The QCD chiral phase transition for different numbers of quark flavours

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- Light mass regime in the Columbia plot: contradicting results?
- No, new data suggest resolution and modified Columbia plot

[Cuteri, O.P., Sciarra, arxiv2107.12739]

CRC-TR 211 Strong-interaction matter under extreme conditions
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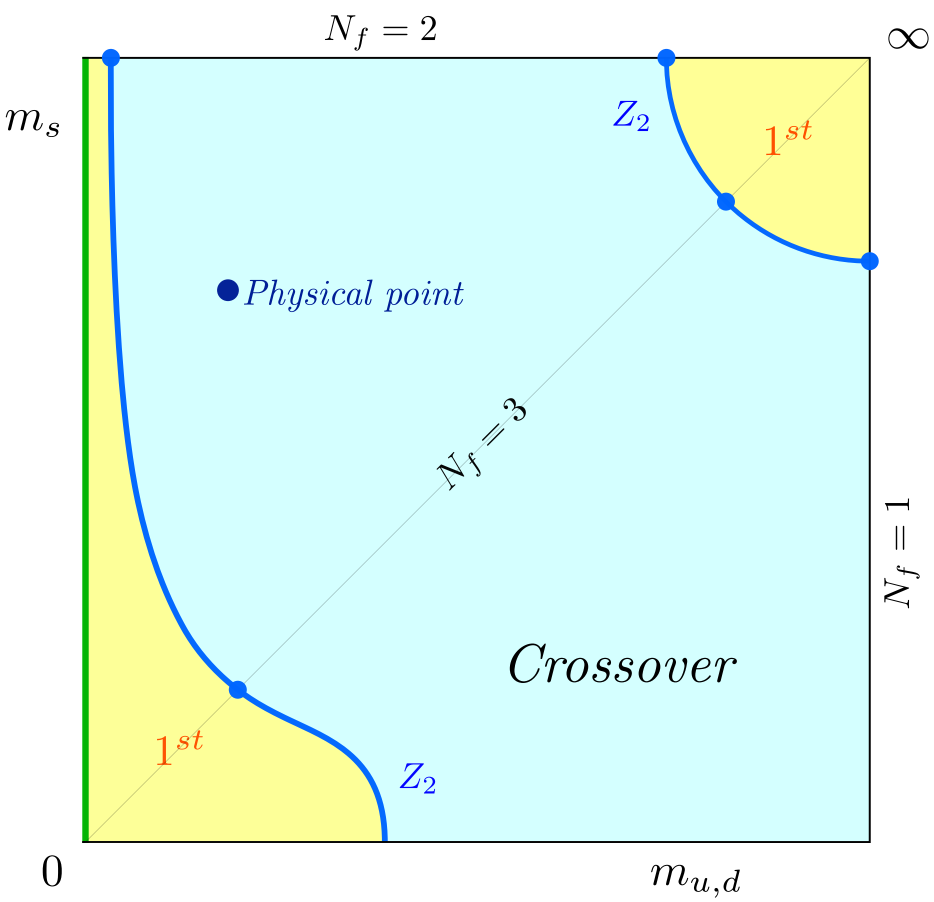
NIC

The nature of the QCD thermal transition

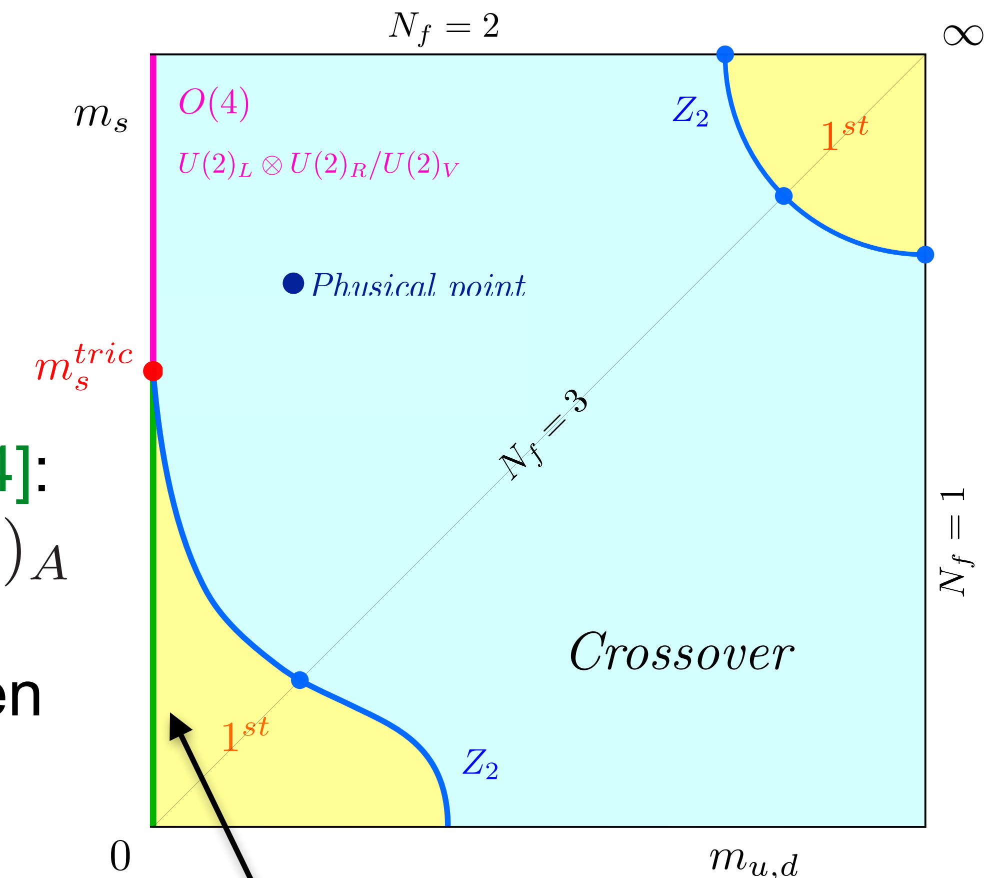
deconfinement p.t.:
breaking of global $Z(3)$ symmetry



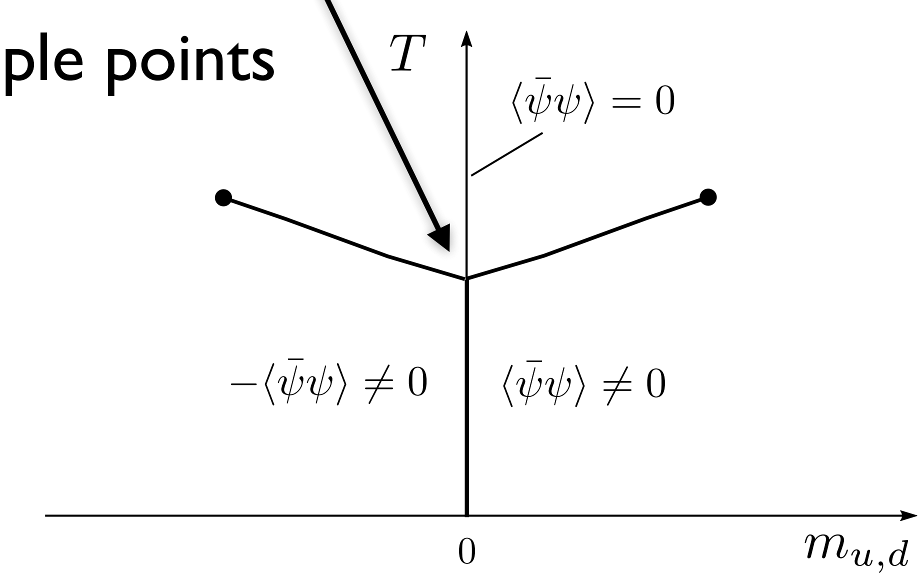
[Pisarski, Wilczek, PRD 84]:
$N_f = 2$ depends on $U(1)_A$

restored | broken

$N_f \geq 3$ 1st order



triple points



chiral p.t.
restoration of global symmetry in flavour space

$$SU(2)_L \times SU(2)_R \times U(1)_A$$

anomalous

Long history, key results

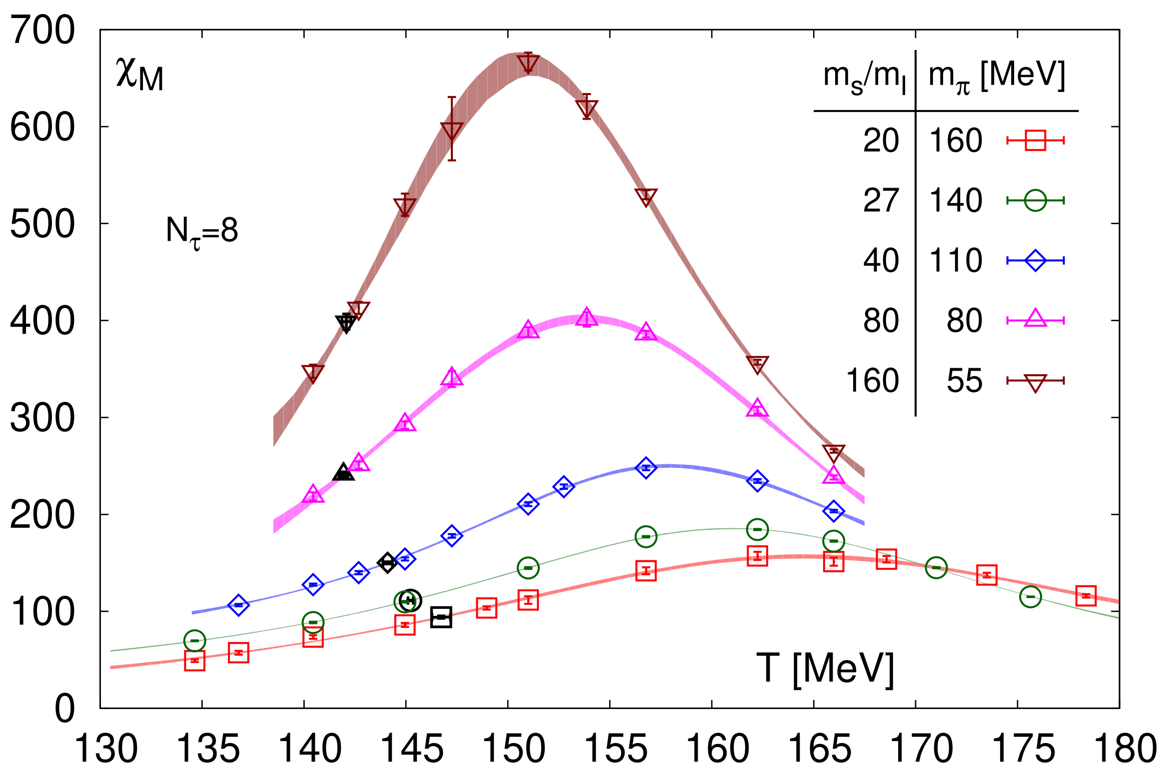
- [Pisarski, Wilczek PRD 84]
  Universality, 3D sigma models: 1st order for $N_f \geq 3$
  $N_f = 2$ depends on $U(1)_A$

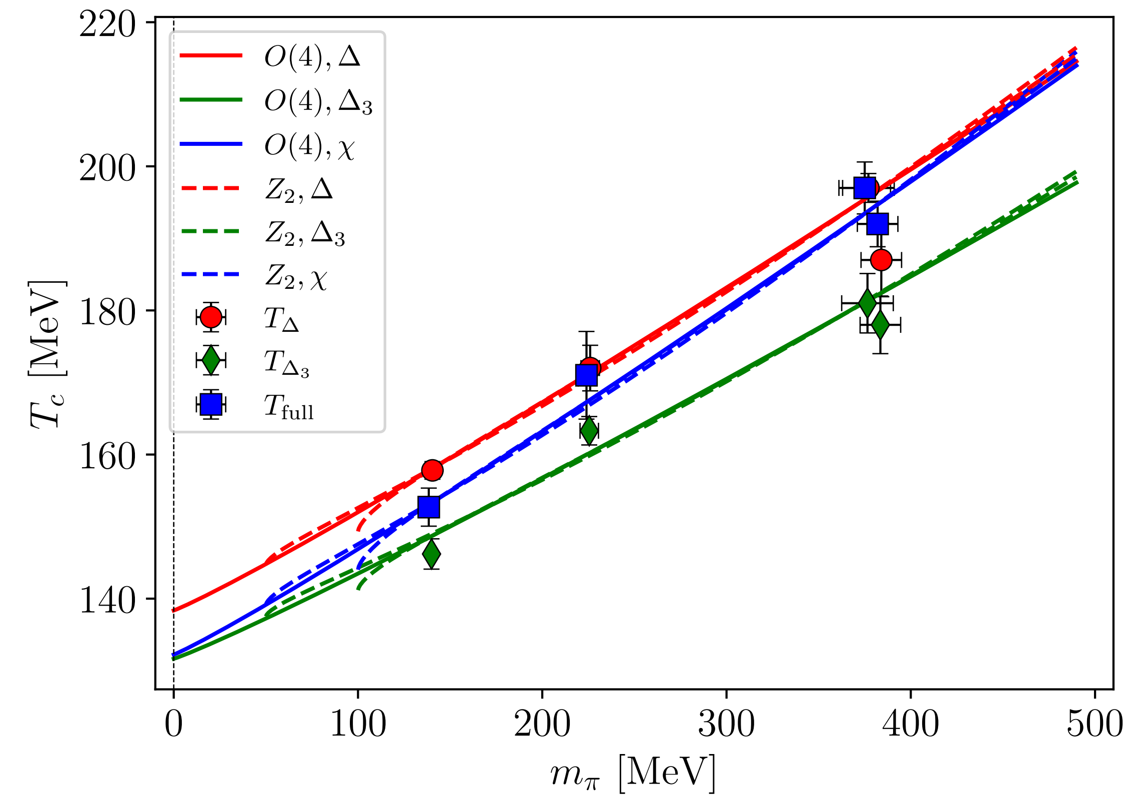
- Shrinking of 1st-order region towards continuum, will something remain?:
  Standard staggered $N_f = 2$ [Bonati et al. PRD 14, Cuteri et al. PoS LAT 18]
  $N_f = 3$ [de Forcrand, O.P. PoS LAT 07]
  $N_f = 4$ [de Forcrand, D'Elia PoS LAT 16]

  O(a)-improved Wilson $N_f = 3$ [Jin et al. PRD 15,17; Kuramashi et al. PRD 20]
  $N_f = 4$ [Ohno et al. PoS LAT 18]

- No 1st order transition seen at all:
  HISQ, $N_f = 3, m_{PS} \geq 50$ MeV [Bazavov et al. PRD 17]

From the physical point to the chiral limit





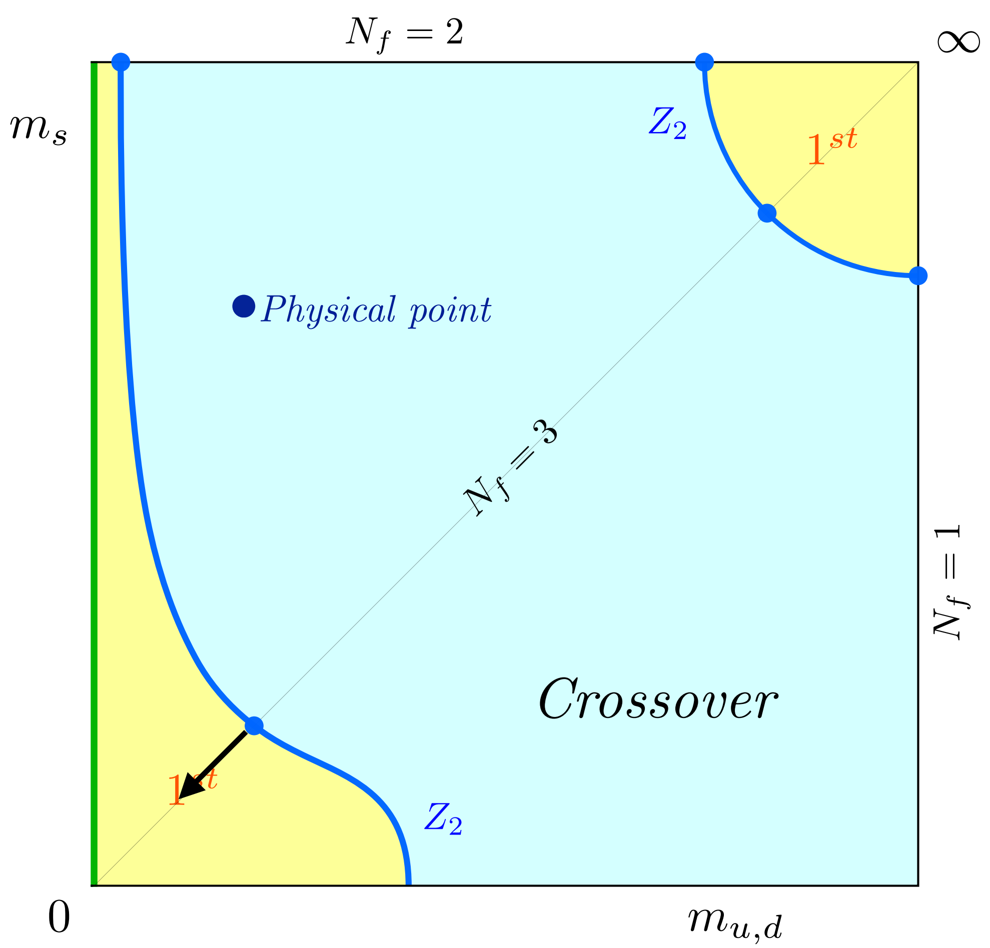
[HotQCD, PRL 19] HISQ (staggered) [Kotov, Lombardo, Trunin, 21] Wilson twisted mass

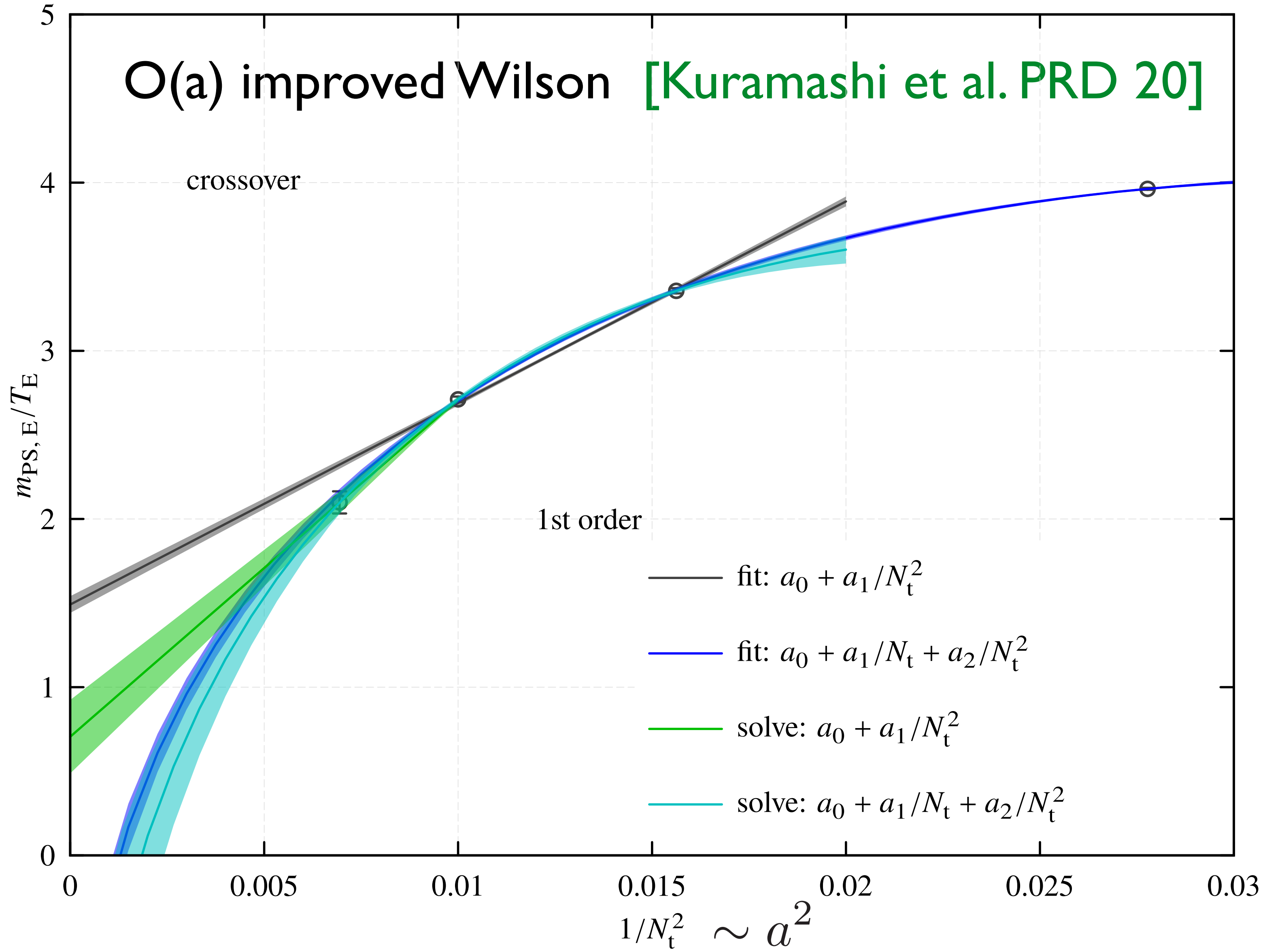
$$T_c^0 = 132^{+3}_{-6}\ \mathrm{MeV} \qquad T_{pc}(m_l) = T_c^0 + K\, m_l^{1/\beta\delta} \qquad T_c^0 = 134^{+6}_{-4}\ \mathrm{MeV}$$

- Keep strange quark mass fixed, crossover gets stronger as chiral limit approached
- Cannot distinguish between Z(2) vs. O(4) exponents, need exponential accuracy!
- Determination of chiral critical temperature possible, but not the order of the transition

The nature of the QCD chiral transition, Nf=3

…has enormously large cut-off effects!



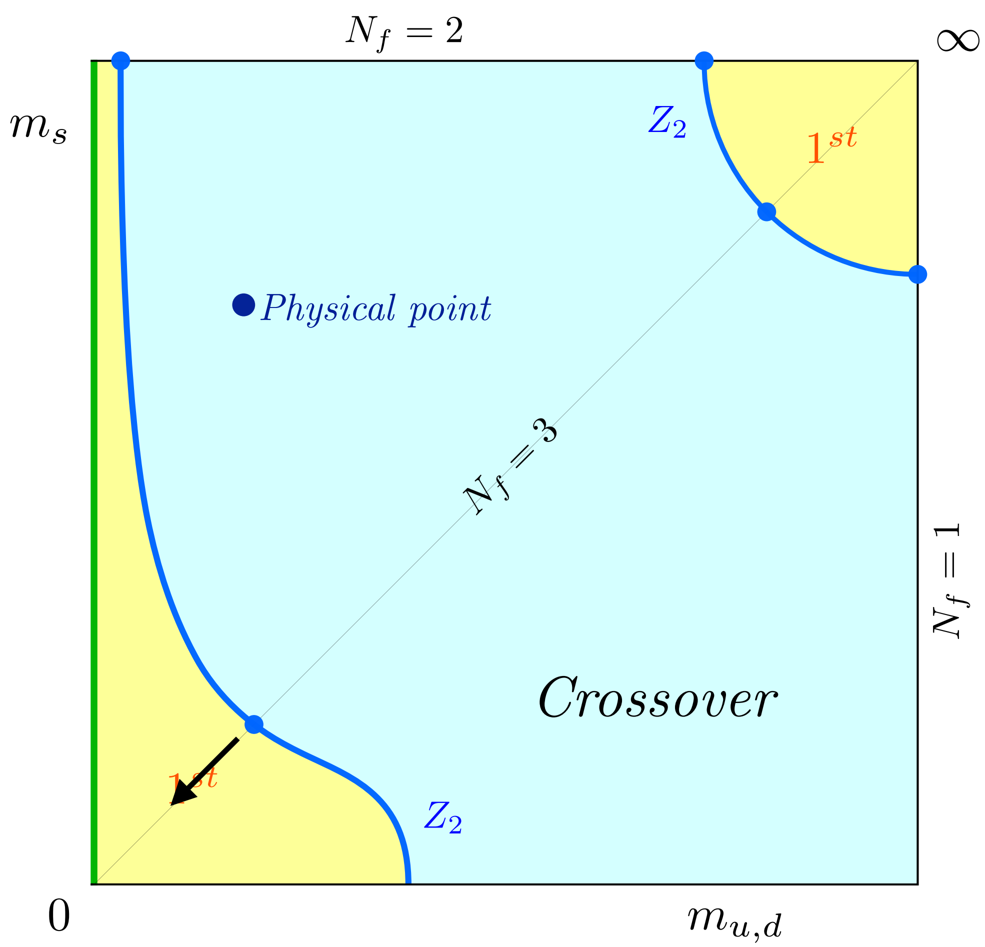


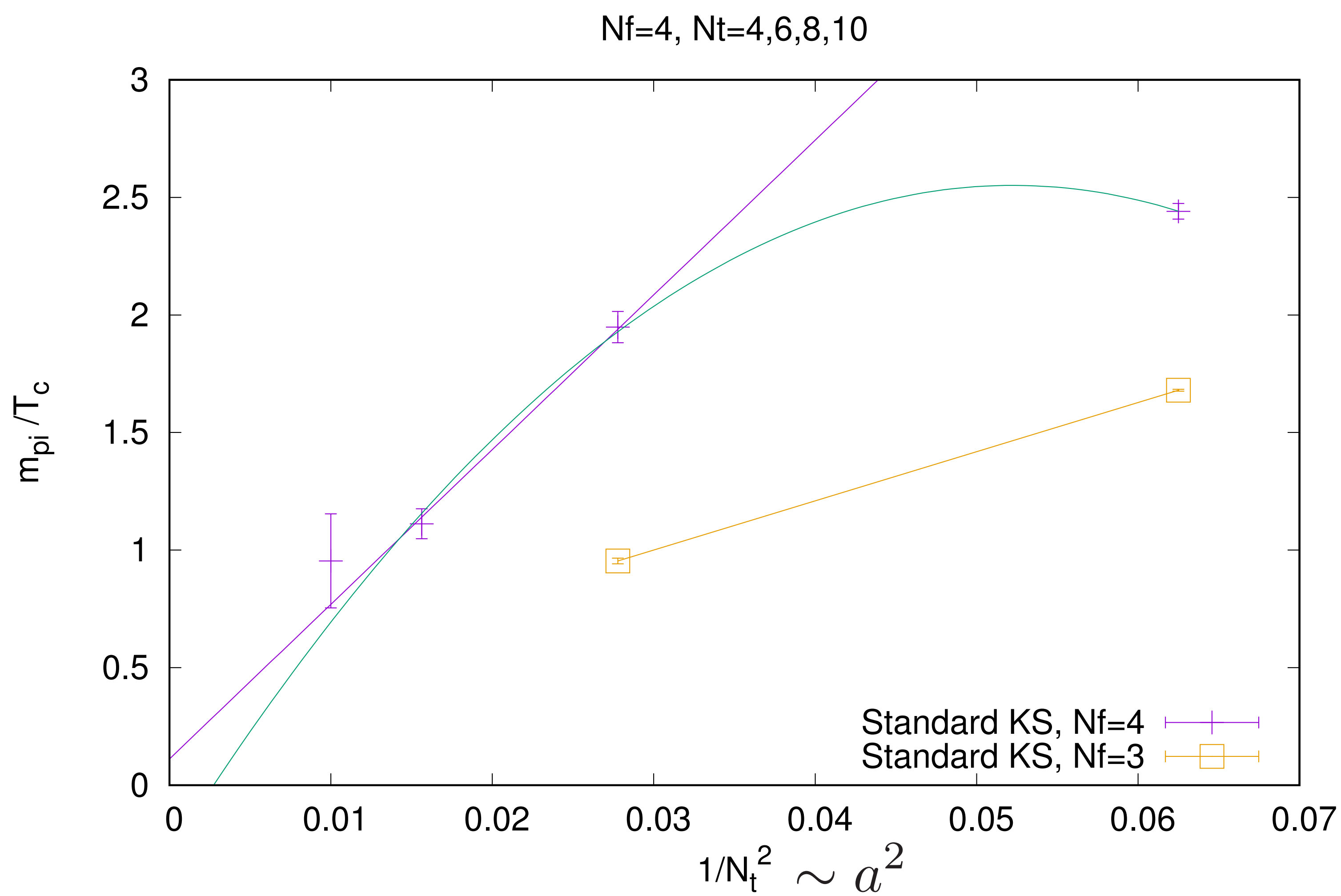
O(a)-improved Wilson:
1st order region shrinks for $a \to 0$

$m_\pi^c \leq 110$ MeV $\quad N_\tau = 4, 6, 8, 10, 12$

The nature of the QCD chiral transition, Nf=3,4

…has enormously large cut-off effects!





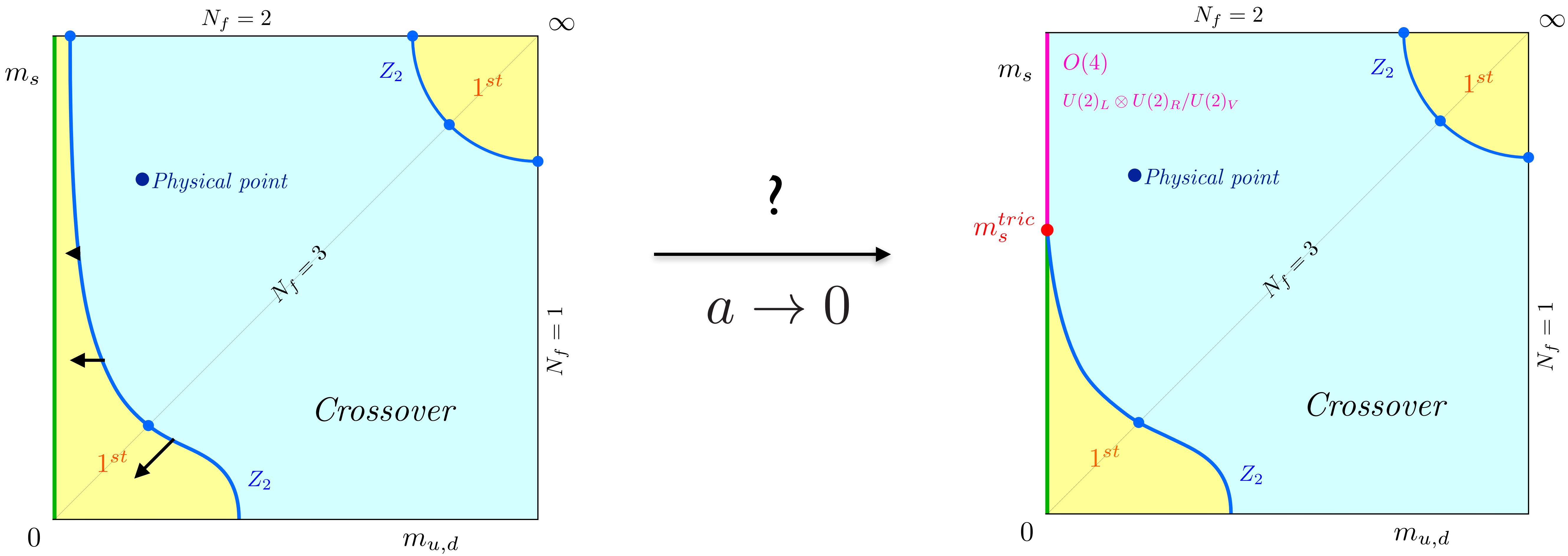
Unimproved staggered:

1st order region shrinks for $a \to 0$, both for $N_f = 3, 4$ [de Forcrand, D'Elia, PoS LAT 17]

No 1st order seen for improved staggered actions, even $N_f = 3$ [HotQCD PRD 17]

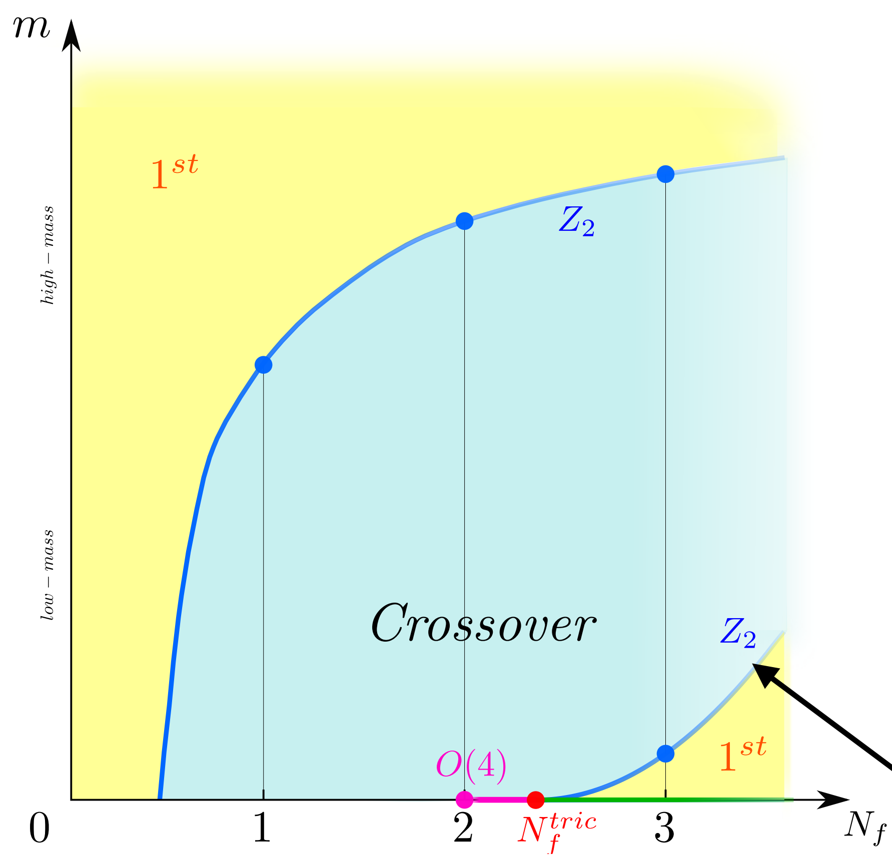
The nature of the QCD chiral transition

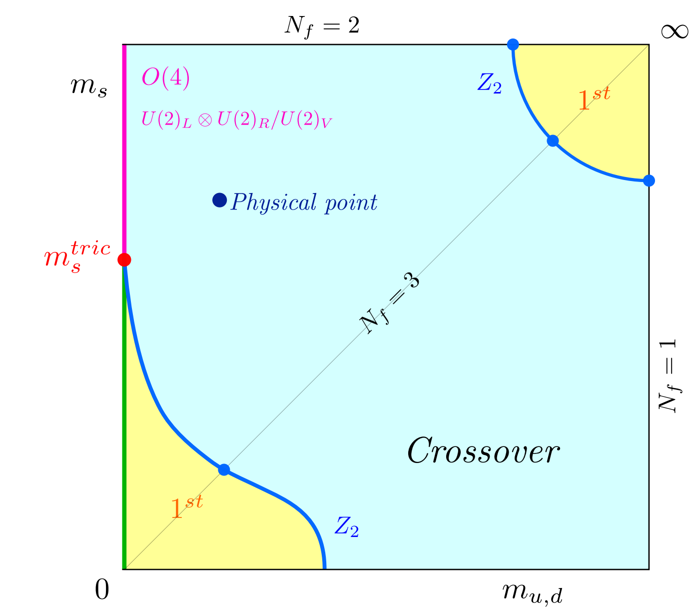
…is elusive, massless limit not simulable!



- Coarse lattices or unimproved actions: 1st order for $N_f = 2, 3$
- 1st order region shrinks rapidly as $a \to 0$
- Improved staggered actions: no 1st order region so far, even $N_f = 3$

Different view point: mass degenerate quarks





$$Z(N_\mathrm{f}, g, m) = \int \mathcal{D}A_\mu \, (\det M[A_\mu, m])^{N_\mathrm{f}} \, e^{-\mathcal{S}_{\mathrm{YM}}[A_\mu]}$$

$$N_\mathrm{f}^c(am) = N_\mathrm{f}^{\mathrm{tric}} + \mathcal{B}_1 \cdot (am)^{2/5} + \mathcal{O}\big((am)^{4/5}\big)$$

- Consider analytic continuation to continuous N_f
- Tricritical point guaranteed to exist if there is 1st order at any N_f
- Known exponents for critical line entering tric. point!
- Continuation to $a \neq 0$: Z(2) surface ends in tricritical line

[Cuteri, O.P., Sciarra PRD 18]

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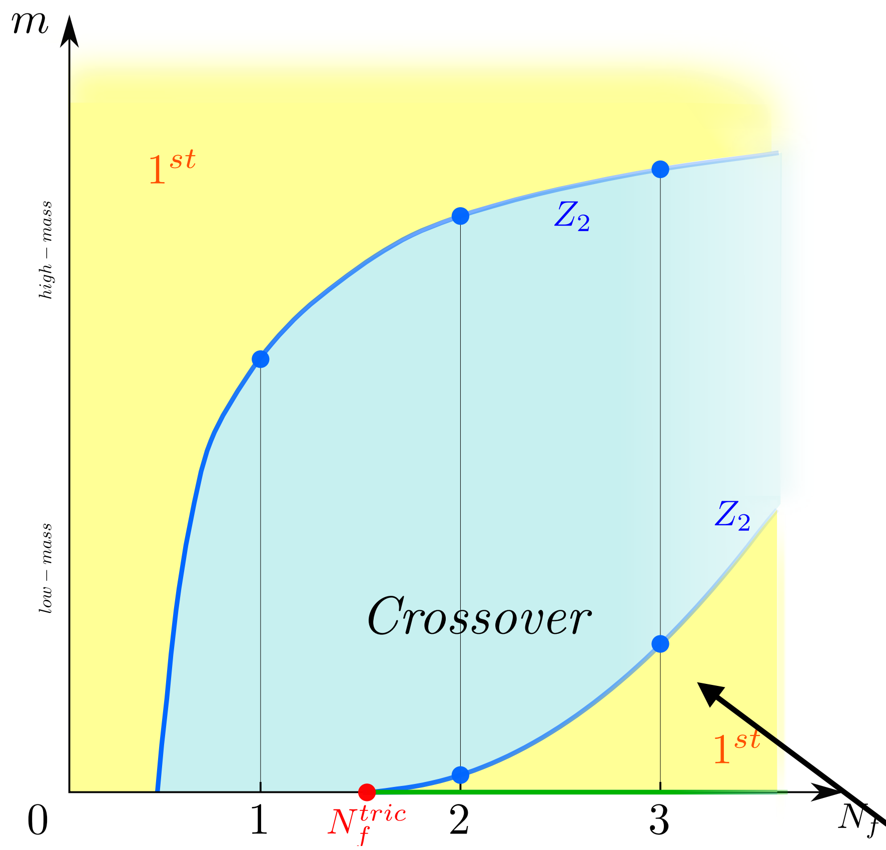

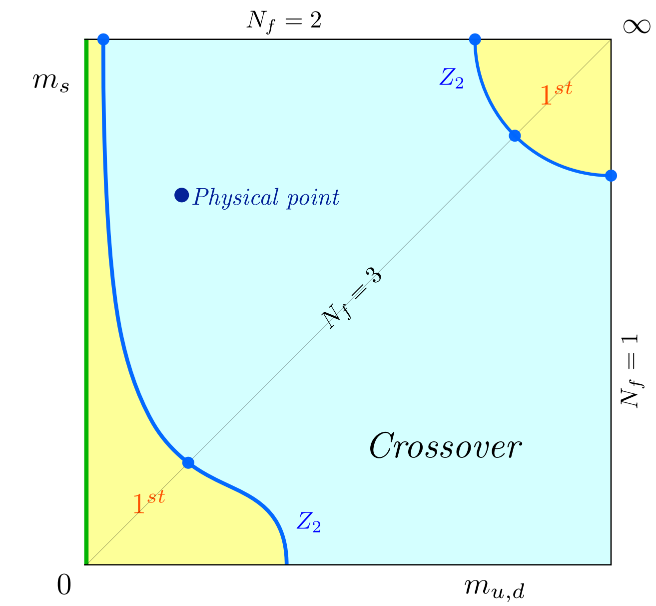

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[Cuteri, O.P., Sciarra PRD 18]

Methodology to determine order of transition

Finite size scaling of generalised cumulants

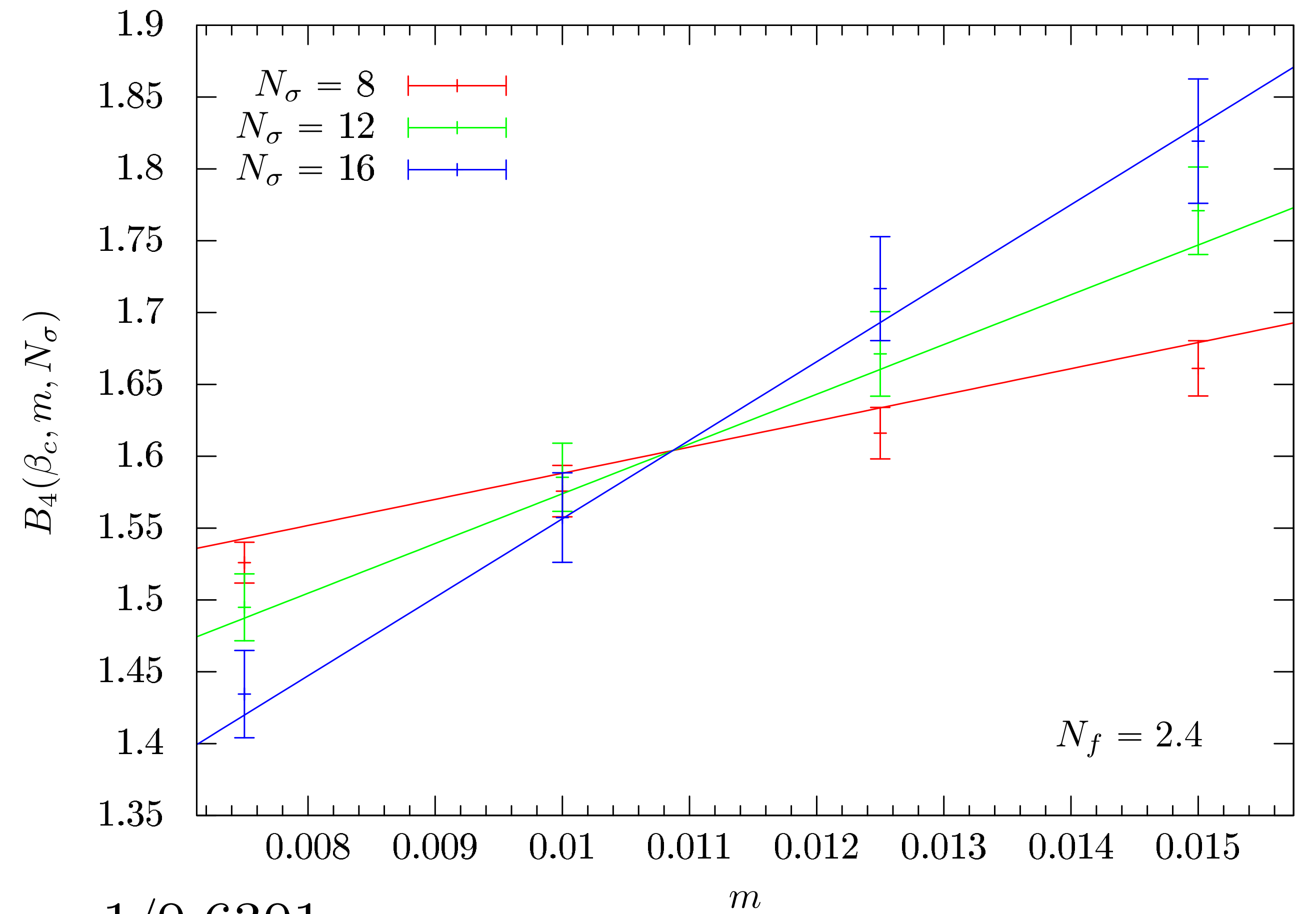
$$B_n = \frac{\langle (\bar\psi\psi - \langle\bar\psi\psi\rangle)^n\rangle}{\langle (\bar\psi\psi - \langle\bar\psi\psi\rangle)^2\rangle^{n/2}}$$

Standard staggered fermions, bare parameters: β, am, N_f, N_τ

(Pseudo-critical) phase boundary: $B_3 = 0$ **3d manifold**

Second-order 3D Ising:

2d chiral critical surface separates 1st order from crossover

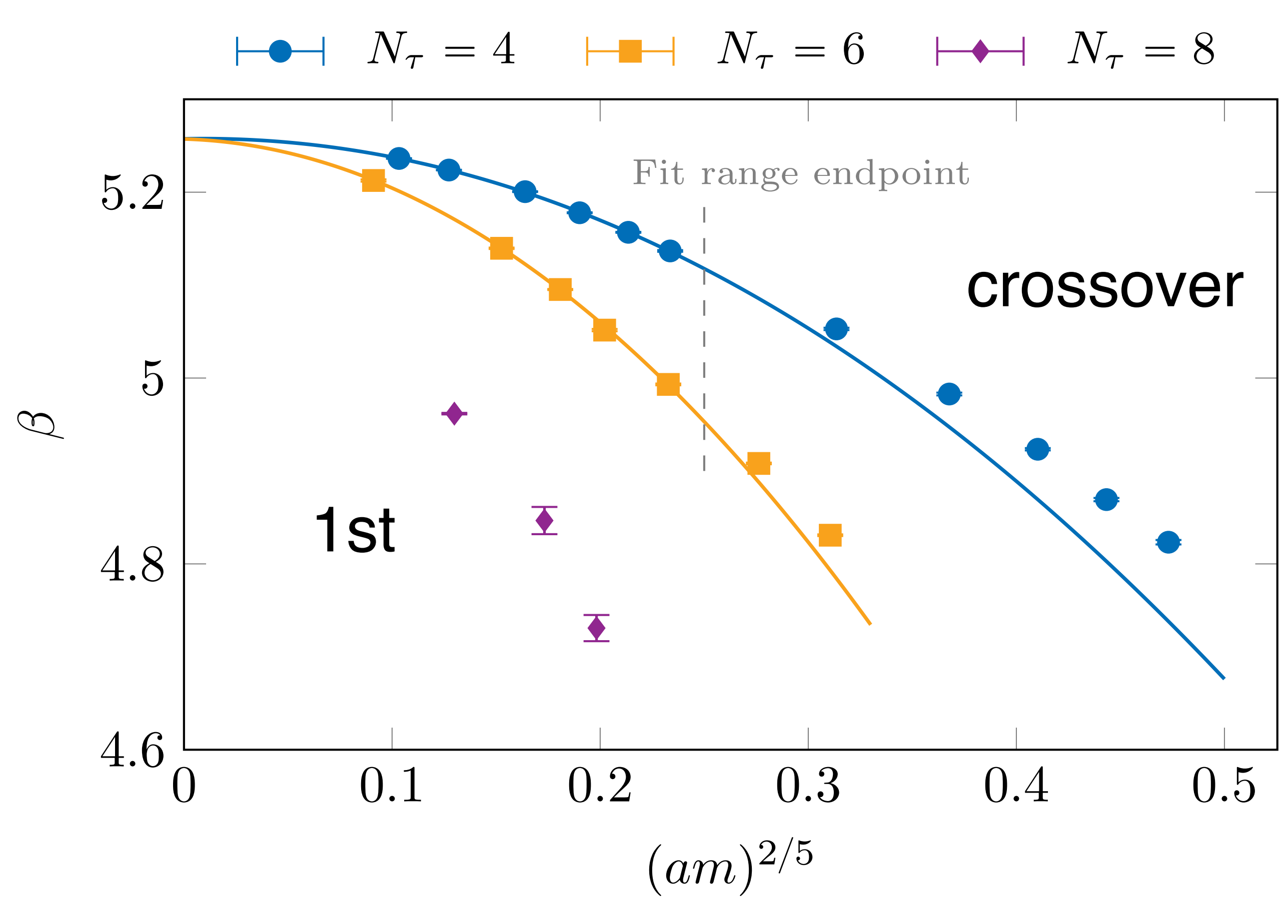


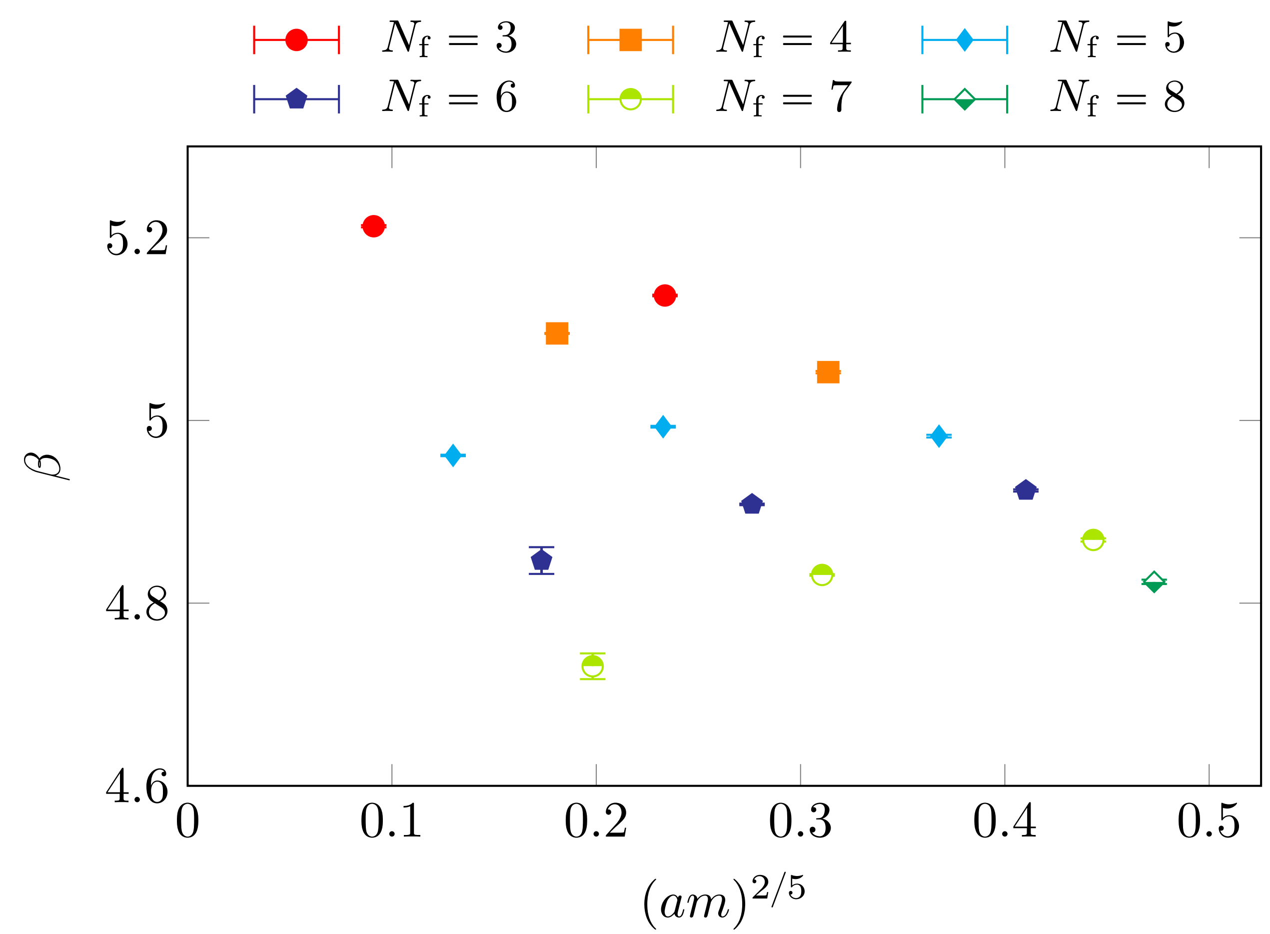
$$B_4(\beta_c, am, N_\sigma) \approx 1.604 + c\,(am - am_c)\,N_\sigma^{1/0.6301}$$

Bare parameter space of unimproved staggered LQCD

[Cuteri, O.P., Sciarra 21]

~120 M Monte Carlo trajectories with light fermions, aspect ratios 3,4,5

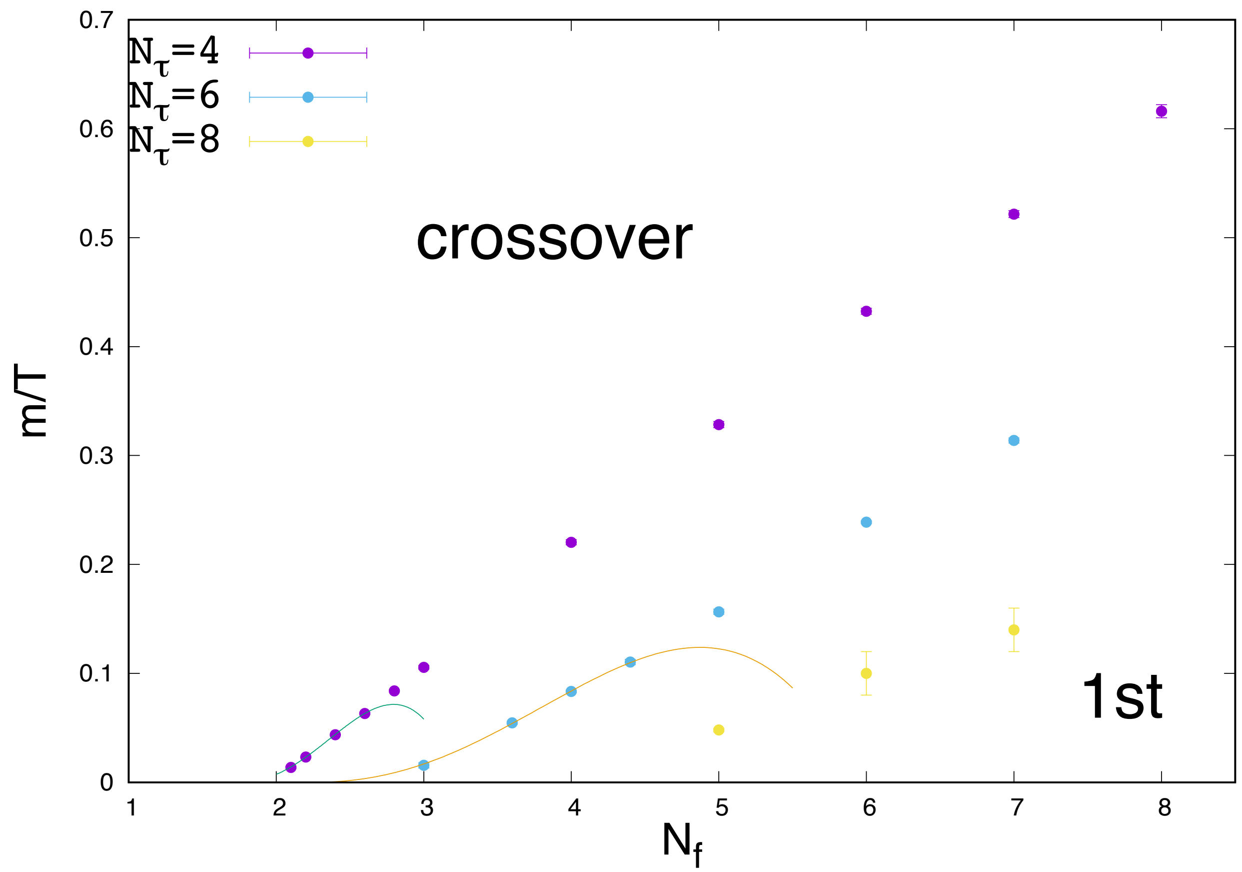


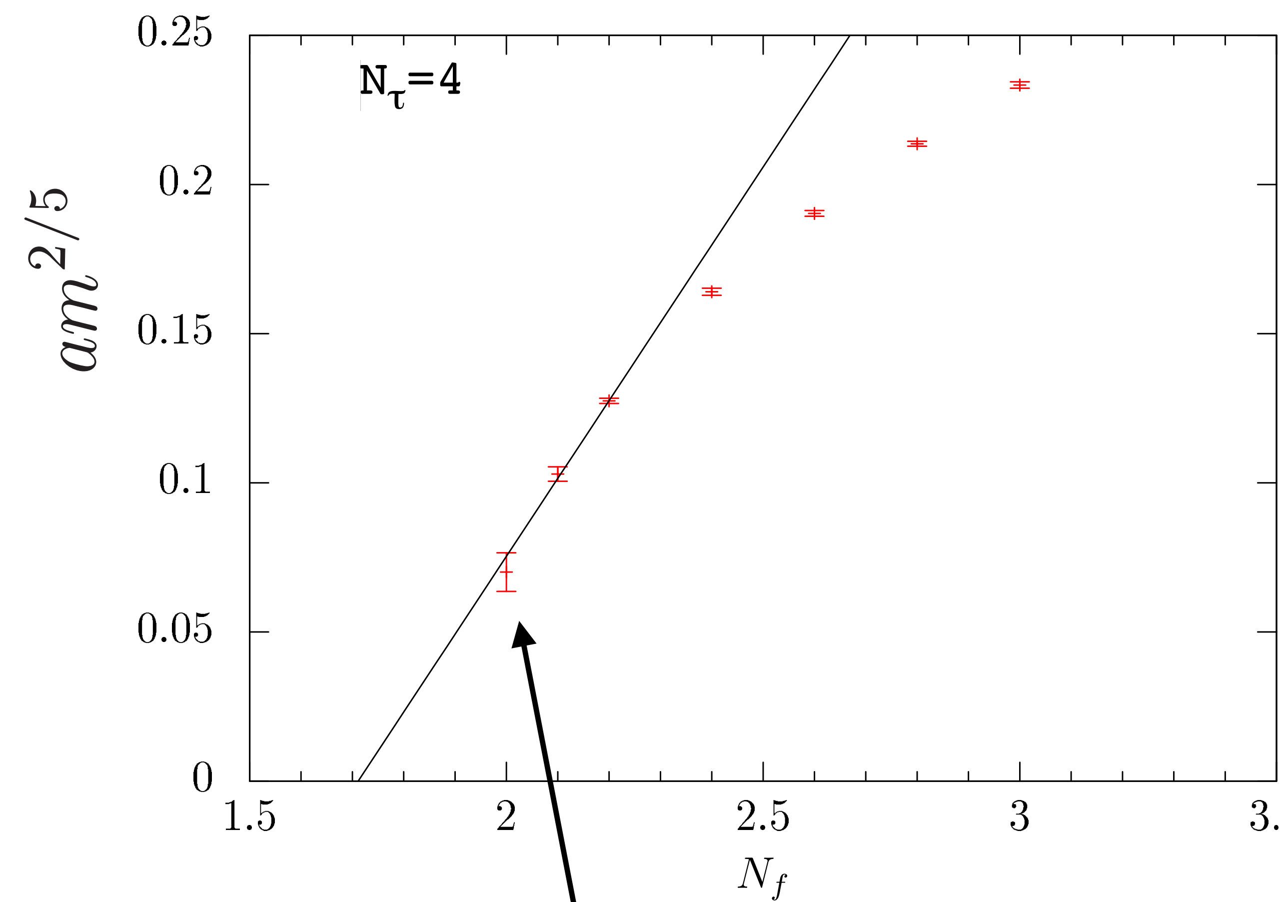


- Tricritical scaling observed in lattice bare parameter space
- Left plot: fixed N_τ , tricritical extrapolation always possible
- Right plot: fixed N_f , existence of tricritical point not guaranteed

Bare parameter space of unimproved staggered LQCD

[Cuteri, O.P., Sciarra 21]

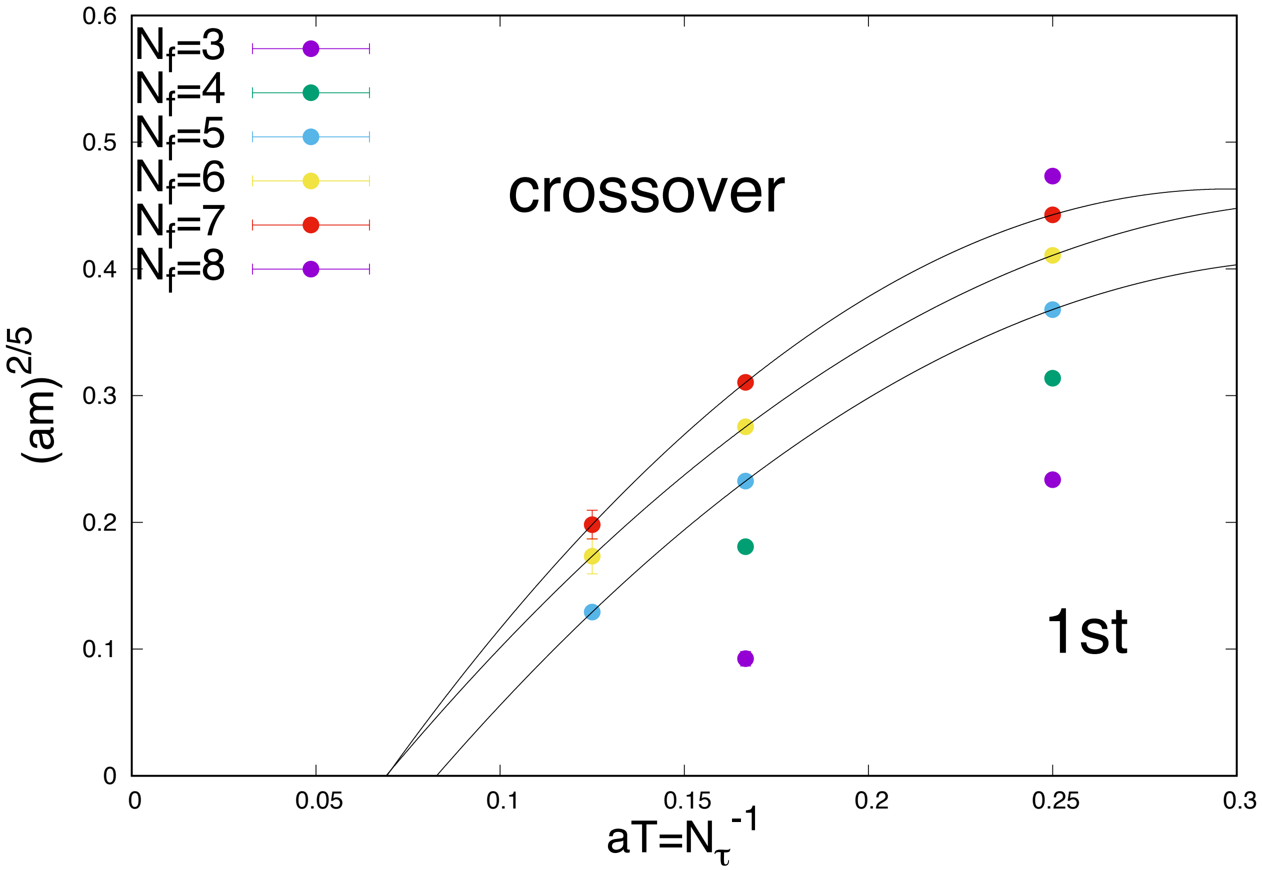




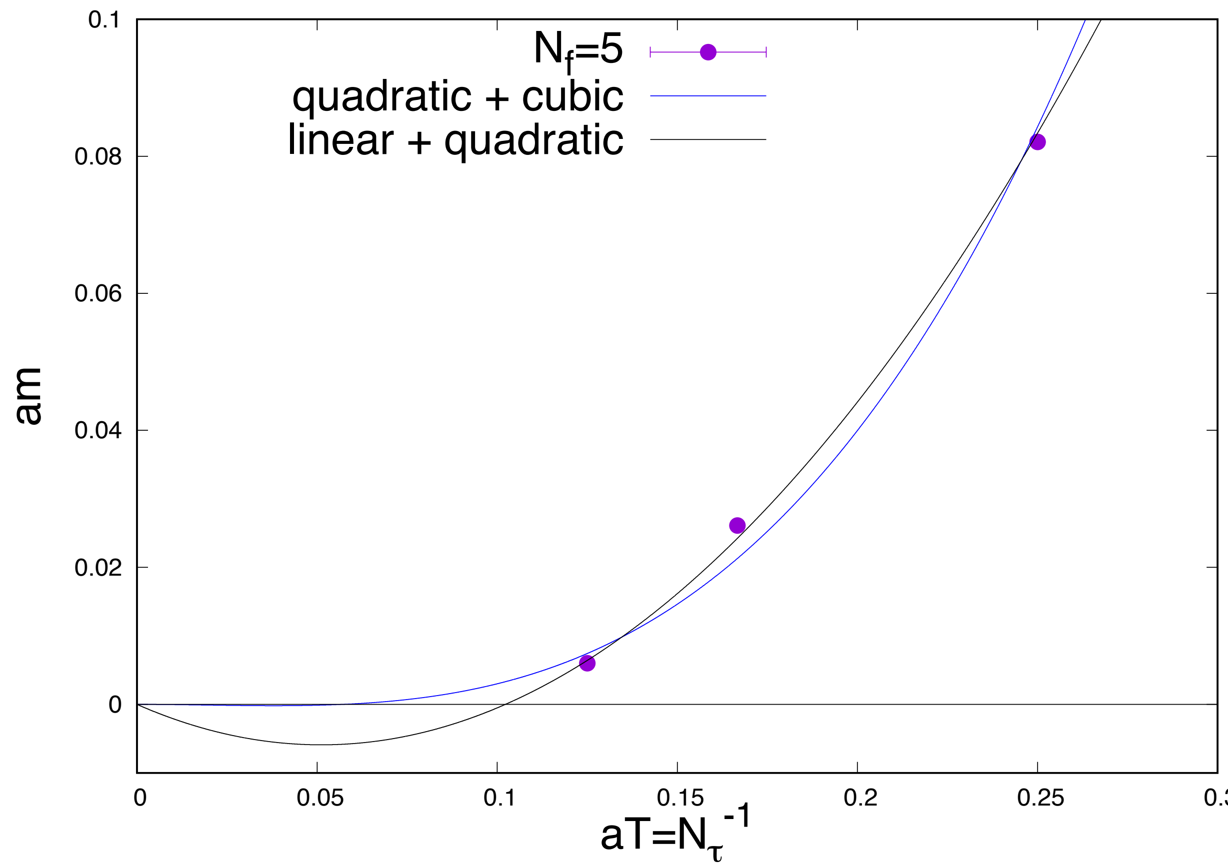
- Tricritical scaling observed also in different variable pairings
- Extrapolated by tric. scaling from finite imaginary μ [Bonati et al. PRD 14]
- Old question: $m_c/T = 0 \text{ or } \neq 0$? Answered for $N_f = 2$
- New question: will N_f^{tric} slide beyond $N_f = 3$?

Bare parameter space of unimproved staggered LQCD

[Cuteri, O.P., Sciarra 21]



1st order scenario does not fit!

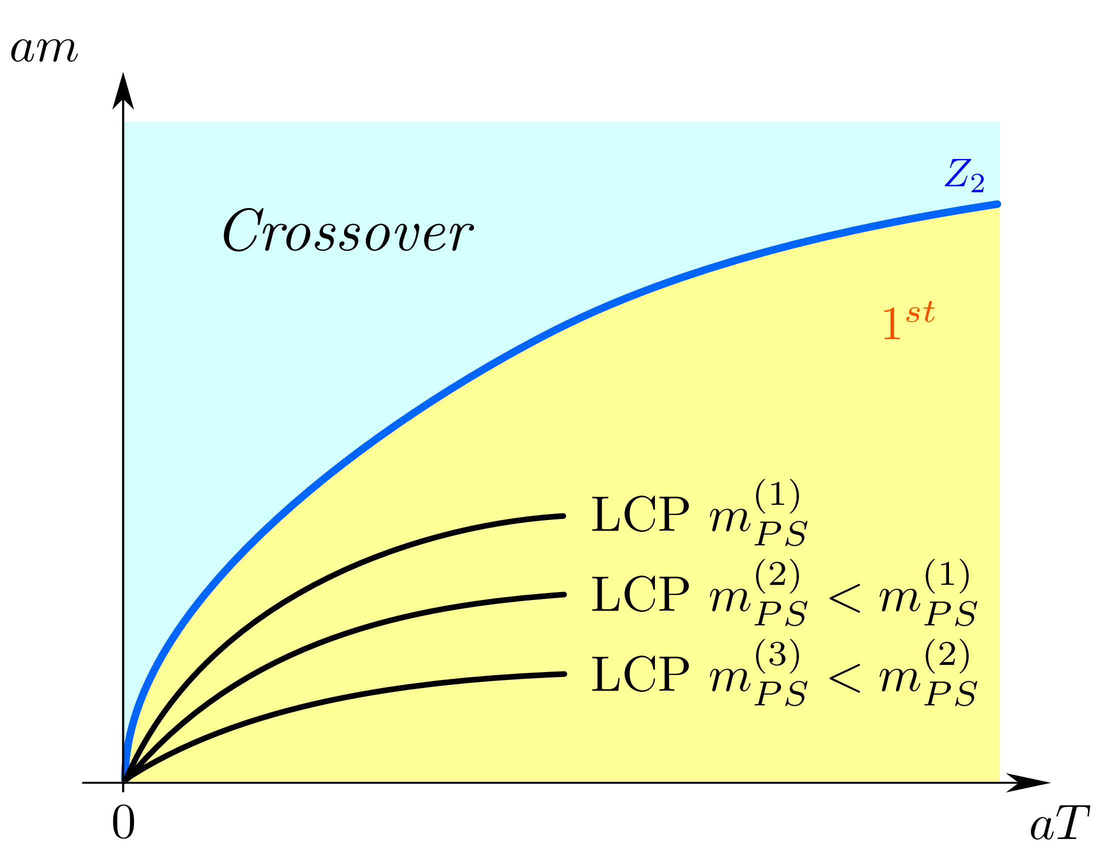


- Tricritical scaling observed also in plane of mass vs. lattice spacing
- Allows extrapolation to lattice chiral limit, tricritical points $N_\tau^{\rm tric}(N_f)$
- 1st order scenario: $m_c(a) = m_c(0) + c_1(aT) + c_2(aT)^2 + \ldots$

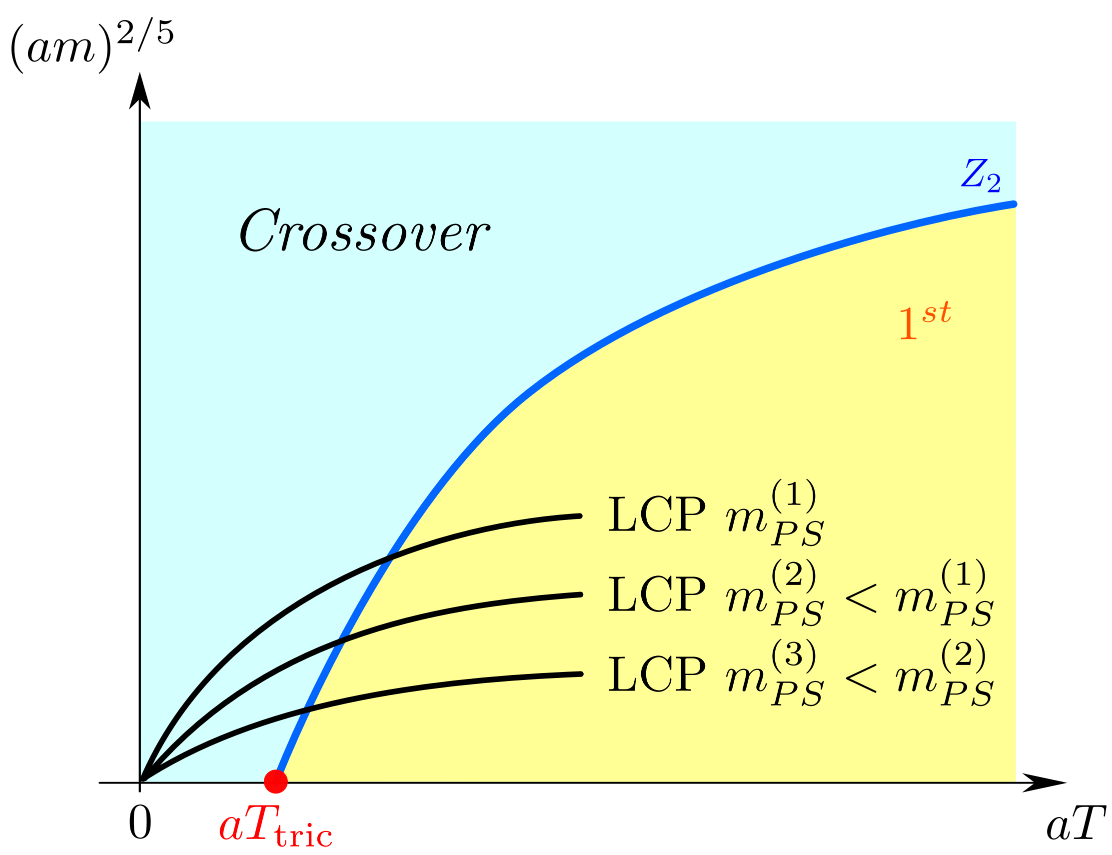
Incompatible with data! $\chi^2_{\rm dof} > 10$

Implications for the continuum

- Finite $N_\tau^{\rm tric}(N_f)$ implies that 1st order transition is not connected to continuum
- Approaching continuum first, then chiral limit:
  Continuum chiral phase transition second-order!



1st order scenario

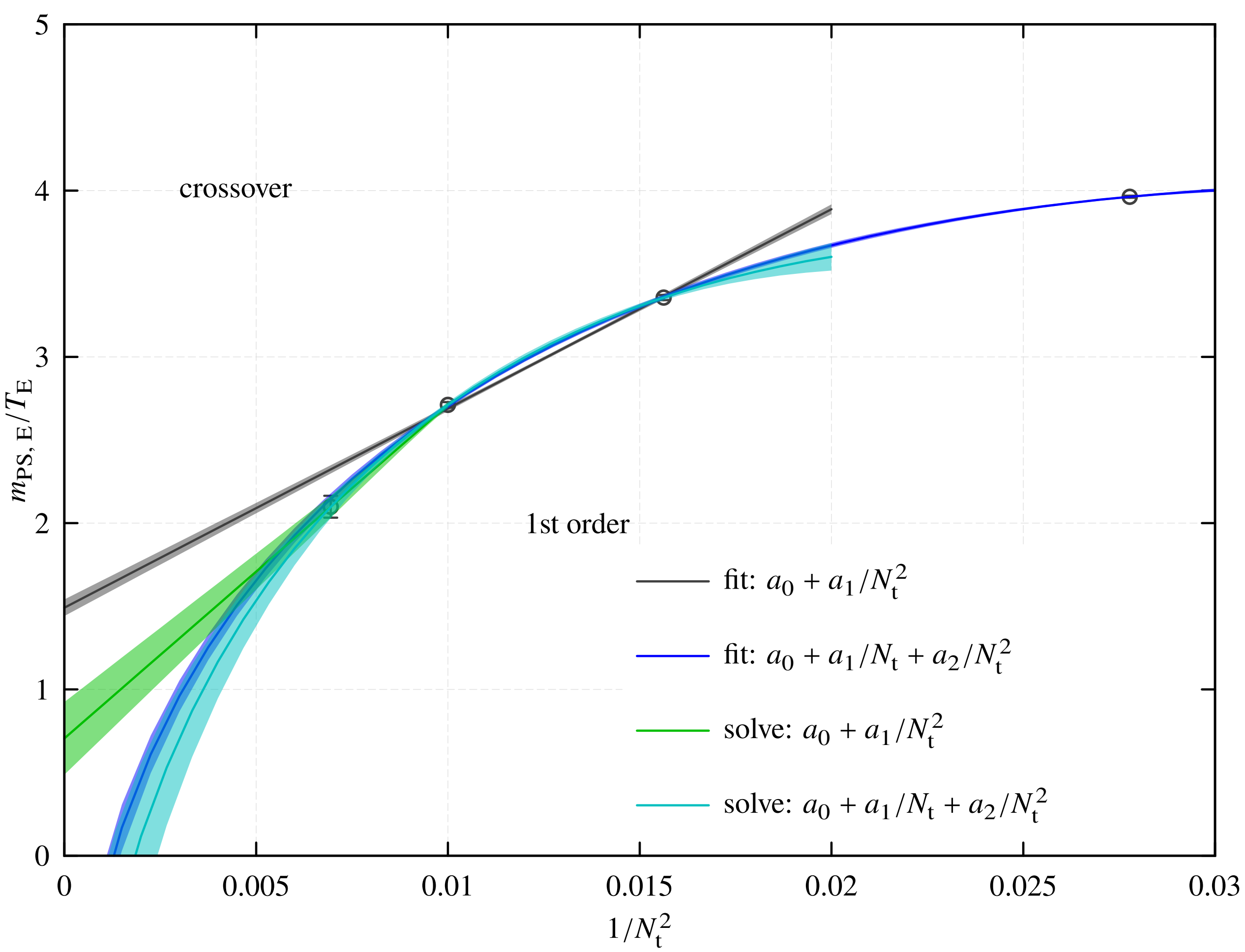


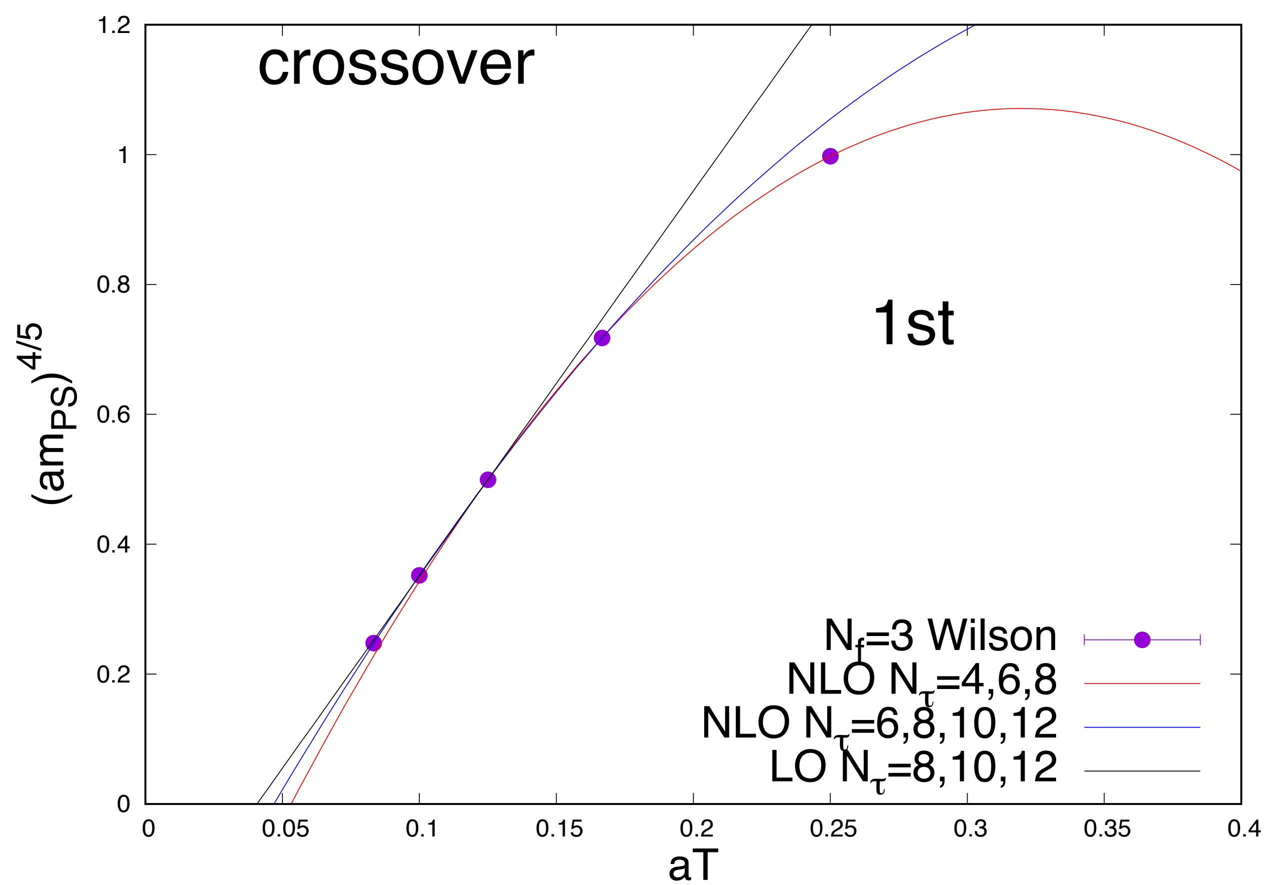
2nd order scenario

Nf=3 O(a)-improved Wilson fermions

[Kuramashi et al. PRD 20] $m_\pi^c \leq 110\,\mathrm{MeV} \quad N_\tau = 4, 6, 8, 10, 12$

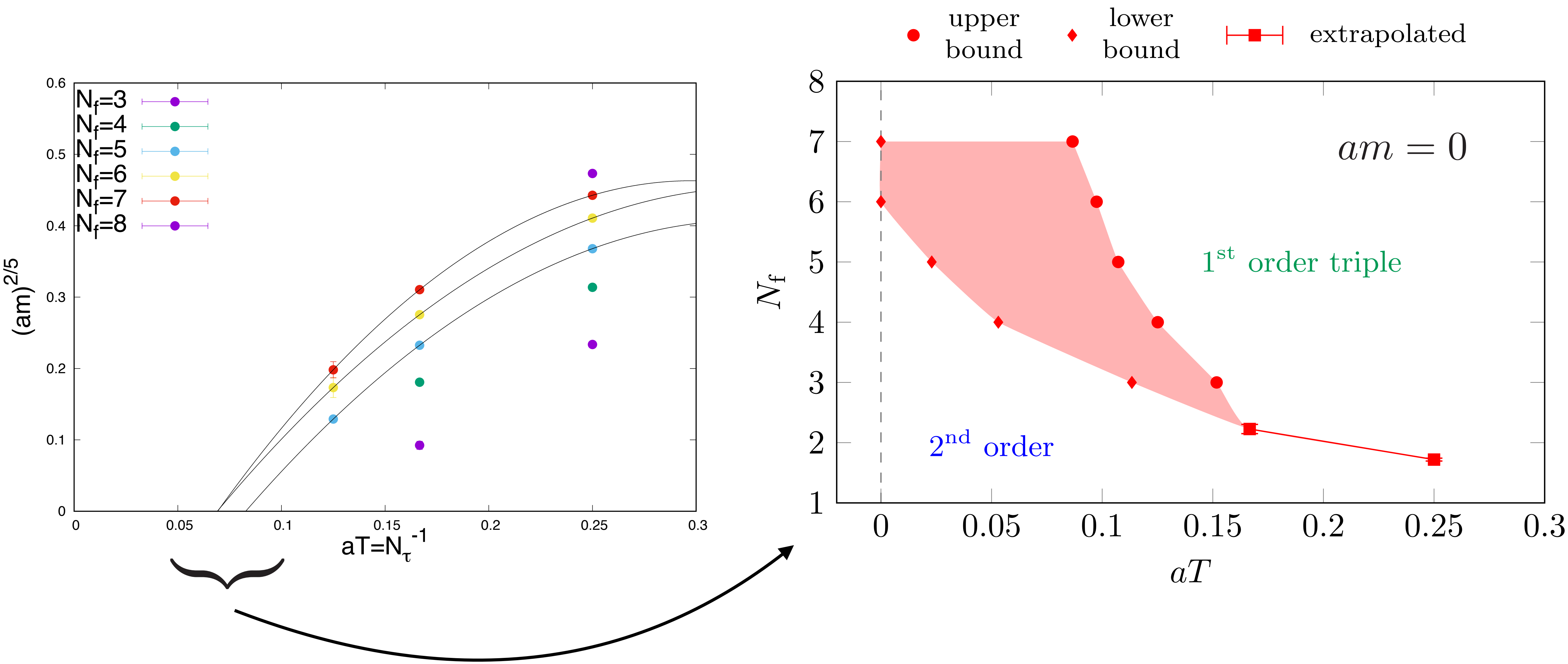
Re-analysis using: $am_{PS}^2 \propto am_q$





Tricritical scaling, Nf=3 consistent with unimproved staggered!

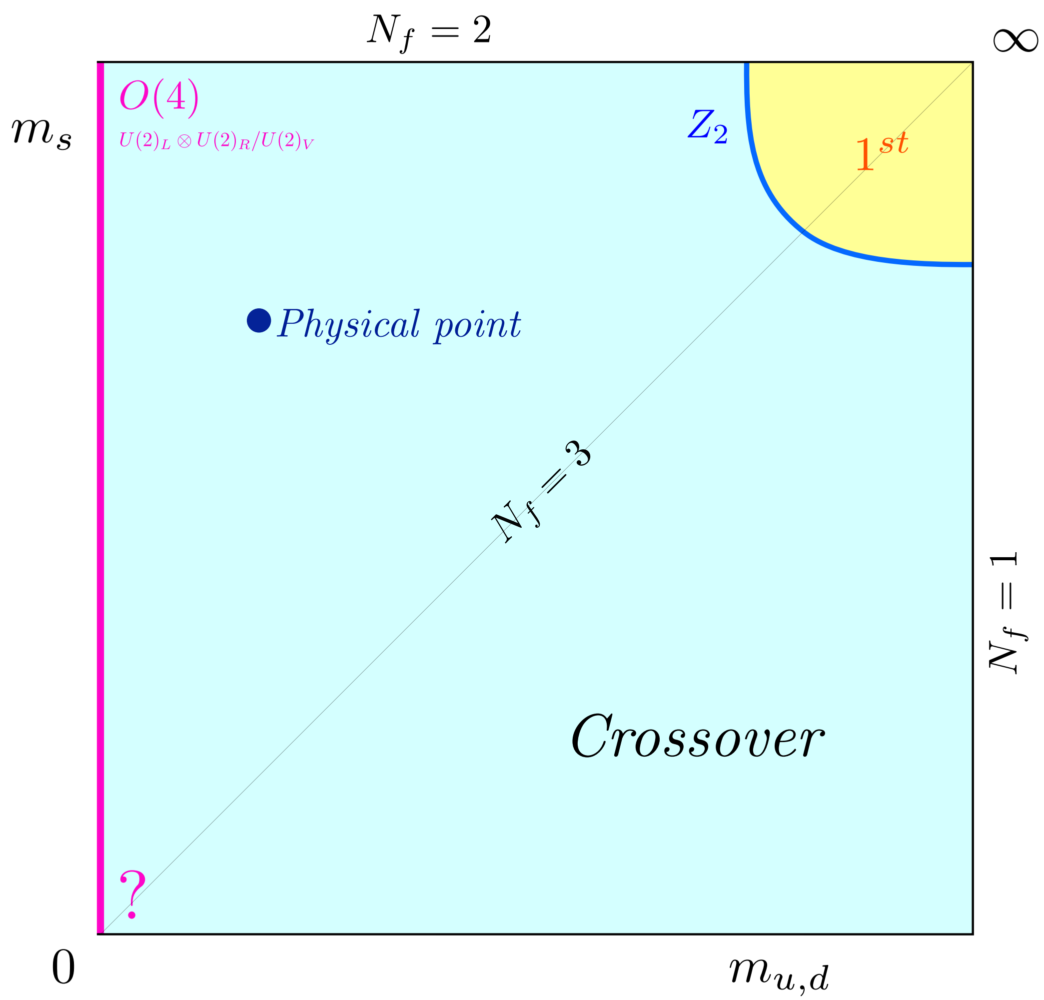
Staggered: tricritical points as function of Nf



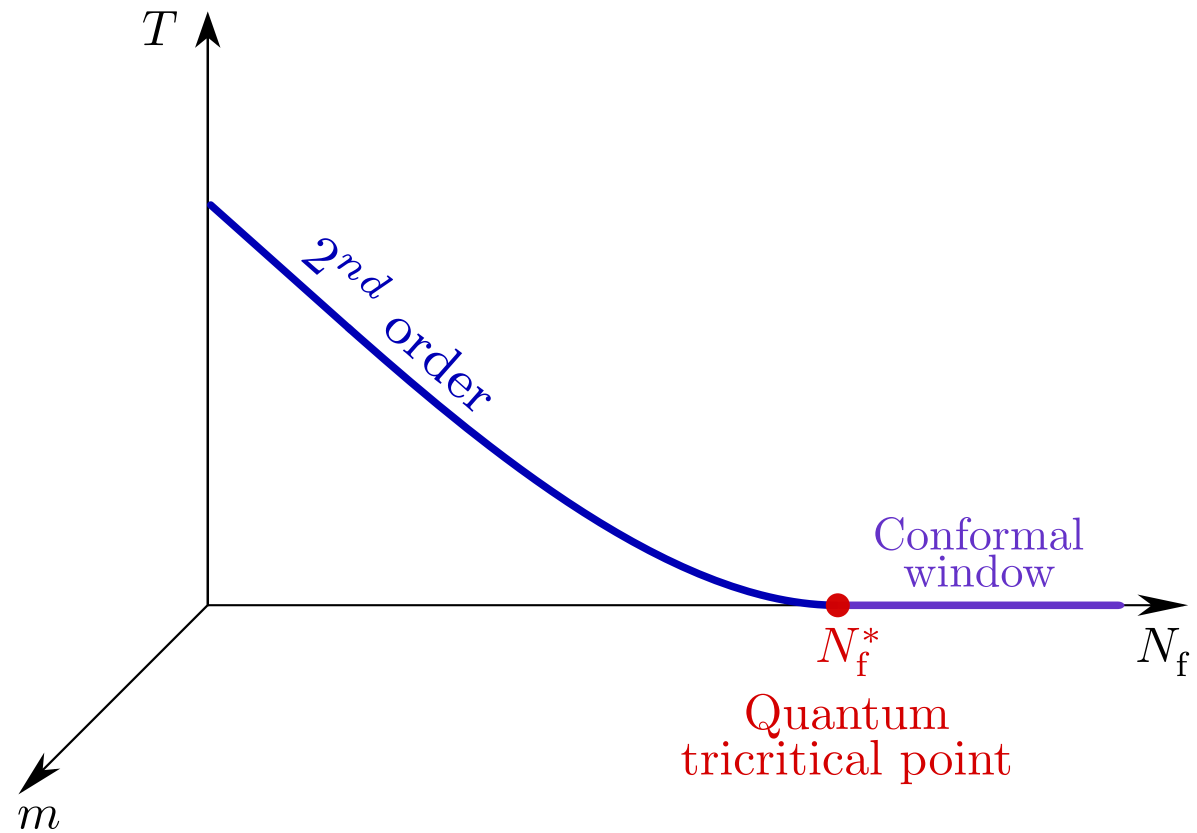
- $N_\tau^{\mathrm{tric}}(N_f)$ increasing function (approximately linear?)
- Tricritical line in the plane of the lattice chiral limit, separates 1st from 2nd

The chiral phase transition for different N_f

[Cuteri, O.P., Sciarra 21]



Conjecture:



The chiral phase transition in the massless limit is likely second-order for all N_f

What about Pisarski, Wilczek?

- Investigated 3d sigma model,
  i.e. ϕ^4 - Ginzburg-Landau-Wilson theory for chiral condensate
- Results based on epsilon expansion about $\epsilon = 1$
- All conclusions confirmed by [Butti, Pelisetto, Vicari, JHEP 03]
  High order perturbative expansion in fixed d
- Support also from simulation of 3d sigma model [Gausterer, Sanielovici, PLB 88]

Suggested resolution:

ϕ^6 term should be included in 3d, renormalisable!

FRG: 3d ϕ^6 has infrared fixed points and 2nd order transitions [Litim, Tetradis, NPB 96]

Conclusions

- Modified strategy to study Columbia plot: lattice bare parameter space
- Knowing order of chiral transition in bare parameter space: Conclusions for continuum approach in correct order of limits
- For unimproved staggered, Nf=2-6, O(a)-improved Wilson Nf=3: 1st order transition region not connected to continuum limit
- Chiral transition second order up to conformal window?
- Check: Wilson Nf=4!
- Domain wall and overlap fermions……