A new order parameter, and the scaling window of the QCD transition

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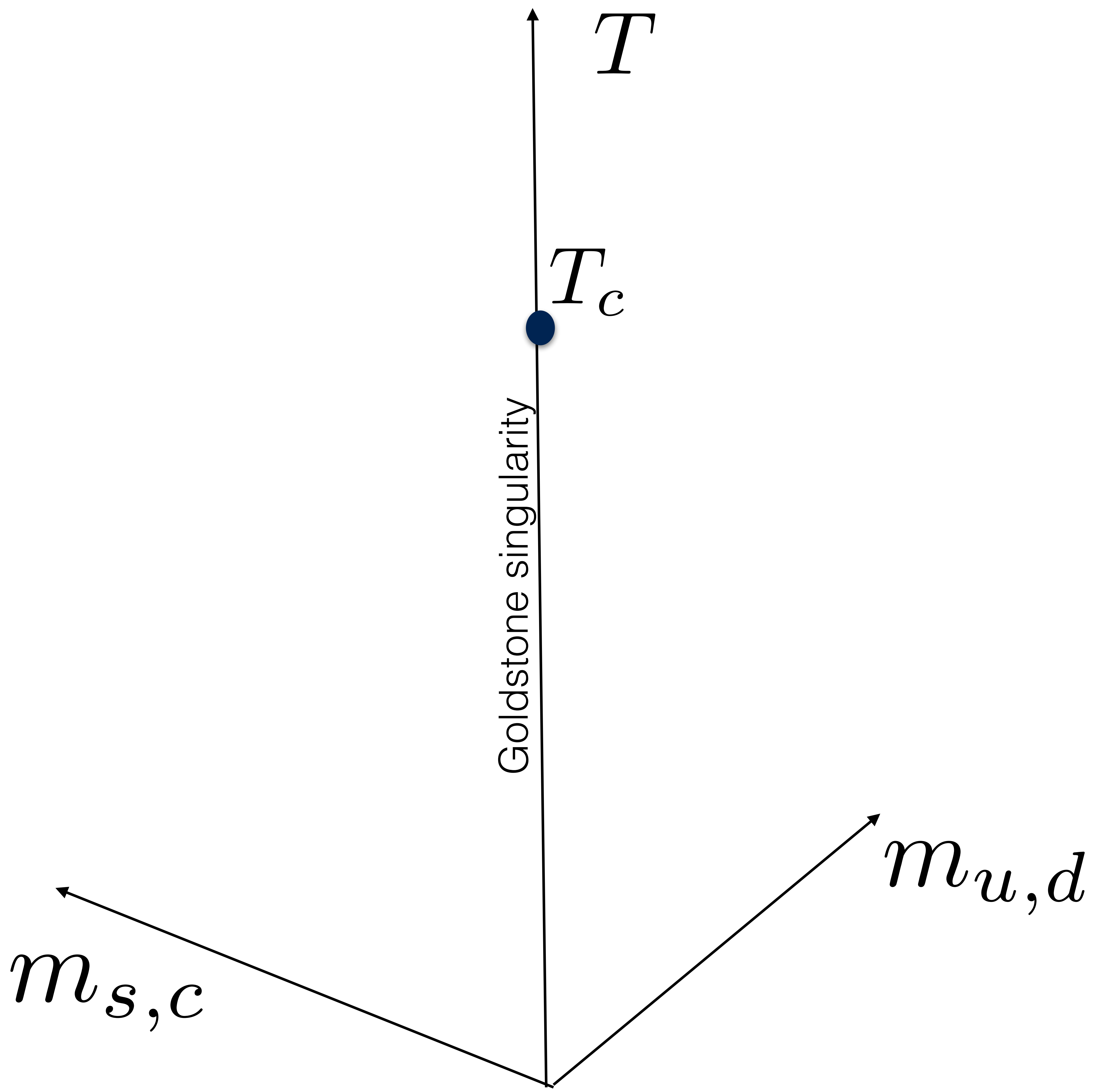
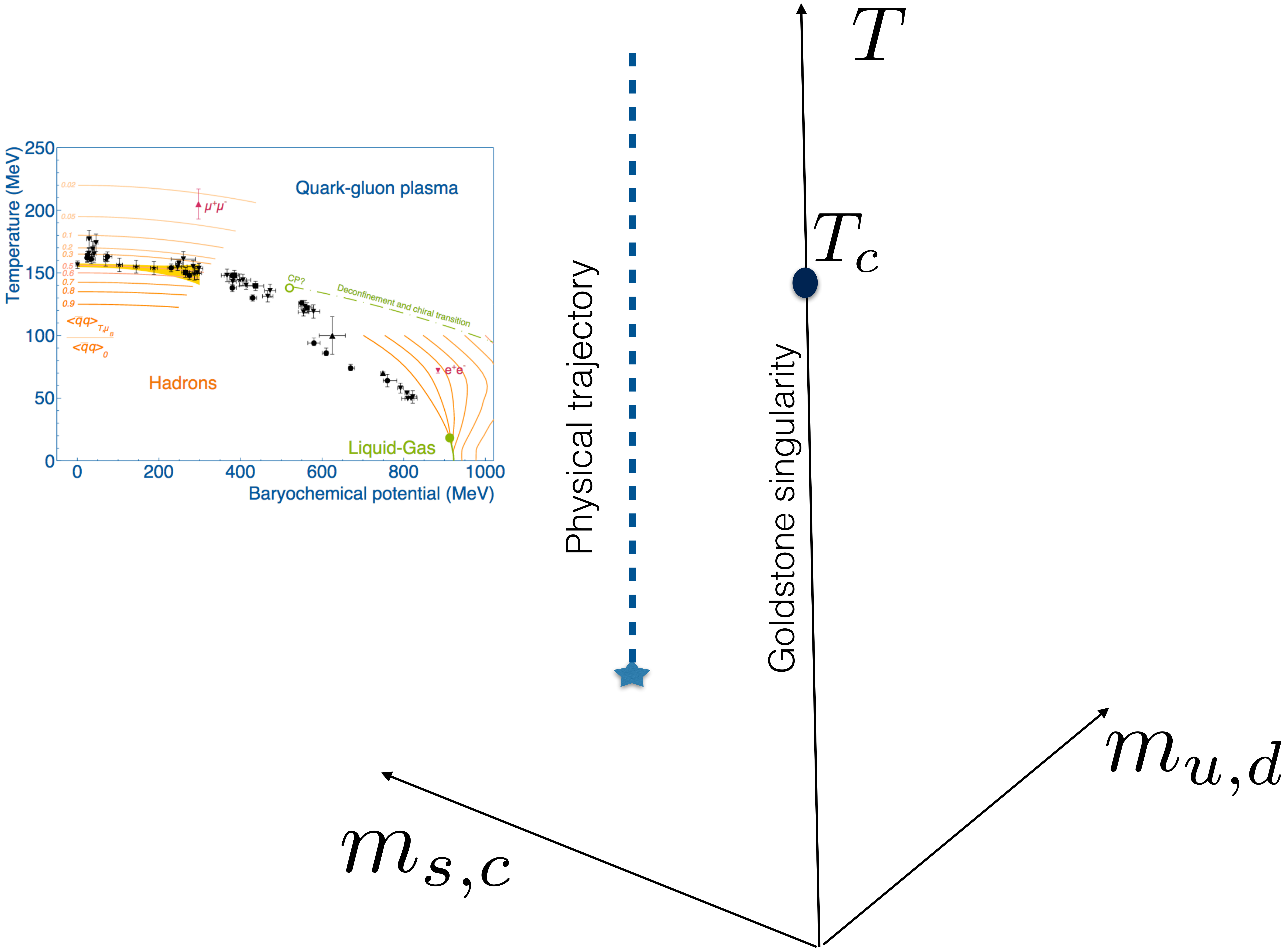
Andrey Yu. Kotov, MpL and Anton Trunin, Phys.Lett. B 2021, in press
Andrey Yu. Kotov, MpL and Anton Trunin, *Symmetry* 13 (2021) 10, 1833

Issues:

— Nature of the phase transition for Nf = 2 (+1)
— Critical temperature in the chiral limit and physical strange mass
— Threshold between sQGP and perturbative QGP?


T


T
T_c
Goldstone singularity
$m_{u,d}$
$m_{s,c}$



T
T_c
Physical trajectory
Goldstone singularity
$m_{u,d}$
$m_{s,c}$
Quark-gluon plasma
Hadrons
Liquid-Gas
CP?
Deconfinement and chiral transition
Temperature (MeV)
Baryochemical potential (MeV)
250
200
150
100
50
0
0
200
400
600
800
1000

Symmetries of QCD

$$\mathcal{L} = \sum_{a=1}^{n} \bar{q}_{La} \not{\partial} q_{La} + \bar{q}_{Ra} \not{\partial} q_{Ra} - m(\bar{q}_{La} q_{La} + \bar{q}_{Ra} q_{Ra}) + \theta \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a + \mathcal{L}_{gauge}$$

With $m = 0$, invariant under

$q_L \to V_L q_L q_R \to V_R q_R$, with $V \in U(n)$

Global symmetry:

$$U(n)_L \times U(n)_R \cong SU(n) \times \; SU(n) \times U(1)_V \times U(1)_A$$

.

Spontaneously Broken, $(n^2 - 1)$ GB ↓

baryon number

↓

Experimental Evidence

Explicitely broken

$N_f = ?$

T=0, no difference, just different #Goldstones

$m_{u,d} = 0$

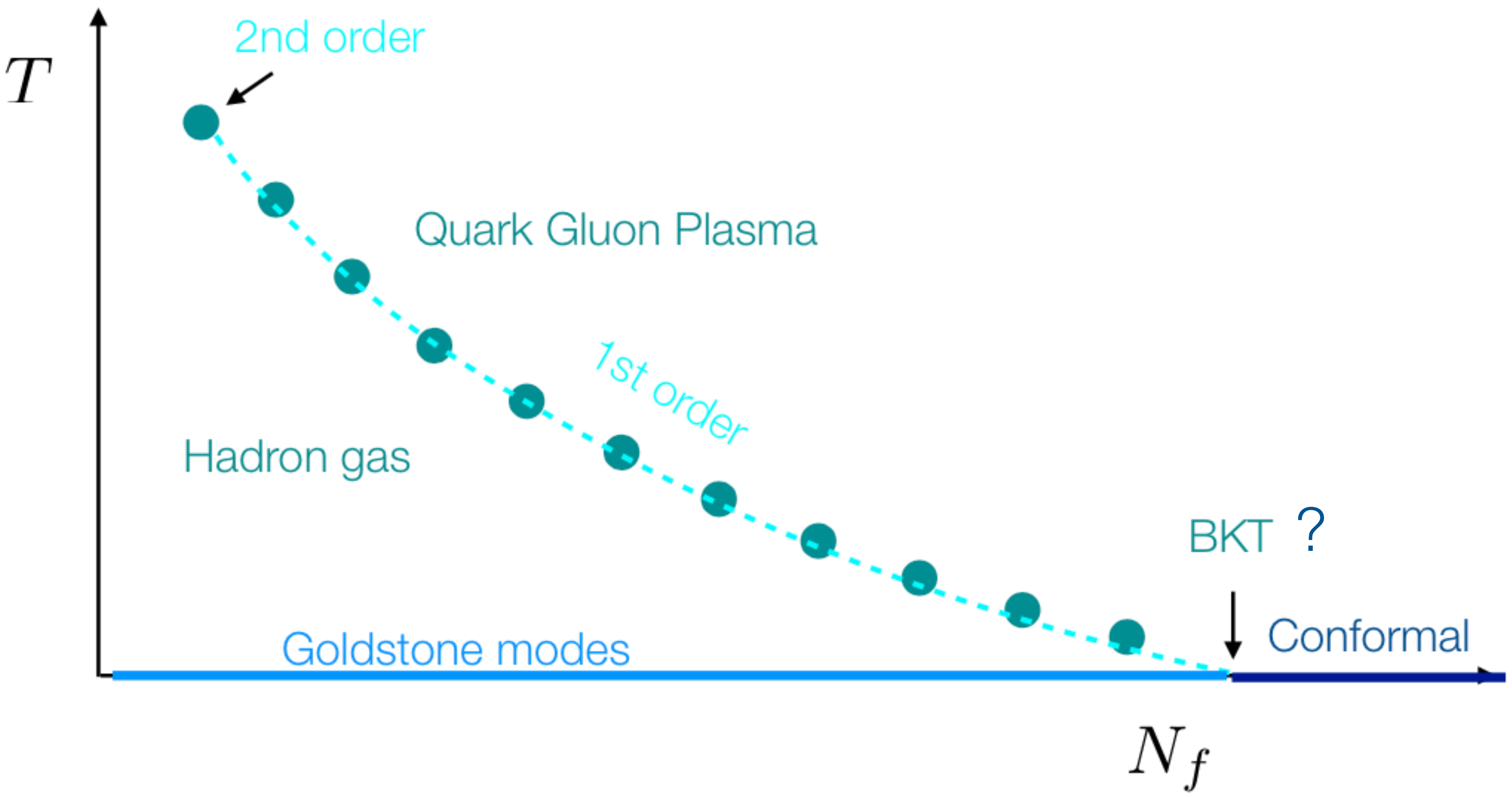
$N_f = 3$

$N_f = 2$

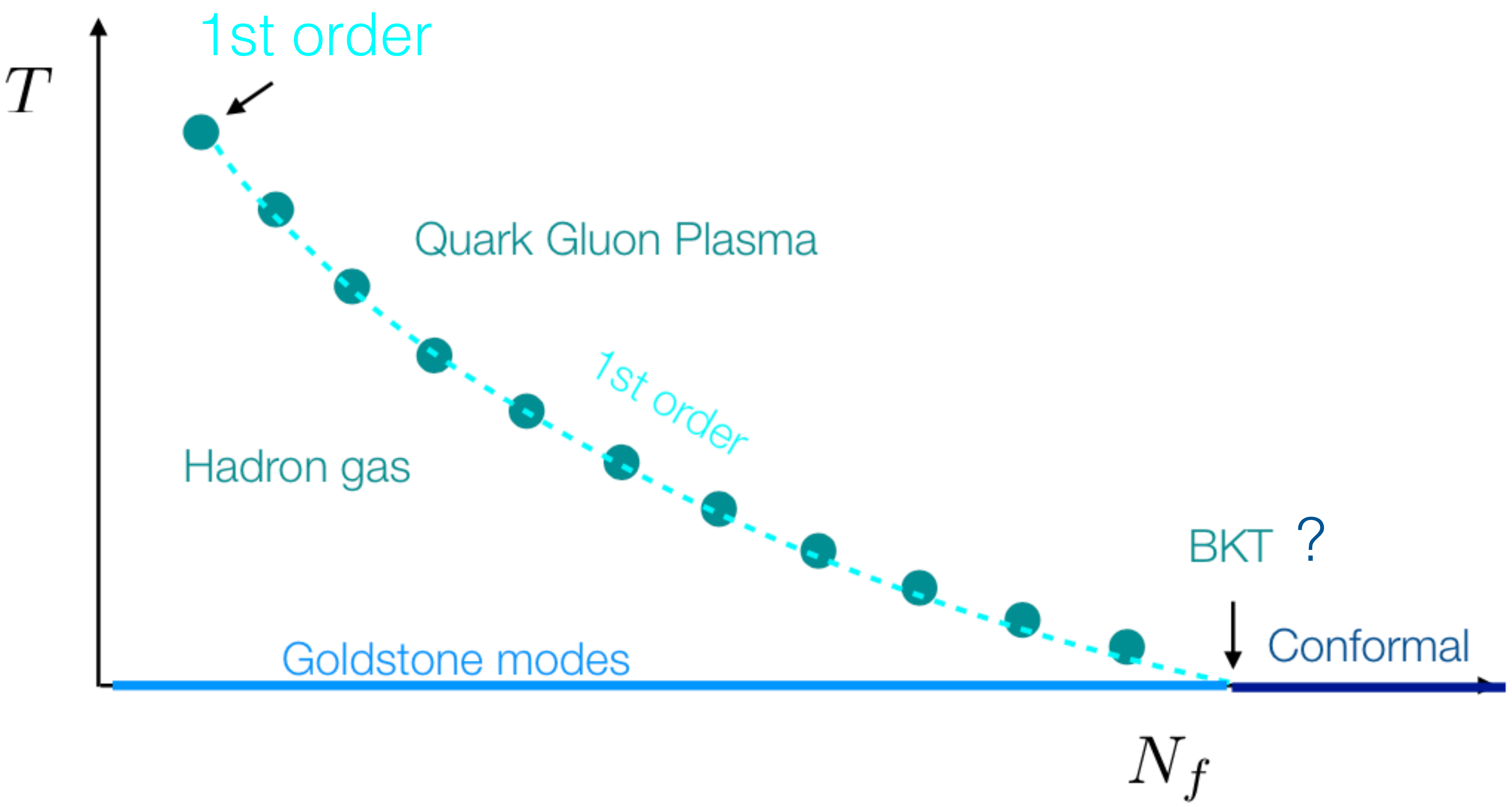
m_s

0 $\quad\longrightarrow\quad$ ∞

Switching on temperature - Scenario 1

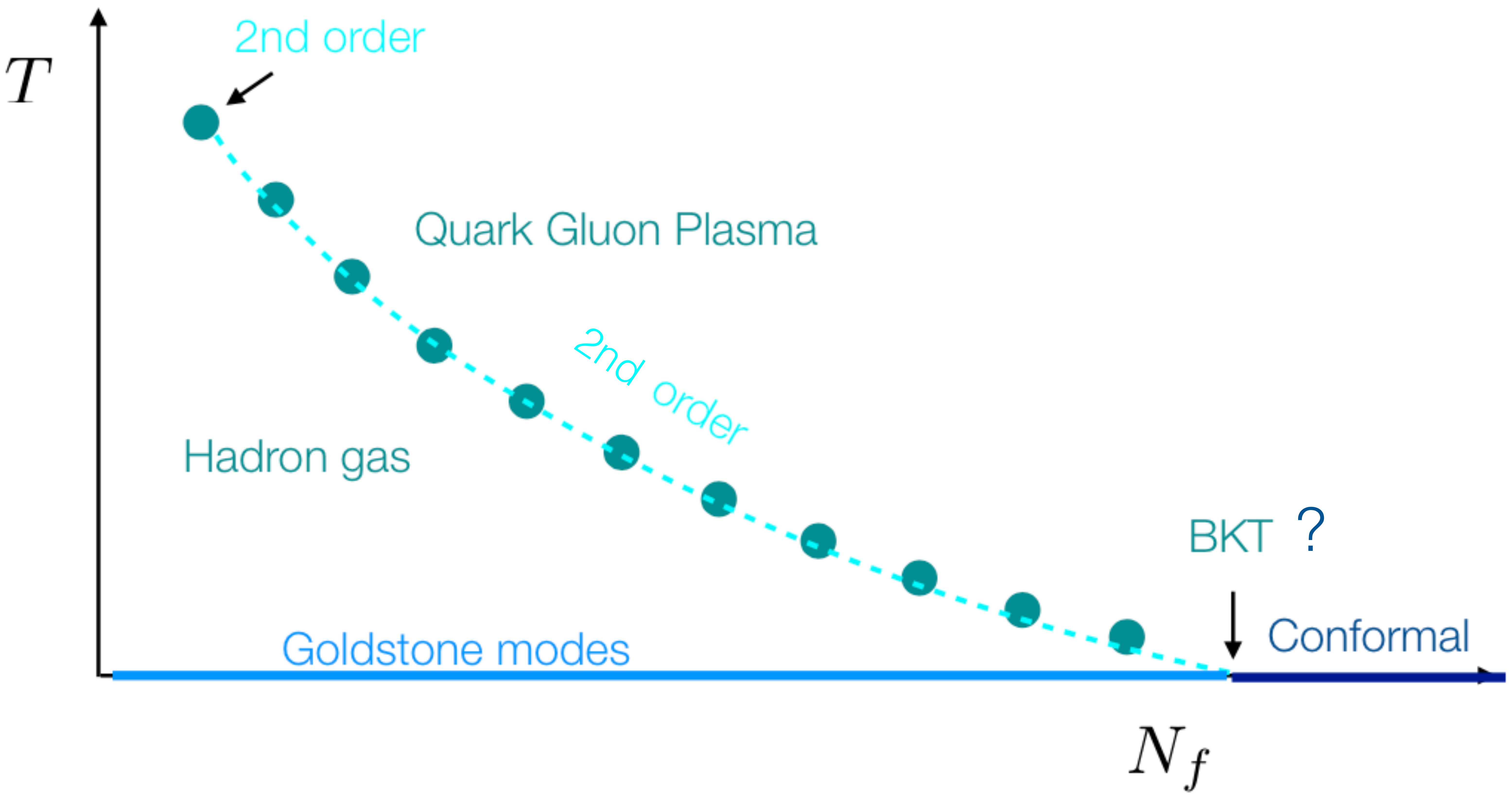


Switching on temperature - Scenario 2

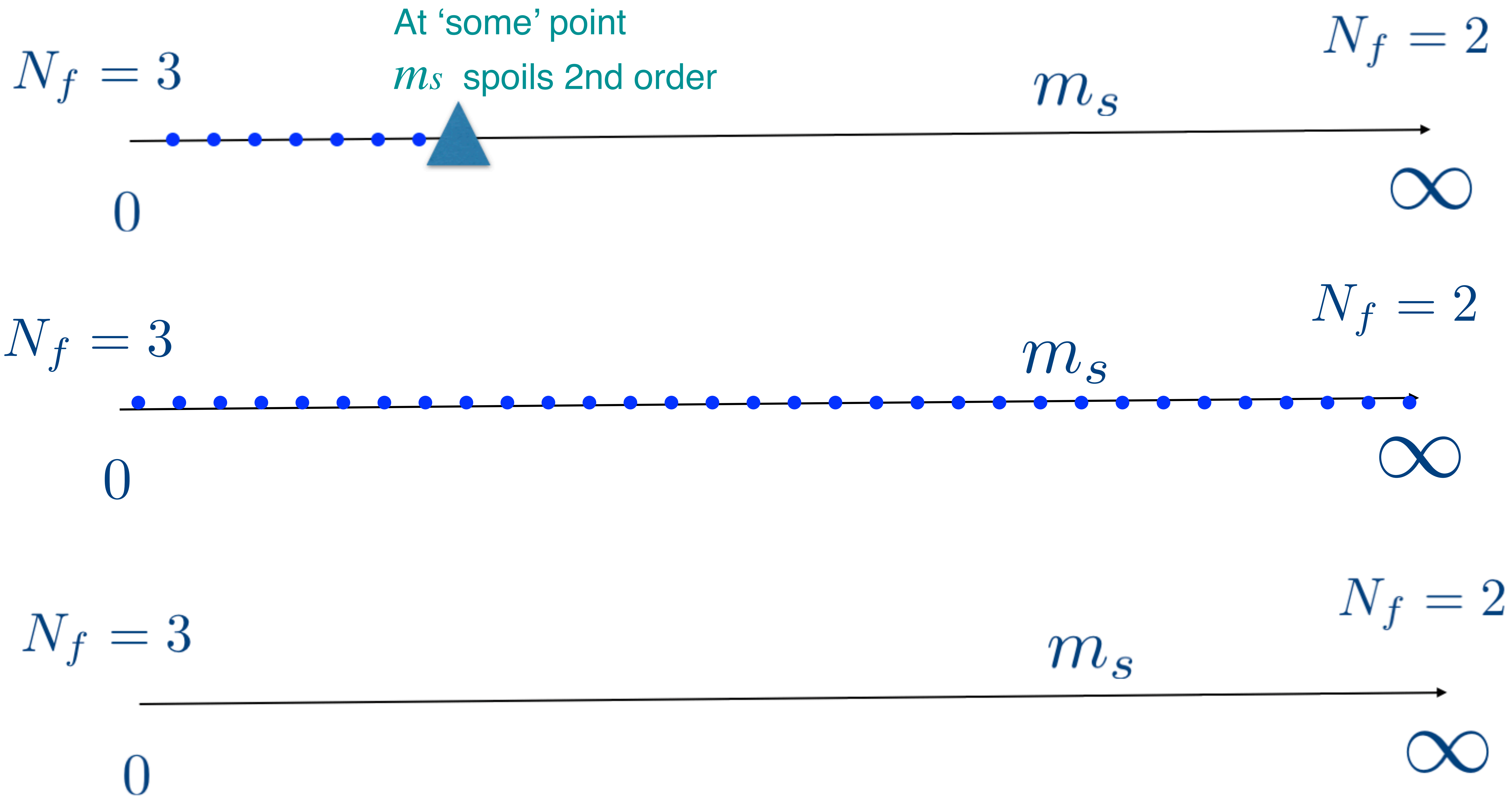


Switching on temperature - Scenario 3

Cuteri, Philipsen, Sciarra

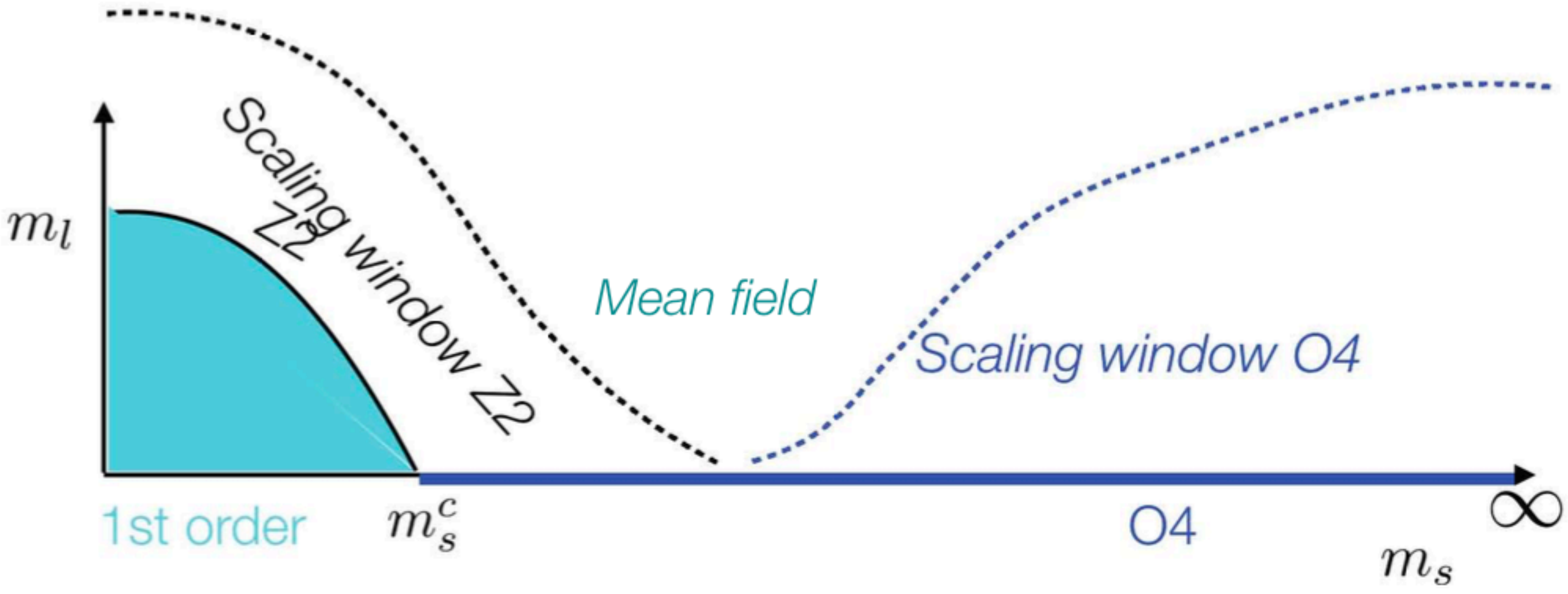


Strange mass as interpolator between Nf=3 and Nf=2

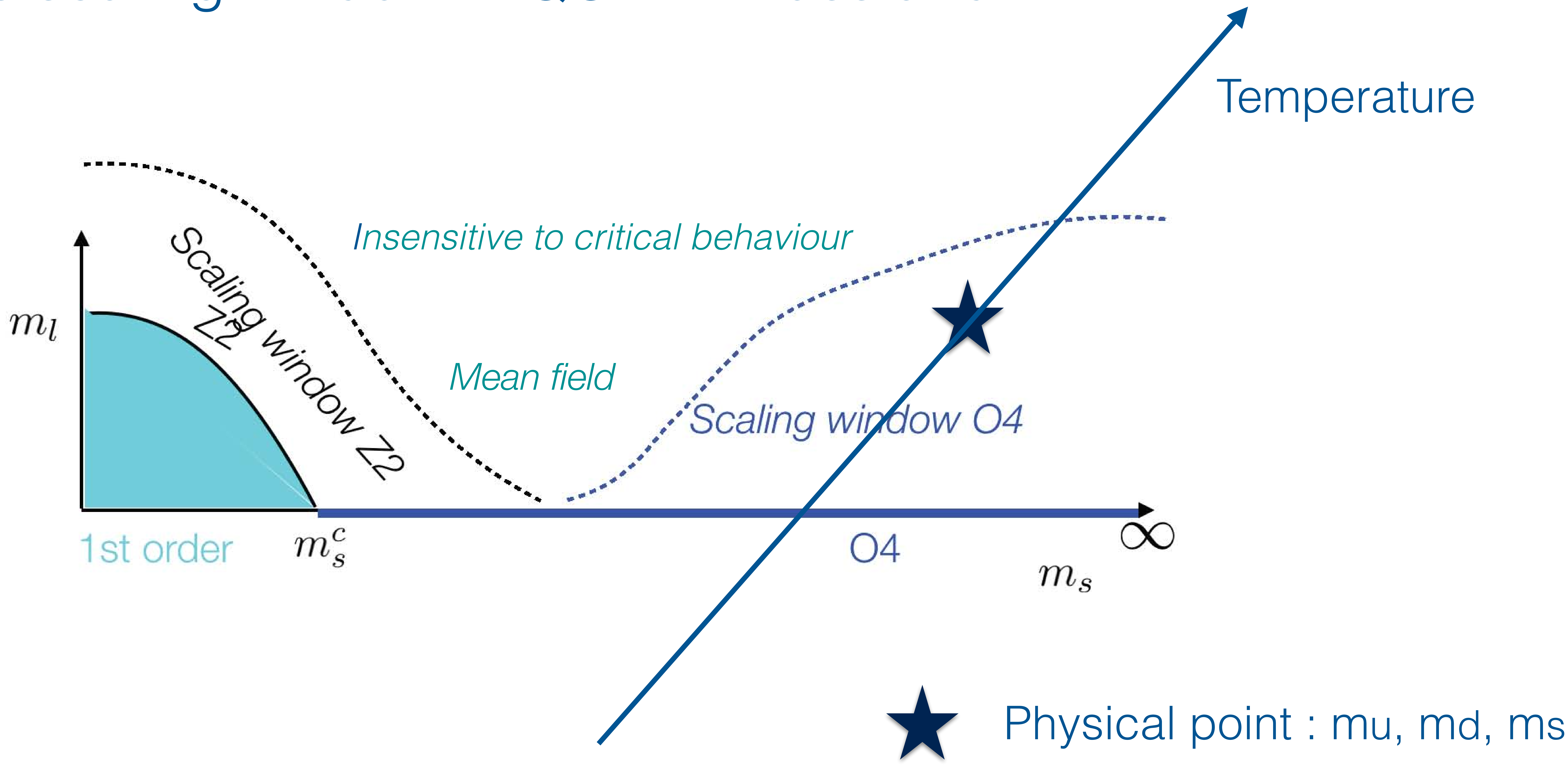


Switching on the light mass: a possible Scenario 1

Shrinking of the scaling window with decreasing ms precursor effect of Nf=3 first order?



Where is the scaling window in QCD in mass and T?



The magnetic equation of State:

$$h = M^{\delta} f(t/M^{1/\beta})$$

$M \equiv \bar{\psi}\psi$, $h \equiv m_q$, $t \equiv T - T_c$, m_q is the quark mass and T_c is the critical temperature

Three strategies to identify the scaling behaviour:

- direct comparison with the Equation of State
- the study of the dependence of the pseudo-critical temperatures on the breaking field, also known as scaling of pseudo-critical temperatures
- definition of RG invariant quantities, which do not depend on the breaking field at the critical point.

Byproduct: critical temperature in the chiral limit

Significant source of scaling violations:

additive linear mass corrections to $\bar{\psi}\psi$

A ‘new’ order parameter

also mentioned in the PhD thesis by Wolfgang Unger

’Beating’ the regular terms/additive renormalization
for more stringent universality checks

$$\Delta_3 \equiv (\bar{\psi}\psi - m\chi_L) \equiv (\bar{\psi}\psi - m\frac{\partial\bar{\psi}\psi}{\partial m}) \quad \equiv m(\chi_T - \chi_L)$$

Transverse and longitudinal susceptibilities

$$\chi_T = \frac{\bar{\psi}\psi}{m}$$

$$\chi_L = \frac{\partial\bar{\psi}\psi}{\partial m}.$$

$$R_\pi \equiv \chi_T^{-1}/\chi_L^{-1}$$

$$\frac{1}{R_\pi(t,m)} = \delta - \frac{x}{\beta}\frac{f'(x)}{f(x)},$$

$$R_\pi(0,m) = \frac{1}{\delta}$$

Kocic, Kogut, MpL;
Karsch, Laermann

Equation of State for Δ_3

- linear terms in m drop in $\Delta_3 \equiv (\bar{\psi}\psi - m\chi_L) \equiv (\bar{\psi}\psi - m\frac{\partial\bar{\psi}\psi}{\partial m})$

Use: $M = h^{1/\delta} f_G(t/h^{1/\beta\delta})$ (parametrization in:

J.Engels and F.Karsch, Phys. Rev. D 85, (2012)

To get EoS for Δ_3

$$\Delta_3 = m^{1/\delta-1} f_G(t/m^{1/\beta\delta}) - 1/\delta m^{1/\delta-1} f_G(t/m^{1/\beta\delta}) + m^{1/\beta\delta+1} f'_G((t/m^{1/\beta\delta})$$

$$\frac{\Delta_3}{m^{1/\delta}} = f_G(x)(1 - 1/\delta) + \frac{x}{\beta\delta} f_G(x)'$$

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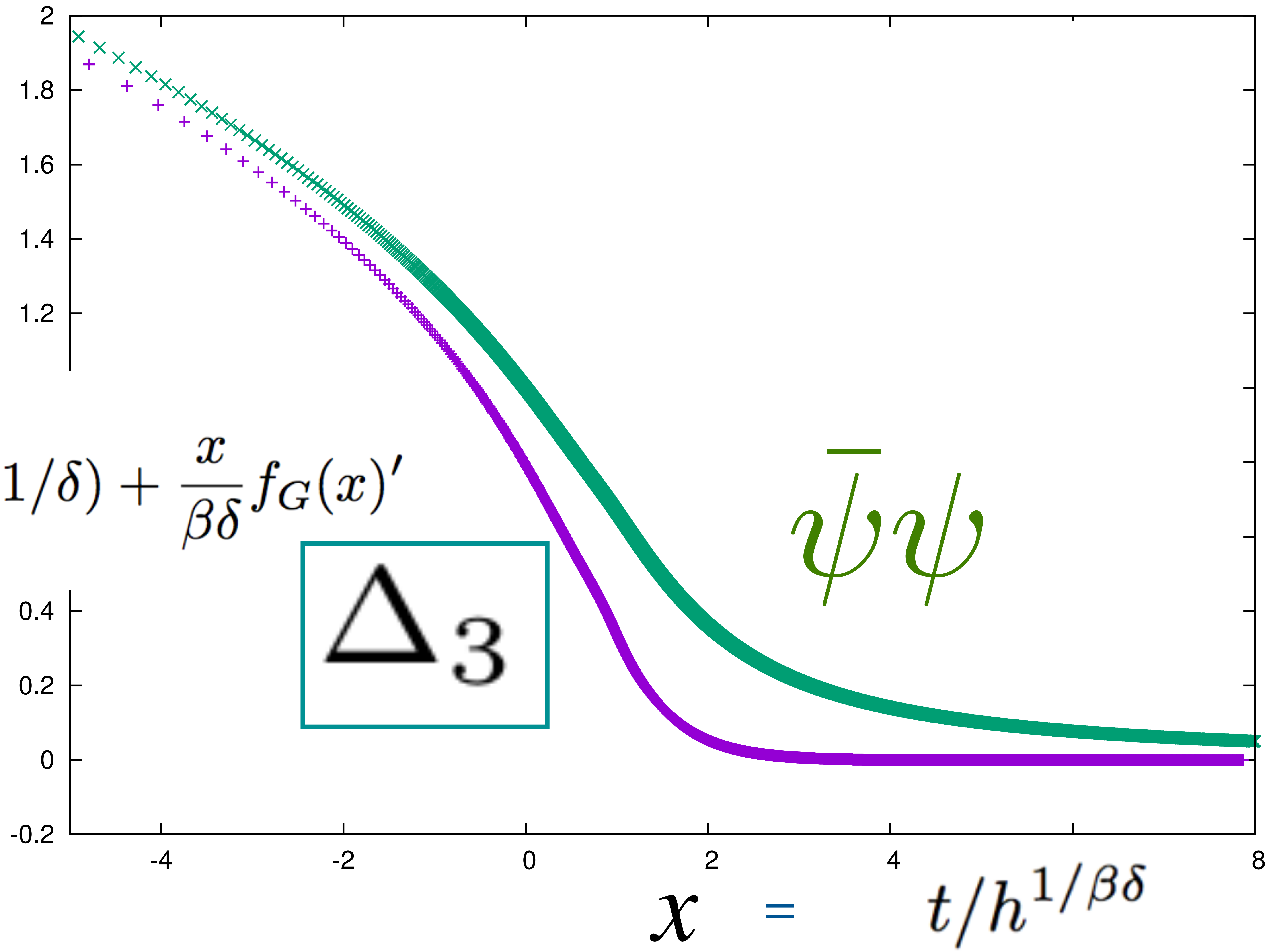
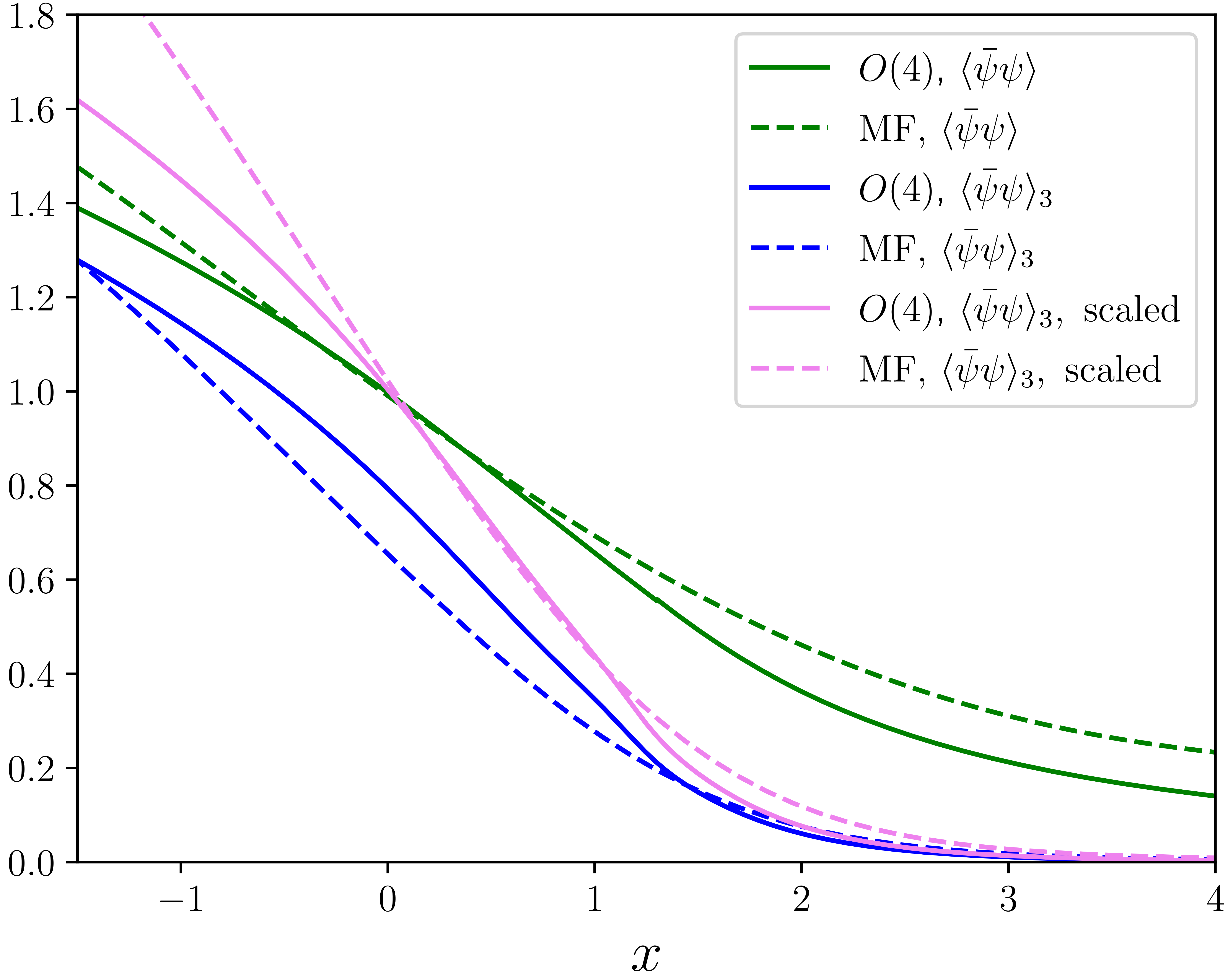
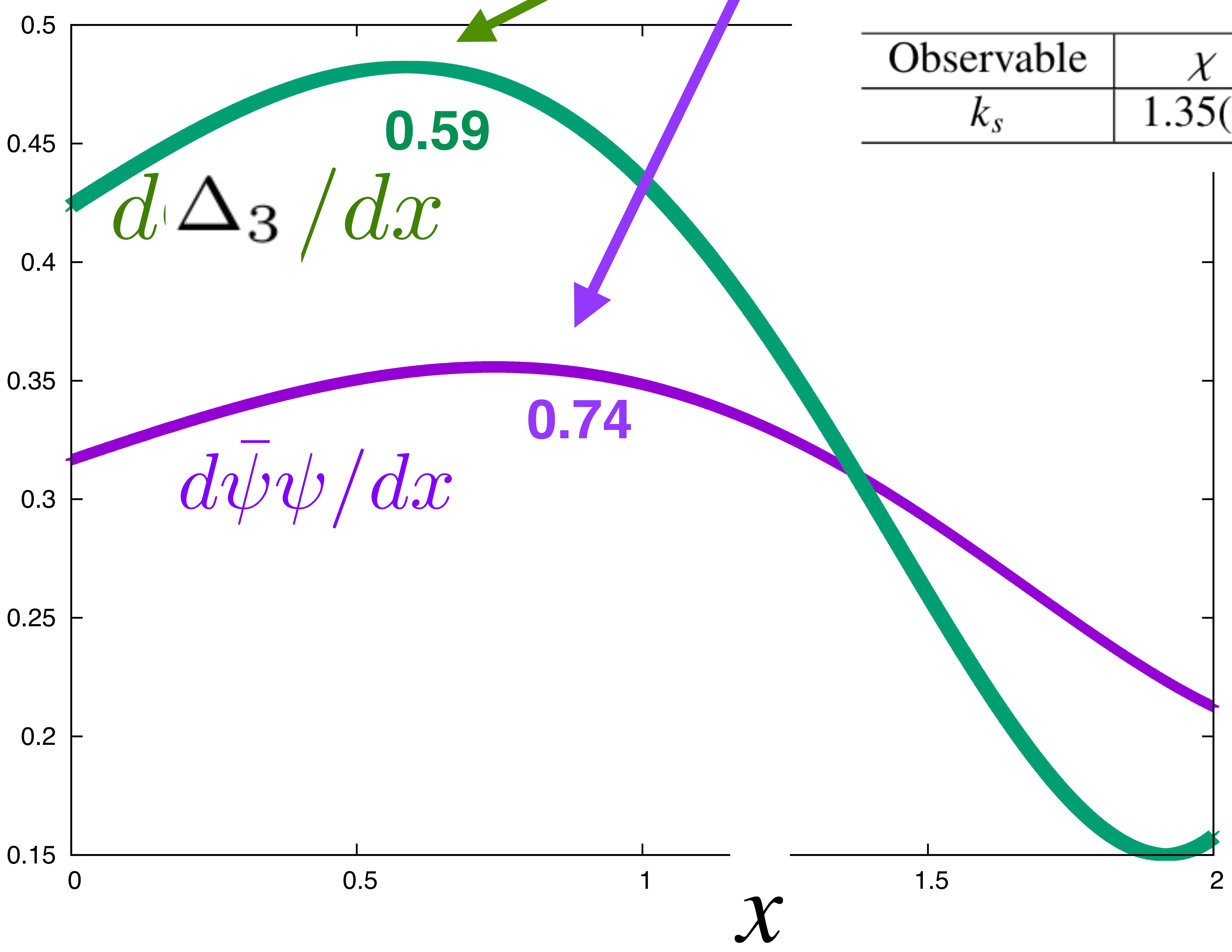



1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−1
0
1
2
3
4
x
$O(4)$, $\langle\bar{\psi}\psi\rangle$
MF, $\langle\bar{\psi}\psi\rangle$
$O(4)$, $\langle\bar{\psi}\psi\rangle_3$
MF, $\langle\bar{\psi}\psi\rangle_3$
$O(4)$, $\langle\bar{\psi}\psi\rangle_3$, scaled
MF, $\langle\bar{\psi}\psi\rangle_3$, scaled

Derivatives: give scaling of pseudo critical temperature Tc with mass

$$T_c = T_c(0) + k_s m_\pi^{2/\beta\delta}$$

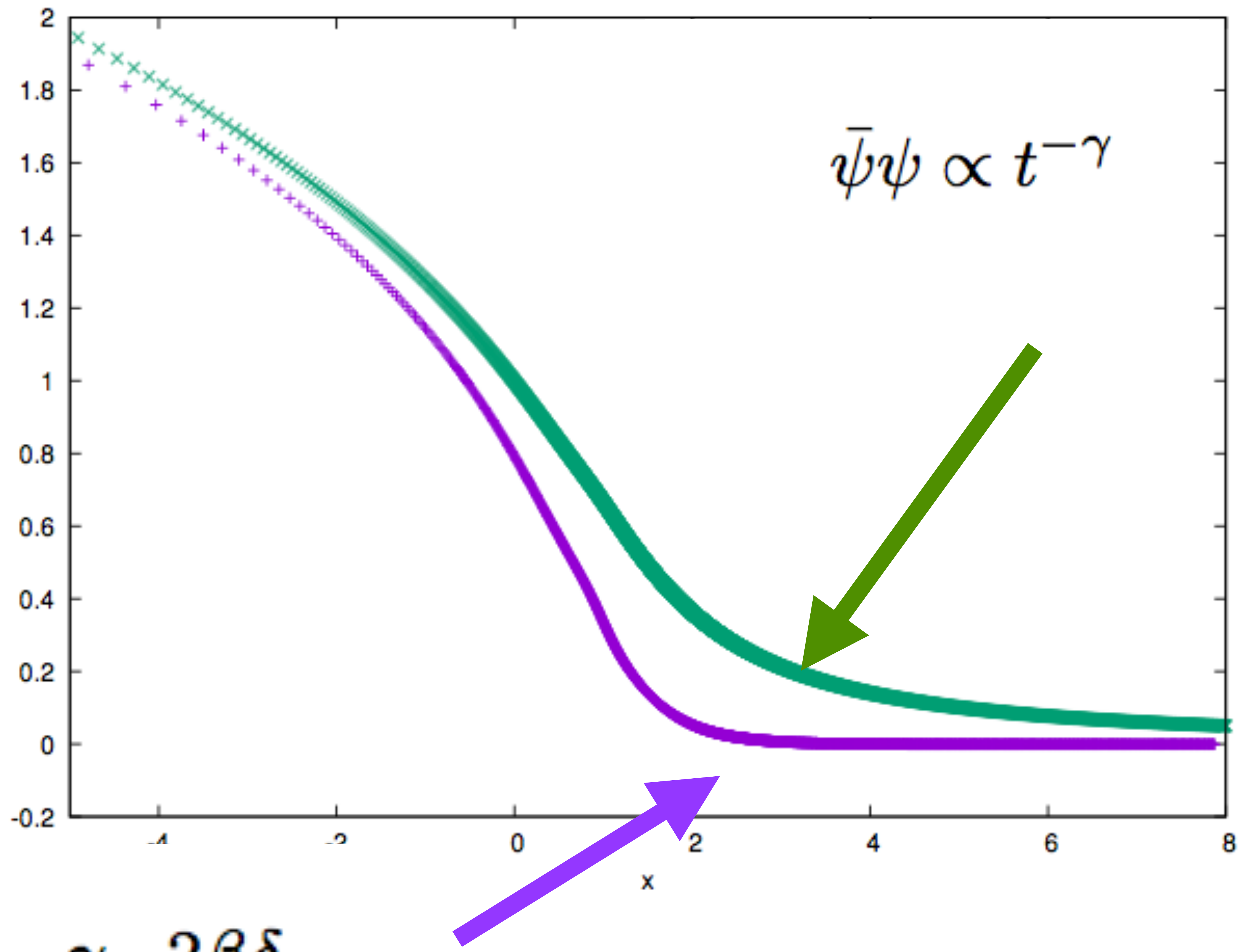
Observable	χ	$\bar{\psi}\psi$	Δ_3
k_s	1.35(3)	0.74(4)	0.59(1)



Asymptotic behavior - high T expansion

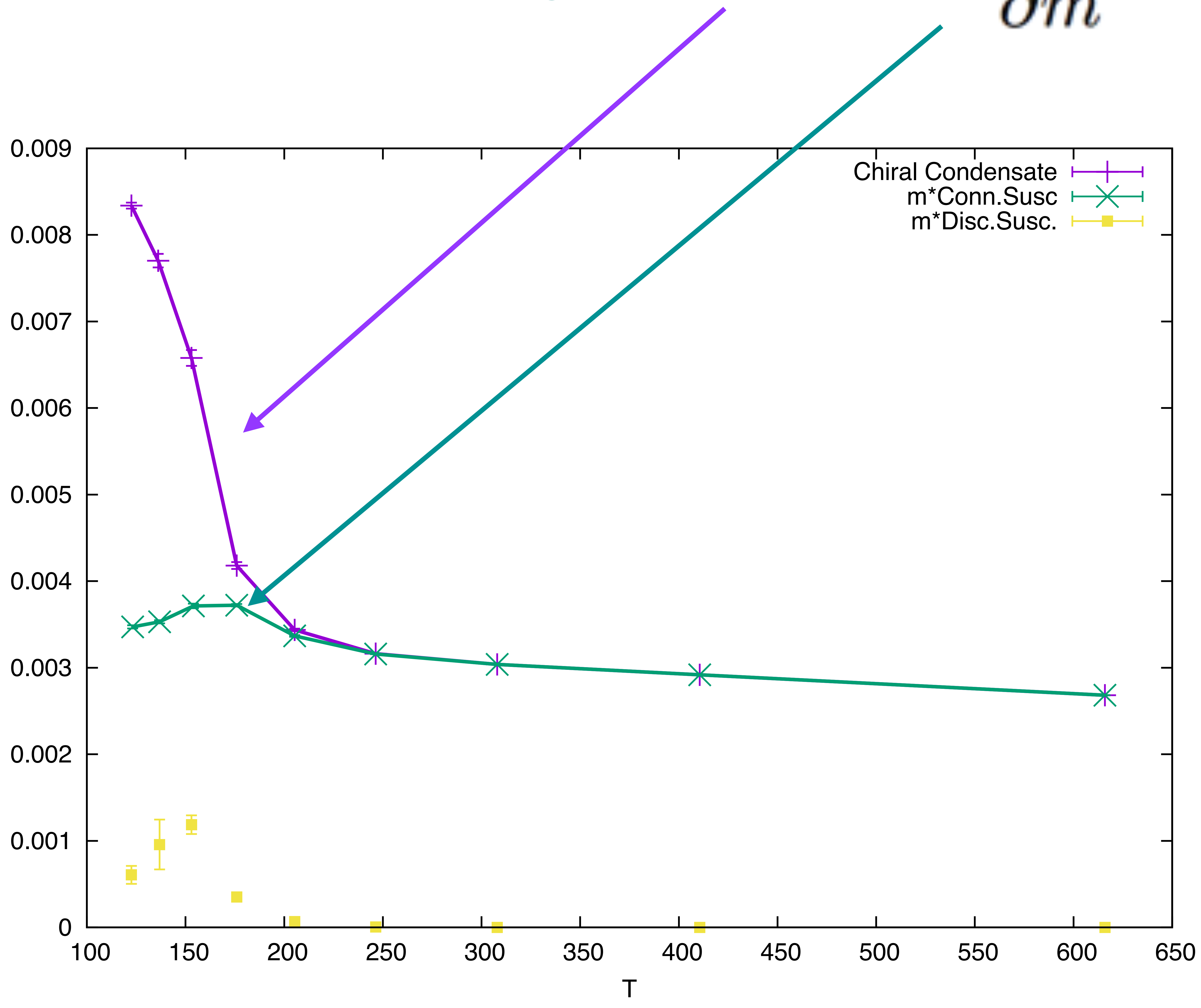
$$f_G(x) = x^{-\gamma} \sum_{n=0}^{\infty} d_n x^{-2n\Delta}$$

again, linear term drops in Δ_3



$$\Delta_3 \propto t^{-\gamma-2\beta\delta}$$

Building $\Delta_3 \equiv (\bar{\psi}\psi - m\frac{\partial \bar{\psi}\psi}{\partial m})$



Results

Setup

Twisted mass - Maximal twist

Nf = 2 +1 + 1, $m_\pi^{phys} < m_\pi < 470$ MeV $a = 0.06 - 0.09$ fm

Fixed scale approach - Temperature range 130 MeV$< T < 500$ MeV

Observables: Chiral condensate and Susceptibility,

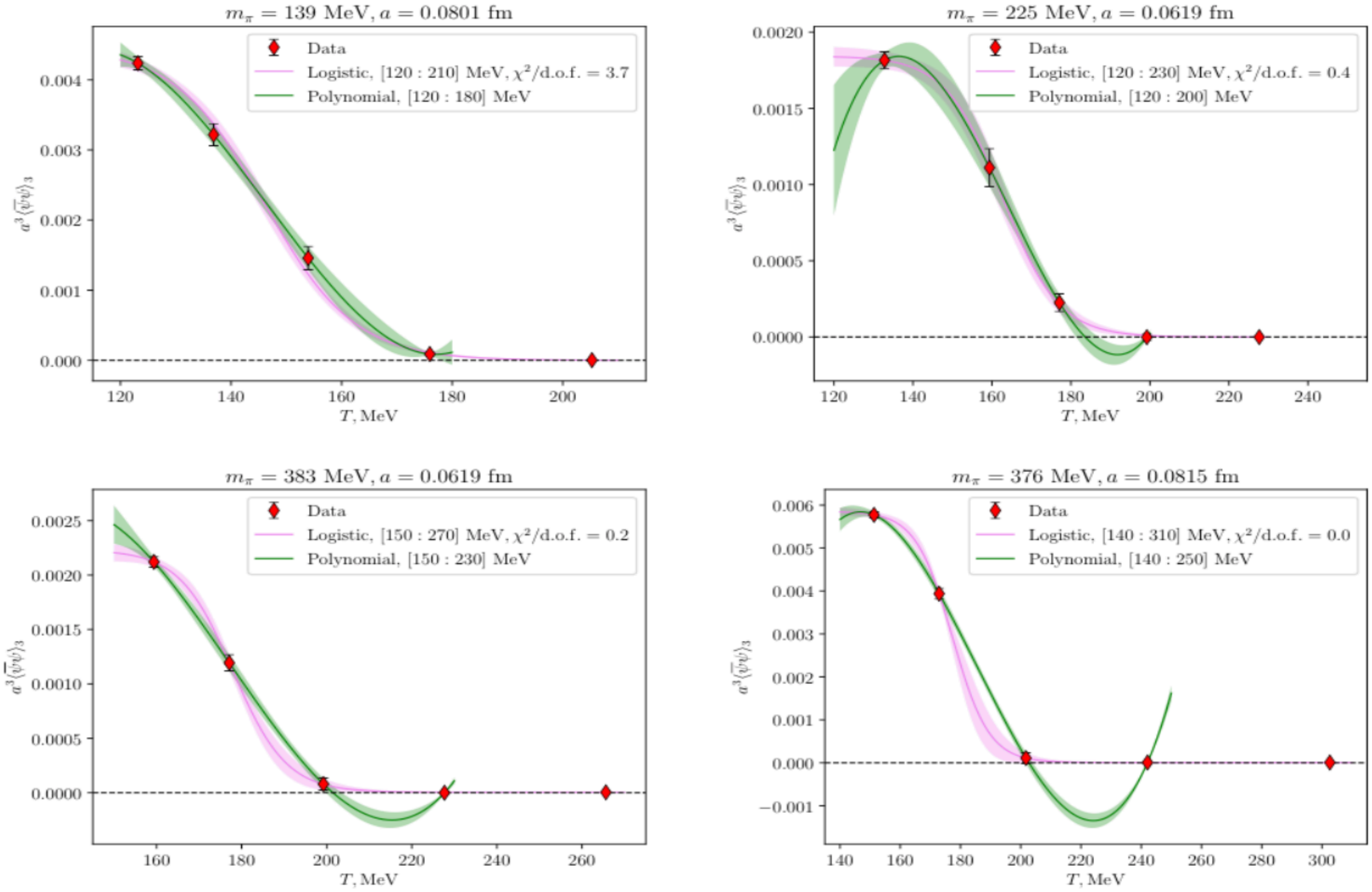
[light mesons' screening masses, η']

Statistics for physical pion mass

N_t	T [MeV]	# conf	N_t	T [MeV]	# conf
20	123(1)	782	10	246(1)	592
18	137(1)	892	8	308(2)	498
16	154(1)	534	6	411(2)	195
14	176(1)	359	4	616(3)	472
12	205(1)	337			

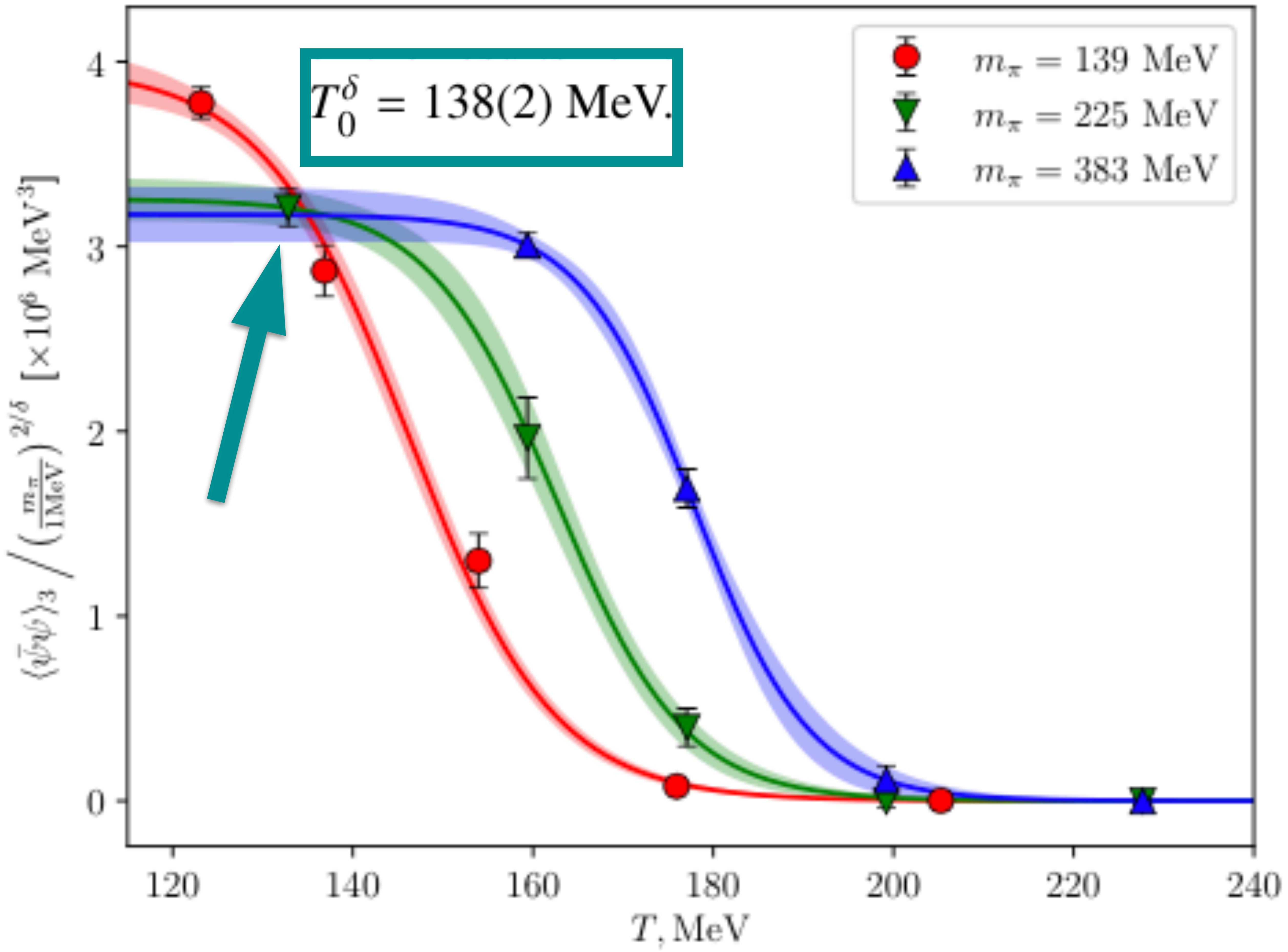
Heavier masses: Burger, Ilgenfritz, MpL, Trunin *Phys.Rev.D* 98 (2018) 9, 094501

Bare Δ_3



$m_\pi = 139$ MeV, $a = 0.0801$ fm
Data
Logistic, [120 : 210] MeV, χ^2/d.o.f. = 3.7
Polynomial, [120 : 180] MeV
$a^3\langle\bar\psi\psi\rangle_3$
T, MeV
$m_\pi = 225$ MeV, $a = 0.0619$ fm
Data
Logistic, [120 : 230] MeV, χ^2/d.o.f. = 0.4
Polynomial, [120 : 200] MeV
$a^3\langle\bar\psi\psi\rangle_3$
T, MeV
$m_\pi = 383$ MeV, $a = 0.0619$ fm
Data
Logistic, [150 : 270] MeV, χ^2/d.o.f. = 0.2
Polynomial, [150 : 230] MeV
$a^3\langle\bar\psi\psi\rangle_3$
T, MeV
$m_\pi = 376$ MeV, $a = 0.0815$ fm
Data
Logistic, [140 : 310] MeV, χ^2/d.o.f. = 0.0
Polynomial, [140 : 250] MeV
$a^3\langle\bar\psi\psi\rangle_3$
T, MeV

Scaling at the critical point: searching for $< \bar{\psi}\psi >_3 (T = T_0) = A m_\pi^{2/\delta}$

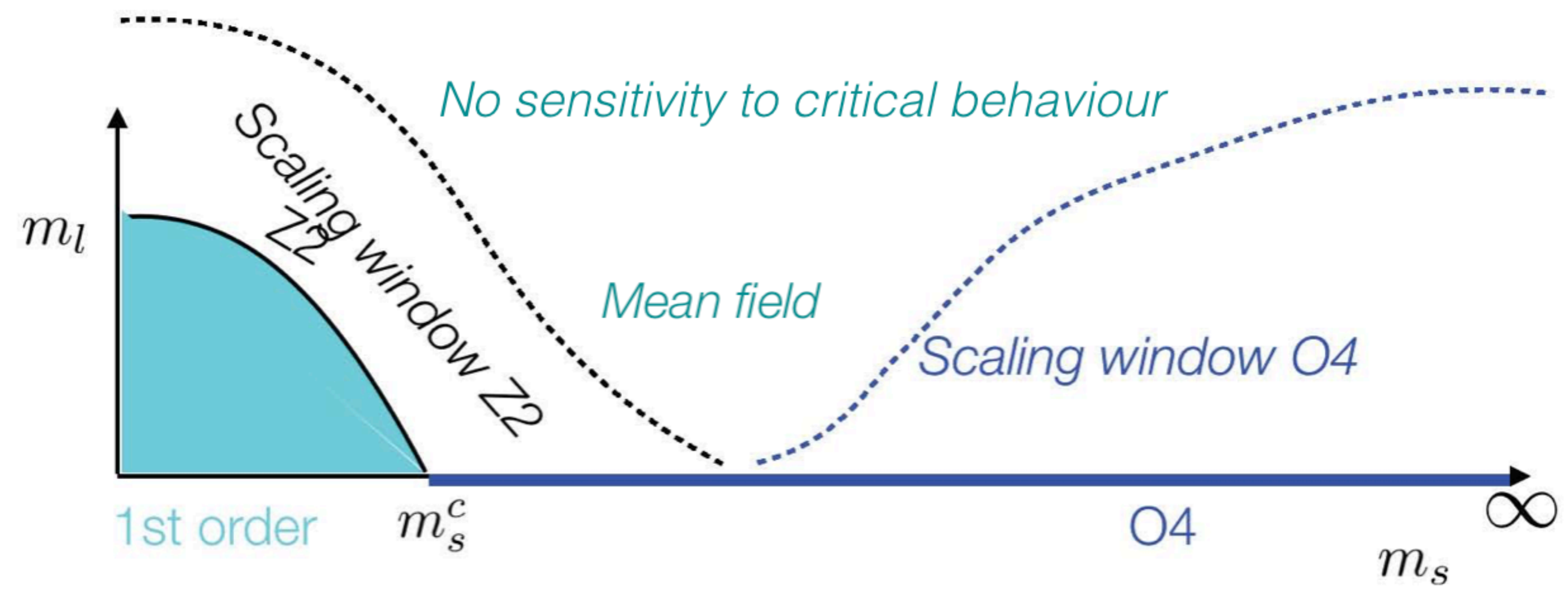


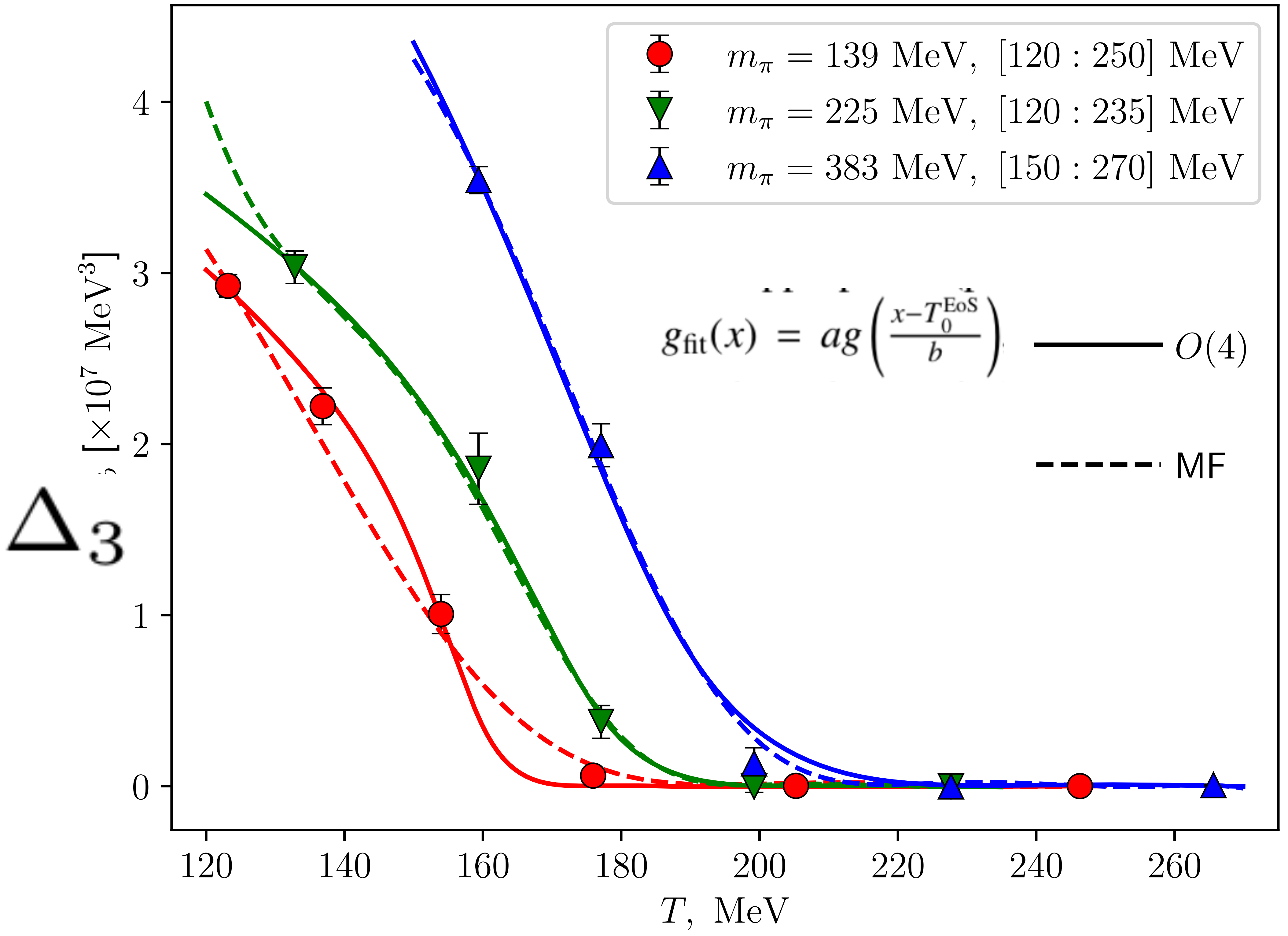
Searching for the scaling window in mass

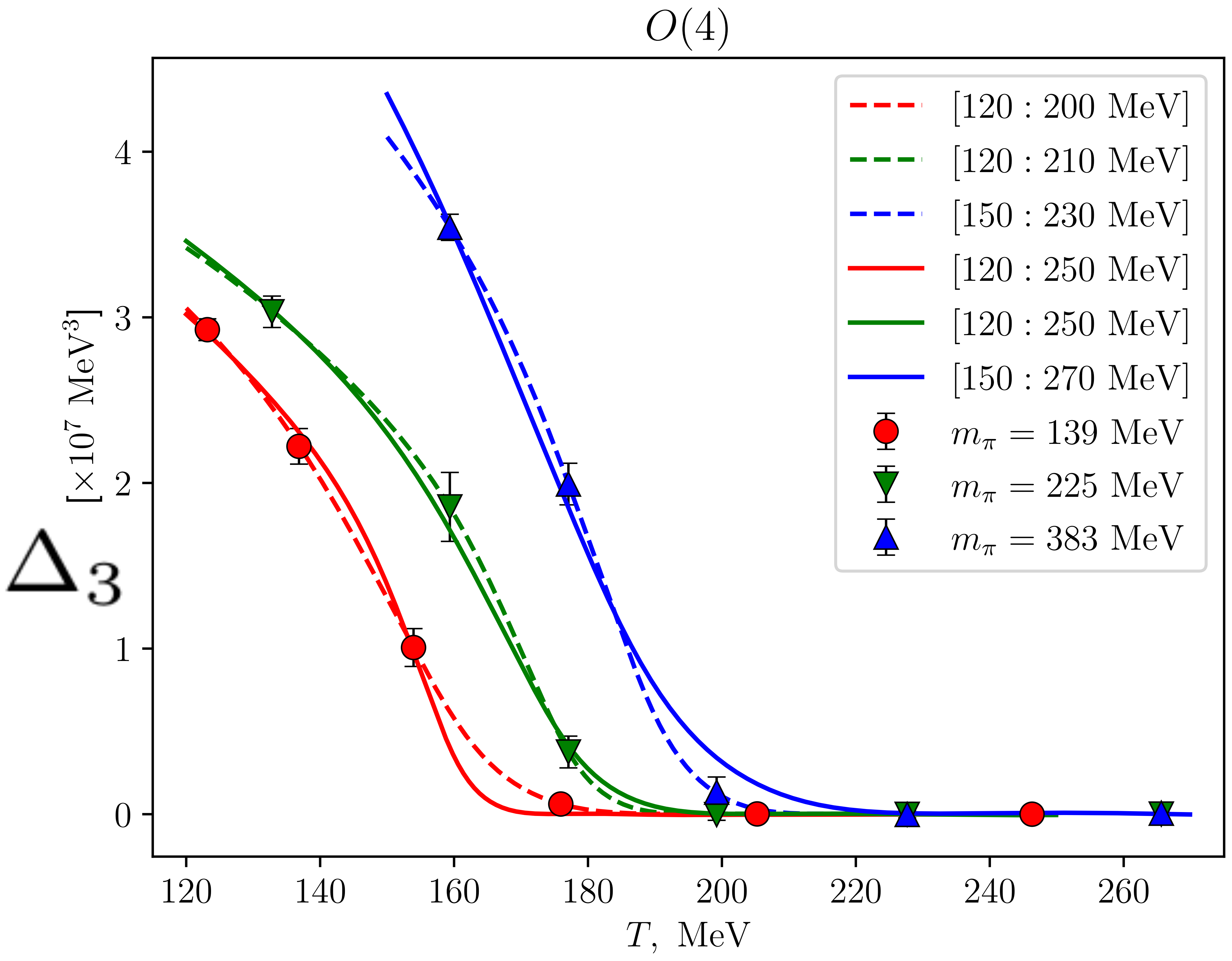
O(4) or mean field?

Unrealistic T0 from O4 at high mass

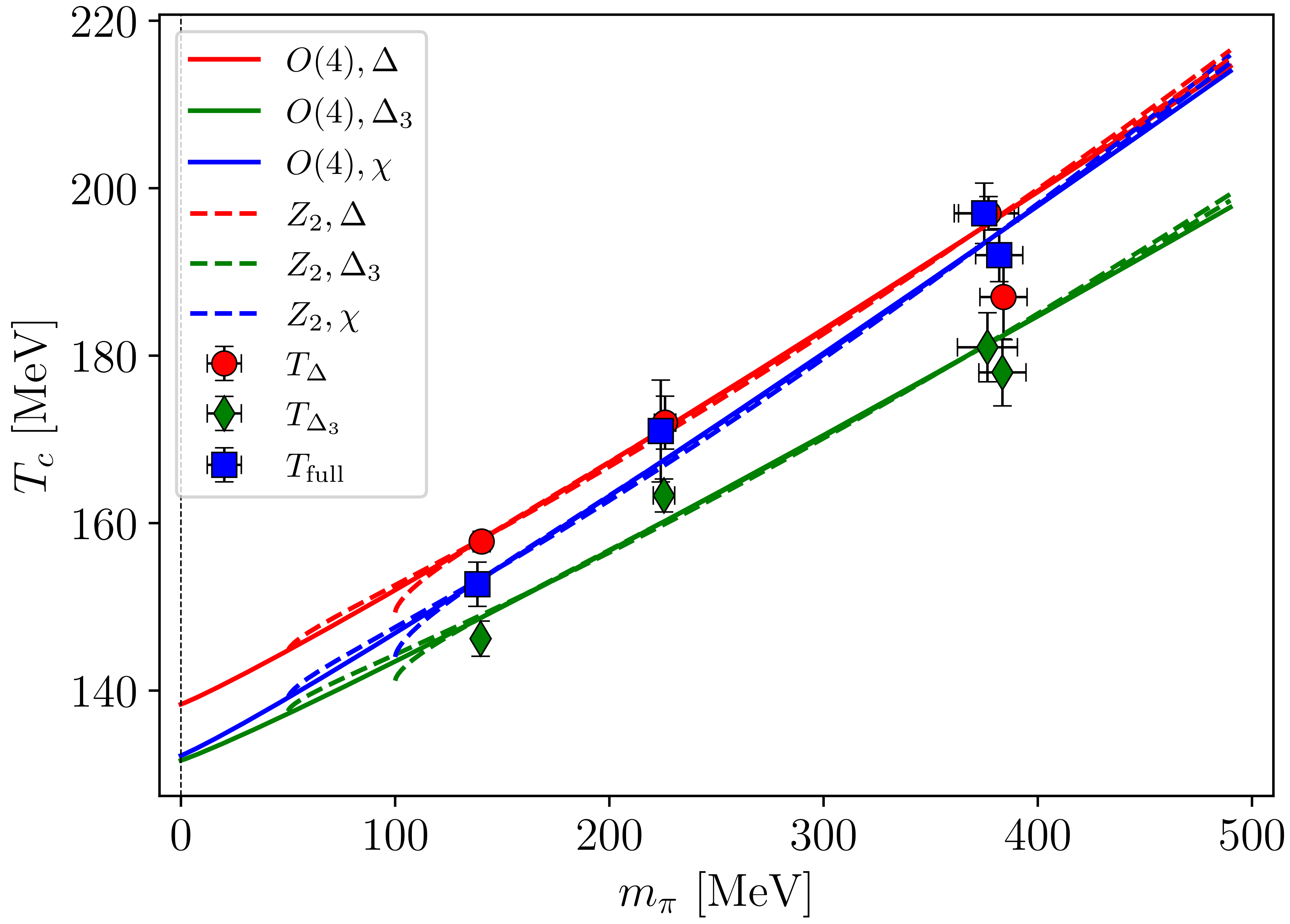
TEOS = 142(2),159(3), 174(2) MeV







Scaling of the pseudo critical temperatures



Check O4:

$$T_c(m_\pi) = T_0 + A z_p m_\pi^{2/\beta\delta}$$

Observable	T_0 [MeV]	$z_p/z_{\bar\psi\psi_3}$	$z_p/z_{\bar\psi\psi_3}$ $O(4)$	z_p $O(4)$
χ	132(4)	1.24(17)	2.45(4)	1.35(3)
$\langle\bar\psi\psi\rangle$	138(2)	1.15(24)	1.35(7)	0.74(4)
$\langle\bar\psi\psi\rangle_3$	132(3)	1	1	0.55(1)

O4 vs Z2

$$T_c(m_\pi) = T_0 + B(m_\pi^2 - m_c^2)^{1/\beta\delta}$$

m_c = 100 MeV still OK

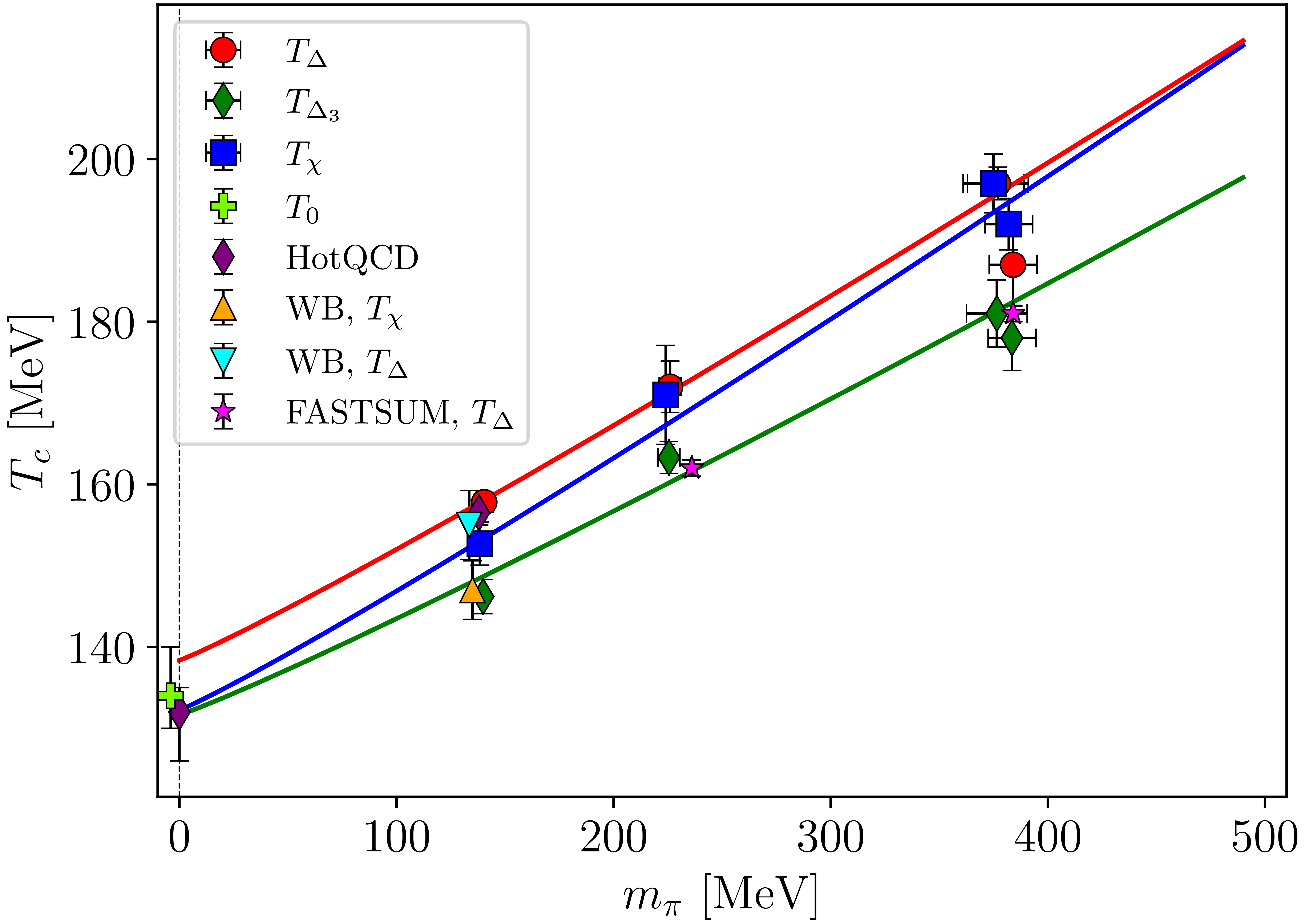
m_c = 0 still OK, indistinguishable from O4

Consistent (not a proof) with O4

Robust extrapolation:

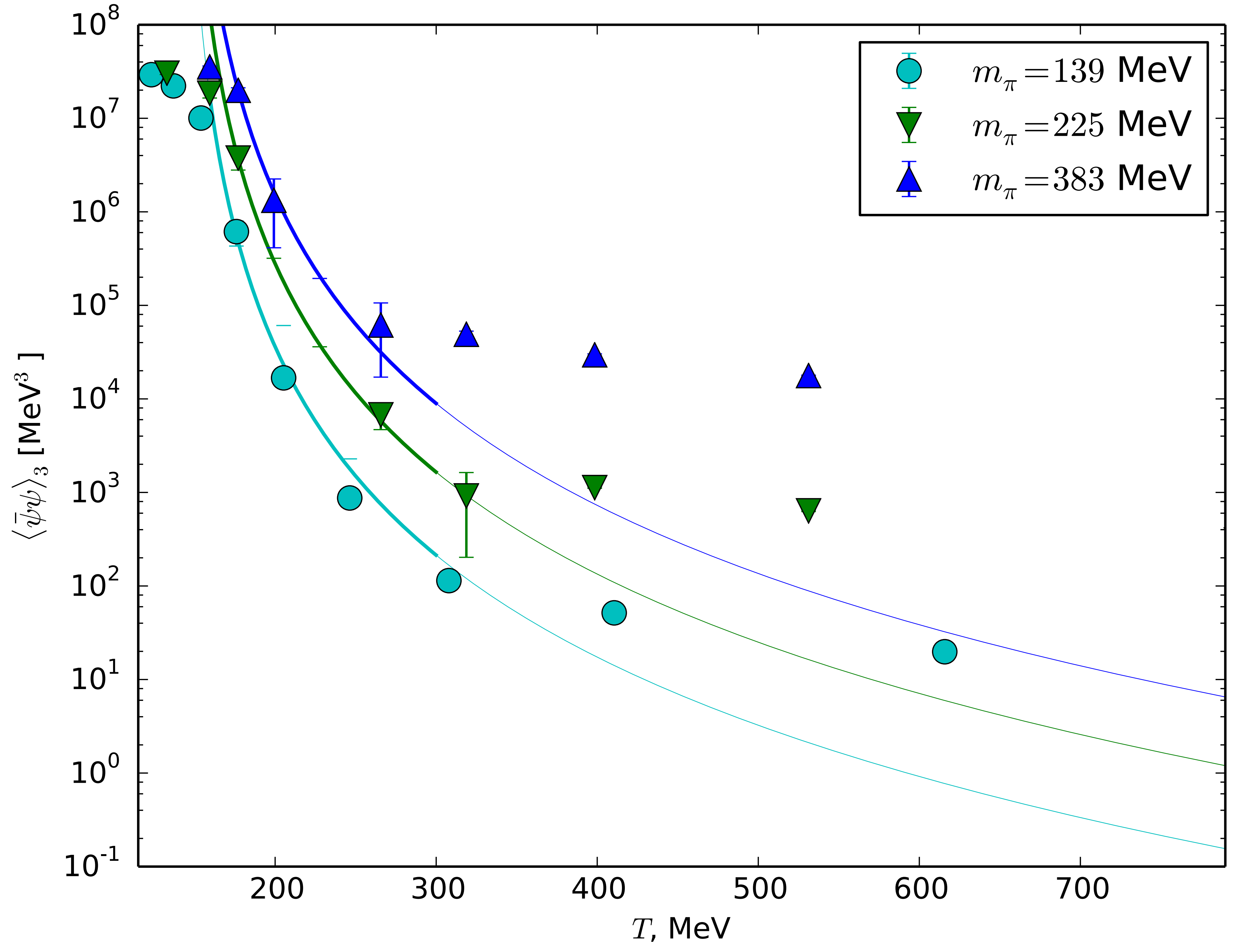
$$T_0 \equiv T_c(m_\pi \to 0) = 134^{+6}_{-4} \text{ MeV}$$

Comparisons: pseudo critical temperatures, and chiral extrapolation



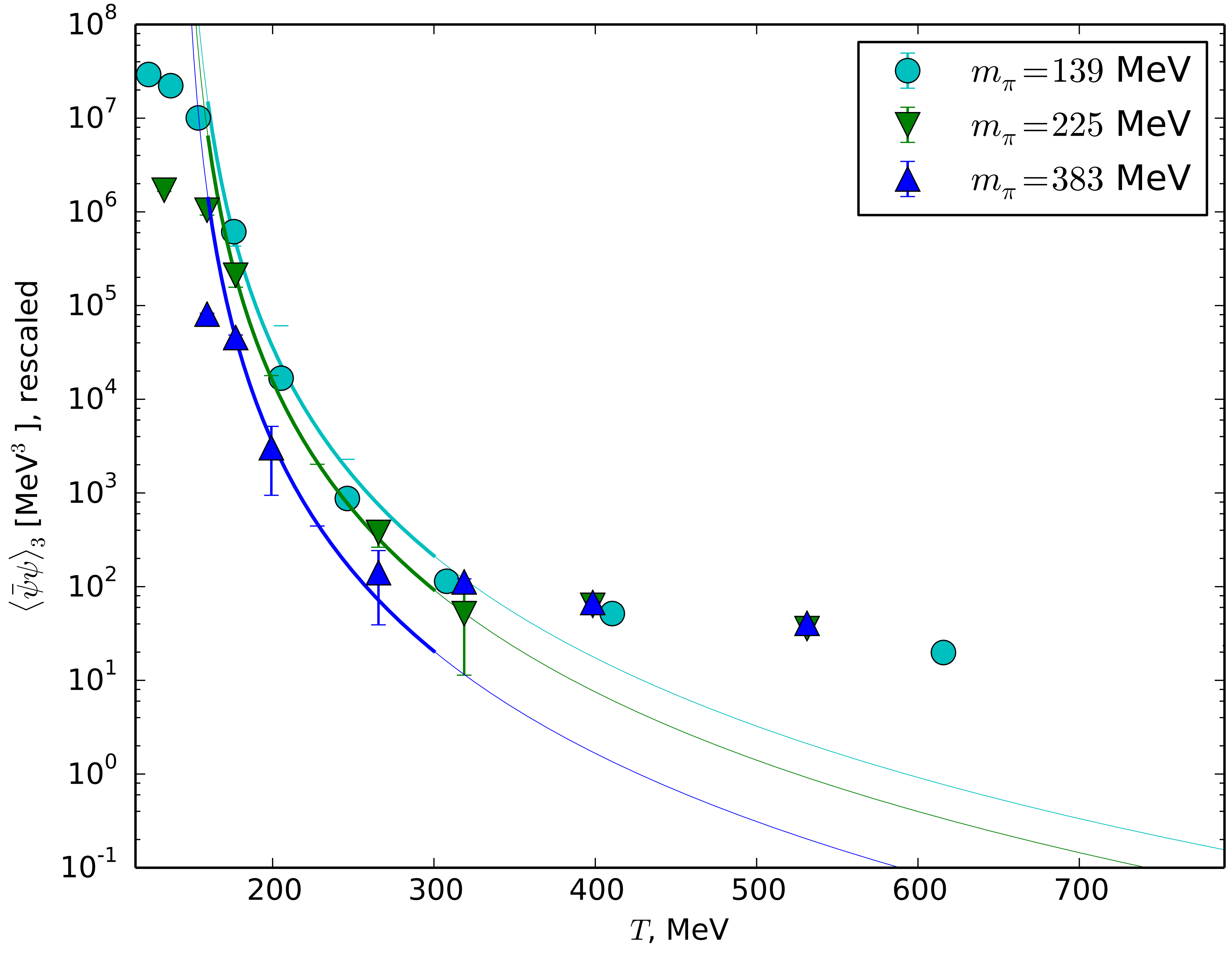
Searching for the scaling window in temperature

‘Forgotten’ microscopic dynamics



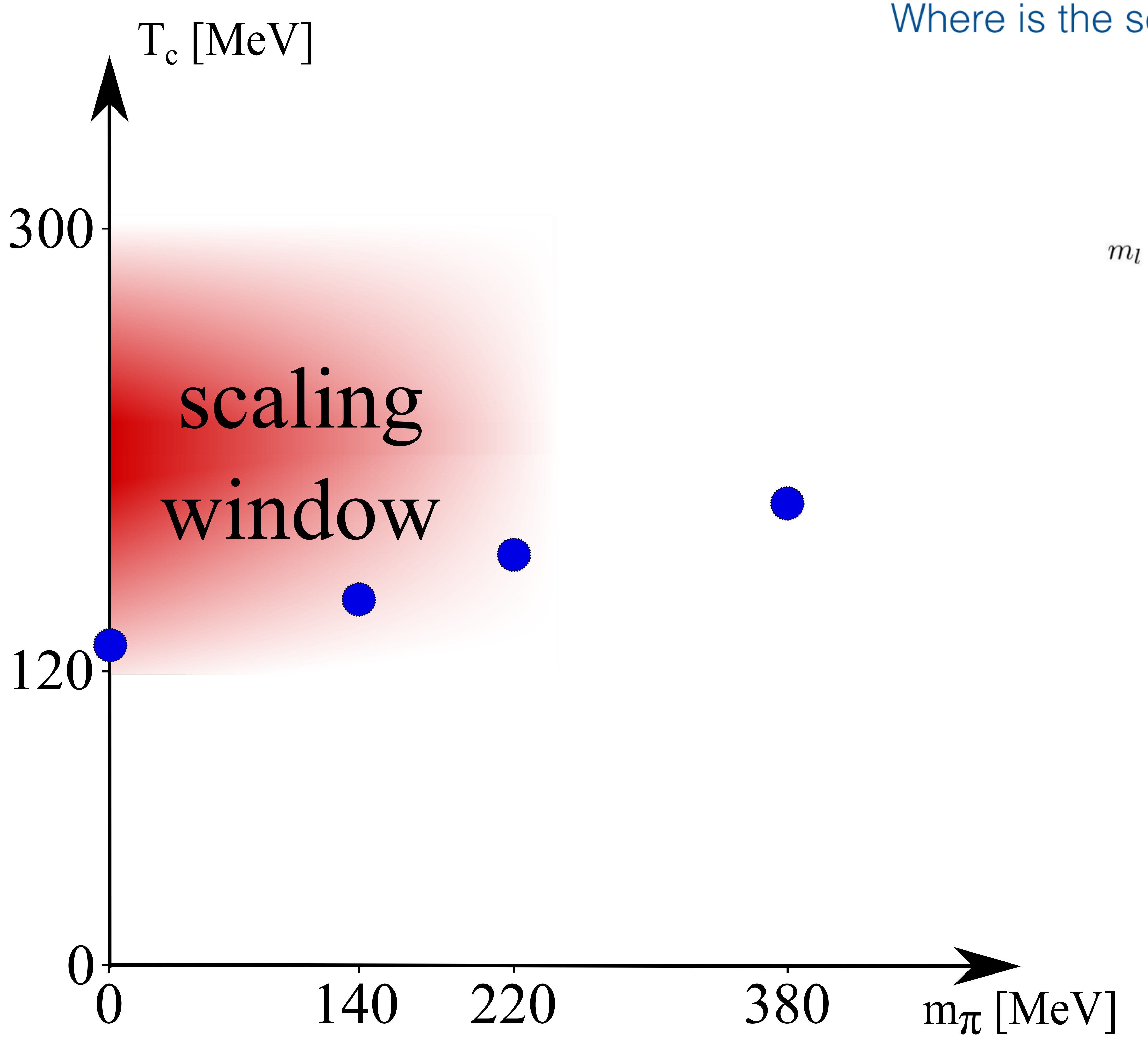
$$\Delta_3 \propto t^{-\gamma-2\beta\delta}$$ T < 300 MeV

‘Forgotten’ critical behaviour..

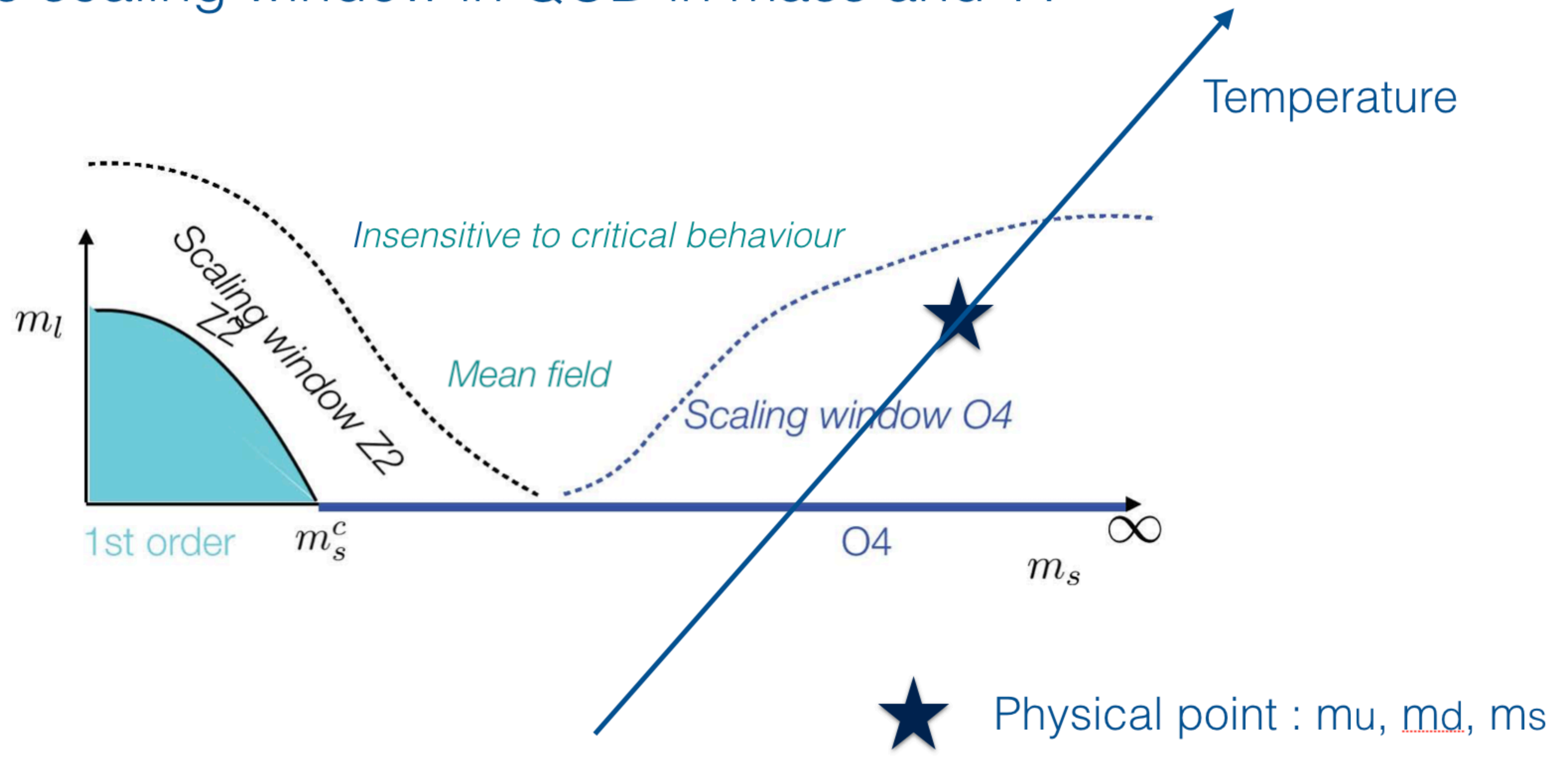


$$\Delta_3 \propto m_\pi^6$$ T > 300 MeV

A sketch of the scaling window for physics strange mass



Where is the scaling window in QCD in mass and T?



Beyond the scaling window: a threshold in the QGP?

A few lattice studies
indicate a possible fast crossover at a
temperature of about T_{YM} :

See talk by L. Glozman

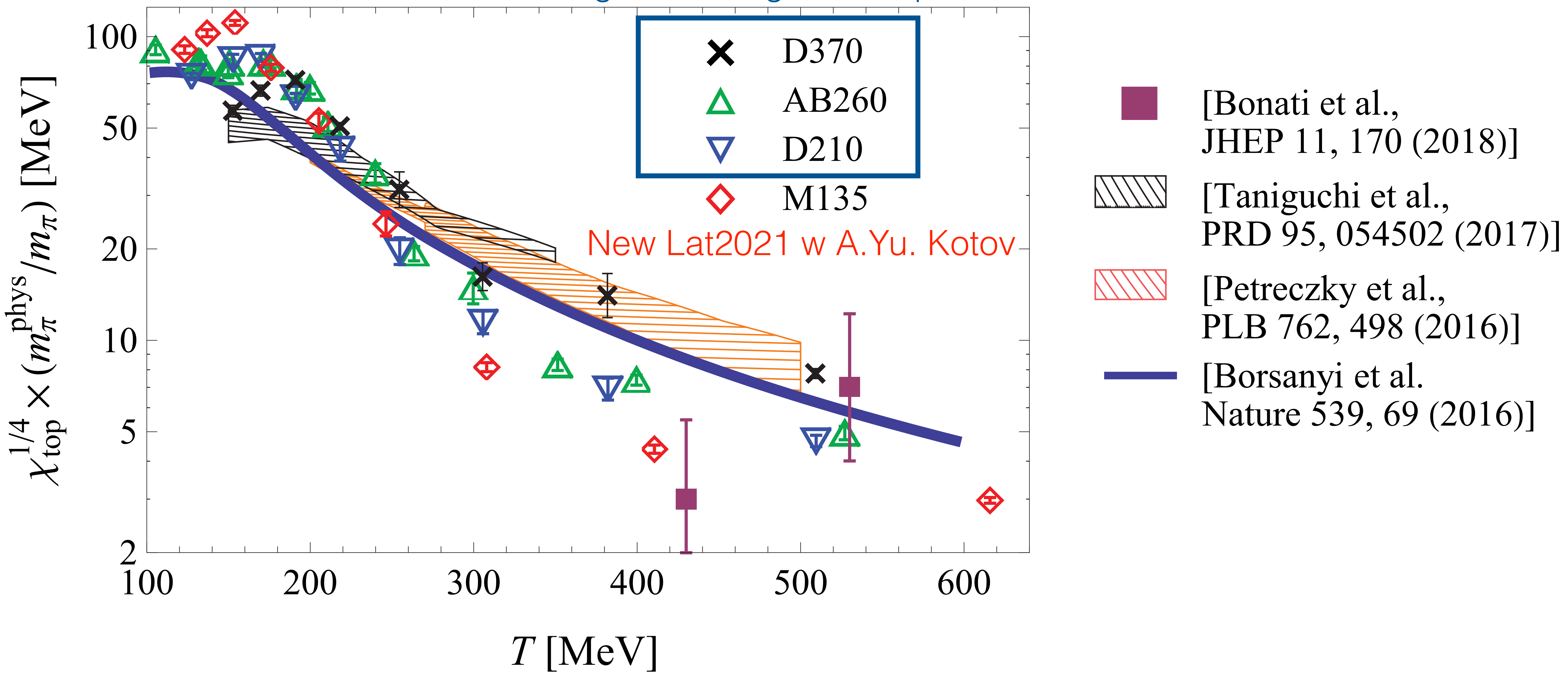
Around the same temperature we observe the limit of the scaling window

Further, we observe a change of behaviour in the topological susceptibility

Results for physical pion mass from rescaling

$T^{4-\beta_0}\left(\frac{m}{T}\right)^{N_f}$

F. Burger, E.-M. Ilgenfritz,MpL, A. Trunin, PRD2018

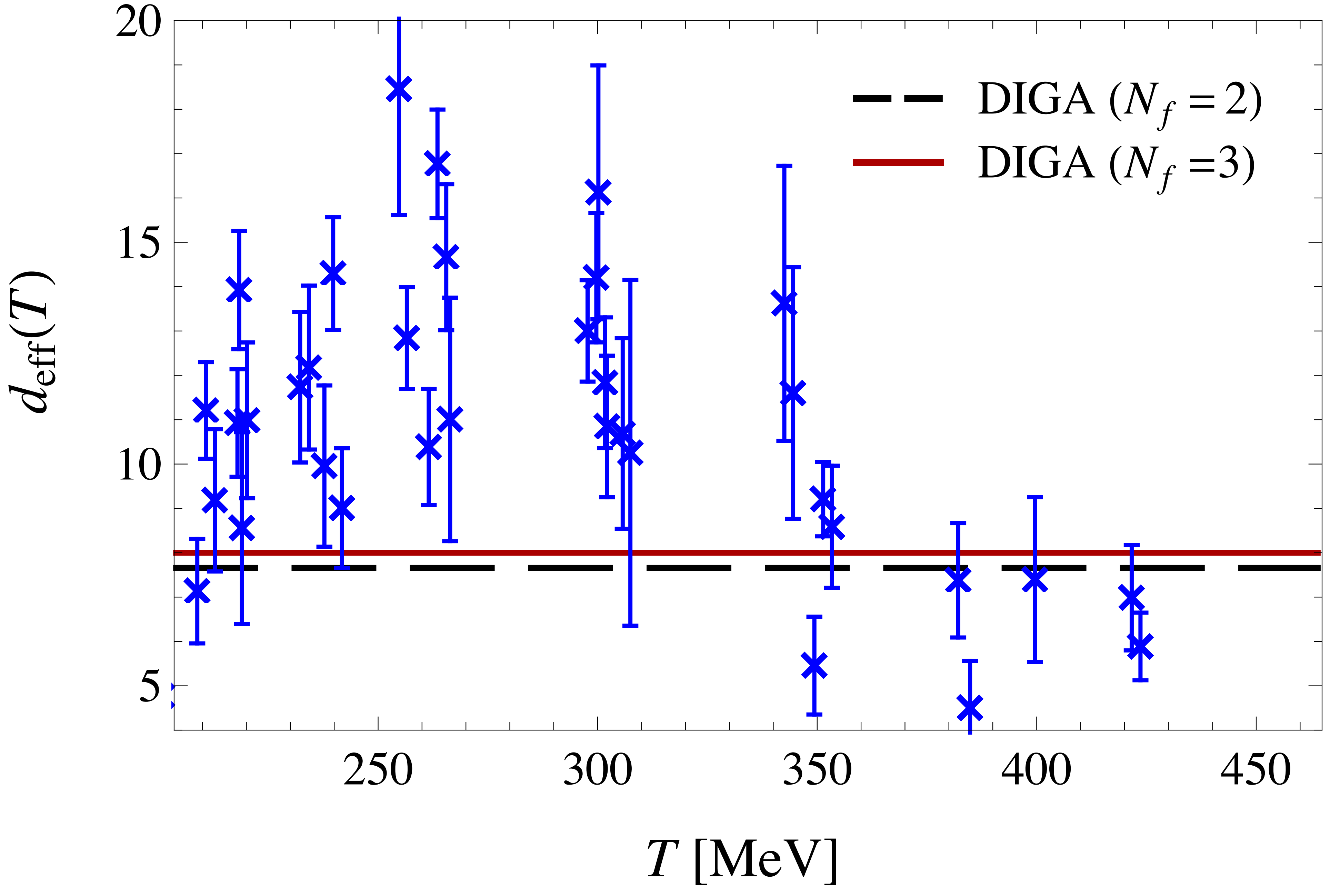


Power-law decay?

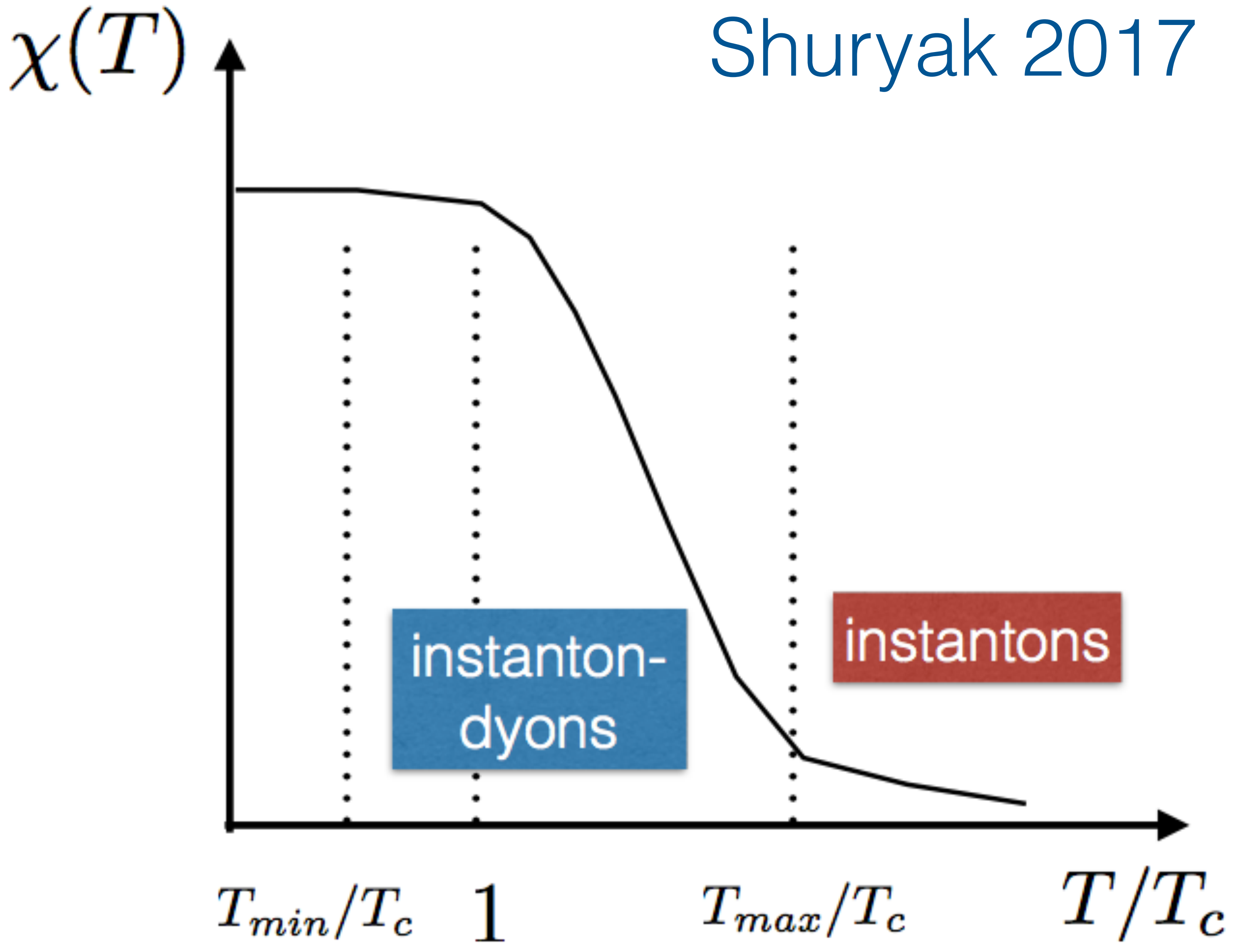
$$\chi^{0.25}(T) = aT^{-d(T)}$$

For instanton gas $\quad d(T) \equiv const \simeq (7 + \frac{N_f}{3})$

$$d(T) = -T\frac{d}{dT}\ln\chi^{0.25}(T)$$

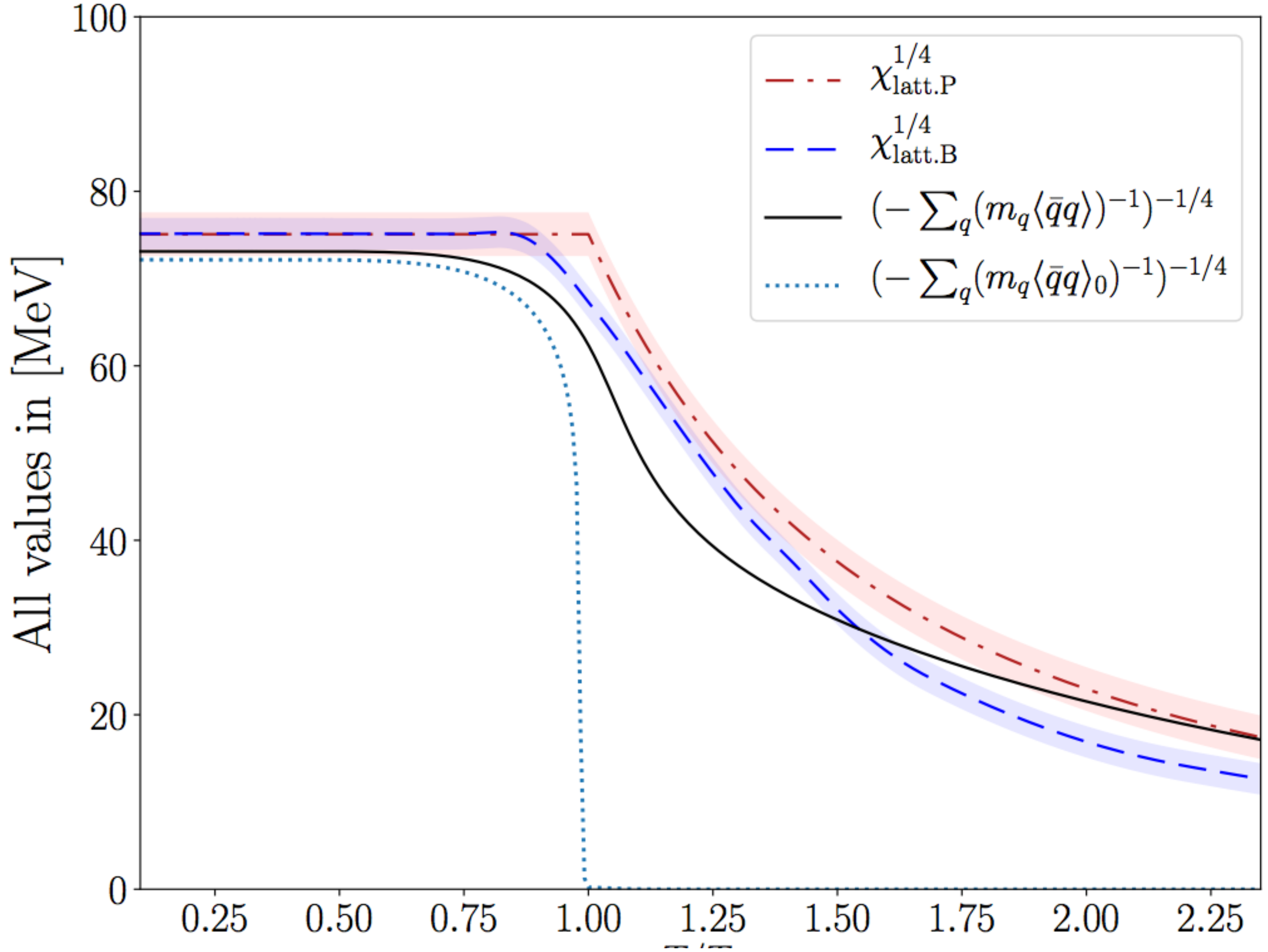


Possibly consistent with instant -dyon? Shuryak 2017



Faster decrease before DIGA sets in

DIGA incompatible with critical scaling ?



Is chiral symmetry driving axial symmetry?

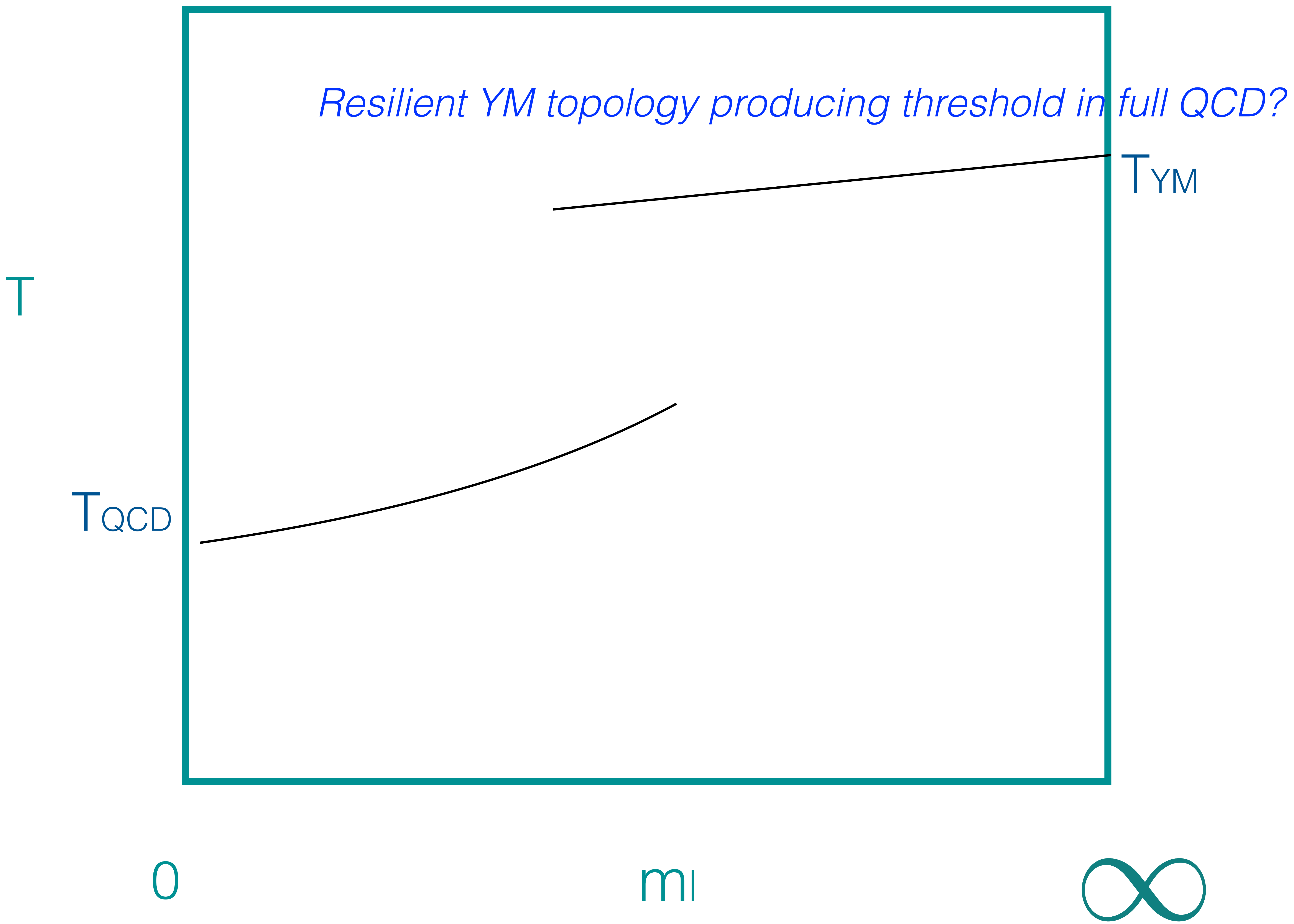
Horvatic et al (2020)

$$\chi_{QCD} = \frac{\chi_{YM}}{1 + a \sum_{i=1}^{N_f} \frac{1}{\mu_i^2(\theta)}} = \chi_{YM} \left(1 - \frac{\chi_{YM}}{\sum_{k=1}^{N_f} (m_i \langle \bar{\psi}\psi \rangle)} \right)^{-1}$$

A natural role for Yang-Mills dynamics??

Di Vecchia, Rossi, Veneziano, Yankielowicz
Gomez-Nicola, ..

…a speculation…



Summary

Consistency with 3D O4 scaling at lower masses, and $T < 300$ MeV -
Apparent O4 scaling at larger masses ruled out by EoS analysis.
Analysis helped by new order parameter

Three different methods to measure critical temperature in the chiral limit
-Conformal scaling
-From EoS analysy
-From the scaling of pseudo critical temperatures
Consistent results for T_0

Upper limit of the scaling window in temperature is close to the observed threshold in the QCD. The same threshold is visible in results for the topological susceptibility.