Three topics

1. χ-spirals & "moat" spectrum
   a. Goldstone bosons disorder χ-spiral ⇒ Quantum π Liquid
   b. How to find QπL on the lattice?

2. Nuclear matter in 1+1 dim's
   @ low energy, soluble & simple, Luttinger liquid

3. χ-sym. spin-1 mesons
   Lattice: @ T: $T_\chi \to \sim (3\text{-}4)\, T_\chi$, spin-1 $\neq$ free q's
   Surprising mass degeneracy – "accidental?"

Rohrhofer, Aoki, Cossu, Fukaya, Glozman, Hashimoto, 1707.01881; 1902.03191; 1909.00927

Review of kink crystals,
χ-spirals

Kink crystal @ $\mu \neq 0$

Basar, Dunne, Thies 0903.1868. Gross-Neveu model:

$$\mathcal{L} = \bar{\psi} i \not\partial \psi + g^2 (\bar{\psi}\psi)^2$$

Soluble as # flavors $N \to \infty$

low T, high μ: kink crystal

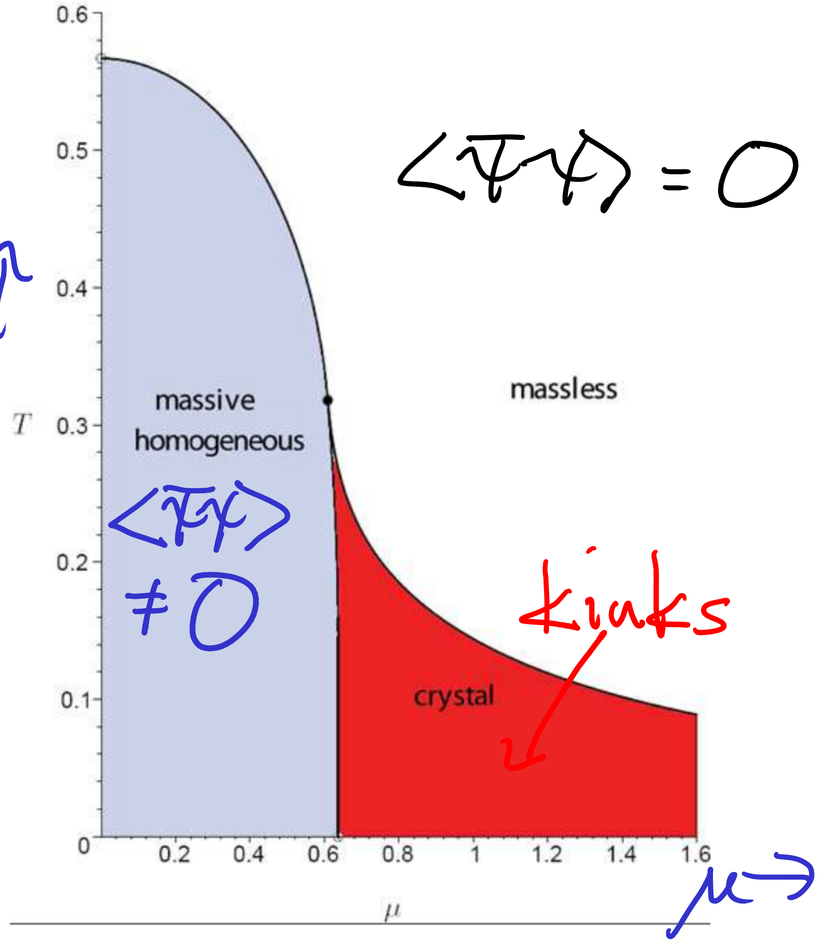
$\bar{\psi}\psi \uparrow$ $z \rightarrow$

Only Z(2) symmetry broken

Lattice: $\approx$ same @ small N

Lenz,Pannullo,Wagner,Wellegehausen,Wipf,2004.00295

$T \uparrow$



$\mu \rightarrow$

χ-spiral @ $\mu \neq 0$

χGN model: U(1) sym.,

$$\mathcal{L} = \bar{\psi} i \not{\partial} \psi + g^2 \left((\bar{\psi}\psi)^2 + (\bar{\psi}\gamma_5\psi)^2 \right)$$

As $N \to \infty$, $\mu \neq 0$, χ-spiral:

$\bar{\psi}\psi \uparrow$

$\bar{\psi}\gamma_5\psi \swarrow$

$z \rightarrow$

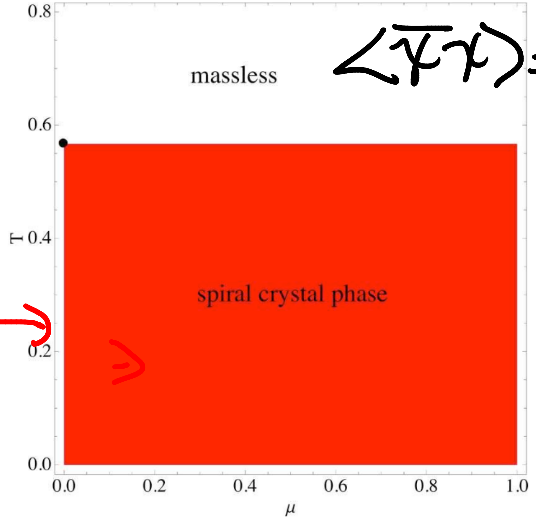
Non-trivial soln. @ $N \to \infty$

Phase diagram odd, special to 1+1 dim.'s

No soluble model $\bar{c}$ Goldstone Bosons (duh, 1+1 dim's)

T ↑

χ-spiral →



$\langle \bar{\psi}\psi \rangle = 0$

$\mu \rightarrow$

Eff. thy's @ $\mu \neq 0$

Take $\vec{\phi}$ = O(N) vector. Usual, in vacuum:

$$\mathcal{L}_\phi = \frac{1}{2}(\partial_0 \vec{\phi})^2 + \frac{1}{2}(\partial_i \vec{\phi})^2 + \frac{m^2}{2}\vec{\phi}^2 + \frac{\lambda}{4}(\vec{\phi}^2)^2 + \frac{\kappa}{6}(\vec{\phi}^2)^3$$

In medium, as $\mu \neq 0$

$$\mathcal{L}_\phi = \frac{1}{2}(\partial_0 \vec{\phi})^2 + \frac{Z}{2}(\partial_i \vec{\phi})^2 + \frac{1}{2M^2}(\partial_i^2 \vec{\phi})^2 + \ldots$$

Critical end-point (CEP) $\lambda = 0$ $\quad \kappa > 0$

1^{st} order line: $\lambda < 0$

Moat spectrum: $Z < 0$, M^2 finite $\quad$ stability

χ spirals

Much work in "π-condensate", χ-spirals. $\vec{\phi} = O(2)$:

$$\vec{\phi} = \phi_0 \left(\cos(p_0 z), \sin(p_0 z) \right) \Rightarrow \vec{\phi}^{\,2} = \phi_0^2$$

Need to satisfy two conditions

$Z<0 \Rightarrow \mathcal{L} \sim \frac{1}{M^2} (\partial_i^2 \phi)^2 + Z (\partial_i \phi)^2 \Rightarrow p_0^2 \sim -Z M^2$

and $V_{eff}(\phi) \sim m_{eff}^2 \phi_0^2 + \lambda \phi_0^4 \Rightarrow \phi_0^2 \sim m_{eff}^2 / \lambda$

IF ϕ_0 and $p_0 \neq 0$, χ-spiral

Also possible: $\phi_0 = 0$, $p_0 \neq 0$: "moat" spectrum

Region $\bar{c}$ moat spectrum large

FPR '19: Use Func. Ren. Group

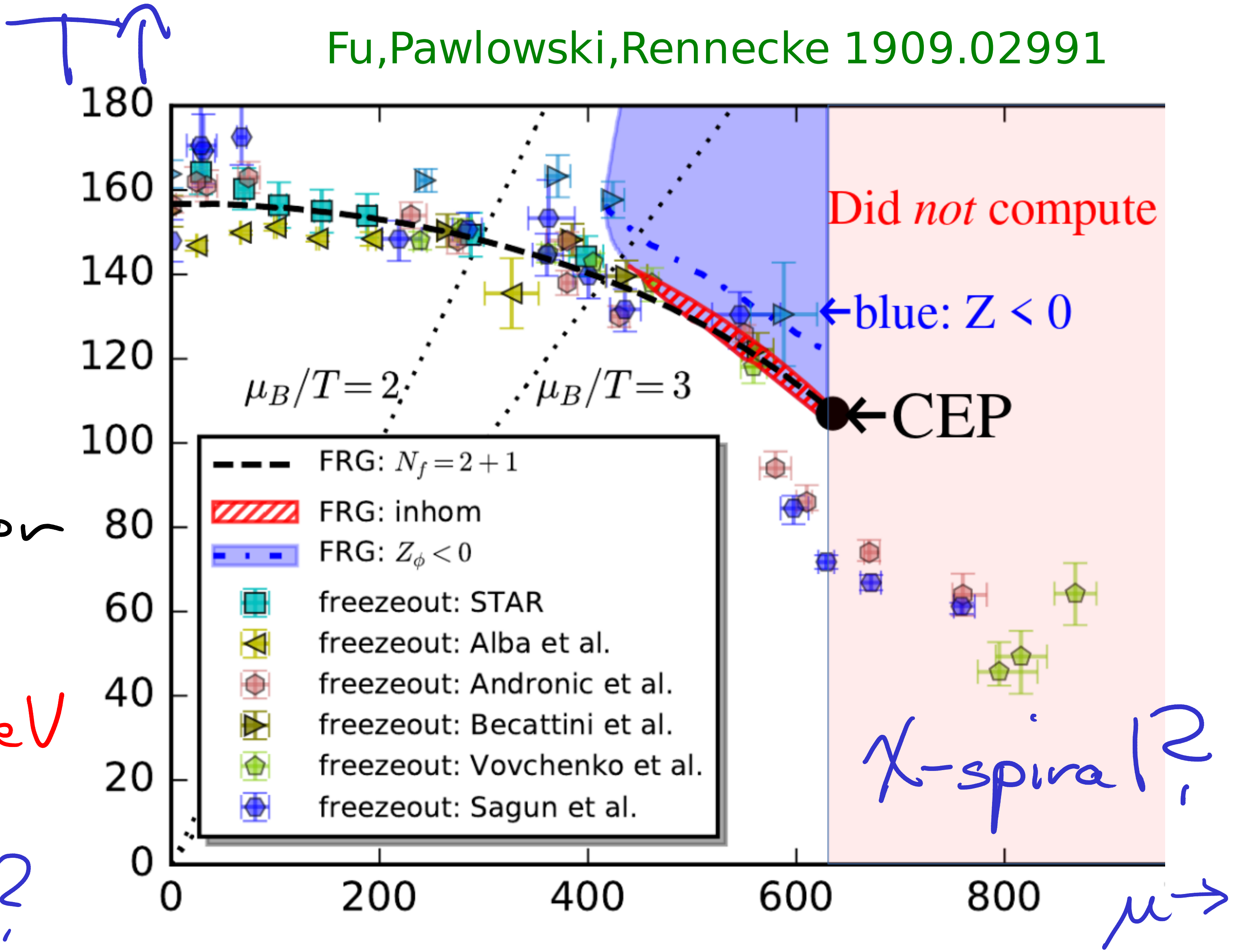
Find large region $\bar{c}$ moat spectrum

χ-restored phase $\bar{c}$ "funny" π-propagator

N.B.: critical region for CEP tiny, ~1 MeV where $m_\sigma \sim m_\pi$

χ-spiral @ low μ?

Fu,Pawlowski,Rennecke 1909.02991



Goldstone Bosons
& χ-spirals

When are there χ-spirals?

Consider eff. thy.:

$$\mathcal{L}_{eff} = \frac{1}{2}(\partial_0 \vec{\phi})^2 + \sum_{n=1}^{\infty} \frac{Z_n}{2} \vec{\phi}(-\partial_i^2)^n \vec{\phi} + \frac{v_n}{2}(\vec{\phi}^2)^n \qquad \vec{\phi} \in O(N)$$

Assume $\quad \vec{\phi}_{cl} = (\phi_0 \cos k_0 z, \ \phi_0 \sin k_0 z, \ \vec{0}), \quad \vec{\phi}_{cl}^{\,2} = \phi_0^2 = \text{const}.$

$= \chi$-spiral.

$$\mathcal{L}_{eff}(\phi_0) = \sum_{n=1}^{\infty} \frac{Z_n}{2} k_0^2 \phi_0^2 + \frac{v_n}{2} \phi_0^{2n}$$

Vary $\bar{c}$ respect to k_0 & ϕ_0 to fix their values.

Goldstone bosons of χ-spirals

Assume $N > 2$. Transverse prop.:

$$\Delta^{-1}(\vec{k}) = \sum_{n=1}^{\infty} Z_n (\vec{k}^2)^n + n v_n \phi_0^{2n-2}$$

So?

$$\Delta^{-1}(k_0 \hat{k}) = \sum_{n=1}^{\infty} Z_n k_0^{2n} + n v_n \phi_0^{2n-2} = \frac{\partial}{\partial \phi_0} \mathcal{L}_{eff} = 0$$

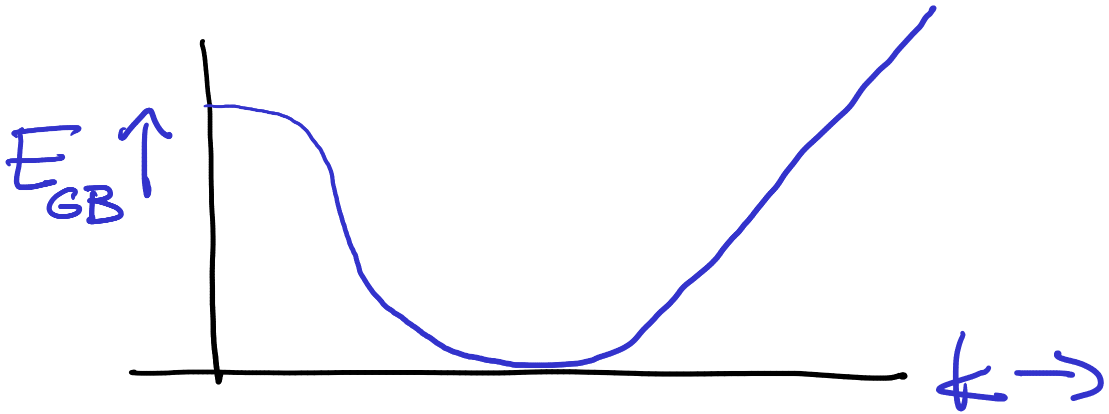
and

$$\frac{\partial}{\partial \vec{k}} \Delta^{-1}(\vec{k}) \Big|_{k=k_0} = \sum_n n Z_n k_0^{2n-1} \hat{k} = \frac{\partial}{\partial k_0} \mathcal{L}_{eff} = 0$$

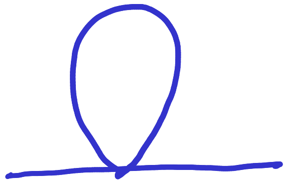
For above χ-spiral, $E_{Goldstone} = 0$ @ $k_0 \neq 0$

GB's disorder X-spirals

Graphically:



So? But:

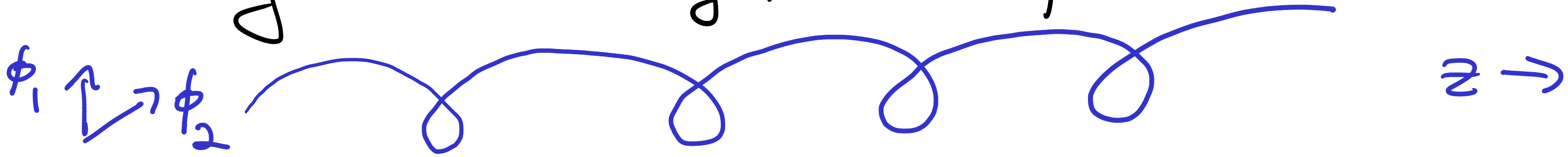
 $\sim \lambda \int d^d k \frac{1}{(k^2 - k_0^2)^2} \sim k_0^{d-1} \int \frac{d\,\delta k}{(\delta k)^2}$

= <u>linear</u> divergence. Conjecture: theorem for <u>all</u> X-spirals

RDP, Valgushev, Tsvelik, 2005.10259

GB's & χ-spirals

Pictorially: classically, a χ-spiral



But if there are Goldstone Bosons $(O(N), N>2)$,
GB's disorder:

N.B.: cannot prove in general. Need to assume $\overline{\Phi}_{cl}^{\,2} = \text{const.}$
Instead of ordered χ-spirals, disorder. Specifically?

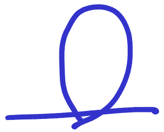
For any model $\bar{c}$ GB's, useless to look for classical χ-spirals

<u>Excludes</u> Gross-Neveu & χGN

Solution @ large N, $T \neq 0$

RDP, Valgushev, Tsvelik: 2005.10259.

introduce constraint field $\omega \sim \bar{\Phi}^2$. As $N \to \infty$, GB's dominate

$m_{eff}^2 \sim \lambda$  : $m_{eff}^2 - m^2 = \lambda N \int d^3k \, \dfrac{1}{(k^2)^2/M^2 + Z k^2 + m_{eff}^2}$

$$= \lambda N \frac{M^{3/2}}{\sqrt{2 m_{eff} + Z M}}$$

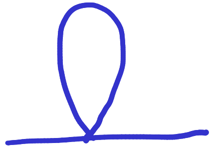
Above for 3-d's, $T \neq 0$ (static mode) Soln:

$Z \to +\infty$ $\quad m_{eff}^2 = m^2 + \lambda N/\sqrt{Z} + \dots$ usual pert. thy.

$Z \to -\infty$ $\quad m_{eff} = -Z\dfrac{M}{2} + \dfrac{16\lambda^2}{M}\dfrac{1}{Z^4} + \dots$

QπL, $T = 0$

Above for $T \neq 0$. Because only higher derivatives in space, changes @ $T=0$.

$m_{eff}^2 \sim \lambda$  $\qquad m_{eff}^2 \sim \lambda N \int d\omega \int d^3k \, \dfrac{1}{\omega^2 + (\vec{k}^2)^2/M^2 + Z\vec{k}^2 + m_{eff}^2}$

$$m_{eff}^2 \sim \lambda N \int \frac{d^3k}{((\vec{k}^2 - k_0^2)^2/M^2)^{1/2}} \sim \lambda N \int \frac{d\delta k}{|\delta k|} \sim \lambda N \log \delta m_{eff}$$

ω-integral softens power IR-div. $\Rightarrow$ log.

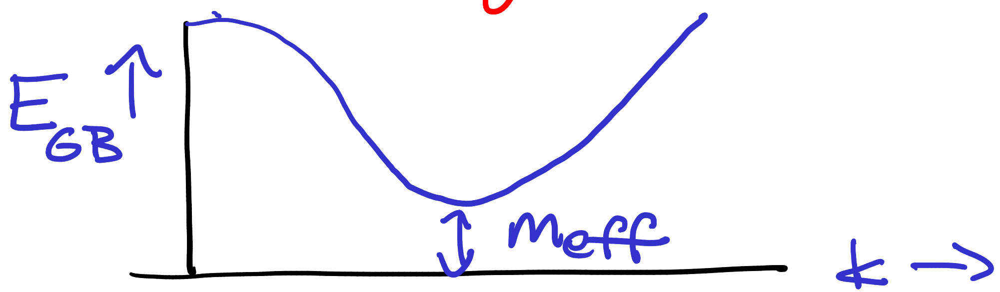
$Z \to -\infty$: $\quad m_{eff} \sim -\dfrac{ZM}{2} + \# \sqrt{-Z}\, M \, e^{-\frac{2^{3/2}\pi^2}{\lambda N}(-Z)^{3/2}} + \ldots$

m_{dyn} @ $T=0$ $\ll$ m_{dyn} @ $T \neq 0$; weaker IR div

RDP, 2101.05813

Quantum π Liquid

GB's get a dynamical mass gap:



$\Delta_{GB}^{-1} = (k^2 + m_+^2)(k^2 + m_-^2)$. Usually, $m_\pm^2$ real.

For small $z > 0$, & $z < 0$, m_+^2 & m_-^2 have imaginary parts

$z \to -\infty$: $m_{imag} \sim \sqrt{-z}\, M$ $\quad m_{real} \sim \lambda / z^2$

$$\langle \phi^i(x) \phi^j(0) \rangle \underset{x \to \infty}{\sim} \delta^{ij} e^{-m_{real} x} \cos(m_{imag} x)$$

$\Rightarrow$ Quantum pion liquid Like Quantum Spin Liquid

QπL on Lattice?

M.P. Lombardo: how to test QπL on lattice? ϕ^4!

If Goldstone bosons essential, need 2+1, 3+1 dim.'s

as GB's always disorder in 1+1 dim.'s anyway!

Narayanan, 2001.09200; Buballa, Kurth, Wagner, & Winstel, 2012.09588

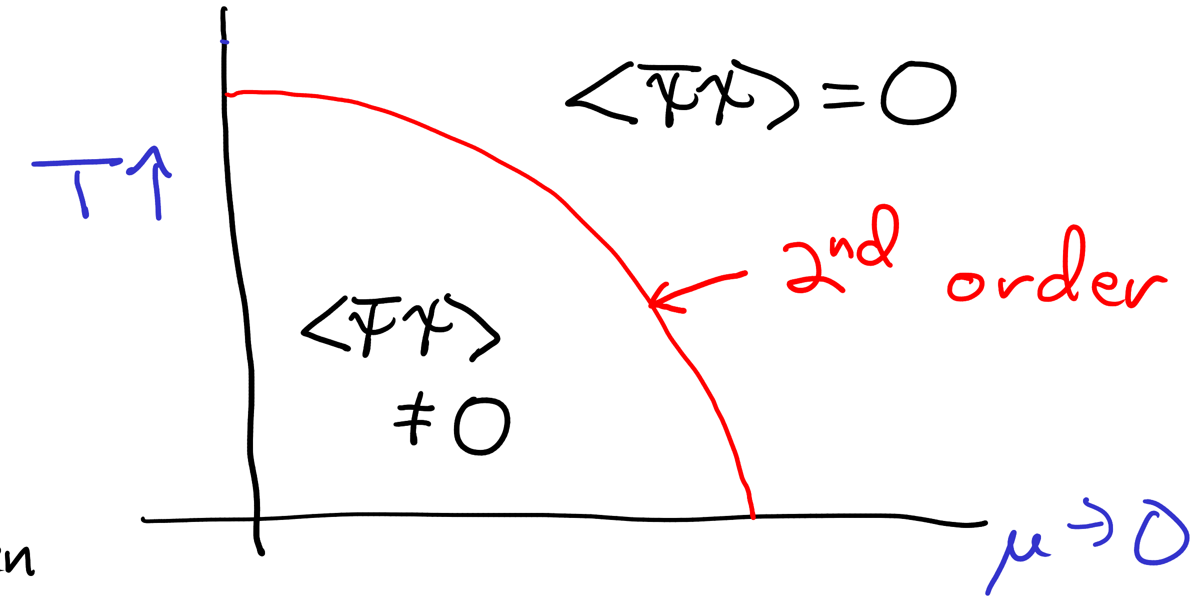
Can NOT use Gross-Neveu models to study χ-spirals in 2+1 dim.'s - ?!

There is χ-transition:

but always 2nd order

in continuum limit, $a \to 0$

And no GB's, $Z(2)$ broken



No χ-spirals for GN in 2+1 dim's

Gross-Neveu:

$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi + \lambda (\bar{\psi}\psi)^2 \Rightarrow \bar{\psi}(i\partial\!\!\!/ + \tilde{\sigma})\psi + \frac{1}{\lambda}\tilde{\sigma}^2$$

Int.'g over ψ:

$$\mathcal{L}_\sigma = \frac{1}{\lambda}\tilde{\sigma}^2 + N_f \operatorname{tr} \ln (i\partial\!\!\!/ + \tilde{\sigma})$$

Expand in momentum:

$$\mathcal{L}_\sigma \approx \tilde{\sigma}^2 \left\{ \underbrace{\left(\frac{1}{\lambda} + N_f \int^{\Lambda} \frac{d^3k}{k\!\!\!/^2}\right)}_{\text{mass term}} + \underbrace{N_f\, p^2 \int^{\Lambda} \frac{d^3k}{(k^2)^2}}_{p^2 \text{ term}} + \cdots \right\}$$

No χ-spirals in 2+1 dim's?

GN: non-renormalizable in pert. thy in $\lambda \sim 1/\Lambda$, is in $\frac{1}{N_f}$

mass term: tune $\lambda N_f \sim \frac{1}{\Lambda}$, get σ_0, $m_f \sim 1$

Vacuum spon'y breaks $Z(2)$, restored @ T_χ, μ_χ.

χ-spiral @ $\Lambda \sim \frac{1}{a} < \infty$, vanishes as $\Lambda \to \infty$, $a \to 0$

Why?

$$\Delta_\chi^{-1} \sim m_f + p^2 \int^{\Lambda} \frac{d^3k}{(k^2)^2} \sim m_f^2 + \frac{p^2}{(T,\mu)} + \ldots$$

Perfectly sensible χ-transition

But NO critical end-point

OR χ-spiral

term $\sim m_f^2 \to 0$ @ (T_χ, μ_χ)

but term $\sim p^2$ always > 0

can't flip the sign

Different phase diagram

With Gross-Neveu, $\tilde{\sigma}$-propagator only @ 1-loop, not tree,

Use model $\bar{c}$ dynamical σ: Pannullo, RDP, Wagner, Winstel, in progress

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ + h\sigma)\psi + \frac{1}{2}(\partial\sigma)^2 + \frac{1}{2}m_\sigma^2\sigma^2 + \frac{\kappa}{4}\sigma^4 + \dots$$

Renormalizable in $h \sim \text{mass}^{1/2}$, $\kappa \sim \text{mass}$

$$\Delta_\sigma^{-1} \sim m_\sigma^2 + h^2 N_f \int \frac{d^3k}{k\!\!\!/^2} + \dots + p^2\Big(1 + \underbrace{h^2 N_f \int \frac{d^3k}{(k^2)^2}}\Big)$$

Depending on h^2, κ, perhaps the sign of $\sim p^2$ may flip

1st order χ, χ-spirals

Need to tune $m_\sigma a \ll 1$. For $Q\pi L$, need GB's $\Rightarrow$ more than GN or χGN

1.b: Exp. signals of "moat" spectrum

with NO χ-spiral

Moat spectrum

RDP & F. Rennecke, 2103.06890 : only moat, NO χ-spiral

$$E^2_{moat} = \frac{(\vec{p}^{\,2})^2}{M^2} + Z\,\vec{p}^{\,2} + m^2_{eff}$$

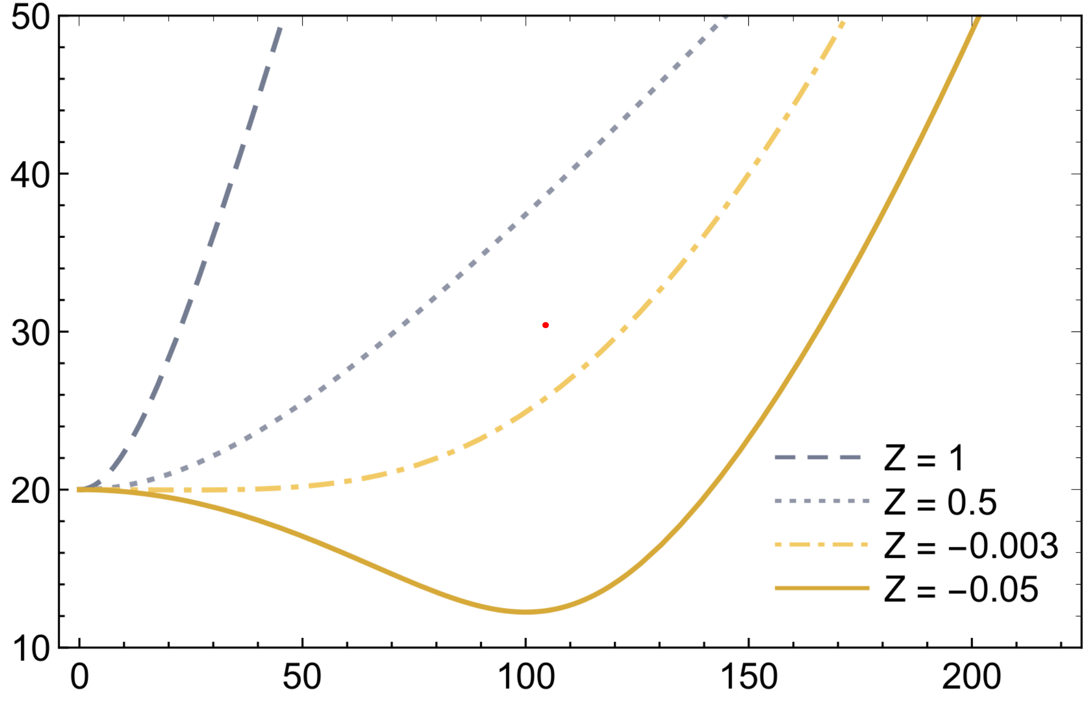
Parameters illustrative

E_{moat} only for $p < 2m_\pi$; else usual

Necessary to avoid soln's $\bar{c}$ Im E_{moat} in boosted frame. (Use blast wave)

NOT a χ-spiral: $\phi_0 = 0$

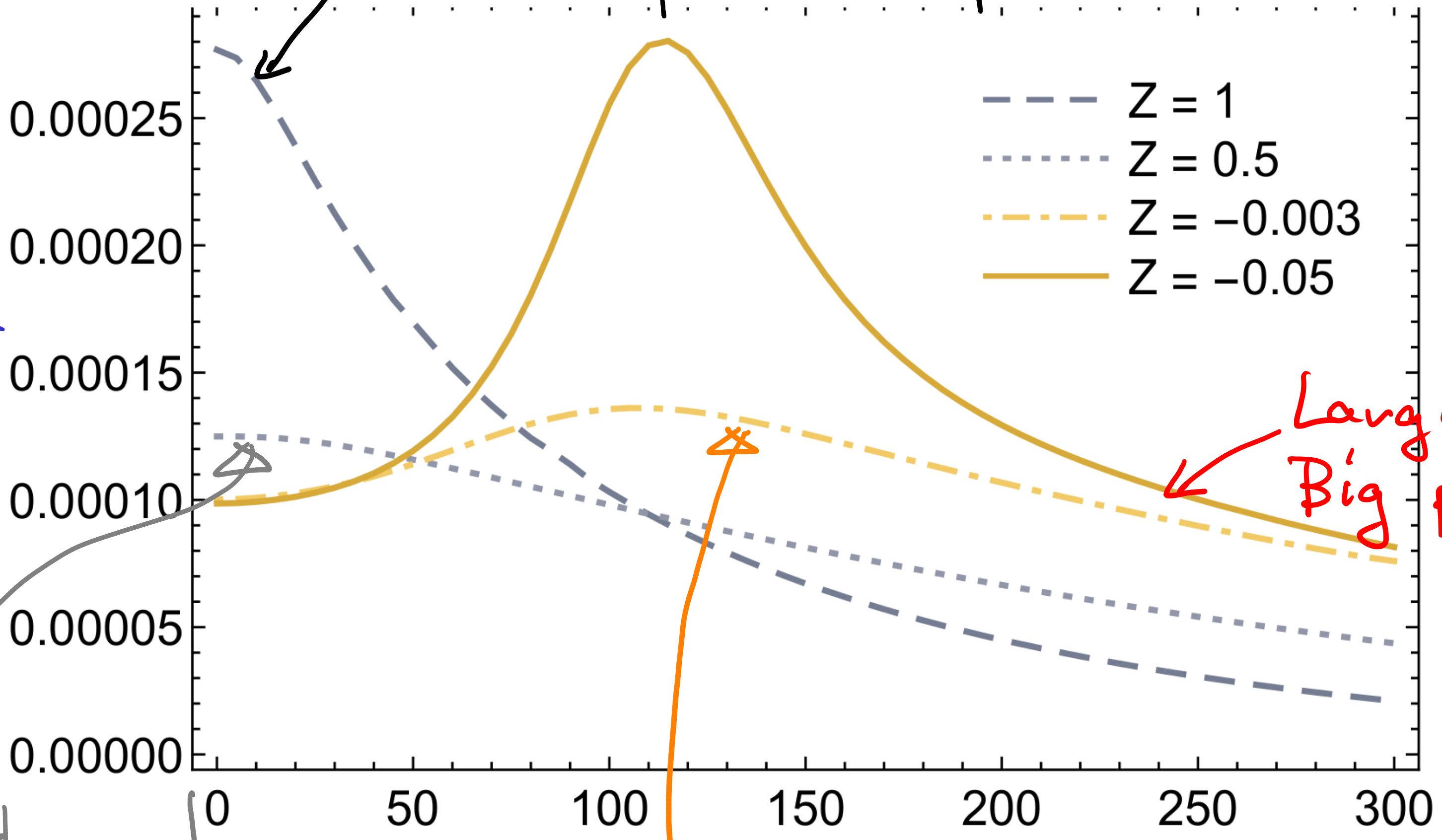
$m_{eff} = 20$ MeV, $M = 1.2$ GeV



Moat: single particle spectra

usual spectrum, $Z = 1$. Max @ $p_t = 0$

$\dfrac{d^3N}{dp_t^2\,dy}$ ↑



p_t →

Large $Z < 0$
Big peak @ $p_t \neq 0$

Even for $Z = 0.5$, much broader, non-thermal spectra

$Z \sim 0$; very broad, small peak @ $p_t \neq 0$

Blast wave para.

Use standard Cooper-Frye:

$$\frac{d^3N}{dp^3} \sim \int d\Sigma_\mu \int dp^0 \, p^\mu \, \Theta(\check{p}_0)\, \delta(\check{p}_0^2 - \underbrace{E^2(\check{p}^2)}_{F(\check{p})})\, n(\check{p}_0)$$

u^μ = velocity of medium @ freezeout

$\check{p}_0 = p_\mu u^\mu \,, \quad \check{p}^2 = (u^\mu u^\nu - g^{\mu\nu}) p_\mu p_\nu$

Can obtain imaginary energies even for $u = 0.3$!

$\Rightarrow$ use E_{moat} only for $p < 2m_\pi$

Zhang et al 1602.01564: $\sqrt{s} = 5$ GeV, $u = 0.3$

$T \sim 115$ MeV, $\mu_B \sim 536$ MeV

$\tau_{freezout} \sim 5$ fm/c

Two point correlations

CEP: genuine multi-particle corr.'s

Moat spectrum: corr.'s from propagation in medium can be large!

$$\left\langle \prod_i \frac{d^3N}{dp_i^3} \right\rangle \sim \prod_i \int d\Sigma_i^\mu \int dp_i^0 \, p_i^\mu \left\langle \prod_i F(p_i) \right\rangle$$

Fluctuations in thermo. variables, $\kappa^\mu = (T(x), \mu_B(x), u^\mu(x))$

Expand to quadratic order, $\kappa \sim \kappa_0 + \delta\kappa$, $\langle \delta\kappa \rangle = 0$

$$\langle F(p_1) F(p_2) \rangle_{\text{connected}} \sim \frac{\partial F}{\partial \kappa_1^\mu} \frac{\partial F}{\partial \kappa_2^\nu} \langle \delta\kappa_1^\mu \, \delta\kappa_2^\nu \rangle$$

Computing fluctuations

Generalize Landau-Lifshitz (S. Floerchinger, unpub.)

$$e^{W(J)} \sim \int \mathcal{D}\kappa \int d\Sigma_\mu \exp\left(\Delta s^\mu + J^\nu \hat{\nu}^\mu \delta\kappa^\nu\right)$$

$\hat{\nu}^\mu$ = normal to Σ^μ

After some computation,

$$\hat{\nu}^\mu \Delta s^\mu = -\frac{1}{2}\delta\kappa_{i\mu} \underbrace{\begin{pmatrix} \hat{u}\,\partial s/\partial T & \hat{u}\,\partial s/\partial\mu_B & s\hat{\nu}^\nu \\ \hat{u}\,\partial s/\partial\mu_B & \hat{u}\,\partial n/\partial\mu_B & n\hat{\nu}^\nu \\ s\hat{\nu}^\mu & n\hat{\nu}^\mu & -\hat{u}(sT+\mu_B n)g^{\mu\nu} \end{pmatrix}}_{\mathcal{F}^{\mu\nu}_{ij}} \delta\kappa_{j\nu}$$

$\hat{u} = u^\mu \hat{\nu}_\mu$

Includes fluc.'s in κ: T, μ_B, u^μ

Two-point fluctuations

Final result

$$n_{12} = \left\langle \frac{d^3N}{dp_1^3} \frac{d^3N}{dp_2^3} \right\rangle \sim \int d\Sigma^\mu \int dp_1^0 \int dp_2^0 \; p_1^\mu \, \hat{\nu}_1 \cdot p_2$$

$$\frac{\partial F(\not{p}_1)}{\partial K_i^\rho} \frac{\partial F(\not{p}_2)}{\partial K_j^\sigma} \left(\mathcal{F}_{ij}^{\rho\sigma} \right)^{-1}$$

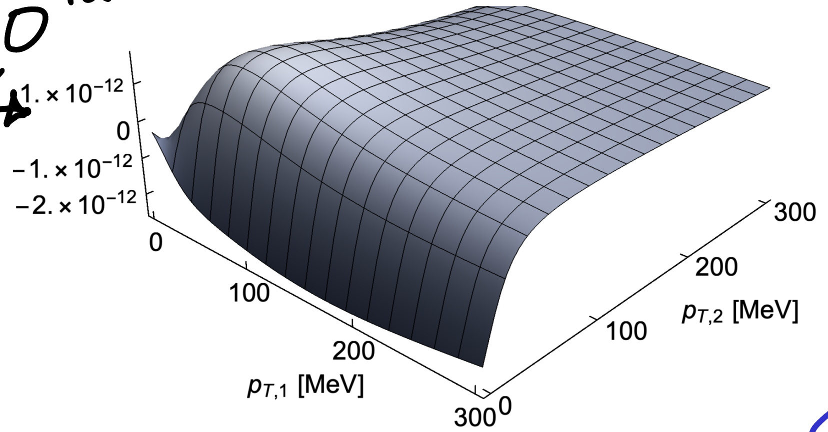
Computed for most optimistic case,

$z = -0.05$, $M = 1.2$ GeV, $m_{eff} = 20$ MeV

Fluc.'s : usual vs moat

Usual

10^{-12}

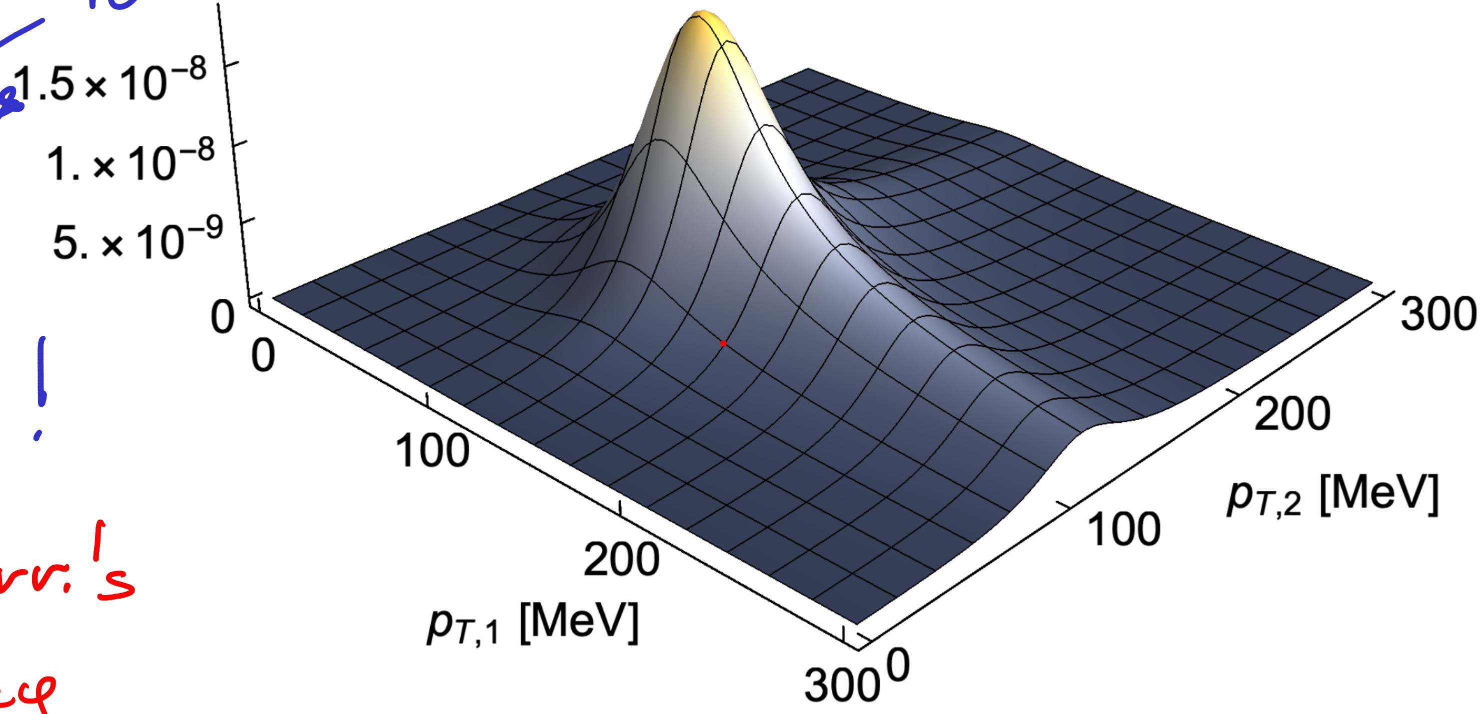


Flat, small

Plot: $\frac{n_{12}}{n_1 n_2}$ vs p_1 & p_2

Moat

10^{-8}



Moat: large peak!

Need to measure corr.'s @ low p_t $\bar{c}$ accuracy

2. Nuclear matter in 1+1 dim.'s

QCD in 1+1 dim.'s

Soluble in several ways.

't Hooft '74: $SU(N_c)$ soluble for massive quarks as $N_c \to \infty$, $m_{qk} \gg g$

Bringoltz 0901.4035: @ $\mu \neq 0$, kink crystal:
$\langle \bar{\psi}\psi \rangle$ oscillates about $\langle \bar{\psi}\psi \rangle \neq 0$

Baluni '80, Steinhardt '80...Armoni, Frishman, Sonnenschein, th/9709097
NJL: Azaria, Konik, Lecheminant, Palmai, Takacs, Tsvelik, 1601.02979
QCD: Lajer, Konik, Tsvelik, RDP, to appear

For arbitrary N_c, # flavors N_f, complicated

At strong coupling, $m_{qk} \ll g$ ($g \sim$ mass in 1+1 dim.'s)
Wess-Zumino-Witten models arise. Sure.

At $\mu \neq 0$, near Fermi surface, just WZW plus Luttinger liquid
Very simple!

Bosonization in 1+1 dim.'s

For small m_{qk}, bosonize.

$N_f = 1$: Abelian bosonization $\quad j_\mu = \bar{\psi}\gamma_\mu\psi = \varepsilon_{\mu\nu}\partial^\nu\phi$

Coleman '74 $\quad \bar{\psi}\psi \sim e^{i\#\phi}$

$N_f \geq 2$: non-Abelian boson'n

Witten '84...James, Konik, Lecheminant,Robinson,Tsvelik, 1703.046002

$N_f = 1$ simplest. Baluni '80: take $A_0 = 0$.

For $A_1 = A$ = $SU(N_c)$ matrix, clever choice of gauge:

A = off-diagonal, $\quad E$ = electric field = diagonal

$A^{aa} = 0$ (no sum!) $\quad E^{ab} = e^a \delta^{ab} \quad a, b = 1 \ldots N_c$

Yeah, so....

QCD_{1+1}, $N_f = 1$

Gauss' law: $\partial_x e^a = j_0^{aa}$ $\quad ig(e^a - e^b) A^{ab} = j_0^{ab}$ $\quad a \neq b$

Integrating out A, get $S_{int} \sim \int\int j_0(x) \frac{1}{|x-y|} j_0(y)$

With Abelian boson'n, $N_c - 1$ sine-Gordon models: (π = conj. mom.)

$$\mathcal{H}_0 = \frac{1}{2} \sum_{a=1}^{N_c} \pi_a^2 + \tilde{m}\left(1 - \cos(2\sqrt{\pi}\, e^a)\right) \qquad \tilde{m} \sim m_{qk}$$

$$\mathcal{H}_g = \sum_{a,b=1}^{N_c} \frac{g^2}{8\pi N_c} (e^a - e^b)^2 + \frac{\Lambda^2}{g} \frac{\sin(2\sqrt{\pi}(e^a - e^b))}{e^a - e^b}$$

from normal ordering, so $\Lambda \sim g$

Still, $N_c - 1$ s-G models: not easy! Plus $\mathcal{H}_g$ from gauge int.'s

QCD_{1+1}, $N_f = 1$, $\mu \neq 0$

Remember $j_0 \sim \partial_z \varphi$. So constant j_0 is like $\varphi \sim \mu z$

Writing $e^a \to \varphi^a$, $\varphi \equiv \frac{1}{\sqrt{N_c}} \sum_{a=1}^{N_c} e^a$, π = conj. mom,

$$\mathcal{H}_{eff}(\varphi) = \frac{1}{2}\left(\pi^2 + (\partial_x \varphi)^2\right) + \tilde{m}\left(1 - \cos\sqrt{\frac{4\pi}{N_c}}\,\varphi\right)$$

At $\mu \neq 0$, only φ matters. All $V(\varphi^a - \varphi^b)$ drop out.

$$\mathcal{L}_{eff} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{\tilde{m}}{2\pi}\cos\left(\sqrt{\frac{4\pi}{N_c}}\,\varphi + 2k_0 x\right)$$

$\hookrightarrow k_0 = \frac{\mu}{N_c}$

Spectra: soliton $\bar{c}$ mass m_s
anti-soliton " "
breathers = soliton + $\overline{\text{soliton}}$

Luttinger liquid

μ = baryon μ. For Fermi sea, $\mu > m_{soliton}$

At low energies,

$$\mathcal{L}_{eff} = \frac{K}{2}\left(\frac{1}{v_F}(\partial_0 \varphi)^2 + v_F(\partial_x \varphi)^2\right)$$

Just single, massless boson $\varphi \sim U(1)$ current

K = Luttinger parameter $= K(\mu)$

v_F = Fermi velocity $= v_F(\mu)$

non-Fermi liquid – excitations near Fermi surface are <u>not</u> baryons, but φ

Soln. of Luttinger liquid

M. Lajer, R. Konik, A. Tsvelik, RDP, to appear

Use Thermodynamic Bethe Ansatz:

$\mu \to m_s$: $K \to 1$, $v_f \to 0$

φ doesn't propagate

$\mu \gg m_s$: $K \to \frac{1}{N_c}$, $v_f \to 1$

φ relativistic, $K(\infty)$?

Soln. of Luttinger liquid

M. Lajer, R. Konik, A. Tsvelik, RDP, to appear

Both K, v_F are functions of μ.

When Fermi surface first appears:

$\mu \to m_{soliton}$: $K \to 1$, $\underbrace{v_F \to 0}_{}$

q doesn't propagate

At high μ:

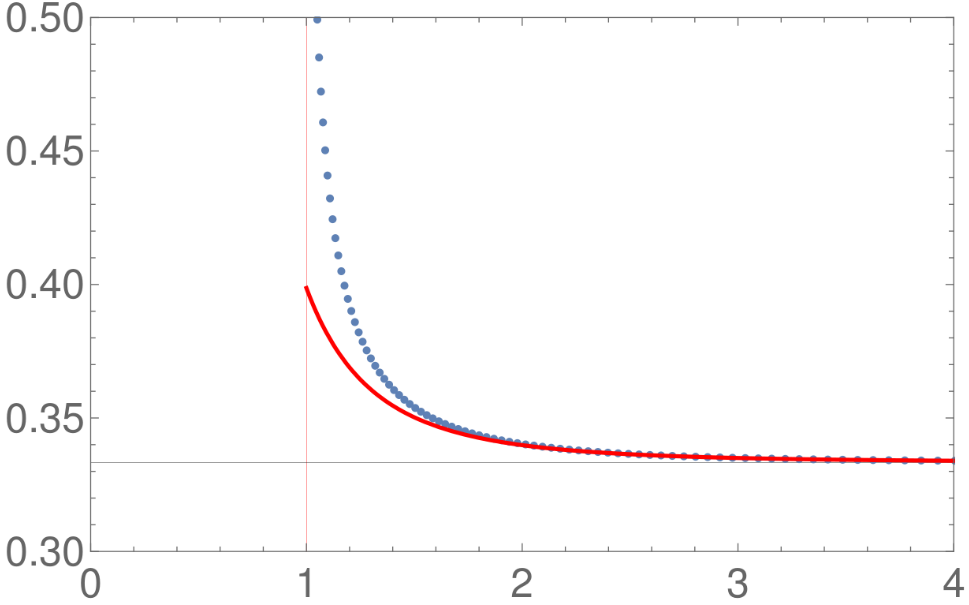
$\mu \gg m_{soliton}$ $\underbrace{K \to \frac{1}{N_c}}_{!}$, $\underbrace{v_F \to 1}_{}$

relativistic propagation

For any μ

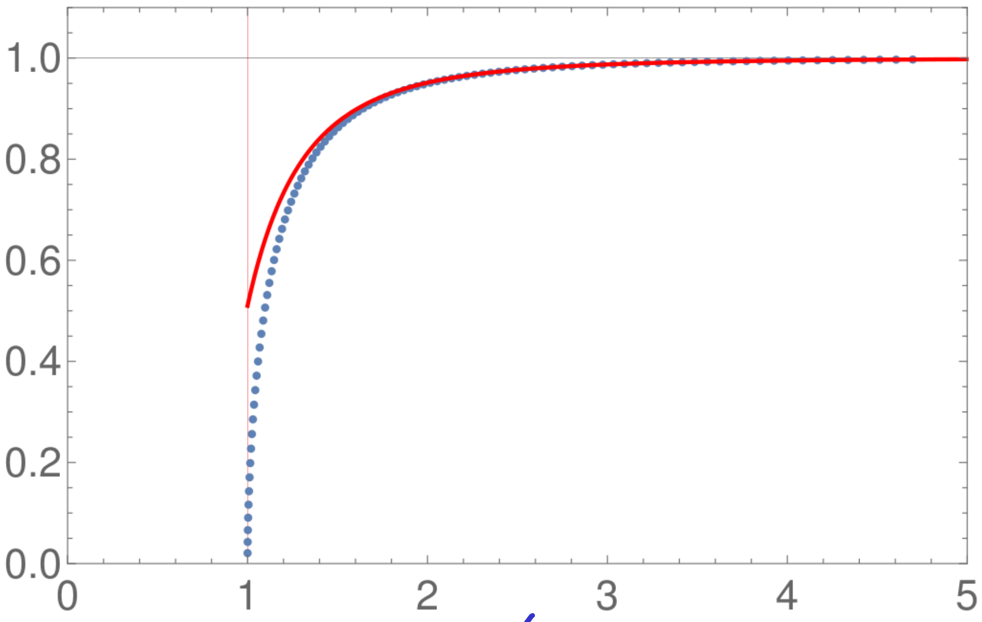
Using Thermodynamic Bethe Ansatz, (blue dots) vs pert. theory in mass (red line) $\underline{N_c = 3, \ N_f = 1}$

$K \uparrow$



$\mu / m_{soliton} \longrightarrow$

$v_F \uparrow$



$\mu / m_s \longrightarrow$

Open questions

1+1 dim.'s: compute for small mass.

$N_f \geq 2$: WZW + Luttinger liquid

Computed K, v_F for $N_c = 3$, $N_f = 2$

NJL: Luttinger liquid only for small mass;

at large, "strange" metal (non-Fermi)

QCD: guess non-Fermi $\forall$ masses

Luttinger liquid " "

Previous work: computed spectrum of baryons = solitons

complicated

Here: near Fermi surface, much simpler

Is Quarkyonic matter a non-Fermi liquid?

In 3+1 dim's: nuclear matter Fermi liquid @ low density

In quarkyonic regime? Can reduce to effectively 1-dim

$$\int \frac{d^4k}{(k^2)^2} \underset{\int d^2k_\perp}{\sim} \int \frac{d^2k}{k^2} \quad \text{qualitative}$$

On Fermi sphere, width Λ_{QCD}:

At large $\mu \sim N_c^{1/4}$, many patches

$\Rightarrow$ effectively 1-dim.

In 3+1 dim's, $\bar{c}$ many patches, perhaps get an effective ϕ,

$$\partial_z \phi \sim j_0 = \text{baryon current!?}$$

Symmetries of higher spin mesons in χ-symmetric phase

Comment on:

Glozman & Pak, 1504.02323
Denissenya, Glozman, Pak, 1508.01413
Catillo & Glozman 1709.01886, 1804.07171
Catillo, Glozman, Lang, 1904.01969
Glozman, 2005.10538

Lattice:
Rohrhofer, Aoki, Cossu, Fukaya, Glozman, Hashimoto, Lang, Prevlosek, 1707.01881
Rohrhofer, Aoki, Cossu, Fukaya, Gattringer, Glozman, Hashimoto, Lang, Prevlosek, 1902.03191
Rohrhofer, Aoki, Glozman, Hashimoto, 1909.00927

χ-symmetric mesons

Take $SU(N_c)$ colors, N_f flavours: $q^{a,i}$ $\quad a = 1,\dots,N_c$, $i = 1,\dots,N_f$

χ-sym $= SU(N_f)_L \times SU(N_f)_R \times U(1)_A$:

$$q_{L,R} = \underbrace{\frac{1}{2}(1 \mp \gamma_5)}_{P_{L,R}}\, q \,, \quad q_{L,R} \to e^{\mp \frac{i\alpha}{2}}\, U_{L,R}\, q_{L,R}$$

$P_L P_R = 0$. $\bar{q}_L q_L = (\bar{q} P_R)(P_L q) = 0$. Only one spin-0:

$$\Phi^{ij} = \bar{q}^i P_L q^j = \bar{q}^i_R q^j_L \,, \quad \Phi \to e^{-i\alpha}\, U_L \Phi U_R^\dagger$$

$\Phi \sim LR \sim$ "heterochiral"

Giacosa, Koenigstein, RDP, 1709.07454

Spin-1 mesons

Usual: "homochiral"

$$L_\mu = \bar{q}_L \gamma_\mu q_L , \quad R_\mu = \bar{q}_R \gamma_\mu q_R$$

$$\to U_L L_\mu U_L^\dagger , \quad R_\mu \to U_R R_\mu U_R^\dagger$$

= adjoint of SU_L & SU_R: $\rho, \omega, \phi ; a_1, f_1 \ldots$

Also:

$$\Phi_\mu = \bar{q}_R \overleftrightarrow{D}_\mu q_L \qquad \Phi_\mu \to e^{-2i\alpha} U_L \Phi_\mu U_R^\dagger$$

heteroχ. Fundamental rep. of $SU_L \times SU_R \times U(1)_A$

$b_1(1235), h_1(1170) \ldots ; \rho(1700), \omega(1650), \ldots$

Mixing for spin-1

L_μ, R_μ, & Φ_μ are in different representations

$\Rightarrow$ no direct mixing. Mass terms:

$$m_1^2 \, \mathrm{tr}\left(L_\mu^\dagger L_\mu + R_\mu^\dagger R_\mu\right), \quad m_2^2 \, \mathrm{tr}\left(\Phi_\mu^\dagger \Phi_\mu\right)$$

Quartic couplings:

$$\lambda_1 \left(\mathrm{tr}\left(L_\mu^\dagger L_\mu + R_\mu^\dagger R_\mu\right)\right)^2$$

$$\lambda_2 \left(\mathrm{tr}\, \Phi_\mu^\dagger \Phi_\mu\right)^2$$

+ <u>many</u> others!

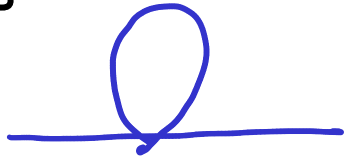
$$\lambda_{mix} \, \mathrm{tr}\left(L_\mu^\dagger L_\mu + R_\mu^\dagger R_\mu\right) \mathrm{tr}\, \Phi_\mu^\dagger \Phi_\mu$$

Will assume λ_{mix} dominates

Spin-1 at high T

For simplicity, assume $T \gg m_1, m_2$. Tadpoles:

$$\delta m^2 \sim$$



$$\sim \lambda T^2$$

So:

$$\delta m^2_{L,R} \sim \lambda_1 T^2 \qquad\qquad \delta m^2_{\Phi} \sim \lambda_2 T^2$$

IF λ_{mix} dominates,

$$\delta m^2_{L,R} = \delta m^2_{\Phi} \sim \lambda_{mix} T^2$$

Accidental degeneracy as $T \to \infty$

Glozman & RDP, in progress

Lattice results

In χ-sym. phase, $T: 300 \to 600$ MeV,

all spin-1 mesons $\neq$ free field!

As $T \neq 0$, mass $V_0 \neq$ mass V_i

Lattice: approx. degeneracy in masses:

$$\text{mass } L_0 = R_0 \approx \text{mass } \Xi_0$$

$$\text{mass } L_i = R_i \approx \text{mass } \Xi_i$$

Accidental degeneracy or extended sym.?

Other spin-1 ?

What about

$$\chi_{\mu\nu} = \bar{q}_R(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu) q_L \to e^{-2i\alpha} U_L \chi_{\mu\nu} U_R^\dagger$$

But: anti-sym. tensor = spin-1 in 4-dim's

$$\chi_{\mu\nu} \to \varepsilon_{\mu\nu\alpha\beta}\,\partial_\alpha \Phi_\beta$$

So $\chi_{\mu\nu} \sim \Phi_\mu$. Both hetero χ. Because of $\varepsilon_{\mu\nu\alpha\beta}$, if $\partial_0 = 0$,

$$\chi_{ij} \sim \varepsilon_{ijk}\Phi_0 \qquad \chi_{0i} \sim \varepsilon_{ijk}\,\partial_j \Phi_k$$

Explains lattice results

Higher spin

Spin 2:

$$L_{\mu\nu} = \bar{q}_L \left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu \right) q_L \quad = \quad \text{homo } \chi$$

$$R_{\mu\nu} = \bar{q}_R \left(\quad '' \qquad\qquad '' \quad \right) q_R$$

and

$$\Phi_{\mu\nu} = \bar{q}_R \overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu \, q_L \quad = \quad \text{hetero } \chi$$

Conjecture mass degeneracy:

masses

$$L_{00},\ R_{00} \approx \Phi_{00}$$
$$L_{0i},\ R_{0i} \approx \Phi_{0i}$$
$$L_{ij},\ R_{ij} \approx \Phi_{ij}$$

Moral?

Certainly: @ $T_\chi \to (3-4)\, T_\chi$, not nearly free q's & g's

Glozman+..., stringy fluid

Semi-QGP: in matrix model, assume T_{deconf} with qks $\approx$ as without

Dumitru, Guo, Hidaka, Altes, RDP, 1011.3820; 1205.0137; RDP, Skokov 1604.00022 + ...

Spin-1 mesons: surprising degeneracy

Accidental? Really constrains couplings

Or extended symmetry (Glozman+...)?

Is T_{deconf} with quarks $\approx$ same as pure glue?