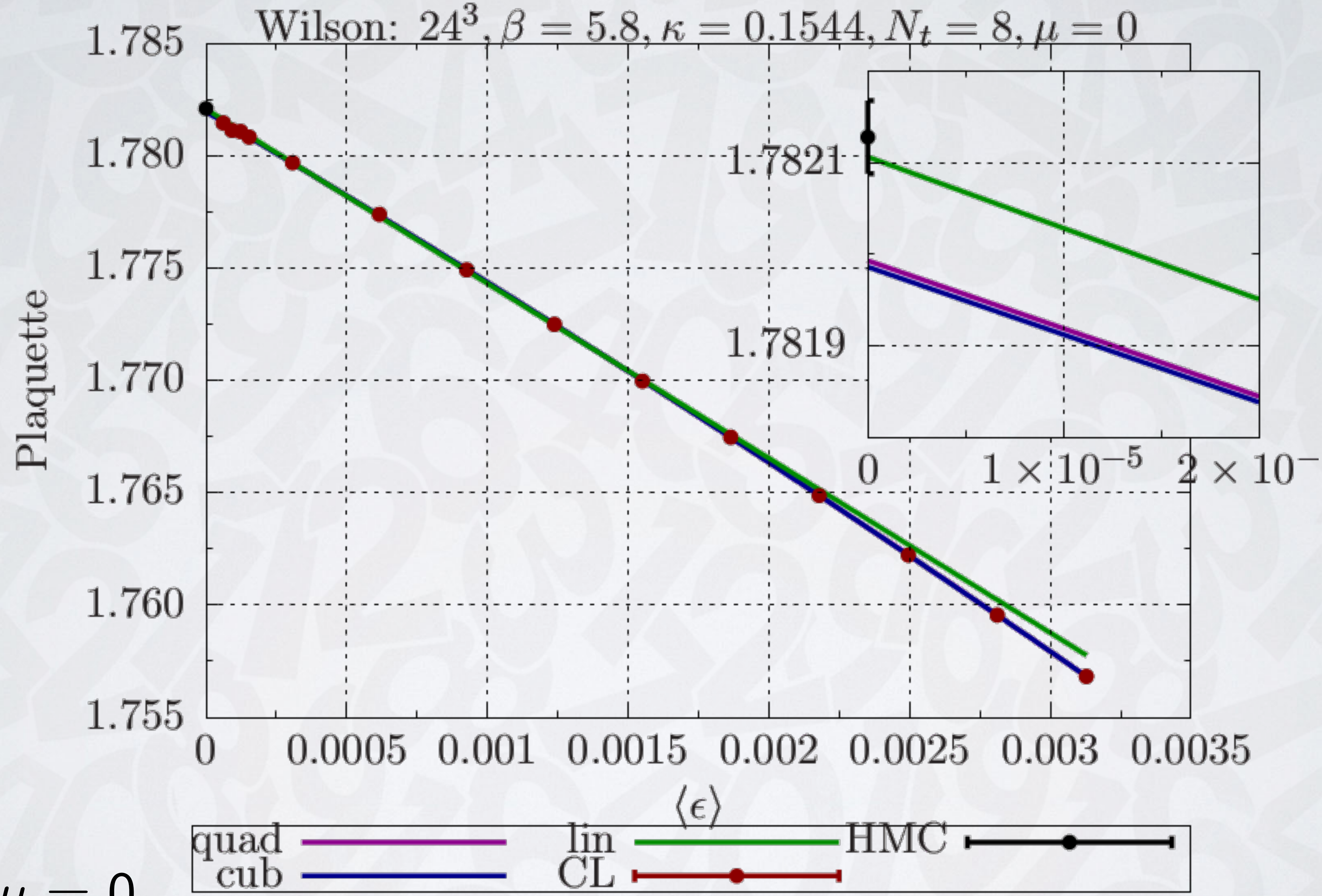
Results

- **Consistency checks @ $\mu = 0$**
  - HMC vs. CL
- **Physics**
  - Fermion density
  - Polyakov loop
  - Chiral condensate
- **Numerics / Stability**
  - Unitarity norm (distance to SU(3))
  - Iterations (Conjugate Gradient)



Wilson @ $m_\pi \sim 500$ MeV

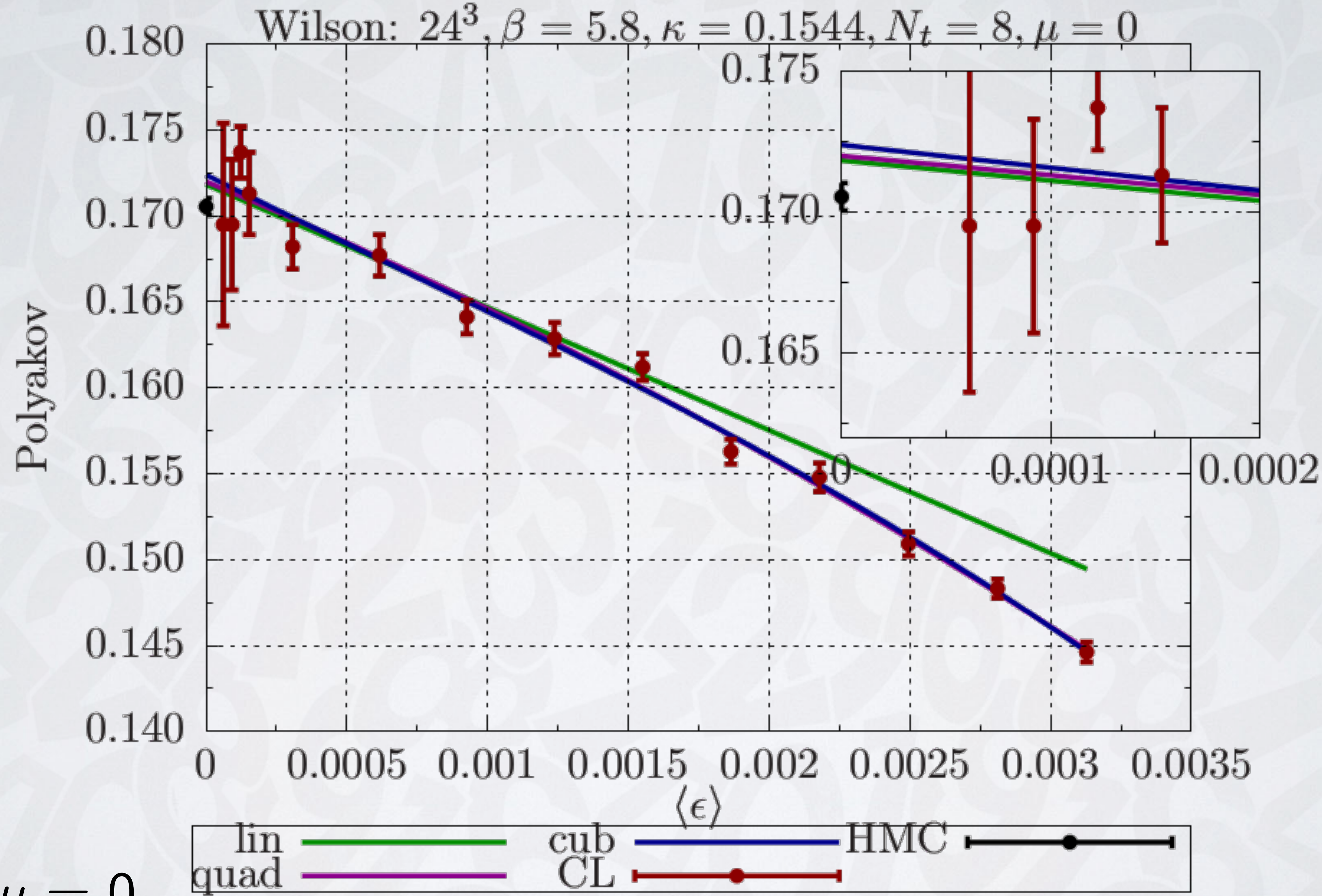
HMC vs CL - deconfined phase



@ $\mu = 0$

$N_t = 8 \leftrightarrow T = 400\ MeV$

HMC vs CL - deconfined phase



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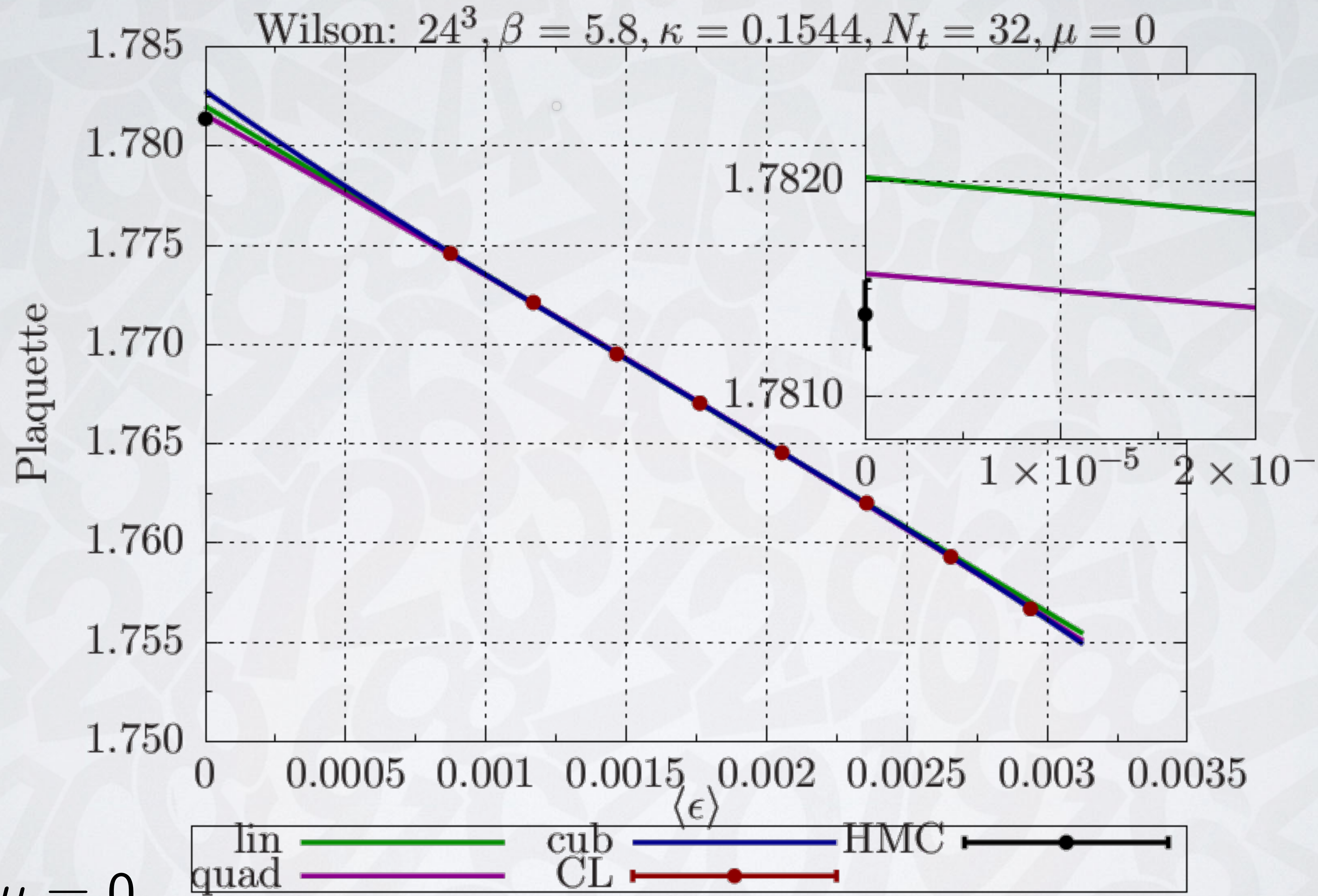
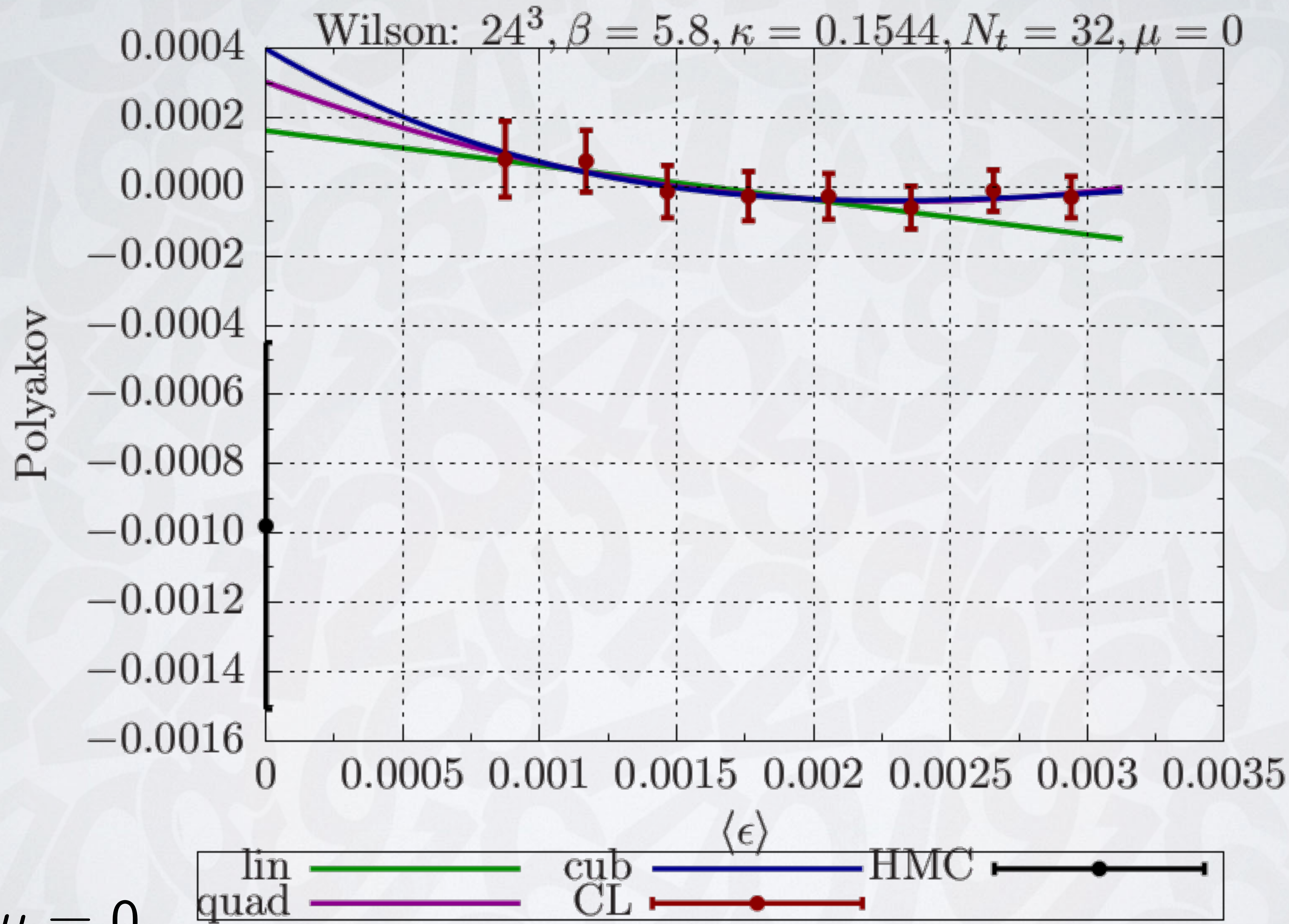


@ $\mu = 0$

$N_t = 32 \leftrightarrow T = 100\ MeV$

HMC vs CL - confined phase



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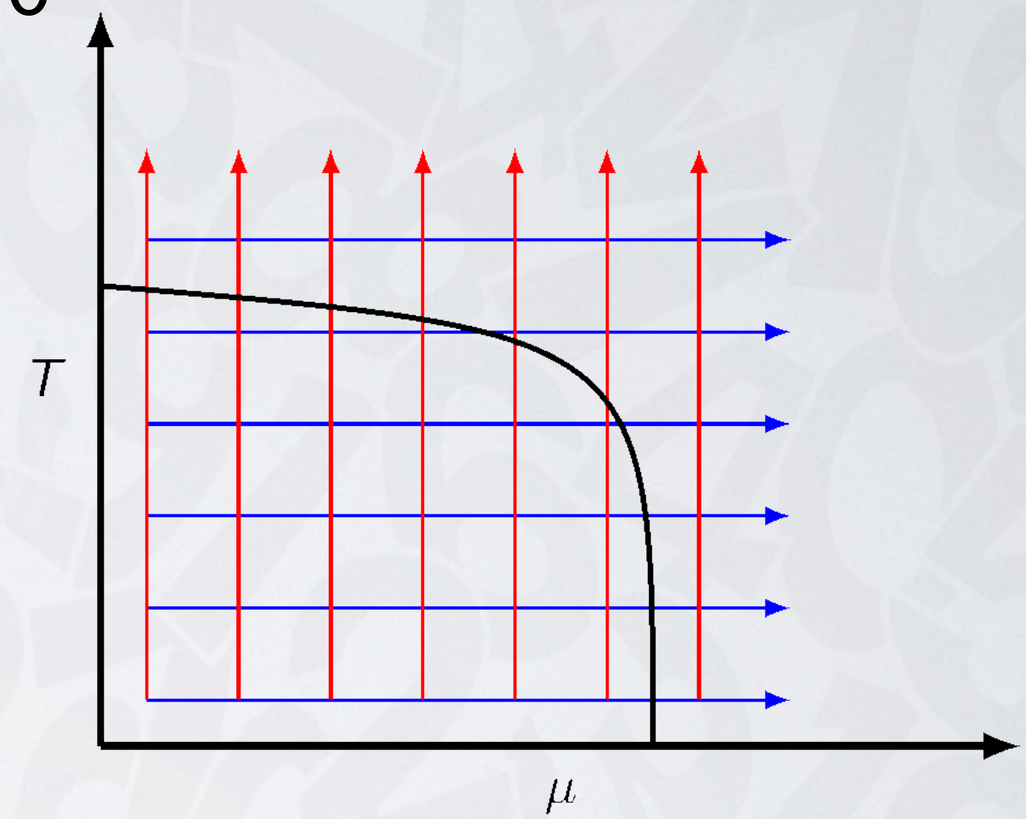

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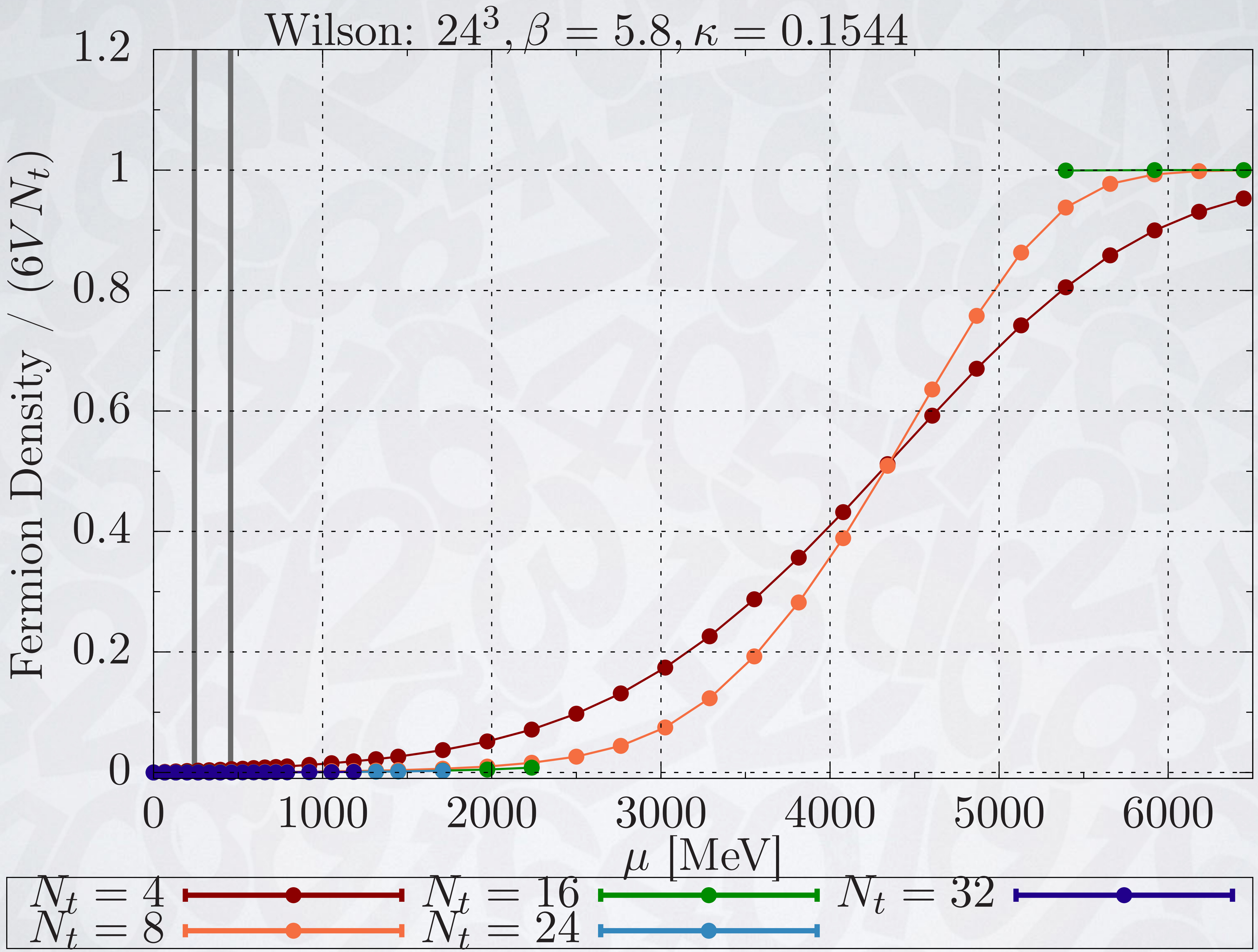
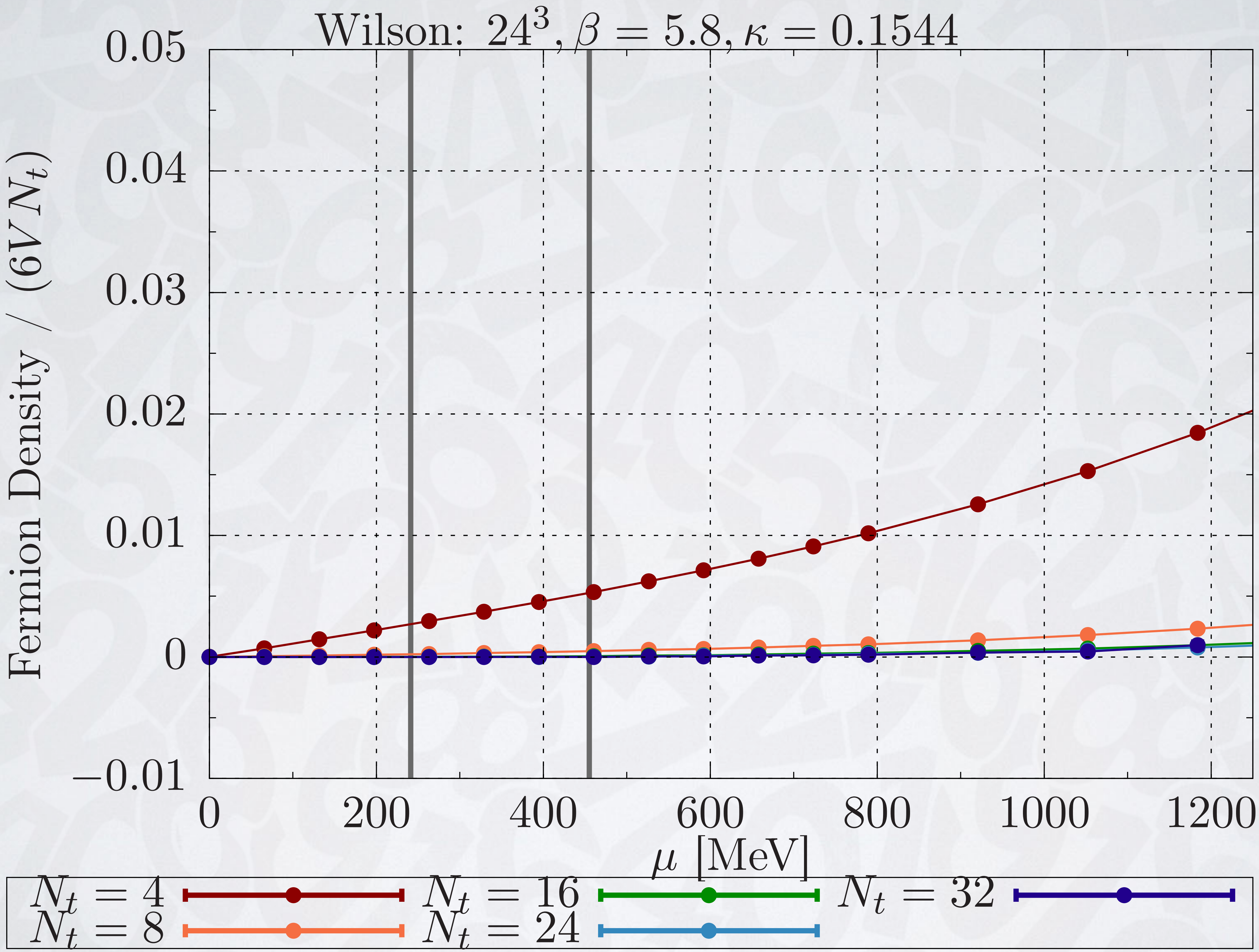
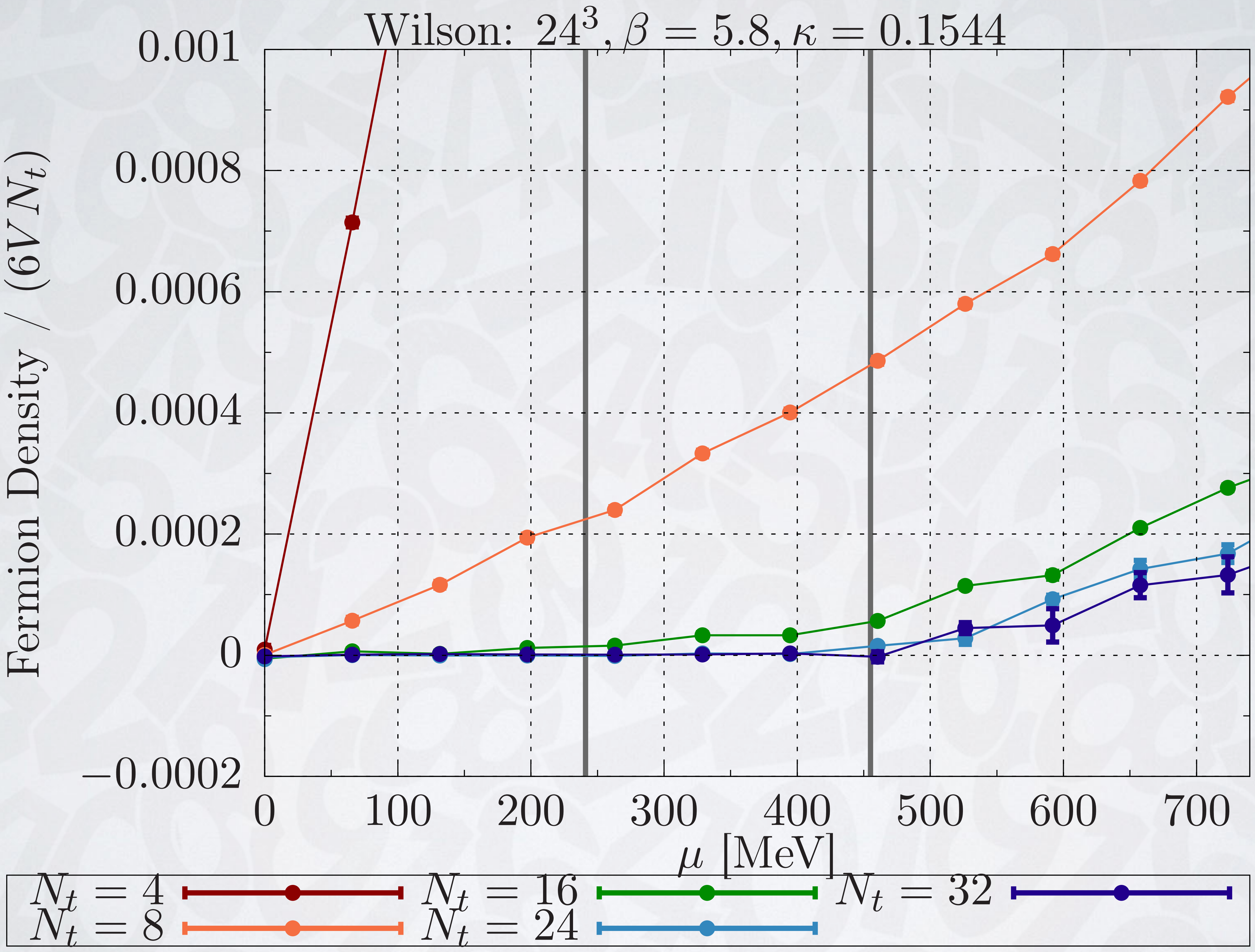
Fermion Density @ large μ



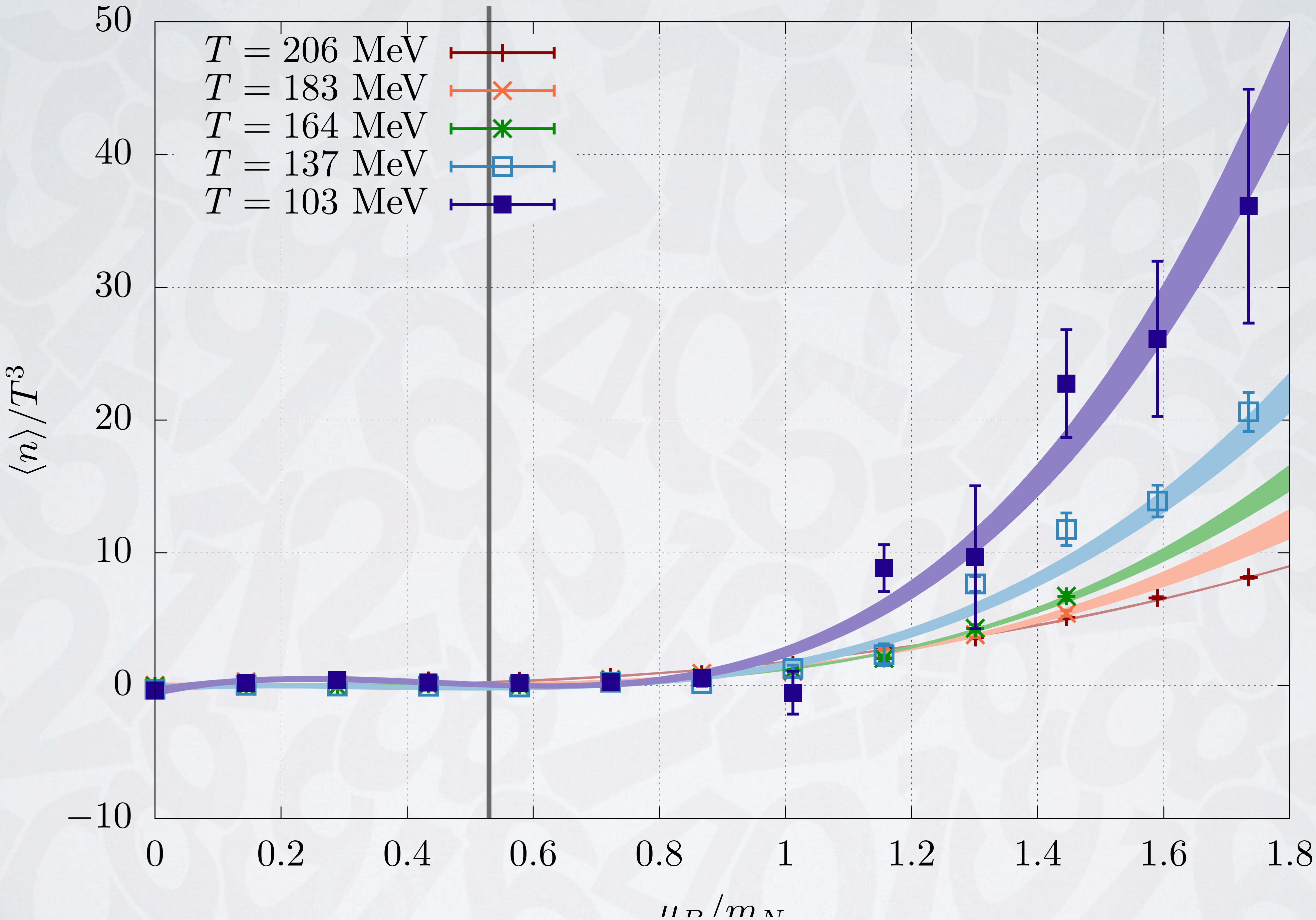
Fermion Density @ medium μ



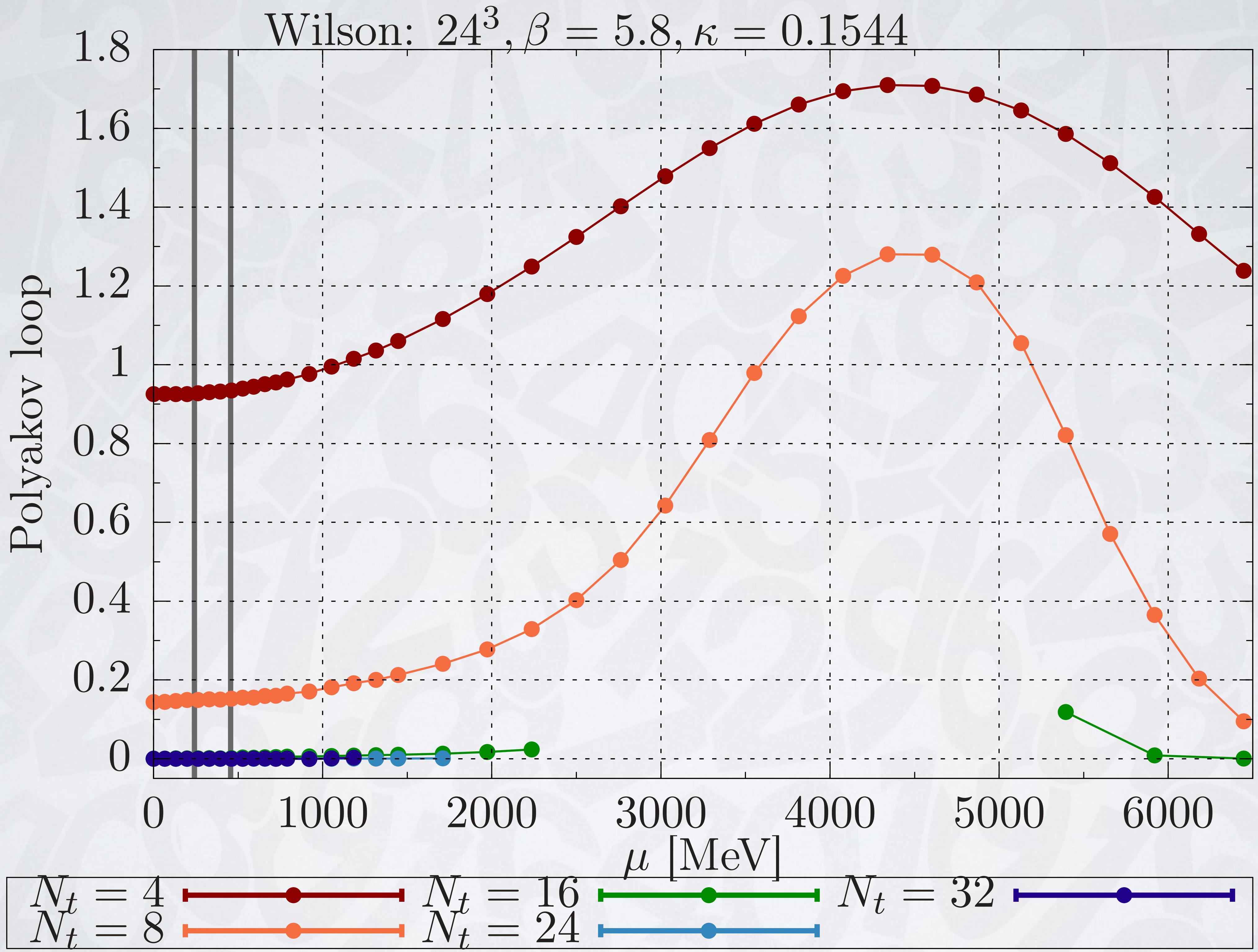
Fermion Density @ small μ



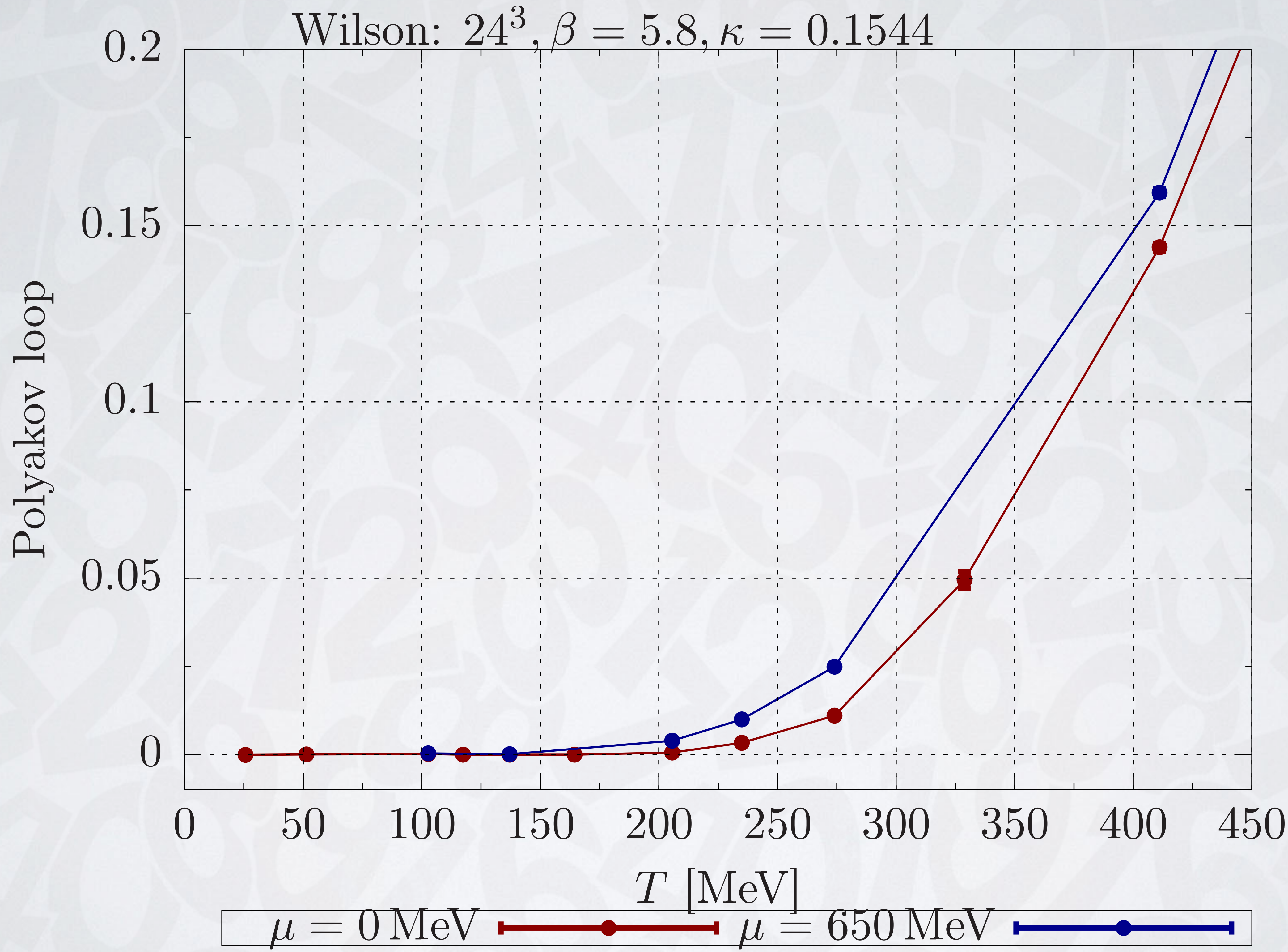
Fermion Density



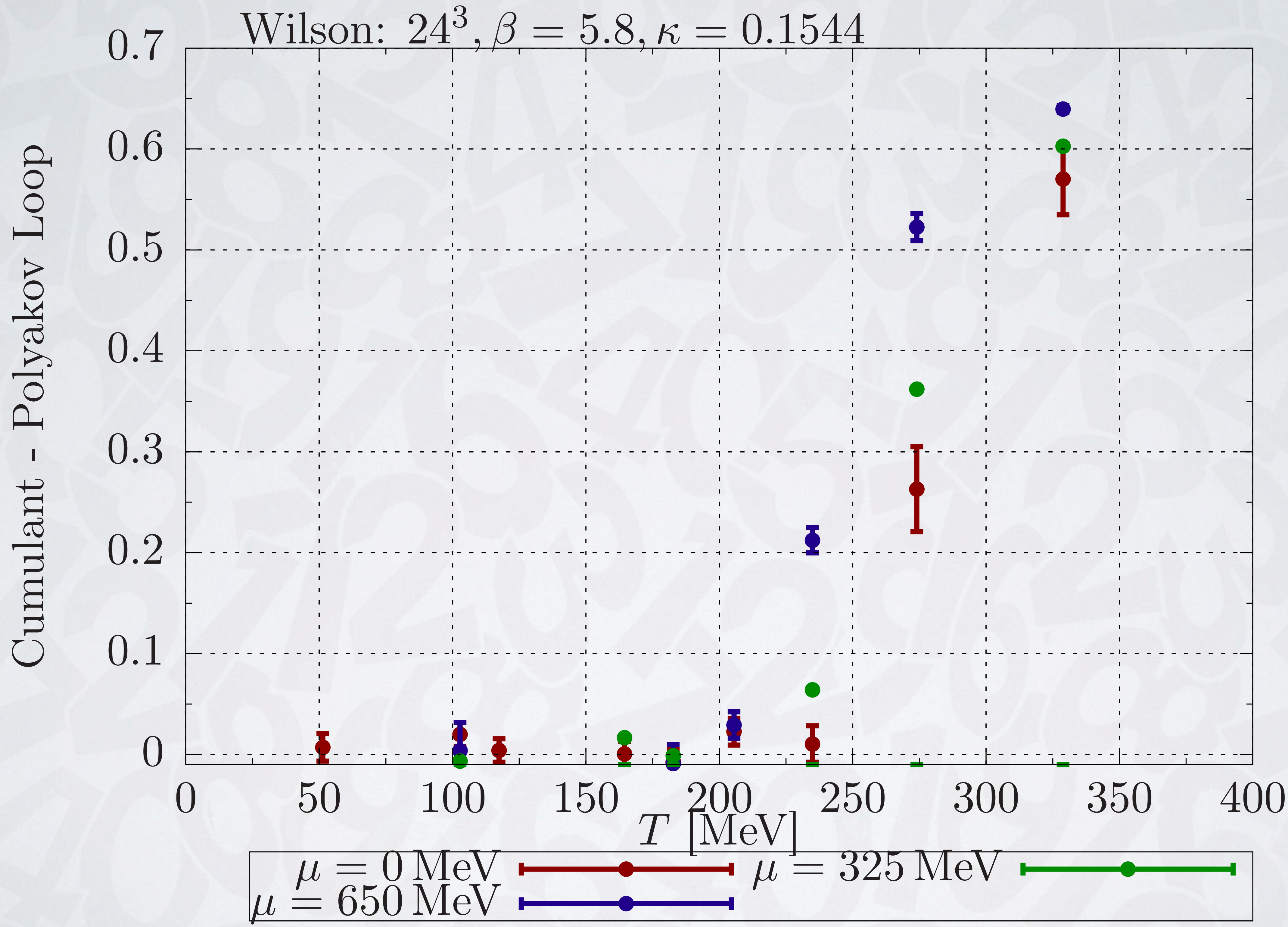
Polyakov Loop @ large μ



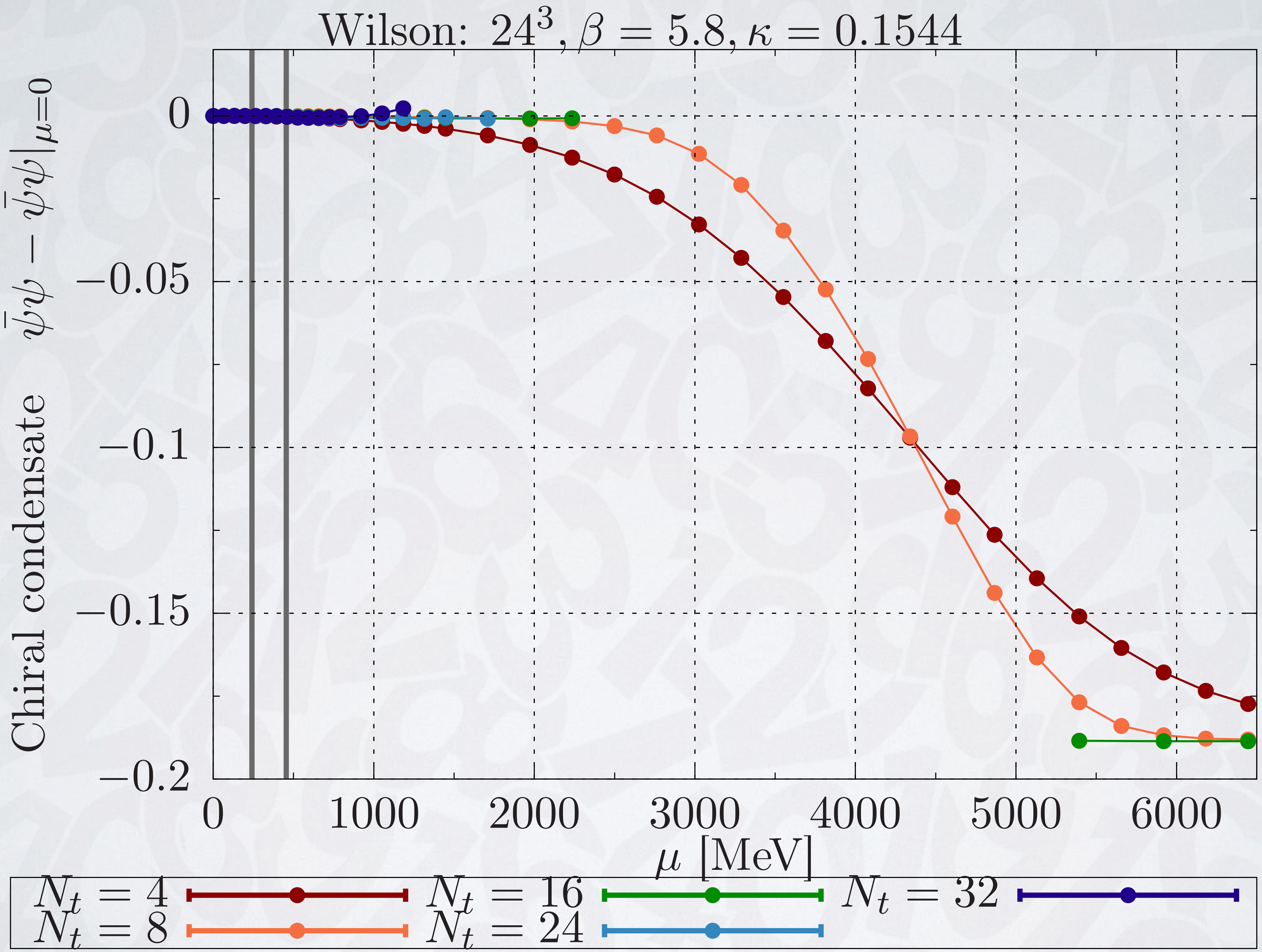
Polyakov Loop @ finite T



Polyakov Loop @ finite T



Chiral Condensate



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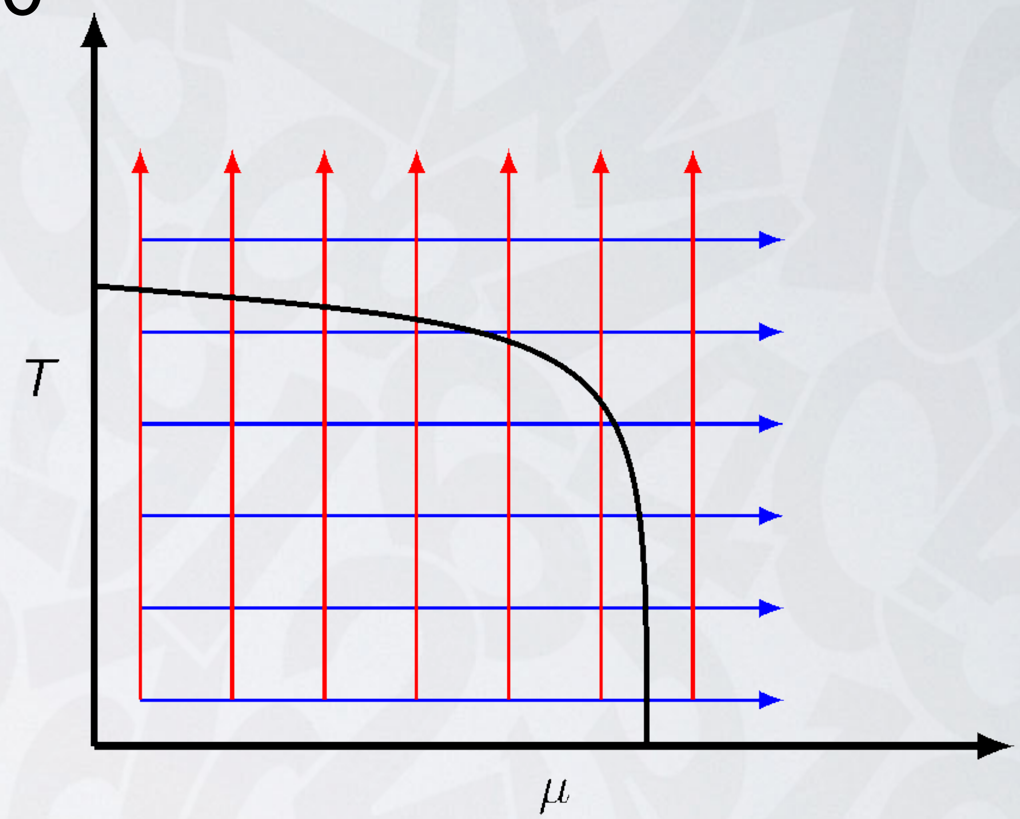
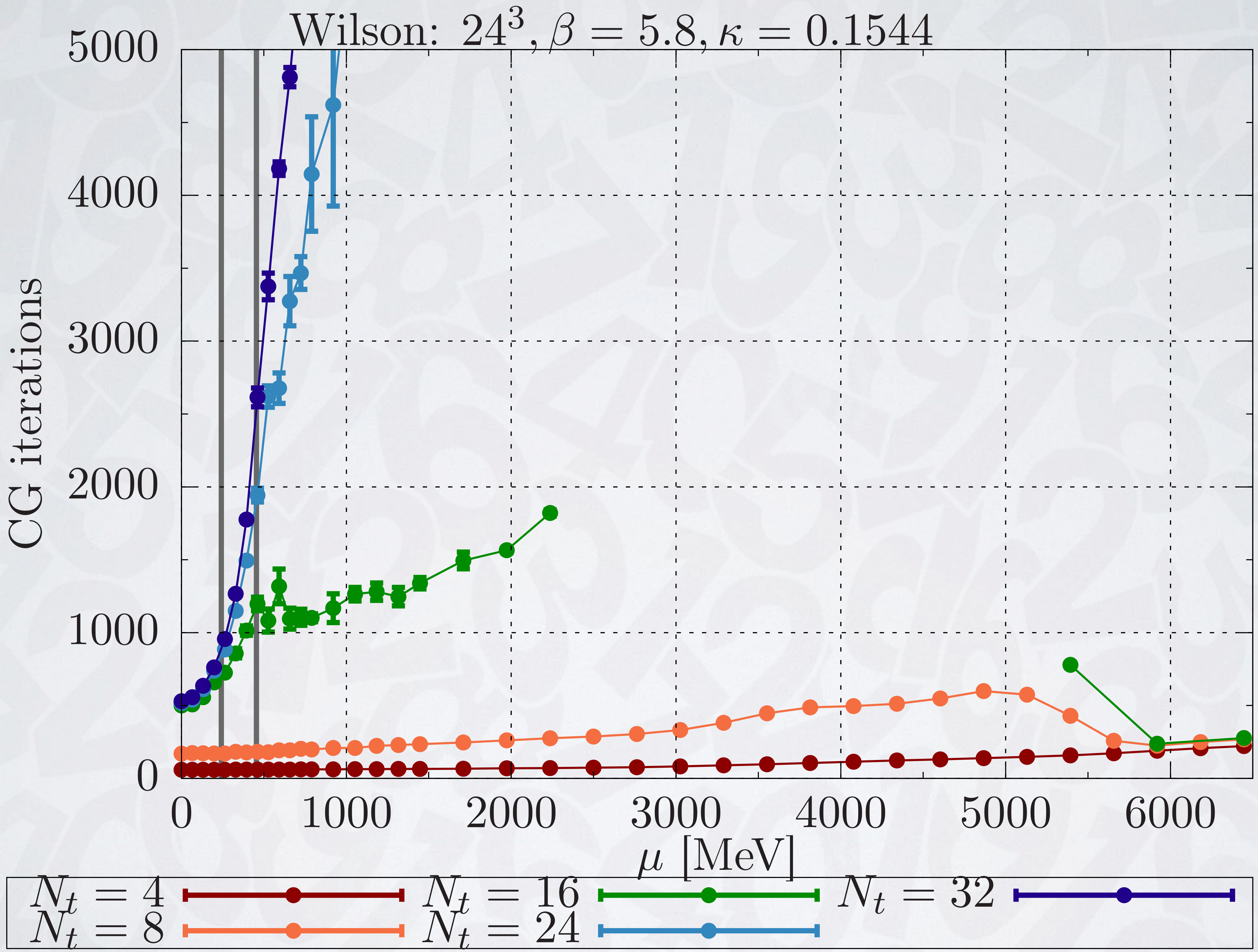
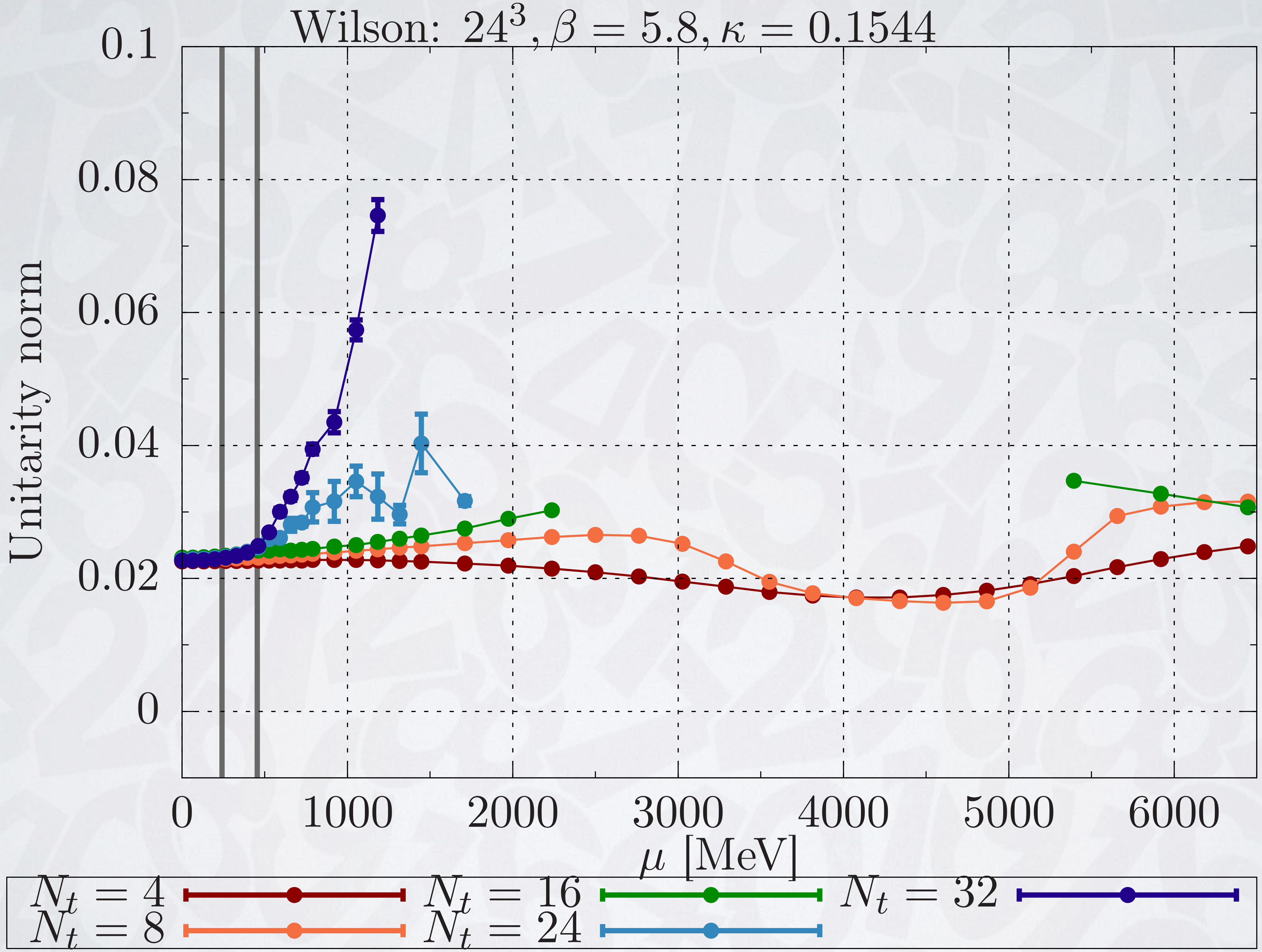


Wilson @ $m_\pi \sim 500$ MeV

CG Iterations

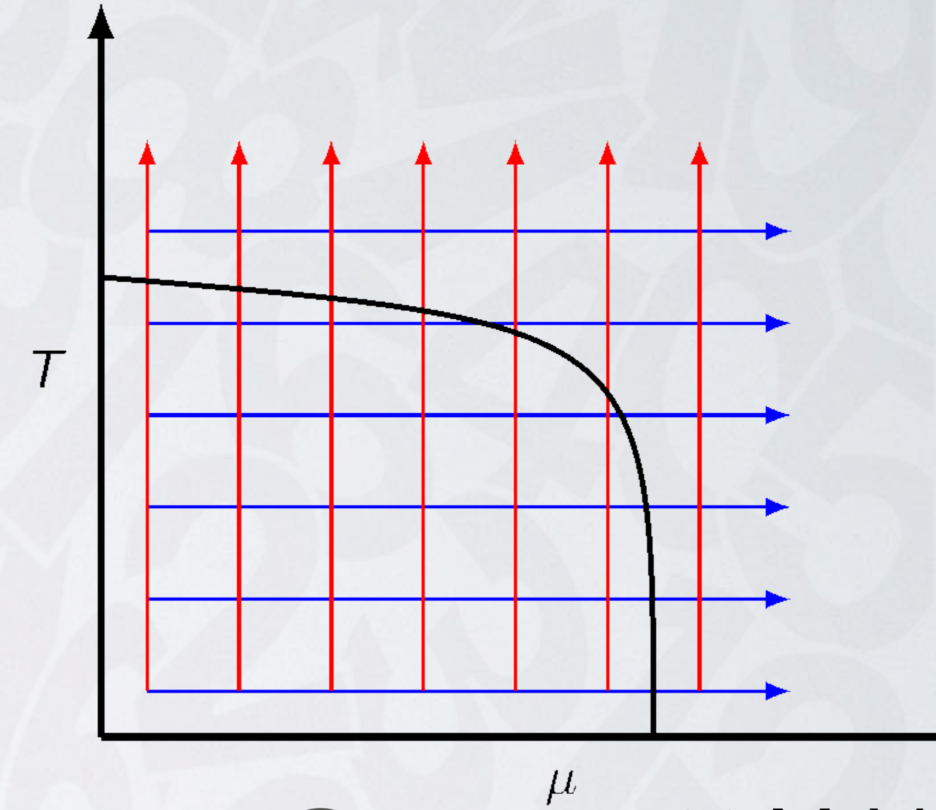


Unitarity Norm



Summary

- **Consistency checks @ $\mu = 0$**
  - Work well in (de-)confined phase
- **Physics**
  - CL feasible at medium m_π
  - Transitions visible
  - Detailed analysis necessary
- **Next?**
  - Zoom in on the transition(s)
  - Lower temperatures (better inverter)
  - Volume scaling (order of phase transitions)



Wilson @ $m_\pi \sim 500\,\text{MeV}$

Questions?

Thank you for your attention!