# NONPERTURBATIVE QUARK-FLAVOR SYMMETRY BREAKING IN TOPOLOGICAL SUSCEPTIBILITY AT HOT QCD

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Based on

- *Phys.Lett.B* 813 (2021) 136044, M.K., Shinya Matsuzaki (Jilin U.), Akio Tomiya(RIKEN BNL)
- *Phys.Rev.D* 103 (2021) 5, 054034, M.K., Shinya Matsuzaki (Jilin U.), Akio Tomiya(RIKEN BNL)
- arXiv:2106.05674[hep-ph], Chuan-Xin Cui (Jilin U.), Jin-Yang Li (Jilin U.), Shinya Matsuzaki (Jilin U.), M.K., Akio Tomiya (RIKEN BNL)

# OUTLINE

- Introduction: Topological susceptibility
- $\chi_{top}$ and U(1)$_A$ rotation with Flavor singlet condition
- Our work
  1. Nonperturbative flavor violation in topological susceptibility
  2. Flavor singlet condition and Anomalous Ward identities for chiral symmetry)
- Summary

# INTRODUCTION

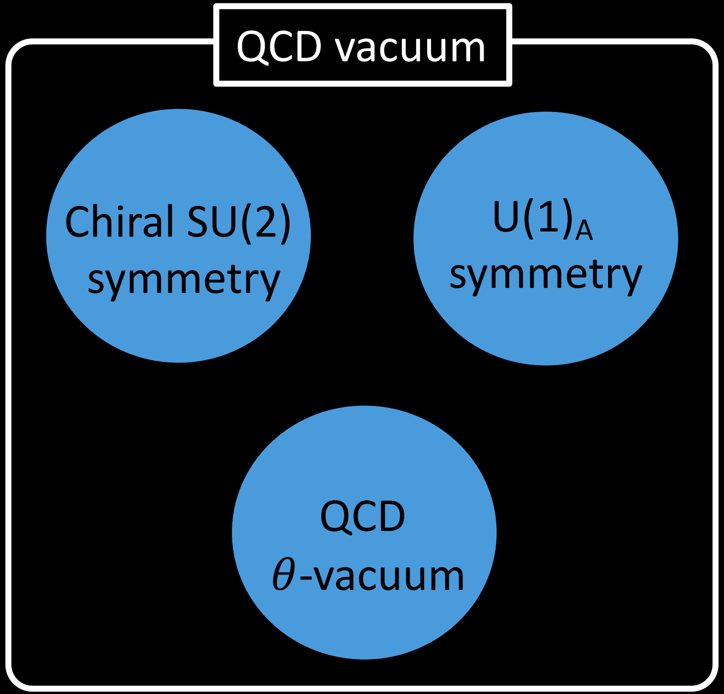
# QCD VACUUM AND $\chi_{top}$

- QCD phase structure is one of the most important subjects, which is related to chiral SU(2) symmetry, $U(1)_A$ and QCD $\theta$-vacuum.

- QCD phase transition can be probed by susceptibilities.

- Chiral SU(2) restoration $\leftrightarrow$ Chiral susceptibilities.
- Effective restoration of $U(1)_A$ $\leftrightarrow$ Axial susceptibilities.

- In this talk, I would like to focus on "Topological susceptibility".




$$\chi_{\text{top}} = \int d^4x \langle Q(x)Q(0)\rangle$$

Topological charge density

$$Q(x) = \frac{g^2}{32\pi^2} G_{\mu\nu}\tilde{G}^{\mu\nu}$$

- Is also a probe for the QCD vacuum.
- Interacts with $\eta'$ meson via $U(1)_A$ anomaly.
- Correlates to axion physics.

# FLAVOR DEPENDENCE ON $\chi_{top}$

$$\chi_{\rm top} = \int d^4x \langle Q(x)Q(0)\rangle$$

Topological charge density

$$Q = \frac{1}{32\pi^2} G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

✓ $\chi_{top}$ at finite temperature has been extensively studied in lattice QCD simulations and chiral effective models.

But…
in $N_f$=2+1 QCD, strange quark dependence on $\chi_{top}$ is unclear…

Does $\chi_{\rm top}$ have quark-flavor dependences?

# FLAVOR DEPENDENCE ON $\chi_{top}$

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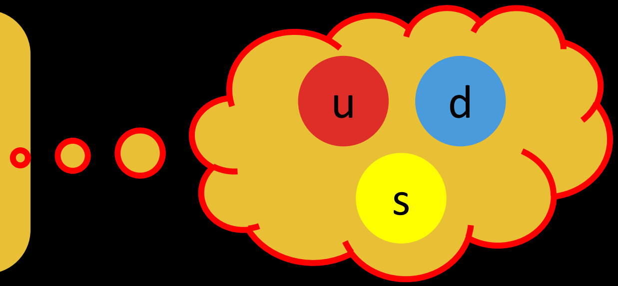


QCD $\theta$-term is flavor independent (or flavor singlet) because gluons do not feel quark flavors.

Quark flavor is invisible in $\chi_{top}$…

Does $\chi_{\text{top}}$ have quark-flavor dependences?

# FLAVOR DEPENDENCE ON $\chi_{top}$

$\chi_{top}$ can be rewritten by quark fields.

$$\chi_{\mathrm{top}} = \int d^4x \langle Q(x)Q(0)\rangle$$

Topological charge density

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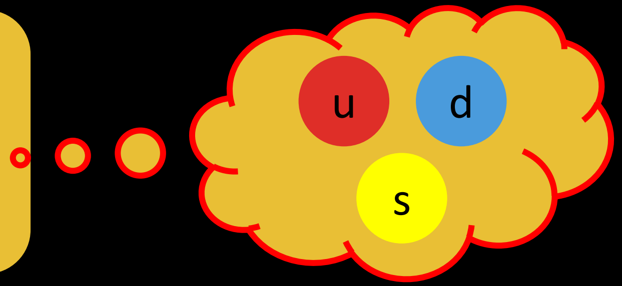


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Topological charge density

$$Q = \frac{1}{32\pi^2} G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

Quarks

u d s

$U(1)_A$ anomaly

$$\partial_\mu j^\mu_{5,a=0} = 2i\sum_f \bar{q}_f m_f \gamma_5 q_f + N_F \frac{1}{32\pi^2} G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

QCD $\theta$-term is flavor independent (or flavor singlet) because gluons do not feel quark flavors.

Under $U(1)_A$ rotation,
QCD $\theta$-term is transferred to quark masses.

$\chi_{\rm top}$ can be expressed by quarks with masses.

$$\chi_{top} = (\text{Quark con.}) + (\text{Pseudoscalar sus.})$$

Quark flavor shows up in $\chi_{\rm top}$

*precise expression will be shown later.

Does $\chi_{\rm top}$ have quark-flavor dependences?
Analysis of $\chi_{top}$ with $U(1)_A$ rotation would give an answer.

# $\chi_{top}$ IN EFFECTIVE MODEL

$\chi_{top}$ with U(1)$_A$ rotation in effective models.

Linear sigma model: $$\chi = \frac{1}{3\sqrt{6}} c\sigma_0^3$$

*PRD* 86 (2012) 105016

NJL model: $$\chi^{(\mathrm{lowest})} = -\frac{K^2}{(3!)^2}(-9)\,\epsilon^{abc}\epsilon^{ijk}\epsilon^{def}\epsilon^{lmn}\,4\left\{\int d^4x N_c \mathrm{tr}\left[S_{di}(x)\gamma_5 S_{al}(x)\gamma_5\right]\right\} \times {N_c}^4 \mathrm{tr}\left[S_{bj}(0)\right]\mathrm{tr}\left[S_{ck}(0)\right]\mathrm{tr}\left[S_{em}(0)\right]\mathrm{tr}\left[S_{fn}(0)\right],$$

*PRC* 63 (2001) 045203

(“$c$” and “$K$” are a model parameter in U(1)$_A$ anomaly term.)

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("$c$" and "$K$" are a model parameter in U(1)$_A$ anomaly term.)

???

QCD:

$$\chi_{top} = \boxed{(\text{Quark con.}) + (\text{Pseudoscalar sus.})}$$

- Are model results consistent with QCD?
- Can we extract flavor dependences from the results?

# $\chi_{top}$ IN EFFECTIVE MODEL

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("$c$" and "$K$" are a model parameter in U(1)$_A$ anomaly term.)

???

QCD:

$$\chi_{top} = \boxed{(\text{Quark con.}) + (\text{Pseudoscalar sus.})}$$

I will show the explicit expression of $\chi_{top}$.

- Are model results consistent with QCD?
- Can we extract flavor dependences from the results?

# $\chi_{top}$ AND U(1)$_A$ ROTATION WITH FLAVOR SINGLET CONDITION

# $\chi_{top}$ AND U(1)$_A$ ROTATION

$$\mathcal{L}_\theta = i\frac{\theta}{32\pi^2} G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

$$\chi_{\rm top} = \int d^4x \langle Q(x)Q(0)\rangle$$

consistent?

$U(1)_A$ rotation, $q^f_{L,R} \to \exp(\mp i\alpha_f/2)\, q^f_{L,R}$

$$\mathcal{L}_\theta = \sum_f \left(m_f \bar{q}_f q_f + \alpha_f m_f\, \bar{q}_f i\gamma_5 q_f + O(\alpha_f^2)\right)$$

$$\chi_{\rm top} = ???$$

$\alpha_f$ has $\theta$-dependence:
$\alpha_u(\theta), \alpha_d(\theta), \alpha_s(\theta)$

Under U(1)$_A$ rotation,
QCD $\theta$-term is transferred to quark masses.

# $\chi_{top}$ AND U(1)$_A$ ROTATION

Flavor singlet

$$\mathcal{L}_\theta = i\frac{\theta}{32\pi^2}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

$$\chi_{\mathrm{top}} = \int d^4x \langle Q(x)Q(0)\rangle$$

inconsistent

$U(1)_A$ rotation, $q^f_{L,R} \to \exp(\mp i\alpha_f/2)\, q^f_{L,R}$

$$\mathcal{L}_\theta = \sum_f \left(m_f\bar{q}_f q_f + \alpha_f m_f\, \bar{q}_f i\gamma_5 q_f + O(\alpha_f^2)\right)$$

$$\chi_{\mathrm{top}} = \;???$$

$\alpha_f$ has $\theta$-dependence:
$\alpha_u(\theta), \alpha_d(\theta), \alpha_s(\theta)$

$\theta$-dependent mass term has flavor dependence.

$\theta$ ??? $\begin{pmatrix}\alpha_u(\theta)\\ \alpha_d(\theta)\\ \alpha_s(\theta)\end{pmatrix}$

flavor singlet

Non singlet

Original $\theta$-term is flavor singlet.
$\theta$-dependent mass term is NOT flavor singlet…

Need to restrict $U(1)_A$ rotation.

# $\chi_{top}$ AND U(1)$_A$ ROTATION

Flavor singlet

$$\mathcal{L}_\theta = i\frac{\theta}{32\pi^2}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

$$\chi_{\rm top} = \int d^4x \langle Q(x)Q(0)\rangle$$

$U(1)_A$ rotation, $q^f_{L,R} \to \exp(\mp i\alpha_f/2)\, q^f_{L,R}$

$$\mathcal{L}_\theta = \sum_f \left(m_f\bar{q}_f q_f + \alpha_f m_f \bar{q}_f i\gamma_5 q_f + O(\alpha_f^2)\right)$$

NOT flavor singlet…

Restrict rotation angles.

Impose ``Flavor singlet condition” to satisfy flavor singlet nature.

$$\alpha_u m_u = \alpha_d m_d = \alpha_s m_s$$

*Phys.Rev.D19, 7 (1979) 2227-2230*

$\theta$-dependent mass term becomes flavor singlet.

$$\mathcal{L}_\theta = \sum_f \left(m_f\bar{q}_f q_f + \theta\bar{m}\,\bar{q}_f i\gamma_5 q_f + O(\alpha_f^2)\right)$$

$$\left(\bar{m} = \frac{m_u m_d m_s}{m_u m_s + m_d m_s + m_u m_d}\right)$$

$\chi_{top}$ with U(1)$_A$ rotation satisfies the flavor singlet nature.

Topological susceptibility with U(1)$_A$ rotation.

$$\chi_{\rm top} = \left(\frac{\langle\bar{q}_l q_l\rangle}{m_l} + \frac{\langle\bar{s}s\rangle}{m_s}\right)\bar{m}^2 + \bar{m}^2\left(\chi_P^{ll} + 2\chi_P^{ls} + \chi_P^{ss}\right)$$

$(m_u = m_d = m_l)$

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*PRD* 103 (2021) 5, 054034, M.K., Shinya Matsuzaki, Akio Tomiya

# ESSENTIAL PROPERTY IN $\chi_{top}$

What is a consequence of flavor singlet nature in $\chi_{top}$ with U(1)$_A$ rotation?

If either of quarks are massless…

$\theta$-dependence can be completely rotated away from the QCD generating functional.

$\boxed{\mathcal{L}_\theta} \to 0$ at $m_f \to 0$

If either of quarks are massless,

$\chi_{\rm top} \to 0$

``Flavor singlet nature" is one of the essential nature in QCD.

Our topological susceptibility satisfies the flavor singlet nature.

Topological susceptibility with flavor singlet nature.

$$\chi_{\rm top} = \left( \frac{\langle \bar{q}_l q_l \rangle}{m_l} + \frac{\langle \bar{s} s \rangle}{m_s} \right) \bar{m}^2 + \bar{m}^2 \left( \chi_P^{ll} + 2\chi_P^{ls} + \chi_P^{ss} \right)$$

$$\bar{m} = \frac{m_u m_d m_s}{m_u m_s + m_d m_s + m_u m_d}$$

$\bar{m} \to 0$ at $m_f \to 0$

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# CONSISTENCY IN $\chi_{top}$

$\chi_{top}$ with U(1)$_A$ rotation in effective models.

Linear sigma model: $\chi = \frac{1}{3\sqrt{6}} c\sigma_0^3$

*PRD 86 (2012) 105016*

Do NOT satisfy the flavor singlet nature…:

$\chi_{top} \neq 0$ at $m_f = 0$.

NJL model:

$$\chi^{(\mathrm{lowest})} = -\frac{K^2}{(3!)^2}(-9)\,\epsilon^{abc}\epsilon^{ijk}\epsilon^{def}\epsilon^{lmn}\,4\left\{\int d^4x N_c \mathrm{tr}\left[S_{di}(x)\gamma_5 S_{al}(x)\gamma_5\right]\right\} \times {N_c}^4 \mathrm{tr}\left[S_{bj}(0)\right]\mathrm{tr}\left[S_{ck}(0)\right]\mathrm{tr}\left[S_{em}(0)\right]\mathrm{tr}\left[S_{fn}(0)\right],$$

*PRC 63 (2001) 045203*

("$c$" and "$K$" are a model parameter in U(1)$_A$ anomaly term.)

- Topological susceptibility in QCD

$$\chi_{\mathrm{top}} = \left(\frac{\langle \bar{q}_l q_l\rangle}{m_l} + \frac{\langle \bar{s}s\rangle}{m_s}\right)\bar{m}^2 + \bar{m}^2\left(\chi_P^{ll} + 2\chi_P^{ls} + \chi_P^{ss}\right)$$

- Flavor singlet nature

$\chi_{top} = 0$ at $m_f = 0$

Can we extract flavor dependences from model results?

Model results do NOT satisfy the flavor singlet nature…

→ Quark mass contributions are not fully reflected in $\chi_{top}$…

We have studied our $\chi_{top}$ based on effective models .

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M.K., Shinya Matsuzaki, Akio Tomiya
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arXiv:2106.05674[hep-ph],
Chuan-Xin Cui, Jin-Yang Li,
Shinya Matsuzaki , M.K.,
Akio Tomiya

By imposing the flavor singlet condition on $\chi_{\text{top}}$,

Topological susceptibility with flavor singlet nature.

$$\chi_{\text{top}} = \left( \frac{\langle \bar{q}_l q_l \rangle}{m_l} + \frac{\langle \bar{s}s \rangle}{m_s} \right) \bar{m}^2 + \bar{m}^2 \left( \chi_P^{ll} + 2\chi_P^{ls} + \chi_P^{ss} \right)$$

- ✓ Quark mass contributions are fully reflected in $\chi_{top}$.
- ✓ We can extract the Quark-flavor/strange quark dependence in $\chi_{\text{top}}$.

Does $\chi_{\text{top}}$ have quark-flavor dependences?

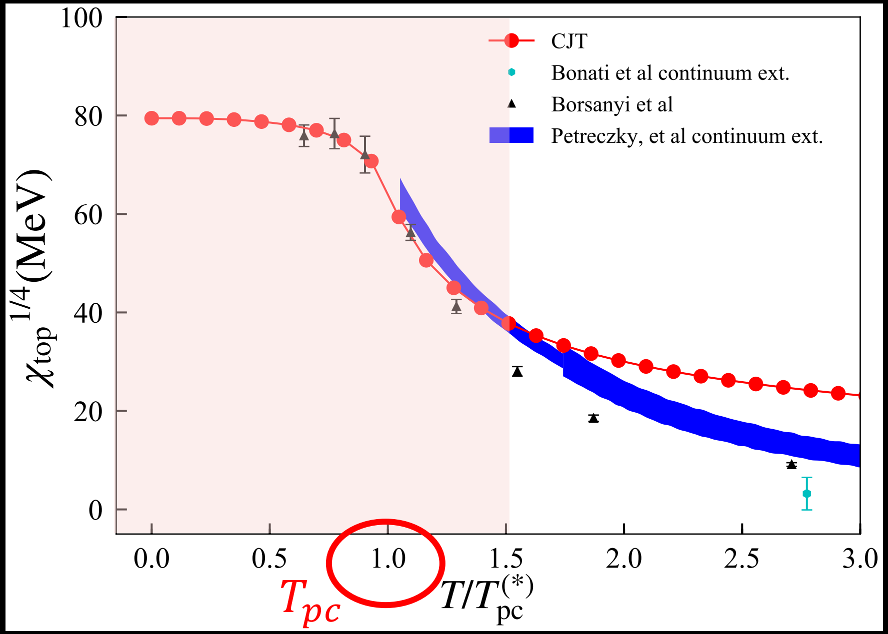
# NONPERTURBATIVE FLAVOR VIOLATION IN TOPOLOGICAL SUSCEPTIBILITY

- *Phys.Lett.B* 813 (2021) 136044, M.K., Shinya Matsuzaki (Jilin U.),   Akio Tomiya(RIKEN BNL)
- *Phys.Rev.D* 103 (2021) 5, 054034, M.K., Shinya Matsuzaki (Jilin U.),   Akio Tomiya(RIKEN BNL)

# $\chi_{top}$ BASED ON EFFECTIVE MODEL

$\chi_{top}$ based on chiral effective model (linear sigma model based on CJT formalism).

$$\chi_{\rm top}^{\rm (eff)} = \left( \frac{\langle \bar{q}_l q_l \rangle^{\rm (eff)}}{m_l} + \frac{\langle \bar{s} s \rangle^{\rm (eff)}}{m_s} \right) \bar{m}^2 + \cancel{O(m_f^2)}$$

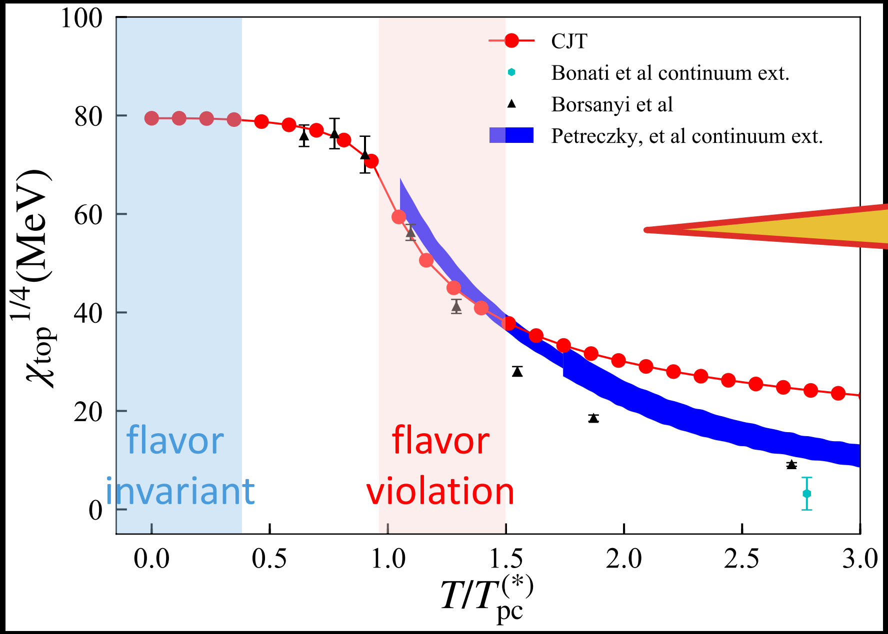



- ✓ $\chi_{\rm top}^{\rm (eff)}$ corresponds to the expression of $\chi_{top}$ in QCD (at the leading order of expansion in $m_q$).
- ✓ When $T/T_{pc} < 1.5$, effective model result is good agreement with those lattice data.
  - C. Bonati et al, JHEP 11, 170 (2018), 1807.07954.
  - *S. Borsanyi et al., Nature 539, no. 7627, 69 (2016).*
  - P. Petreczky et al, Phys. Lett. B 762, 498-505 (2016)

# FLAVOR VIOLATION IN $\chi_{top}$

$\chi_{top}$ based on chiral effective model
(linear sigma model based on CJT formalism).

$$\chi_{\rm top}^{(\rm eff)} = \boxed{\left( \frac{\langle \bar{q}_l q_l \rangle^{(\rm eff)}}{m_l} + \frac{\langle \bar{s}s \rangle^{(\rm eff)}}{m_s} \right)} \bar{m}^2 + \cancel{O(m_f^2)}$$




$\chi_{top}$ is affected by flavor violation

- At vacuum ($T$=0),
  quark condensates are well degenerated:

  $$\langle \bar{l}l \rangle \simeq \langle \bar{s}s \rangle \qquad \left( \langle \bar{q}_l q_l \rangle = \langle \bar{u}u \rangle + \langle \bar{d}d \rangle = 2\langle \bar{l}l \rangle \right)$$

- At around $T \sim T_{pc}$,
  flavor breaking occurs in quark condensates:

  $$\langle \bar{l}l \rangle \ll \langle \bar{s}s \rangle$$

# FLAVOR VIOLATION IN $\chi_{top}$

$\chi_{top}$ based on chiral effective model
(linear sigma model based on CJT formalism).

$$\chi_{\rm top}^{(\rm eff)} = \left( \frac{\langle \bar{q}_l q_l \rangle^{(\rm eff)}}{m_l} + \boxed{\frac{\langle \bar{s} s \rangle^{(\rm eff)}}{m_s}} \right) \bar{m}^2 + \cancel{O(m_f^2)}$$

Make a comparison

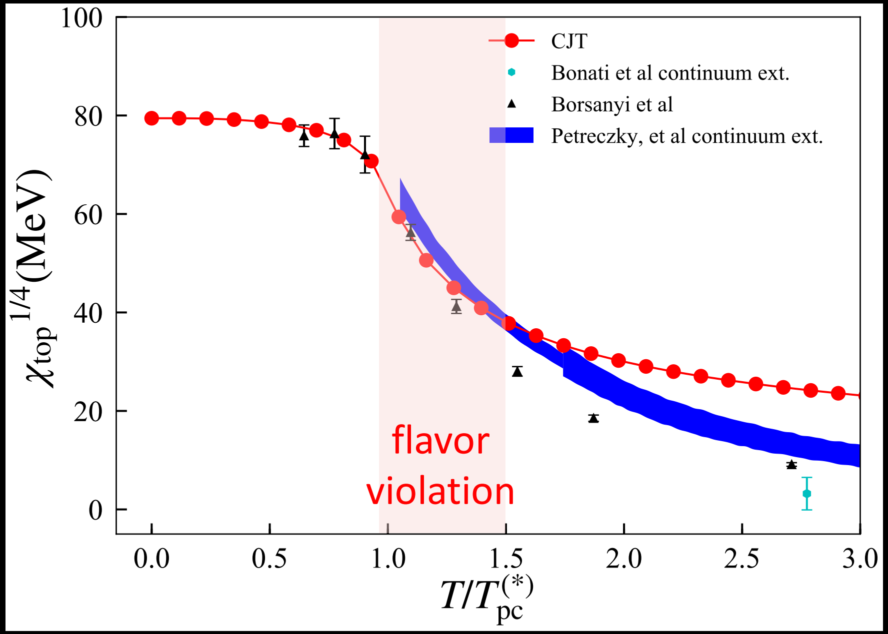
to get s-quark dep.

$\chi_{top}$ in the three-flavor universal limit:

$$\chi_{\rm top}^{\rm 3fl} = \left( \frac{2\langle \bar{l} l \rangle}{m_l} + \frac{\boxed{\langle \bar{l} l \rangle}}{m_s} \right) \bar{m}^2 + O(m_f^2)$$

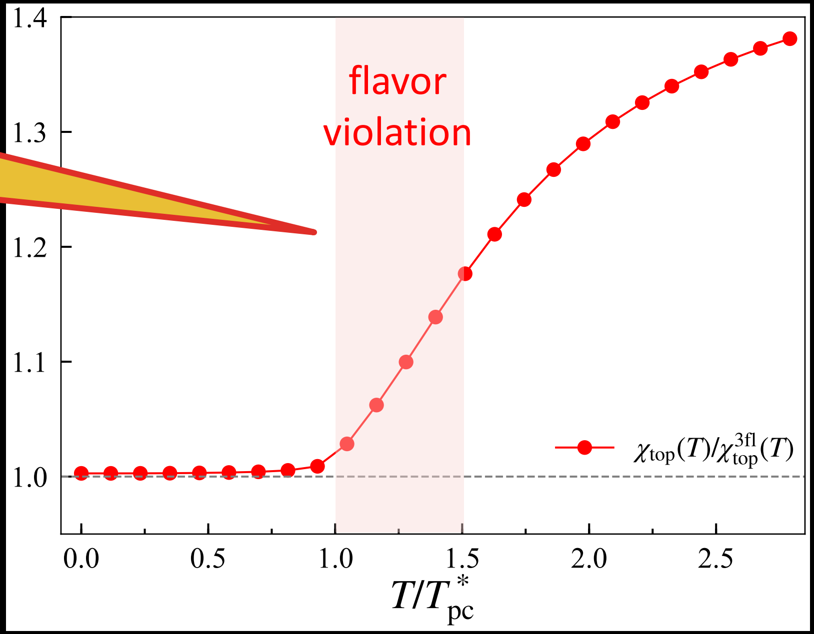
$$\left[ \langle \bar{q}_l q_l \rangle = \langle \bar{u} u \rangle + \langle \bar{d} d \rangle = 2\langle \bar{l} l \rangle \right]$$

$\chi_{\rm top}(T)/\chi_{\rm top}^{\rm 3fl}(T)$ : ``strange quark contribution"




$\langle \bar{s} s \rangle$ is eminent at around $T \sim T_{pc}$.

Flavor violation in $\chi_{\rm top}$



## Flavor singlet condition
## and
## Anomalous Ward identities for chiral symmetry

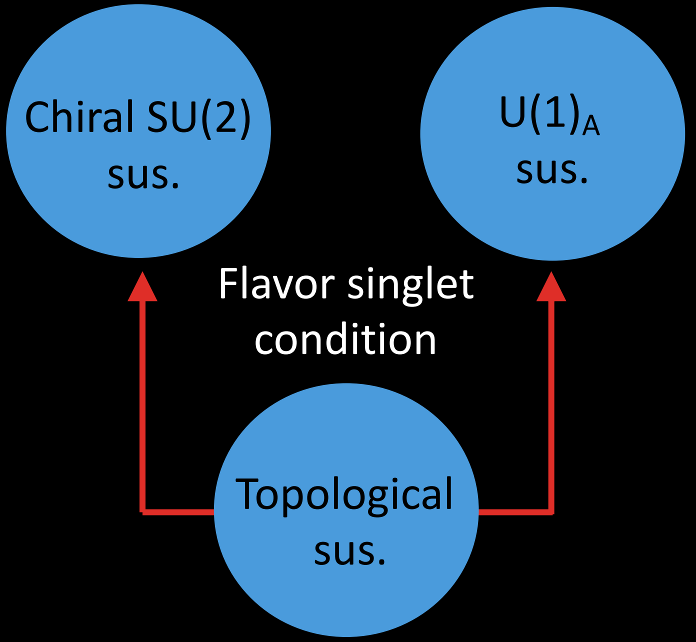
- arXiv:2106.05674[hep-ph], Chuan-Xin Cui (Jilin U.), Jin-Yang Li (Jilin U.), Shinya Matsuzaki (Jilin U.), M.K., Akio Tomiya (RIKEN BNL)

# $\chi_{top}$ AND WARD IDENTITY

- Susceptibilities give the information on QCD phase transition.

- Chiral SU(2) restoration $\leftrightarrow$ Chiral susceptibilities.
- Effective restoration of $U(1)_A$ $\leftrightarrow$ Axial susceptibilities.
- QCD $\theta$-vacuum vacuum $\leftrightarrow$ Topological susceptibility

- Susceptibilities are related to each other in anomalous Ward identities (AWI).




(Chiral SU(2) sus.) = ($U(1)_A$ sus.) + (Topological sus.)

More precisely… $\chi_{\eta-\delta} = \chi_{\pi-\delta} + \frac{4}{m_l^2}\chi_{\mathrm{top}}$

- Flavor singlet condition should be taken into account for AWI.
- $\chi_{top}$ with flavor singlet nature is correlated to QCD phase structure.

# SUSCEPTIBILITIES

(Chiral SU(2) sus.) = (U(1)$_A$ sus.) + (Topological sus.)

$$\chi_{\eta-\delta} = \chi_{\pi-\delta} + \frac{4}{m_l^2}\chi_{top}$$

+

Flavor singlet condition

$$\alpha_u m_u = \alpha_d m_d = \alpha_s m_s$$

- ✓ Flavor singlet is involved in AWI.
- ✓ Strange quark dependence is fully taken into account for AWI.

| Magnitude of susceptibilities | Topological susceptibility | | U(1)$_A$ susceptibility | | Chiral SU(2) susceptibility |
|---|---|---|---|---|---|
| 3-flavor symmetric limit in NJL model ($m_l = m_s$) | $\frac{4}{m_l^2}\chi_{top}$ | $\approx$ | $\chi_{\pi-\delta}$ | $\approx$ | $\chi_{\eta-\delta}$ |
| 2+1 favor in lattice QCD data and NJL ($m_l = 5$ MeV, $m_s = 140$ MeV) | $\frac{4}{m_l^2}\chi_{top}$ | $\approx$ | $\chi_{\pi-\delta}$ | $\gg$ | $\chi_{\eta-\delta}$ |

We have studied

- AWI in $N_f$=3 and $N_f$=2+1 QCD
- magnitude of susceptibilities.
  (would characterize the vacuum structure of QCD)

arXiv:2106.05674[hep-ph],
Chuan-Xin Cui, Jin-Yang Li,
Shinya Matsuzaki , M.K.,
Akio Tomiya

There is an unbalance in magnitude among susceptibilities due to the flavor symmetry violation.

This unbalance may be a new aspect for QCD vacuum structure.

# SUMMARY

# SUMMARY 1

0

Flavor dependence

$$\chi_{\rm top} = \int d^4x \langle Q(x)Q(0) \rangle$$

Previous studies have a lack of the underlying property of QCD...

1

Considered $\chi_{\rm top}$ under $U(1)_A$ rotation.

BUT

Flavor singlet is spoiled by $U(1)_A$ rotation...

???

$\theta$ flavor singlet

$$\begin{pmatrix} \alpha_u(\theta) \\ \alpha_d(\theta) \\ \alpha_s(\theta) \end{pmatrix}$$

Non singlet

2

$\theta$-dependent mass term with $U(1)_A$ rotation

$$\mathcal{L}_\theta(m_f)$$

+

Flavor singlet condition

$$\alpha_u m_u = \alpha_d m_d = \alpha_s m_s$$

$$\mathcal{L}_\theta = \sum_f \alpha_f m_f \bar{q}_f i\gamma_5 q_f + \cdots$$

$$\mathcal{L}_\theta = \sum_f \theta \bar{m} \bar{q}_f i\gamma_5 q_f + \cdots$$

Mass term keeps flavor singlet.

3

Topological susceptibility with flavor singlet nature.

$$\chi_{\rm top} = \left( \frac{\langle \bar{q}_l q_l \rangle}{m_l} + \frac{\langle \bar{s}s \rangle}{m_s} \right) \bar{m}^2 + \bar{m}^2 \left( \chi_P^{ll} + 2\chi_P^{ls} + \chi_P^{ss} \right)$$

Flavor singlet nature: if either of quarks are massless, $\chi_{top}$ vanishes.

$\bar{m} \to 0$ at $m_f \to 0$.

$$\bar{m} = \frac{m_u m_d m_s}{m_u m_s + m_d m_s + m_u m_d}$$

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# SUMMARY 2

Does $\chi_{\rm top}$ have quark-flavor dependences?

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- ✓ To answer this question, we evaluated $\chi_{top}$ with the flavor singlet nature.
- ✓ Quark mass contributions are fully reflected in $\chi_{top}$.

Finite T effect on $\chi_{top}$
based on chiral effective model

$$\chi_{\rm top}^{(\rm eff)} = \boxed{\left( \frac{\langle \bar{q}_l q_l \rangle^{(\rm eff)}}{m_l} + \frac{\langle \bar{s}s \rangle^{(\rm eff)}}{m_s} \right)} \bar{m}^2 + \cancel{O(m_f^2)}$$

- Model result with flavor singlet condition satisfies the expression of $\chi_{top}$ in QCD.
- Strange quark becomes eminent at around chiral crossover:
  Flavor violation in $\chi_{top}$ is induced by $\langle \bar{l} l \rangle \ll \langle \bar{s}s \rangle$ at around $T \sim T_{pc}$,
  (This result would be model independent.)

- Flavor singlet condition should be taken into account for anomalous Ward identities.
- QCD vacuum structure should be addressed by using $\chi_{top}$ with flavor singlet nature .

arXiv:2106.05674[hep-ph],
Chuan-Xin Cui, Jin-Yang Li,
Shinya Matsuzaki , M.K.,
Akio Tomiya

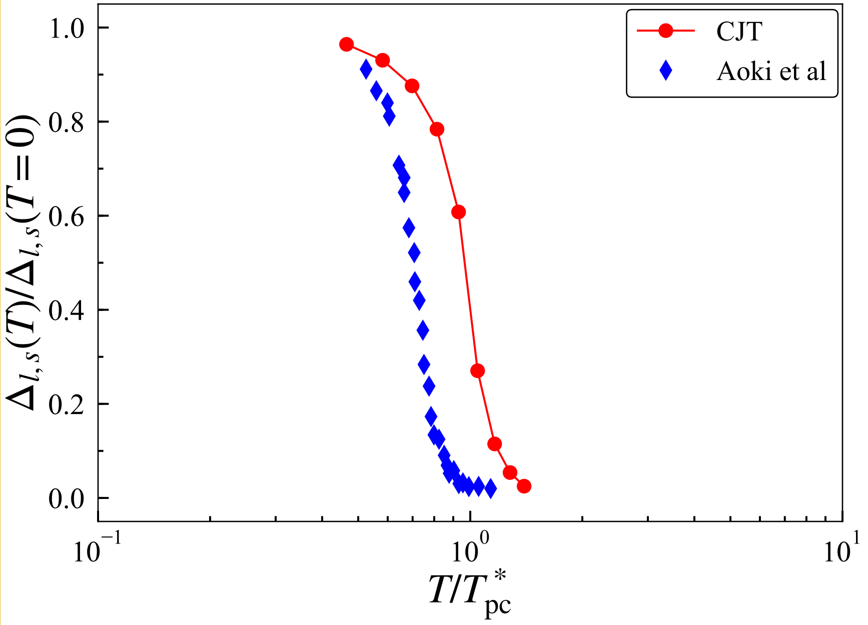
# THANK YOU

# CHIRAL CONDENSATE IN LSM

- Subtracted chiral condensate in comparison with the lattice QCD data

*Y. Aoki, S. Borsanyi, S. Durr, Z. Fodor, S. D. Katz, S. Krieg et al., JHEP 06 (2009) 088.*

$$\Delta_{l,s}(T) = \langle \bar{l}l \rangle - \frac{2m_l}{m_s}\langle \bar{s}s \rangle$$




- Pseudo-critical temperature

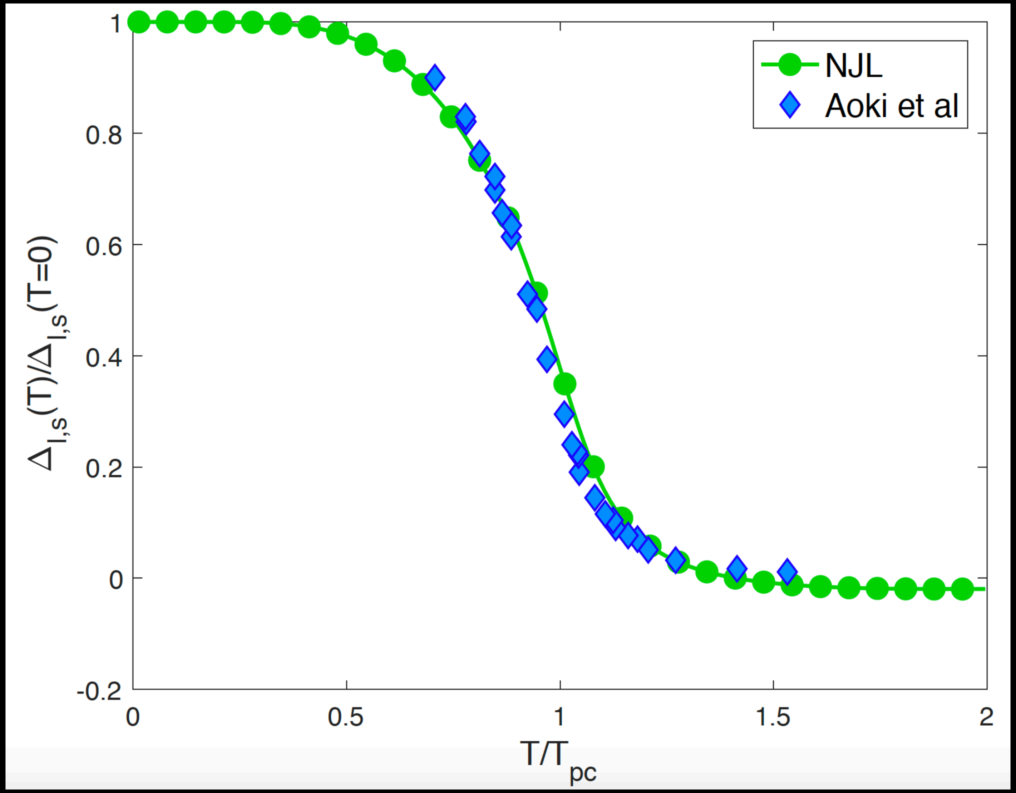
$T_{pc}$ reads from the inflection point of the chiral condensate

CJT result: $T^*_{pc} \simeq 215\,\mathrm{MeV}$

Lattice QCD result: $T_{pc} \simeq 155\,\mathrm{MeV}$

- Although the deviation from the lattice QCD data is read as about 30% around $T \simeq T^*_{pc}$, the CJT analysis qualitatively supplies the chiral crossover.

# CHIRAL CONDENSATE IN NJL



# $\chi_{top}$ IN MODELS

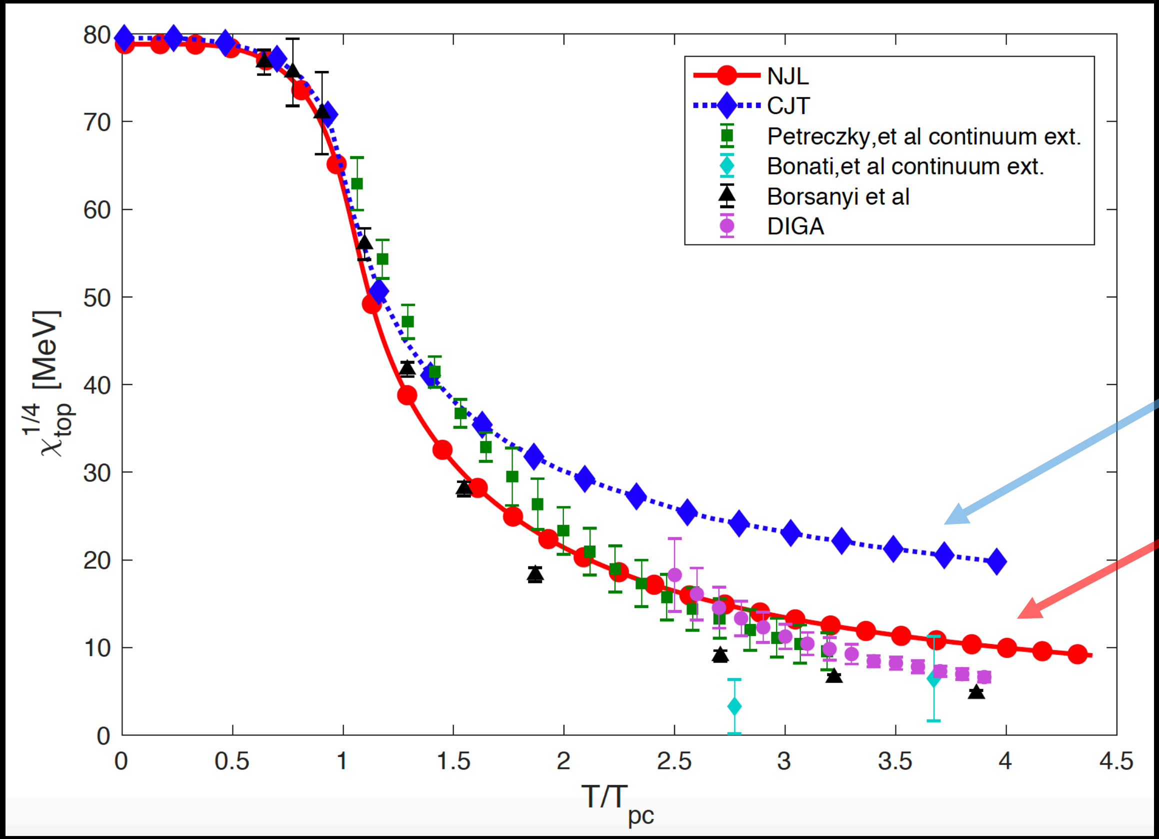
Effective model + Flavor singlet condition

$$\chi_{\rm top}^{\rm (LSM)} = \left( \frac{\langle \bar{q}_l q_l \rangle}{m_l} + \frac{\langle \bar{s}s \rangle}{m_s} \right) \bar{m}^2$$

*Phys.Lett.B* 813 (2021) 136044, M.K., Shinya Matsuzaki, Akio Tomiya
*Phys.Rev.D* 103 (2021) 5, 054034, M.K., Shinya Matsuzaki, Akio Tomiya

$$\chi_{\rm top}^{\rm (NJL)} = \left( \frac{\langle \bar{q}_l q_l \rangle}{m_l} + \frac{\langle \bar{s}s \rangle}{m_s} \right) \bar{m}^2 + \bar{m}^2 \left( \chi_P^{ll} + 2\chi_P^{ls} + \chi_P^{ss} \right)$$

in preparation…




Linear sigma model

NJL

- C. Bonati et al, JHEP 11, 170 (2018), 1807.07954.
- *S. Borsanyi et al., Nature 539, no. 7627, 69 (2016).*
- P. Petreczky et al, Phys. Lett. B 762, 498-505 (2016)

# $\chi_{top}$ WITHOUT FLAVOR SINGLET CONDITION

QCD + Flavor singlet condition

$$\chi_{\rm top}^{\rm (QCD)} = \left(\frac{\langle \bar{q}_l q_l \rangle}{m_l} + \frac{\langle \bar{s}s \rangle}{m_s}\right)\bar{m}^2 + \bar{m}^2\left(\chi_P^{ll} + 2\chi_P^{ls} + \chi_P^{ss}\right)$$

*Phys.Rev.D* 103 (2021) 5, 054034, M.K., Shinya Matsuzaki, Akio Tomiya

Light-quark and strange-quark simultaneously contribute to $\chi_{top}$.

✓ Consequence of flavor singlet nature.

QCD + ~~Flavor singlet condition~~

$$\chi_{\rm top} = \boxed{\frac{1}{4}\left(m_l \langle \bar{q}_l q_l \rangle + m_l^2 \chi_P^{ll}\right)} \to 0$$

$$= \boxed{\left(m_s \langle \bar{s}s \rangle + m_s^2 \chi_P^{ss}\right)} \to 0?$$

*JHEP* 03 (2016) 186, *Phys.Rev.D* 97 (2018) 7, 074016

l- and s-quark dependence are separated in $\chi_{top}$…

✓ Does not satisfy the flavor singlet nature…
($\chi_{top}$ does not vanish at $m_l = 0, m_s \neq 0$.)

# MODEL INDEPENDENCE OF $\chi_{top}$

QCD + Flavor singlet condition

$$\chi_{\rm top}^{\rm (QCD)} = \left(\frac{\langle \bar{q}_l q_l \rangle}{m_l} + \frac{\langle \bar{s}s \rangle}{m_s}\right)\bar{m}^2 + \bar{m}^2\left(\chi_P^{ll} + 2\chi_P^{ls} + \chi_P^{ss}\right)$$

*Phys.Rev.D* 103 (2021) 5, 054034, M.K., Shinya Matsuzaki, Akio Tomiya

Consistent
(model independent)

Effective model + Flavor singlet condition

$$\chi_{\rm top}^{\rm (LSM)} = \left(\frac{\langle \bar{q}_l q_l \rangle}{m_l} + \frac{\langle \bar{s}s \rangle}{m_s}\right)\bar{m}^2$$

*Phys.Lett.B* 813 (2021) 136044, M.K., Shinya Matsuzaki, Akio Tomiya
*Phys.Rev.D* 103 (2021) 5, 054034, M.K., Shinya Matsuzaki, Akio Tomiya

$$\chi_{\rm top}^{\rm (NJL)} = \left(\frac{\langle \bar{q}_l q_l \rangle}{m_l} + \frac{\langle \bar{s}s \rangle}{m_s}\right)\bar{m}^2 + \bar{m}^2\left(\chi_P^{ll} + 2\chi_P^{ls} + \chi_P^{ss}\right)$$

QCD + ~~Flavor singlet condition~~

$$\chi_{\rm top} = \frac{1}{4}\left(m_l\langle \bar{q}_l q_l \rangle + m_l^2\chi_P^{ll}\right) = \left(m_s\langle \bar{s}s \rangle + m_s^2\chi_P^{ss}\right)$$

*JHEP* 03 (2016) 186, *Phys.Rev.D* 97 (2018) 7, 074016

Inconsistent
(model dependent)

Effective model + ~~Flavor singlet condition~~

Expressions of $\chi_{top}$ are model-dependent …

*Phys.Rev.D* 86 (2012) 105016

*Phys.Rev.C* 63 (2001) 045203