Exploring QCD towards the chiral limit

Anirban Lahiri

UNIVERSITÄT BIELEFELD
Faculty of Physics

CRC-TR 211
Strong-interaction matter under extreme conditions

Overview

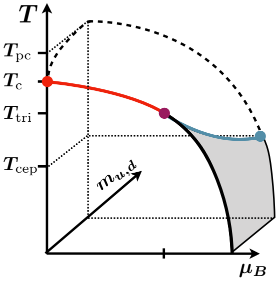
CRC-TR 211
Strong-interaction matter under extreme conditions

UNIVERSITÄT BIELEFELD
Faculty of Physics

Overview

Qualitative discussion

- For massless quarks chiral symmetry is exact and symmetry breaking can only happen through a phase transition.



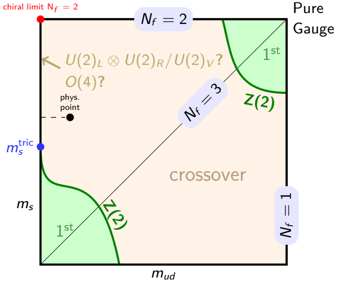
Karsch, arXiv:1905.03936.

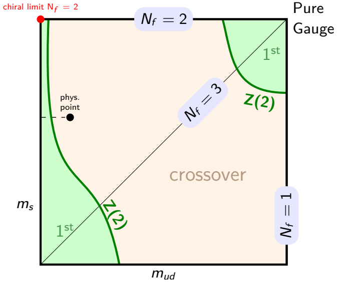
- $T = 0$ transition is of first order.
- Phase transition at $\mu_B = 0$ is expected to be of second order belonging to $SU(2) \times SU(2) \simeq O(4)$ universality class.
  [Pisarski and Wilczek. Phys. Rev. D29, 338, 1983.]
- Tricritical point in the $T - \mu_B$ plane: three phase (two broken with opposite sign of magnetization and on restored) coexistence ends and second order line also terminates from the other side. [Phys. Rev. D58, 096007 (1998).]
- CEP shifts to larger μ_B and smaller T with increasing mass.
  [Hatta and Ikeda. Phys. Rev. D67, 014028, 2003.]

- In case effective restoration of anomalous $U_A(1)$, the chiral transition can be of first order. [Pisarski and Wilczek. Phys. Rev. D29, 338, 1983.]

Questions to be answered

- Key question: What is the chiral transition temperature, T_c^0?
- Possibly another question: What is the nature of the chiral phase transition?
- Two possible scenarios: [O. Philipsen and C. Pinke. Phys. Rev. D93, 114507, 2016.]

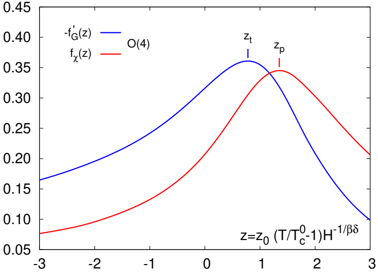


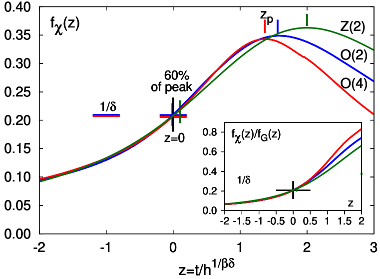


- $N_f = 3$: No direct evidence of 1^{st} order transition down to $m_\pi = 80$ MeV. Scaling argument pushes it further to $m_\pi = 50$ MeV. A. Bazavov *et. al.* Phys. Rev. D95, 074505 (2017).
- Few other possible scenarios proposed. Gupta, J. Phys. G: Nucl. Part. Phys. 35 (2008) 104018.

Overview

Scaling functions: Some intriguing facts





Mass scaling of the pseudo-critical estimators for any fixed z_X (in absence of sub-leading contributions):

$$T_X(H) = T_c^0 \left(1 + \frac{z_X}{z_0} H^{1/\beta\delta}\right)$$

Our approach: Use z_X at or close to 0. We choose to work with $X = \delta$ and 60:

$$\frac{H\chi_M(T_\delta, H)}{M(T_\delta, H)} = \frac{1}{\delta}$$

$$\chi_M(T_{60}, H) = 0.6\chi_M^{max}$$

Dependence on quark mass ($H = m_l/m_s$) reduced by two orders of magnitude

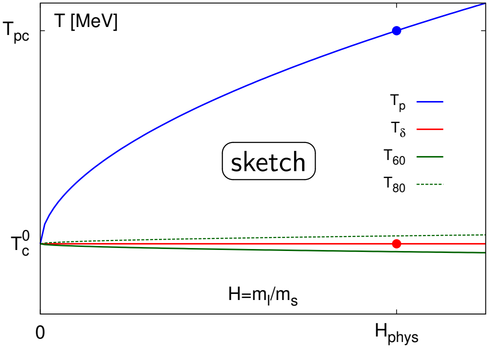
Improved estimators: basic philosophy

Mass scaling of the pseudo-critical estimators for any fixed z_X (in absence of sub-leading contributions):

$$T_X(H) = T_c^0 \left(1 + \frac{z_X}{z_0} H^{1/\beta\delta}\right)$$

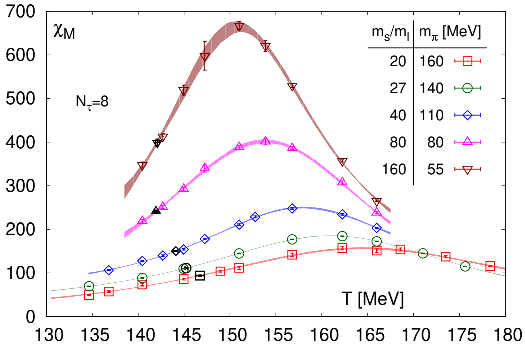
- Our approach: Use z_X at or close to 0.
- Because of the reduced variation w.r.t. H, up to the regular contributions, the pseudo-critical temperatures defined by the improved estimators at any finite value of H, *e.g.* H_{phys}, already gives a close estimate of T_c^0.

- We choose to work with $X = \delta$ and 60.



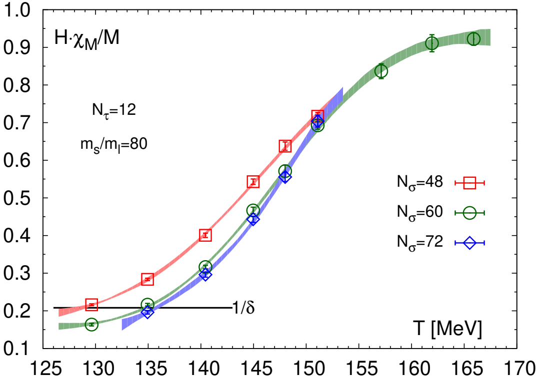
Chiral susceptibility

- No direct evidence of a 1^{st}-order phase transition down to $m_\pi = 80$ MeV.
- The increase of χ_M^{max} is apparently consistent with $H^{1/\delta-1}$ with $\delta \approx 4.8$.
- Precise determination of δ is not possible with the present data.
- Preliminary analyses with H_c being a free parameter gives a quite uncertain estimate of H_c with 0 within the range.



- Saturating trend of T_{60} towards chiral limit even at $N_\tau = 8$ already puts this as an improved estimator.
- There is no strong evidence for H_c being non-zero.

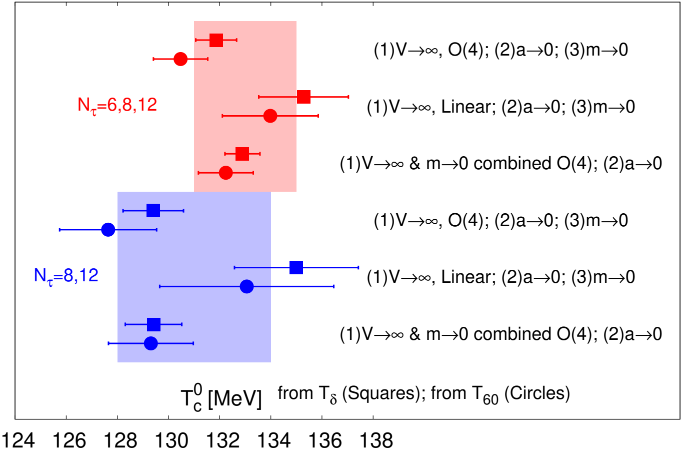
Ratio



- The intersection point of the ratio with the line at $1/\delta$ defines $T_\delta(H, L)$.
- $T_\delta(H, L)$ increases towards thermodynamic limit.
- Results for fixed H have been extrapolated to thermodynamic limit using $O(4)$ as well as $1/V$ ansatz.
- Then continuum and chiral extrapolation has been performed.

- We also tried, for a fixed N_τ, a joint chiral and thermodynamic limit extrapolation using $O(4)$ finite size scaling function and then took the continuum limits and this "improper limit" produces compatible results.

T_c^0: A single number

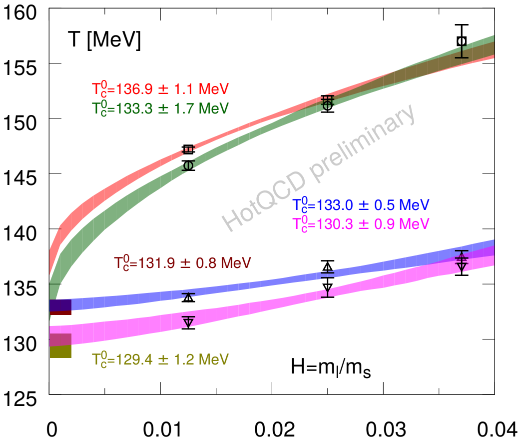


Final number we have quoted: $T_c^0 = 132^{+3}_{-6}$ MeV.

HotQCD; Phys. Rev. Lett. 123, 062002 (2019).

Preliminary comparison with conventional estimator

- Disclaimer: All T_{pc} numbers and T_{δ} for $H = 1/27$ are not infinite volume extrapolated.
- A little tension can be seen for T_{pc} calculation for $H = 1/40$.
- Still compares well.
- In thermodynamic limit, as we have seen earlier, T_{pc} will presumably increase which may pull down T_c^0, more closer to the current estimate.
- Stability of new estimators are vivid.



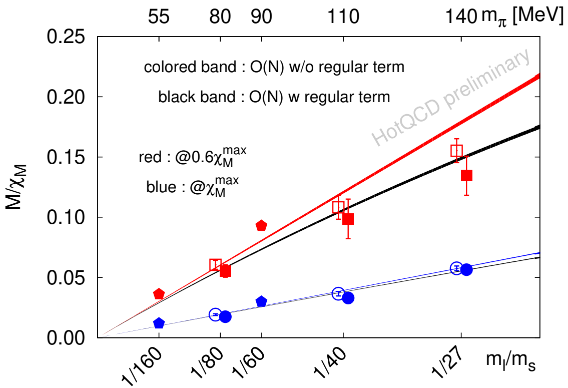
Kaczmarek *et. al.*, arXiv:2010.15593.

- A recent determination with twisted-mass fermion in the fixed scale approach; $T_c^0 = 134^{+6}_{-4}$ MeV. Kotov *et. al.*, arXiv:2105.09842 [hep-lat].

Order of the chiral transition: work in progress....

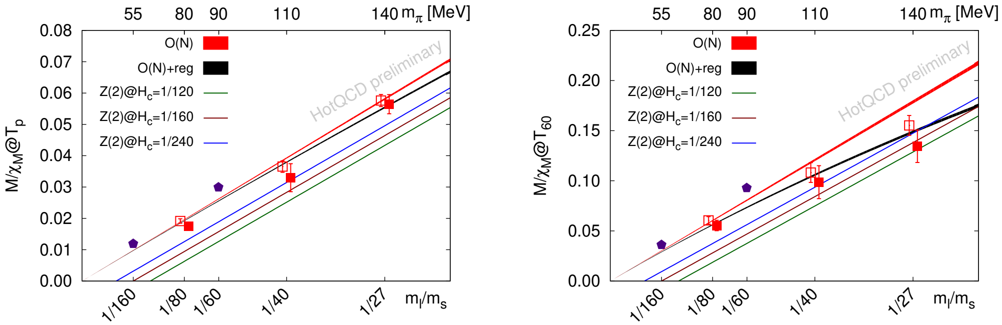
$$\frac{M}{\chi_M} = H \frac{f_G(z)}{f_\chi(z)}$$

- For small H the data seems to be linear.
- Lines are NOT fitted curves rather expectations for $O(2)$ and $O(4)$.
- Regular term $\propto H^{2-1/\delta}$.
- Coefficient of the regular term is NOT fitted, rather taken from MEoS fits.



- $Z(2)$ transition, at some finite H_c, will results into a sudden drop in the ratio $\Rightarrow$ 1^{st} order transition is unlikely for $m_\pi > 55$ MeV.
- Additional low H measurements: slope can be directly determined as a fit parameter.

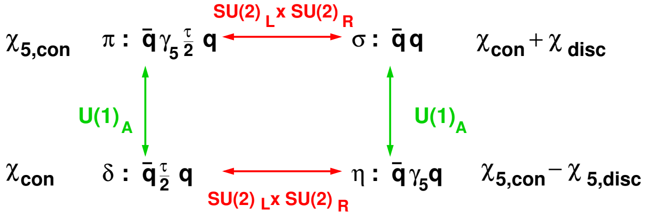
Order of the chiral transition: work in progress....



- $Z(2)$ lines are schematic: $\frac{M}{\chi_M} = (H - H_c)\,\frac{f_G(z)}{f_\chi(z)}$
- If M is not exactly order parameter then the $Z(2)$ lines will have a curvature.
- Mixing becomes weak as H_c becomes small.
- Our calculation seems to favor $O(N)$ compared to $Z(2)$. Kaczmarek *et. al.*, arXiv:2010.15593.
- A recent calculation about the order of chiral PT for various flavors.

Cuteri *et. al.*, arXiv:2107.12739 [hep-lat].

Effective restoration of $U_A(1)$

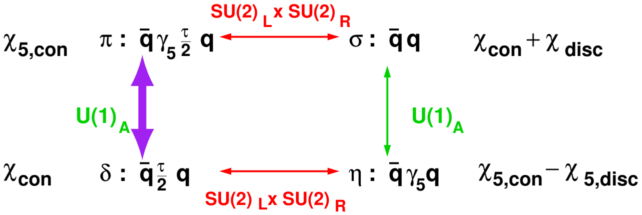


[HotQCD, Phys. Rev. D86, 094503 (2012).]

$$\chi_H = \int d^4x \left[\langle O_H(x) O_H(0)\rangle - \langle O_H(x)\rangle\langle O_H(0)\rangle\right]$$

"Order parameter" for effective restoration of $U_A(1)$.

Effective restoration of $U_A(1)$



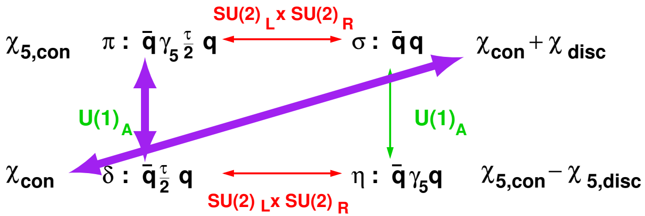
[HotQCD, Phys. Rev. D86, 094503 (2012).]

$$\chi_H = \int d^4x \, [\langle O_H(x) O_H(0)\rangle - \langle O_H(x)\rangle\langle O_H(0)\rangle]$$

"Order parameter" for effective restoration of $U_A(1)$.

- $m_\pi \stackrel{?}{=} m_{a_0}$ or more generally $\chi_\pi \stackrel{?}{=} \chi_{a_0}$.

Effective restoration of $U_A(1)$



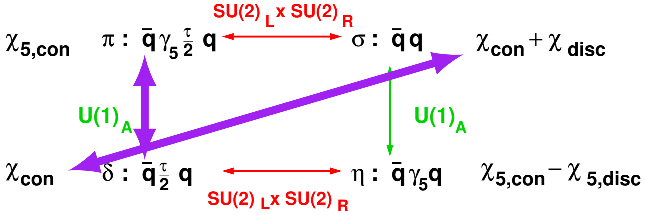
[HotQCD, Phys. Rev. D86, 094503 (2012).]

$$\chi_H = \int d^4x \left[\langle O_H(x) O_H(0)\rangle - \langle O_H(x)\rangle\langle O_H(0)\rangle\right]$$

"Order parameter" for effective restoration of $U_A(1)$.

- $m_\pi \overset{?}{=} m_{a_0}$ or more generally $\chi_\pi \overset{?}{=} \chi_{a_0}$.
- In the chirally symmetric phase, $\chi^{\text{disc}}_{\bar{\psi}\psi} \overset{?}{=} 0$.

Effective restoration of $U_A(1)$



[HotQCD, Phys. Rev. D86, 094503 (2012).]

$$\chi_H = \int d^4x \, [\langle O_H(x) O_H(0)\rangle - \langle O_H(x)\rangle\langle O_H(0)\rangle]$$

"Order parameter" for effective restoration of $U_A(1)$.

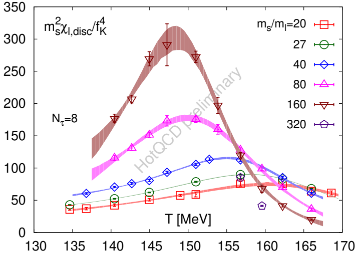
- $m_\pi \stackrel{?}{=} m_{a_0}$ or more generally $\chi_\pi \stackrel{?}{=} \chi_{a_0}$.
- In the chirally symmetric phase, $\chi^{\text{disc}}_{\bar{\psi}\psi} \stackrel{?}{=} 0$.

- Even using same observable, contradictory findings between different fermion formulation, for high T. Ding *et. al.*, Phys. Rev Lett. 126, 082001, 2021. JLQCD, arXiv:2103.05954 [hep-lat].
- Effective restoration of $U_A(1)$ needs more attention near the chiral transition temperature!
- For temperatures higher but close to the transition temperature, the chiral extrapolation is more subtle and yet inconclusive.

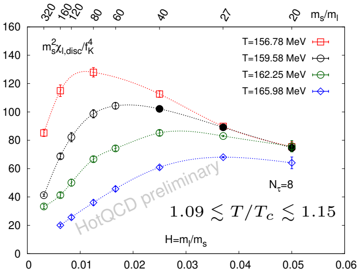
Kaczmarek *et. al.*, arXiv:2003.07920 [hep-lat]. Dentinger *et. al.*, arXiv:2102.09916 [hep-lat].

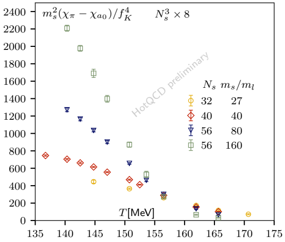
See my Lattice 2021 talk for a more complete overview of the status.

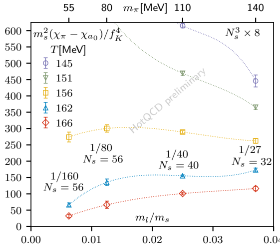
Effective restoration of $U_A(1)$



- $T \leqslant T_c$: divergence in the chiral limit, monotonically.
- $T \gtrsim T_c$: highly non-monotonic approach to chiral limit.
- Towards the chiral limit the aspect ratio has to be increased to keep the finite volume effect under control.
- Continuum extrapolation for $T \gtrsim T_c$ is tricky.
- Similar finding through studies of $\chi_\pi - \chi_{a_0}$.







Dentinger *et. al.*, arXiv:2102.09916 [hep-lat].

Overview

Gluonic observables towards chiral limit

CRC-TR 211 Strong-interaction matter under extreme conditions

- Polyakov loop closes in time direction due to periodic boundary condition.
- Polyakov loop on lattice:

$$L(\mathbf{n}) = \frac{1}{N_c} \mathrm{Tr} \prod_{j=0}^{N_\tau - 1} U_4(\mathbf{n}, j); \quad P = \frac{1}{N_\sigma^3} \sum_{\mathbf{n}} L(\mathbf{n})$$

- Free energy of a static test charge, F_Q, is related to the P by $\langle P \rangle = e^{-\beta F_Q}$.
- In a confined phase (free) energy of a test quark will be infinite resulting a vanishing Polyakov loop, where as both of them are finite in a deconfined phase.
- Polyakov loop can act as an order parameter for the confinement-deconfinement transition in the limit of infinite quark mass. McLerran and Svetitsky, Phys. Rev. D24, 450 (1981).
- It is important to understand its relation to any possible symmetry breaking.

UNIVERSITÄT BIELEFELD Faculty of Physics

Gluonic observables towards chiral limit

- Wilson's RG approach: thermodynamics in the vicinity of a critical point can be described by an effective Hamiltonian.
- Two types of operators: ones which respect the symmetry and others don't; termed as energy-like and magnetization-like.
- Being gluonic, Polyakov loop (PL) and heavy quark free energy (HQFE) are both expected to be energy-like operators w.r.t. chiral phase transition.

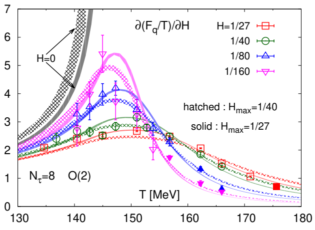
$$F_q(T,H)/T = AH^{(1-\alpha)/\beta\delta} f_f'(z) + f_{\text{reg}}(T,H)$$

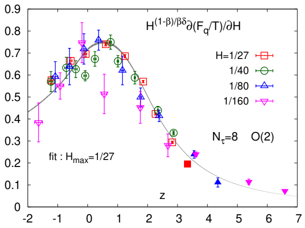
- HQFE doesn't diverge at chiral critical point, so importance of the regular terms could be higher. Let's calculate the mixed susceptibility

$$\frac{\partial F_q(T,H)/T}{\partial H} = -AH^{(\beta-1)/\beta\delta} f_G'(z) + \frac{\partial f_{\text{reg}}(T,H)}{\partial H}$$

which has a divergent behavior.

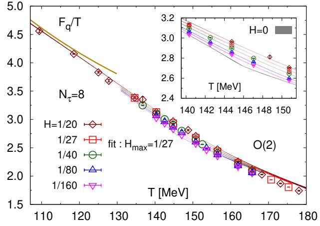
Gluonic observables towards chiral limit





$$\frac{F_q(T,H)}{T} \sim \begin{cases} a^-(T) + Ap_s^-(T)\ H & , T < T_c \\ a^r_{0,0} + Aa_1\ H^{(1-\alpha)/\beta\delta} & , T = T_c \\ a^+(T) + p^+(T)\ H^2 & , T > T_c \end{cases}$$

- Fit with singular terms only.
- Determined singular part compares well with other determinations.

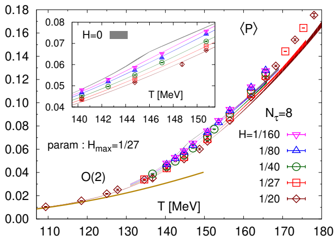


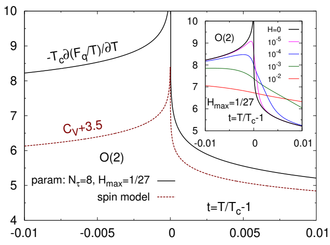
- For $T < T_c$, linear H dependence of HQFE is general, due to Goldstone effect.

Bazavov *et. al.*, Phys. Rev. D87, 094505 (2013).
Brambilla *et. al.*, Phys. Rev. D97, 034503 (2018).
Megías *et. al.*, Phys. Rev. Lett. 109, 151601 (2012).

- Regular part is then determined from HQFE keeping the singular part fixed at the value determined from $\partial(F_q/T)/\partial H$ fit. Clarke *et. al.*, Phys. Rev. D103, L011501 (2021).

Gluonic observables towards chiral limit





- Polyakov loop behaves as an energy-like observable towards chiral limit.
- No inflection point can be identified in the chiral crossover region.
- In the chiral limit: Clarke *et. al.*, Phys. Rev. D103, L011501 (2021).

$$T_c \frac{\partial(F_q(T,0)/T)}{\partial T} = a^r_{1,0}\left(1 + R^{\pm}|t|^{-\alpha}\right)$$

- Peak develops only in a very tiny interval around T_c towards chiral limit.
- Peak height is non-universal.
- Identifying a peak in C_V is hard because of the rising regular background in QCD.

Gupta and Sharma, PoS CPOD2014 (2015) 011.

See Lattice 2021 talk by David A. Clarke for more update.

Conserved charge fluctuations towards chiral limit

- μ_B does not break chiral symmetry explicitly.
- for finite μ_B definition of the $O(4)$ scaling fields to the leading order:

$$t = \frac{1}{t_0}\left(\frac{T - T_c^0}{T_c^0} + \kappa_B \left(\frac{\mu_B}{T}\right)^2\right) \quad \text{and} \quad h = \frac{1}{h_0}\frac{m_l}{m_s}$$

- Close to chiral limit: $\frac{\partial}{\partial T} \sim \frac{\partial}{\partial \mu_B^2} \Rightarrow \chi_2^B$ is expected to be energy-like observable.

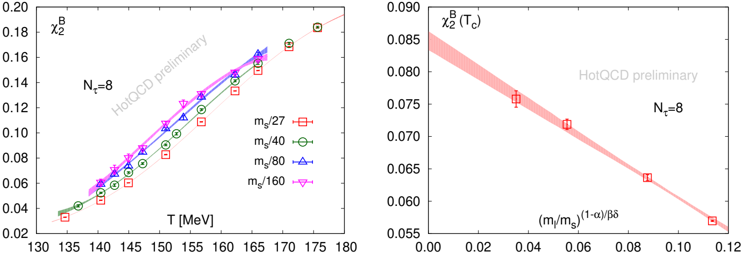
$$\chi_2^B(T,H) = -A\kappa_2^B H^{(1-\alpha)/\beta\delta} f_f'(z) + \text{regular terms.}$$

- We check this at $T = T_c$. Sarkar *et. al.*, arXiv:2011.00240.

$$\chi_2^B(T_c,H) = -A\kappa_2^B H^{(1-\alpha)/\beta\delta} f_f'(0) + \text{constant regular term.}$$

Conserved charge fluctuations towards chiral limit

At $T = T_c$, $\chi_2^B(T_c, H) = -A\kappa_2^B H^{(1-\alpha)/\beta\delta} f_f'(0) +$ constant regular term.

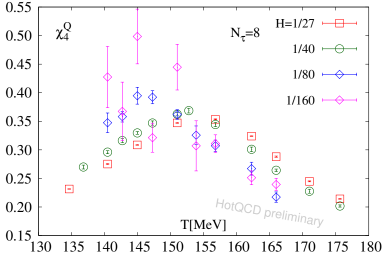


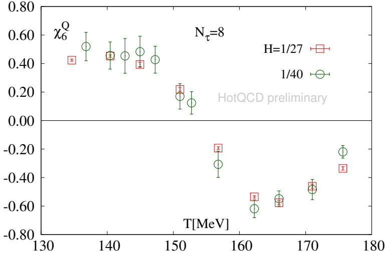
- Linear fit in $H^{(1-\alpha)/\beta\delta}$ works quite well.
- Singular part vanishes at the chiral limit.

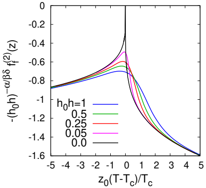
Sarkar *et. al.*, arXiv:2011.00240.

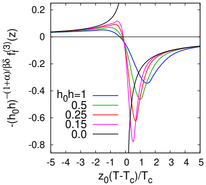
- $\chi_2^B(T_c, 0) - \chi_2^B(T_c, H)$ gives the singular part for any finite mass.
- Ratio of singular parts of of conserved charge X and Y is same as κ_2^X/κ_2^Y.
- Preliminary calculations: $\kappa_2^B/\kappa_2^S = 1.0$ and $\kappa_2^Q/\kappa_2^B = 2.6$.
- Consistent with physical mass results for ratios of κs. [HotQCD; PLB 795 15 (2019)].

Conserved charge fluctuations towards chiral limit







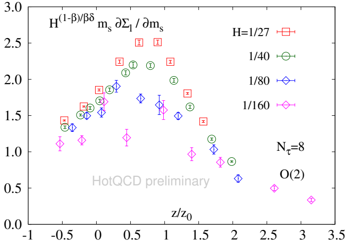


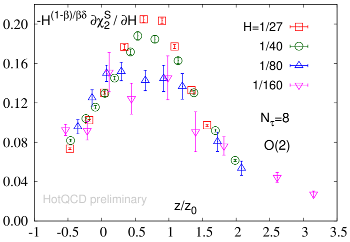
Friman *et. al.*, Eur. Phys. J. C71 1694 (2011).

Given $t = \frac{1}{t_0}\left(\frac{T-T_c^0}{T_c^0} + \kappa_X \left(\frac{\mu_B}{T}\right)^2\right)$

- $\chi_4^X \sim H^{-\alpha/\beta\delta} f_f^{(2)}(z) \Rightarrow$ not divergent for $H \to 0$.
- $\chi_6^X \sim H^{-(\alpha+1)/\beta\delta} f_f^{(3)}(z) \Rightarrow$ moderate divergence.
- Characteristic cusp for χ_4 seems to develop.
- B-channel is more noisy compared to that of Q.

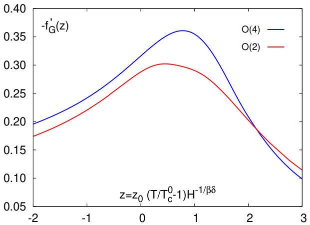
‘Mixed’ susceptibilities





- m_s does not break the chiral symmetry in the light sector $\Rightarrow \langle\bar\psi\psi\rangle_s$ is energy-like observable.
- Since $\langle\bar\psi\psi\rangle_s \sim H^{(1-\alpha)/\beta\delta} f_f'(z)$, so $\frac{\partial\langle\bar\psi\psi\rangle_s}{\partial H} \sim -H^{(\beta-1)/\beta\delta} f_G'(z)$, divergent in the chiral limit.
- Similarly, $\frac{\partial\chi_2^S}{\partial H} \sim -H^{(\beta-1)/\beta\delta} f_G'(z)$.

- Although the singular contributions are similar, various observables could differ w.r.t. the regular terms.



See Lattice 2021 talk by Mugdha Sarkar for more details and update.

Curvature in the chiral limit

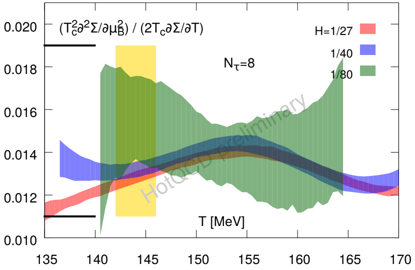
Given $t = \frac{1}{t_0}\left(\frac{T-T_c^0}{T_c^0} + \kappa_B \left(\frac{\mu_B}{T}\right)^2\right)$

$$\kappa_B^{H=0} = \frac{T}{2}\frac{\partial^2\Sigma/\partial\mu_B^2}{\partial\Sigma/\partial T}\bigg|_{T=T_c^0}$$

with $\Sigma = \frac{2m_s}{f_K^4}\left(\langle\bar{\psi}\psi\rangle_l - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_s\right)$

- One can also calculate κ_B^H by following some physical condition over the $T-\mu_B$ plane; *e.g.* following the inflection point of Σ gives

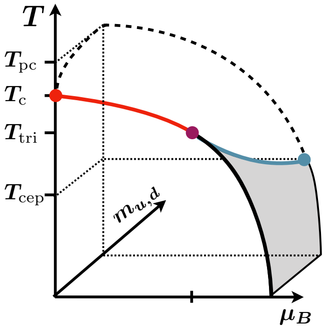
$$\kappa_B^H = \frac{T}{2}\frac{\frac{\partial^2}{\partial T^2}\frac{\partial^2\Sigma}{\partial\mu_B^2}}{\frac{\partial^3\Sigma}{\partial T^3}}\Bigg|_{T=T_{\rm pc}^H}$$



- Similar calculations for $\kappa_Q^{H=0}$ and $\kappa_S^{H=0}$ are in progress.
- Compare $H \to 0$ limit of κ_B^H to $\kappa_B^{H=0}$.

Overview

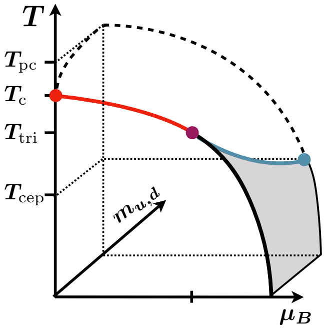
Summary and Outlook



- Progressively increasing evidence of (2+1)-flavor chiral transition to be of second order belonging to $O(4)$ universality class.
- In the chiral limit at $\mu_B = 0$: $T_c^0 = 132^{+3}_{-6}$ MeV.
- CEP is unlikely to be found for $T > 130$ MeV and correspondingly for $\mu_B < 400$ MeV.
- Ongoing calculations about the curvatures of the (pseudo-)critical lines is going to be important.
- Calculations of energy-like observables seems to provide important information.

- $U_A(1)$ restoration is one of the deciding calculation.
- Phase transition in RW plane indirectly constrain the Columbia plot and phase diagram for $\mu_B = 0$.

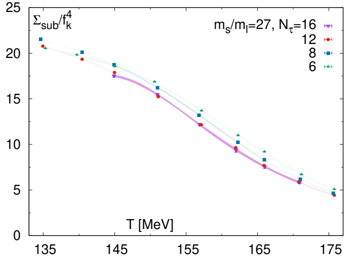
Summary and Outlook



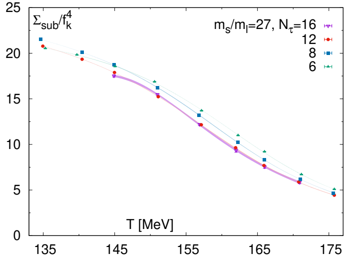
- Progressively increasing evidence of (2+1)-flavor chiral transition to be of second order belonging to $O(4)$ universality class.
- In the chiral limit at $\mu_B = 0$: $T_c^0 = 132^{+3}_{-6}$ MeV.
- CEP is unlikely to be found for $T > 130$ MeV and correspondingly for $\mu_B < 400$ MeV.
- Ongoing calculations about the curvatures of the (pseudo-)critical lines is going to be important.
- Calculations of energy-like observables seems to provide important information.

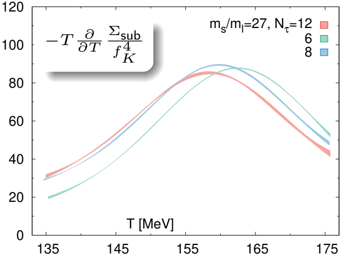
- $U_A(1)$ restoration is one of the deciding calculation.
- Phase transition in RW plane indirectly constrain the Columbia plot and phase diagram for $\mu_B = 0$.

Scaling at physical Pion mass?

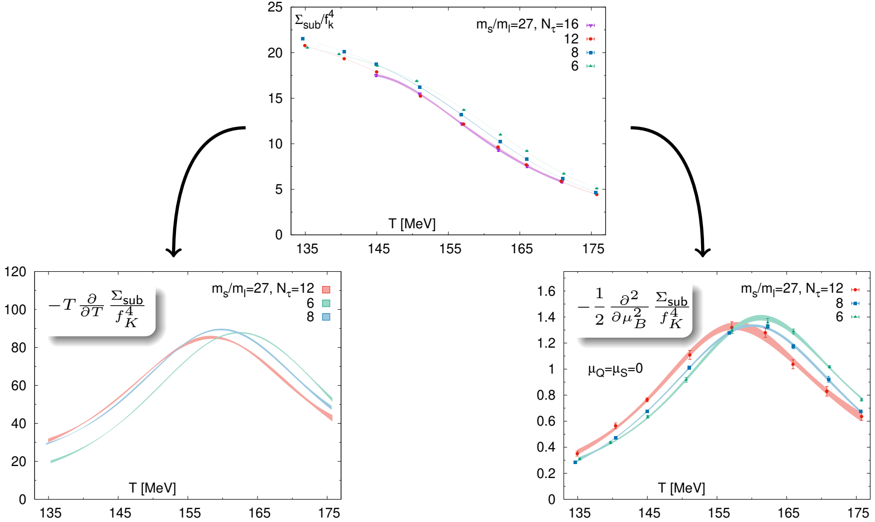


Scaling at physical Pion mass?



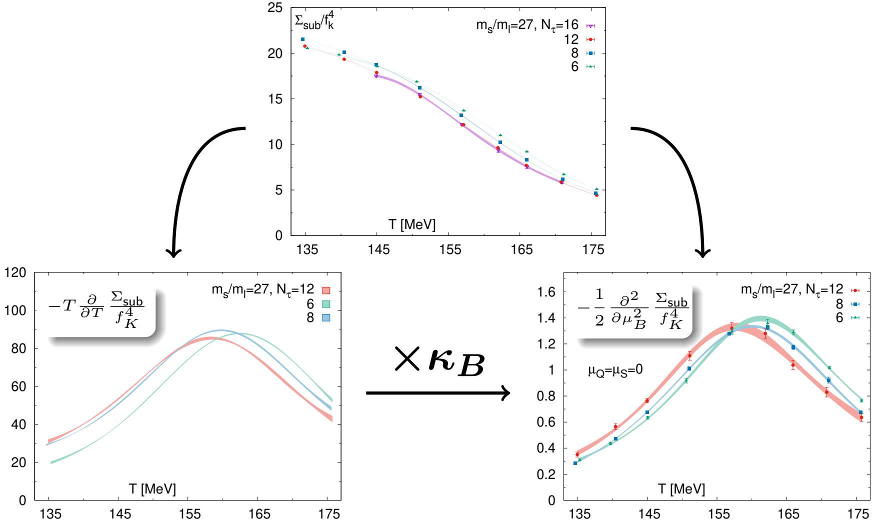


Scaling at physical Pion mass?



Scaling at physical Pion mass?

CRC-TR 211 Strong-interaction matter under extreme conditions

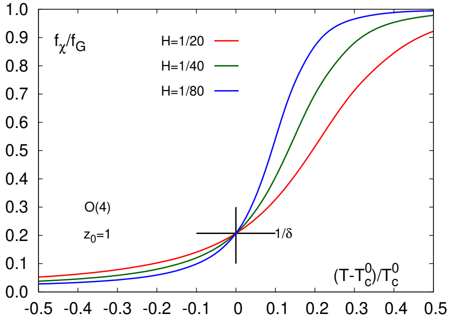


- Scaling expectation already holds for physical Pion.

UNIVERSITÄT BIELEFELD Faculty of Physics

Scaling functions: Some intriguing facts

$$\frac{f_\chi(z)}{f_G(z)} = \left\{ \begin{array}{ccc} 0 & , & z \to -\infty \\ 1/\delta & , & z = 0 \\ 1 & , & z \to +\infty \end{array} \right\}$$



- Behavior of $H\chi_M/M$ is like Binder cumulant at critical point. [F. Karsch and E. Laermann. Phys. Rev. D50, 6954, 1994.]
- Ratio is expected to have a constant value at the crossing point, $z = 0$, *i.e.* in chiral limit at T_c^0.
- Determine temperature $T_\delta(H)$ which satisfies:

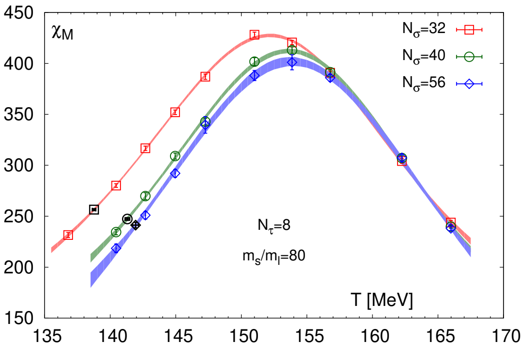
$$\frac{H\chi_M\left(T_\delta, H\right)}{M\left(T_\delta, H\right)} = \frac{1}{\delta} \Rightarrow T_c^0 = \lim_{H\to 0} T_\delta(H)$$

.

- Uniqueness of the crossing point gets spoiled in presence of regular terms.

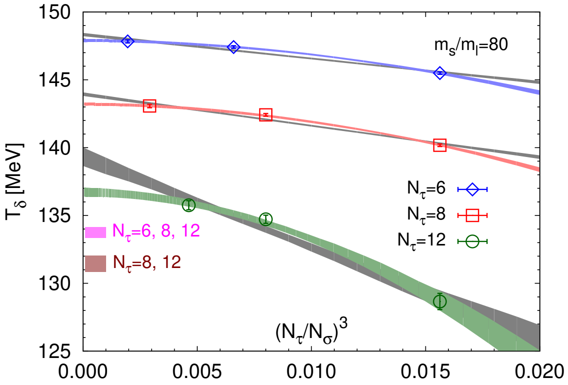
No evidence for 1^{st} order transition

- Volume dependence of χ_M is studied for $H = 1/80$ which corresponds to $m_\pi = 80$ MeV.
- χ_M^{max} is NOT proportional to volume.
- χ_M^{max} seems to saturate towards thermodynamic limit.
- T_{pc} and T_{60} increase towards thermodynamic limit.



- Possibility for 1^{st} order phase transition can be ruled out at $m_\pi = 80$ MeV for $N_\tau = 8$.
- Similar results are also obtained for $N_\tau = 6$ and 12.

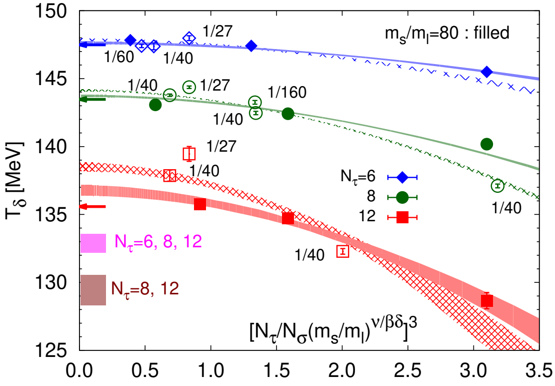
T_c^0 in continuum: 'Proper' limits



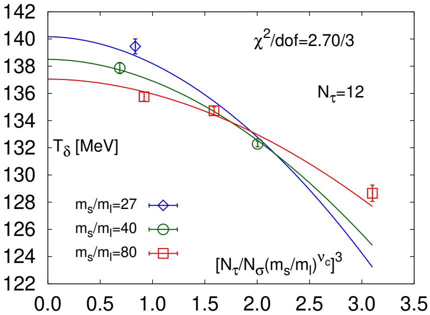
- Results for fixed H have been extrapolated to thermodynamic limit.
- Systematic uncertainty comes in form of difference between $O(4)$ and $1/V$ extrapolations.
- Continuum extrapolation are performed with(out) $N_\tau = 6$ results which is another source of systematic uncertainty.

Chiral extrapolation: $T_X(H) = T_c^0 \left(1 + \frac{z_X}{z_0} H^{1/\beta\delta}\right) + c_X H^{1-1/\delta+1/\beta\delta}$

T_c^0 in continuum: 'Improper' limits



- Results for fixed N_τ have been extrapolated to thermodynamic limit and chiral limit simultaneously using $O(4)$ scaling functions.



- Continuum extrapolation are performed with(out) $N_\tau = 6$ results which is another source of systematic uncertainty.

Symmetry transformations