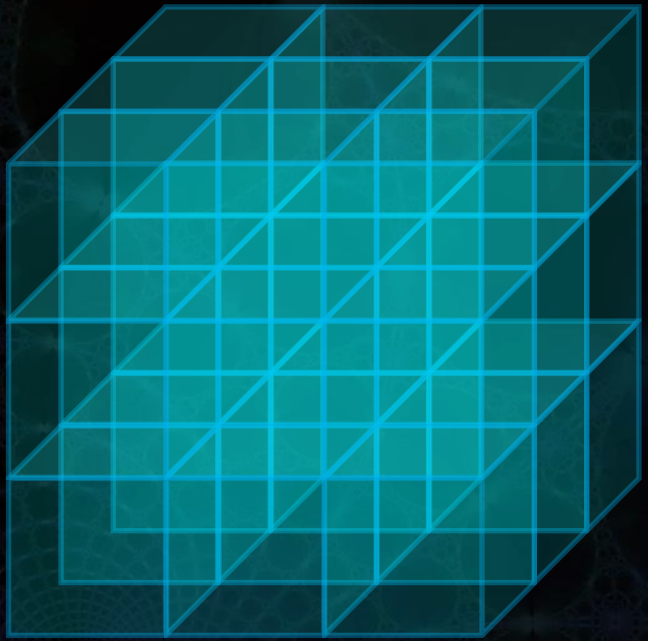
YITP workshop “QCD phase diagram and lattice QCD”

Axial U(1) symmetry at high temperature with chiral fermions



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Yoshifumi Nakamura (RIKEN R-CCS), Christian Rohrhofer (Osaka U.)

Talk by S. Hashimoto (Wed.) → Lattice chiral symmetry should be treated carefully

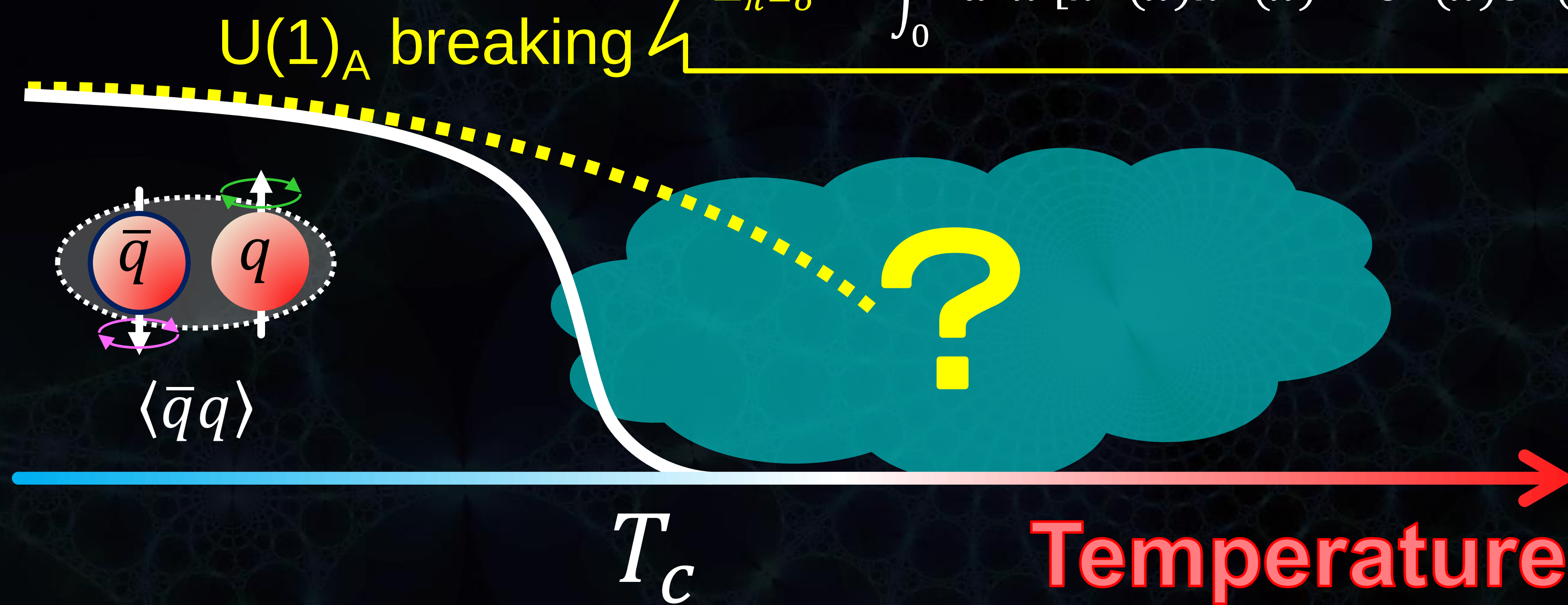
Does the $U(1)_A$ anomaly disappear/survive above T_c?

- Above T_c, chiral symmetry breaking via $\langle \bar{q} q \rangle$ disappears

  ⇒How about <u>$U(1)_A$ symmetry breaking?</u>

An "order parameter" is, for example,

$$\Delta_{\pi-\delta} = \int_0^\infty d^4x \, [\pi^a(x)\pi^a(x) - \delta^a(x)\delta^a(x)]$$



Lattice study with chiral fermion

by JLQCD Collaboration (2012-2020) ⇒ $U(1)_A$ anomaly is suppressed

	valence/sea quark	Setup
G. Cossu et al. PRD87,114514 (2013)	OV on OV (Topology fixed sector)	Nf=2
A. Tomiya et al. PRD96, 034509 (2017)	DW on DW OV on DW OV on (reweighted) OV	Nf=2, 1/a=1.7GeV (a=0.11fm)
S. Aoki et al. arXiv:2011.01499 arXiv:2103.05954	OV on DW OV on (reweighted) OV	Nf=2, 1/a=2.6GeV (a=0.076fm) (Finer lattice)
In progress (JLQCD, 2021)	DW on DW OV on (reweighted) OV	Nf=2+1, 1/a=2.453GeV (a=0.08fm)

Publications for N_f=2 (for JLQCD 2012-2020)

- At T∼1.1T_c, $U(1)_A$ and topological susceptibilities are strongly suppressed near the physical quark mass [arXiv:2011.01499]
- At 220<T<500MeV, $SU(2)_{cs}$ and SU(4) symmetries emerge [arXiv:1902.03191,1909.00927] ⇒Talk by L. Glozman [Thu.]
- Chiral susceptibility is dominated by $U(1)_A$ anomaly [arXiv:2103.05954] ⇒Poster by H. Fukaya [Wed.]

⇒How about N_f=2+1 ?

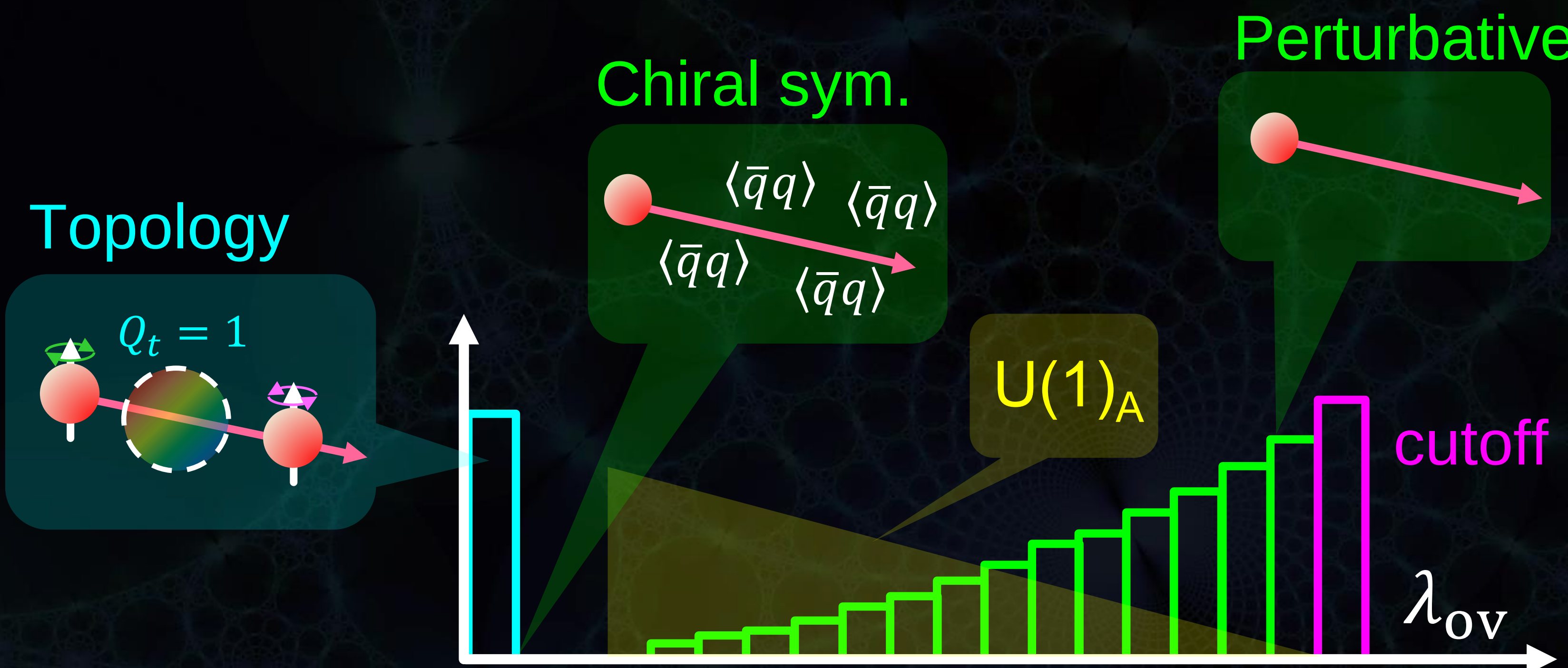
Outline

1. Introduction
2. N_f=2+1 results at T=1.3T_c and 1.1T_c
   - 2-1: Dirac spectrum
   - 2-2: Topological susceptibility
   - 2-3: $U(1)_A$ susceptibility
   - 2-4: Mesonic correlators
3. Summary

Lattice setup (generated mainly on Fugaku)

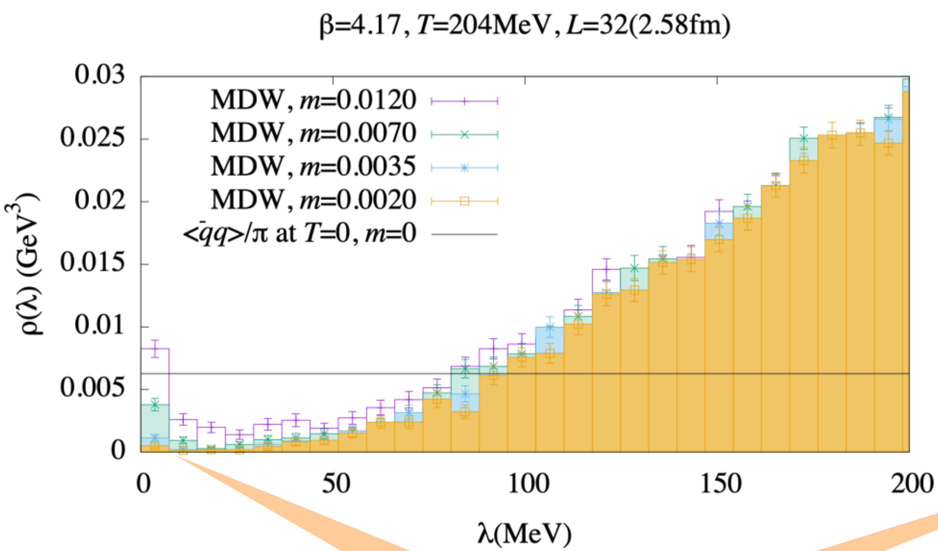
- Nf=2+1 Möbius-DW / overlap fermions
- 1/a=2.453GeV (a=0.08fm)
- L=32 (2.58fm)
- T=204MeV (1.3Tc), 175MeV (1.1Tc)
- m_q=5MeV (phys. pt.), 9, 17, 29MeV
- m_s=100MeV (phys. pt.)

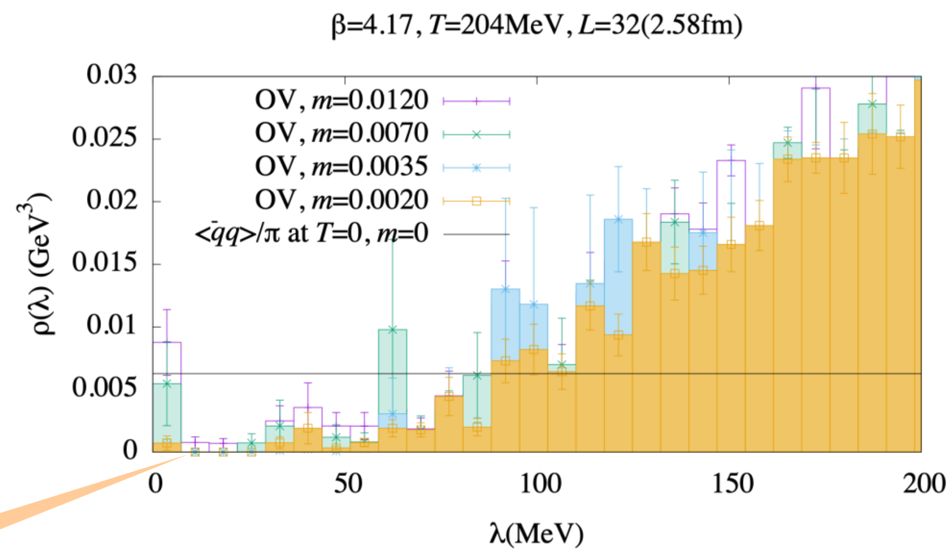
Dirac spectrum and QCD physics at different scales



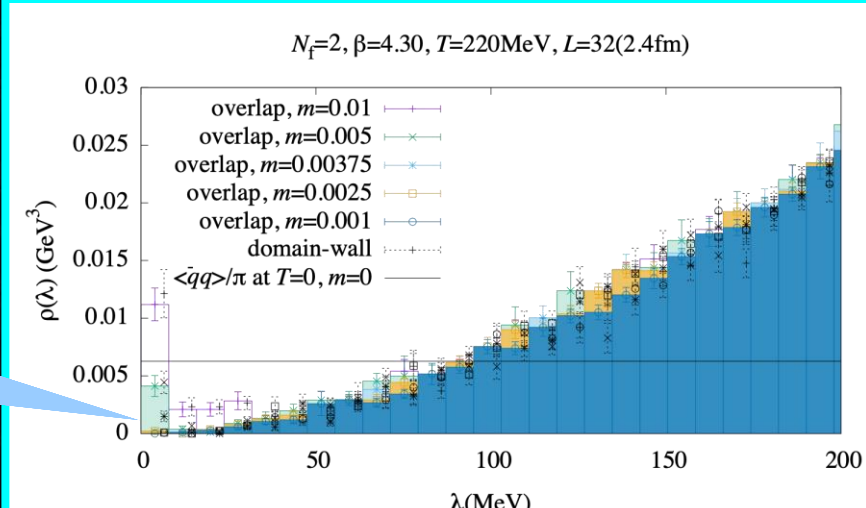
JLQCD, preliminary (2021)

Dirac spectrum at T = 204MeV(1.3Tc)





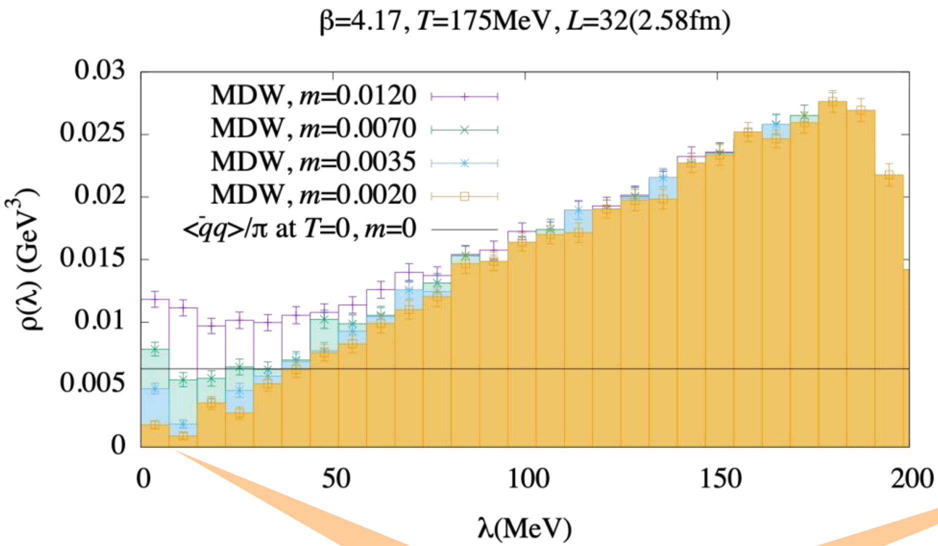
At physical m_q, lower eigenmodes are strongly suppressed

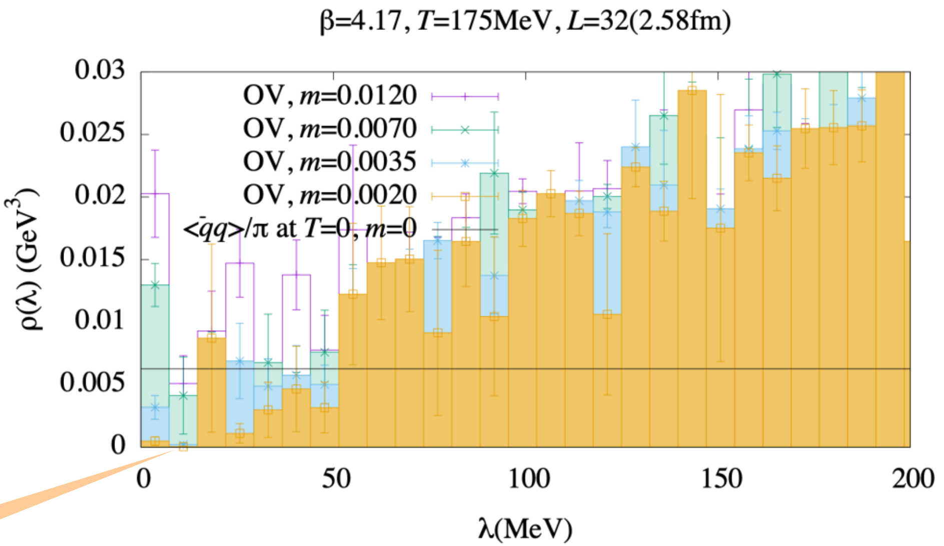


N_f=2+1 is consistent with N_f=2 (T=220MeV)

Nf=2, JLQCD, 2011.01499

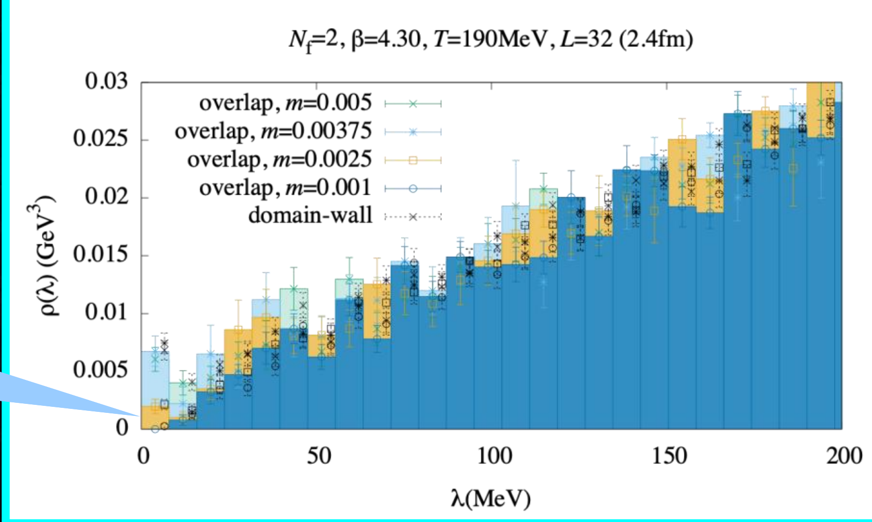
Dirac spectrum at T = 175MeV(1.1Tc)





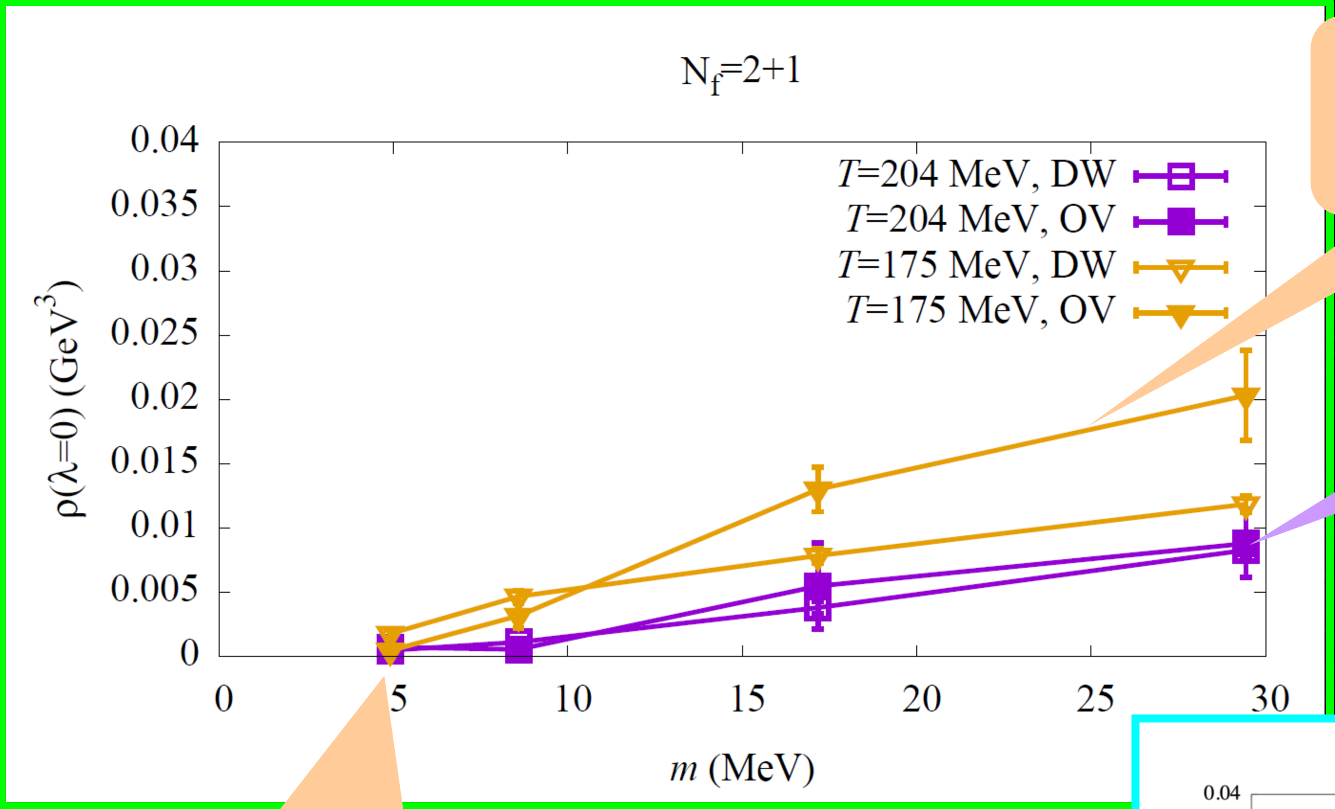
At physical m_q, lower eigenmodes are strongly suppressed

N_f=2+1 is consistent with N_f=2 (T=190MeV)



Nf=2, JLQCD, 2011.01499

Lowest bin of Dirac spectrum

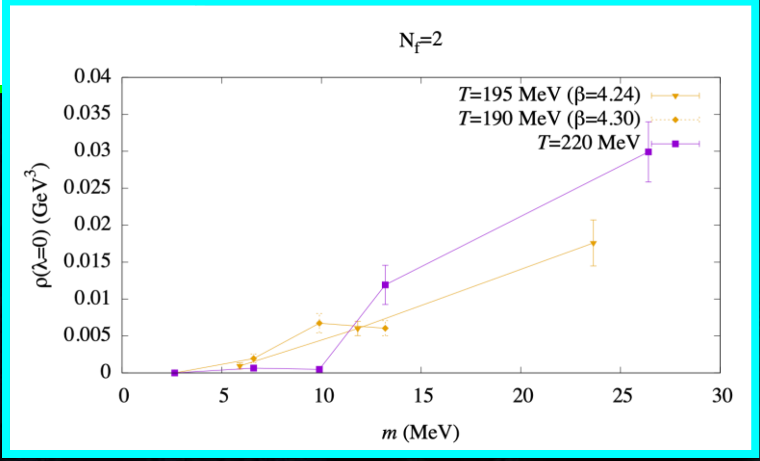


T=175MeV
(1.1T_c)

T=204MeV
(1.3T_c)

Lowest bin of Dirac spectrum is strongly suppressed (related to χ_t and $U(1)_A$)

Nf=2, JLQCD, 2011.01499

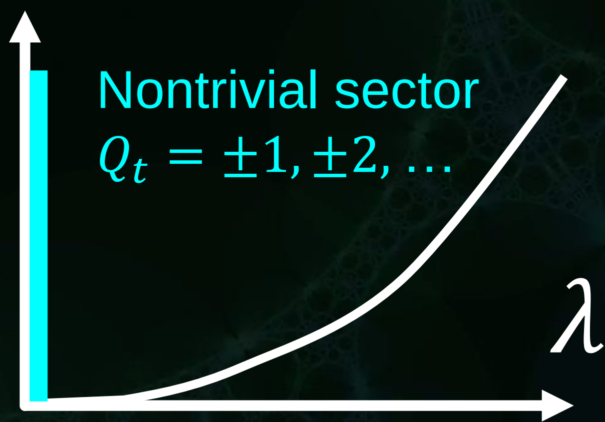


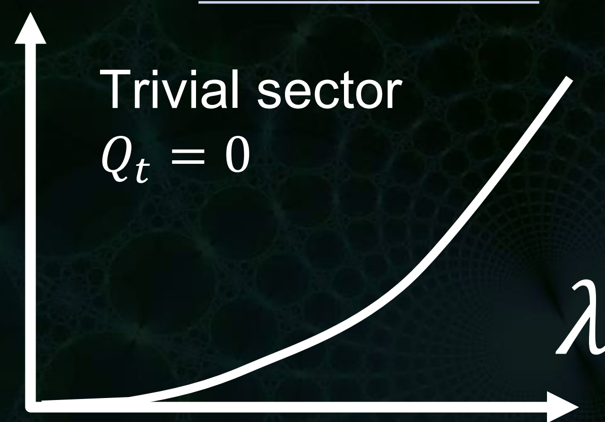
Topological susceptibility and zero mode of Dirac spectra

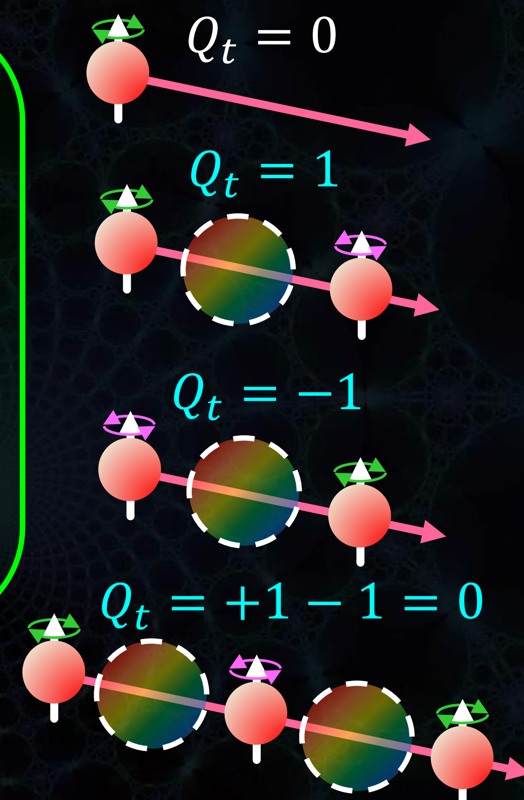
$$\chi_t \equiv \frac{\langle Q_t^2 \rangle}{V}, \qquad Q_t = n_+ - n_-$$

Topological charge Q_t is related to #of Dirac zero mode（Index theorem）

$\rho(\lambda)$



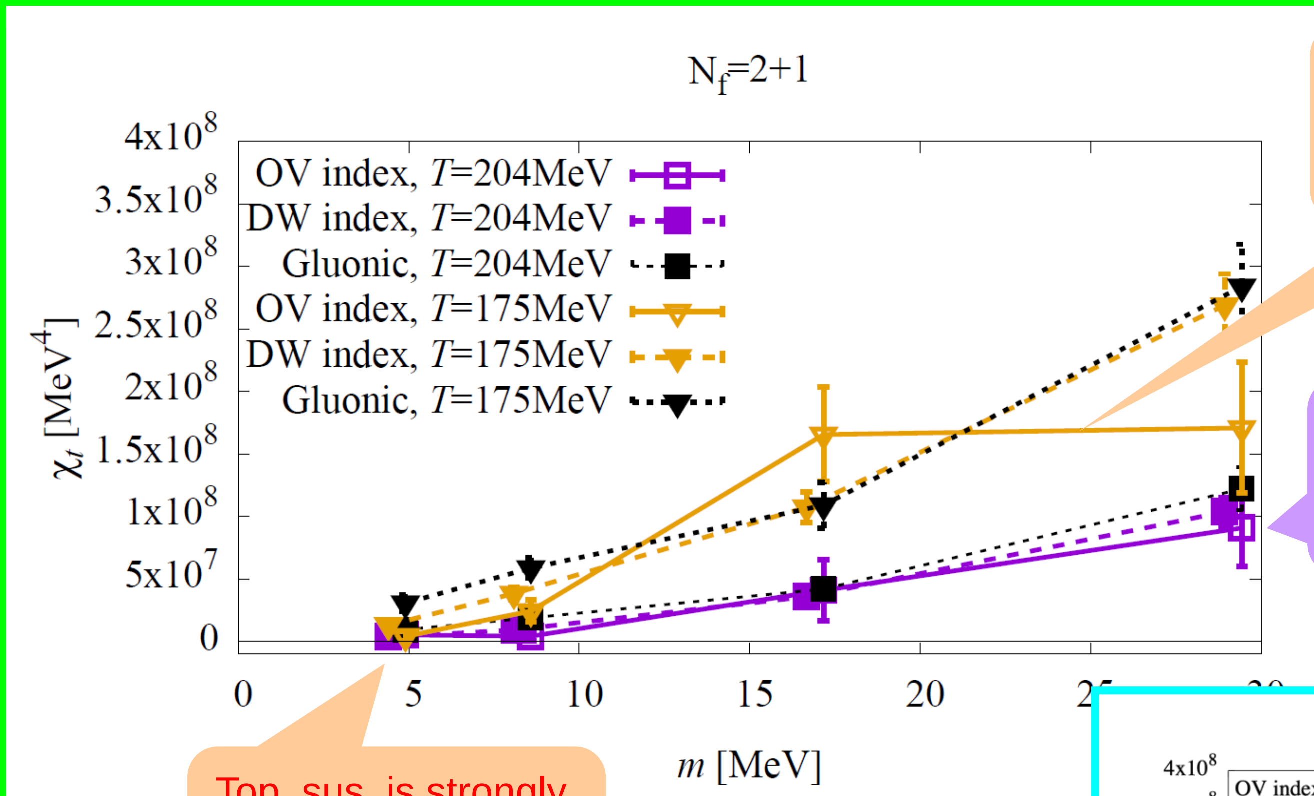




Cf.) Gluonic definition: $Q_t \equiv \frac{g^2}{32\pi^2} \int d^4x\, G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}$

JLQCD, preliminary (2021)

Topological susceptibility

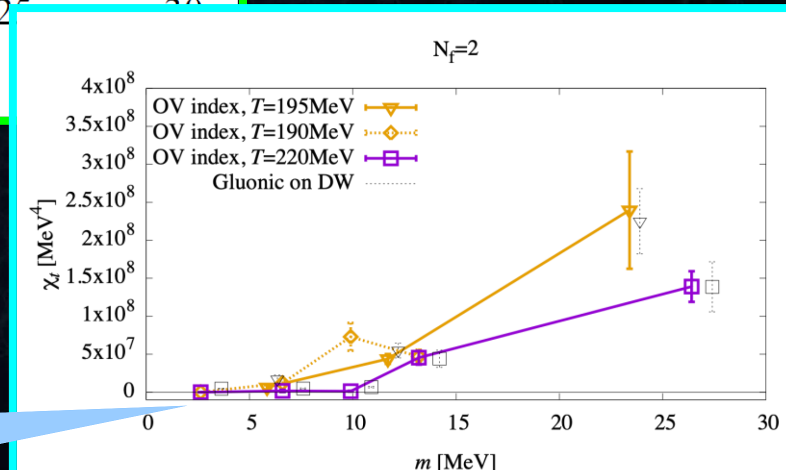


T=175MeV ($1.1T_c$)

T=204MeV ($1.3T_c$)

Top. sus. is strongly suppressed already at 1.1Tc

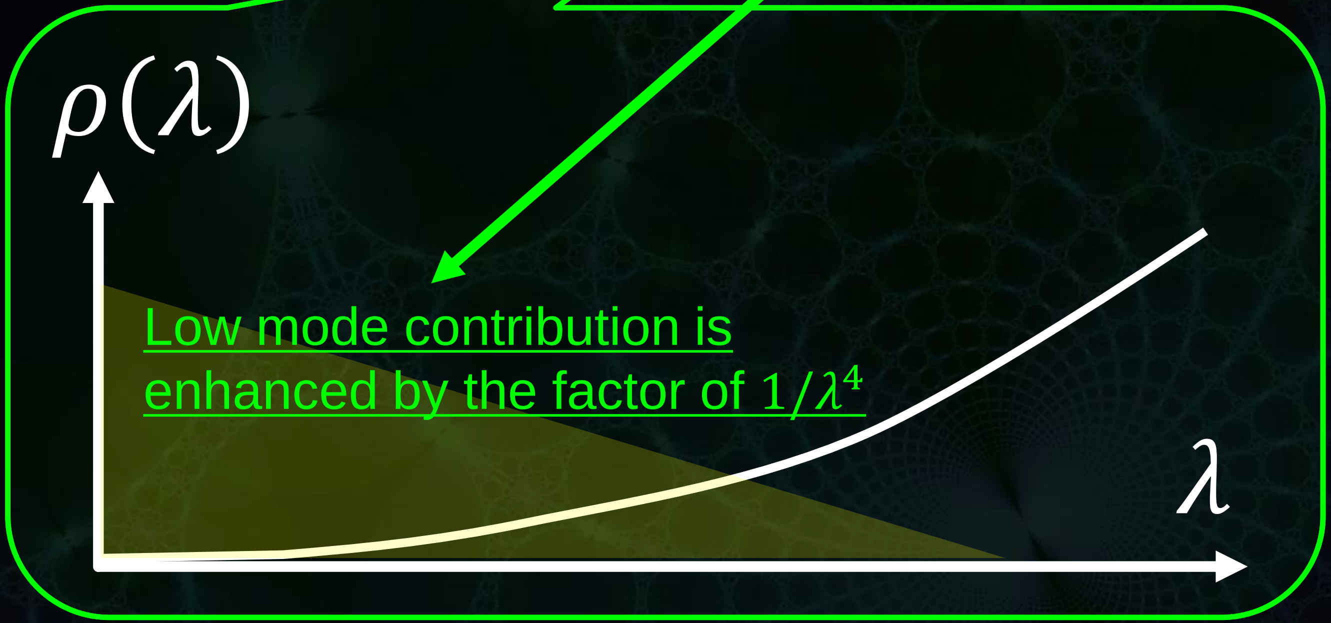
Nf=2, JLQCD, 2011.01499



quantitatively similar to $N_f=2$

U(1)$_A$ susceptibility and low eigenmodes of Dirac spectrum

$$\Delta_{\pi-\delta} = \int_0^\infty d\lambda\, \rho(\lambda)\, \frac{2m^2}{(\lambda^2+m^2)^2}$$

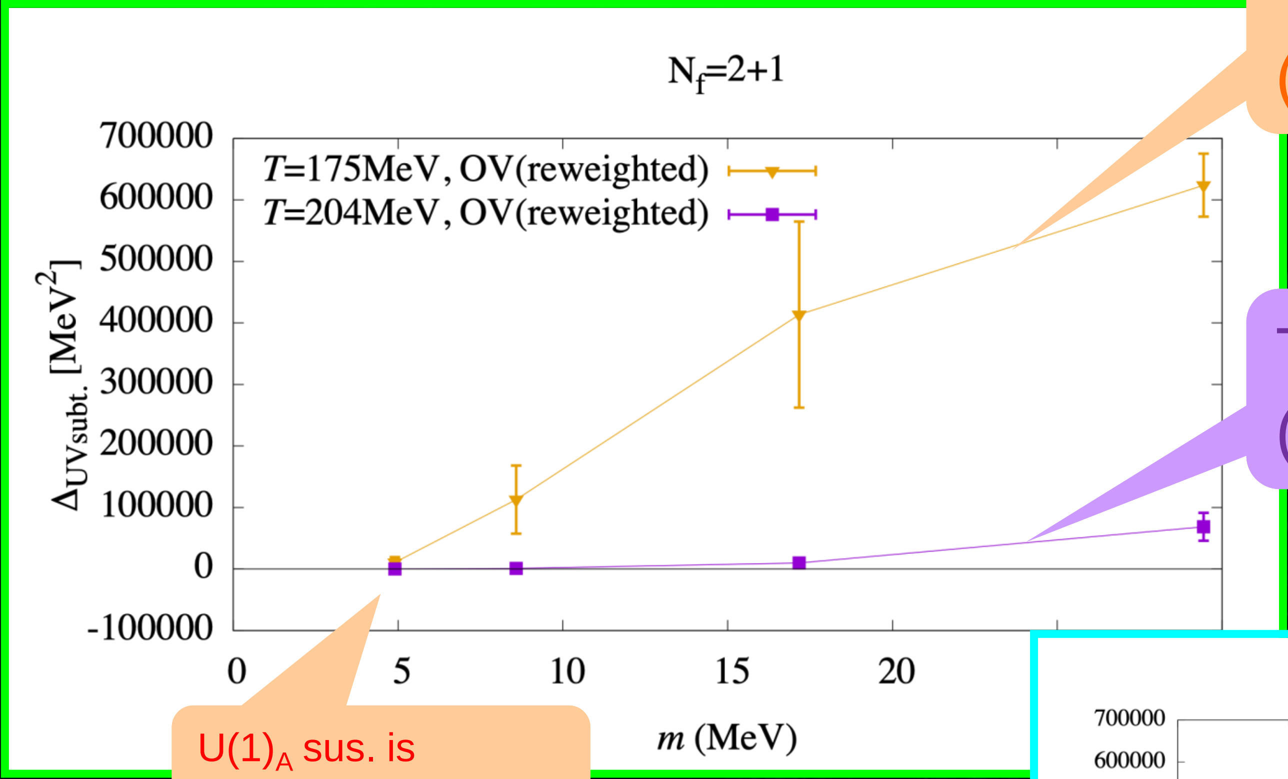


Cf.) Banks-Casher relation: $\langle \bar{q}q \rangle = \lim_{m\to 0} \int_0^\infty d\lambda\, \rho(\lambda)\, \frac{2m}{\lambda^2+m^2}$

Note: precise definition (subtracting zero mode and UV div.) → backup

JLQCD, preliminary (2021)

$U(1)_A$ susceptibility

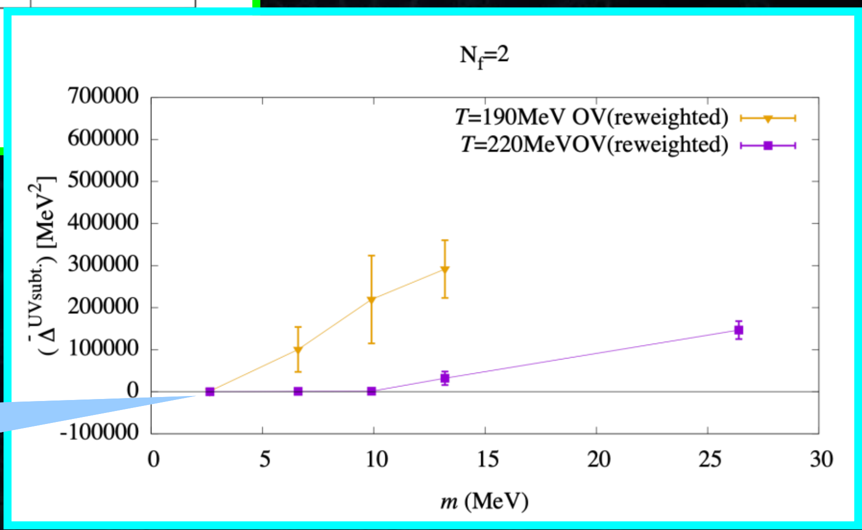


T=175MeV (1.1T_c)

T=204MeV (1.3T_c)

$U(1)_A$ sus. is strongly suppressed

Nf=2, JLQCD, 2011.01499



quantitatively similar to N_f=2

Spatial mesonic correlators

- We use iso-triplet spatial correlators in z-direction

$$C_\Gamma(z) = -\sum_{x,y,t} \langle \bar{u}\Gamma d(x,y,z,t)\, \bar{d}\Gamma u(0,0,0,0)\rangle$$

$$\Gamma = \gamma_5(\mathbf{PS}), 1(\mathbf{S}), \gamma_{1,2}(\mathbf{V}), \gamma_5\gamma_{1,2}(\mathbf{A}), \gamma_4\gamma_3(\mathbf{T}), \gamma_5\gamma_4\gamma_3(\mathbf{X})$$

Scalar correlator seems to be noisy

$U(1)_A$ $SU(2)_L \times SU(2)_R$ $U(1)_A$

Tensor and aXial-tensor are U(1)A partners

$$C_\Gamma(z) = -\int d\omega\, \rho_\Gamma(\omega) e^{-\omega z}$$

$\rho_\Gamma(\omega)$ is spatial spectral function

Ansatz 1 (an isolated pole):

$$\rho_\Gamma(\omega) \equiv \delta(\omega - m_{scr})$$

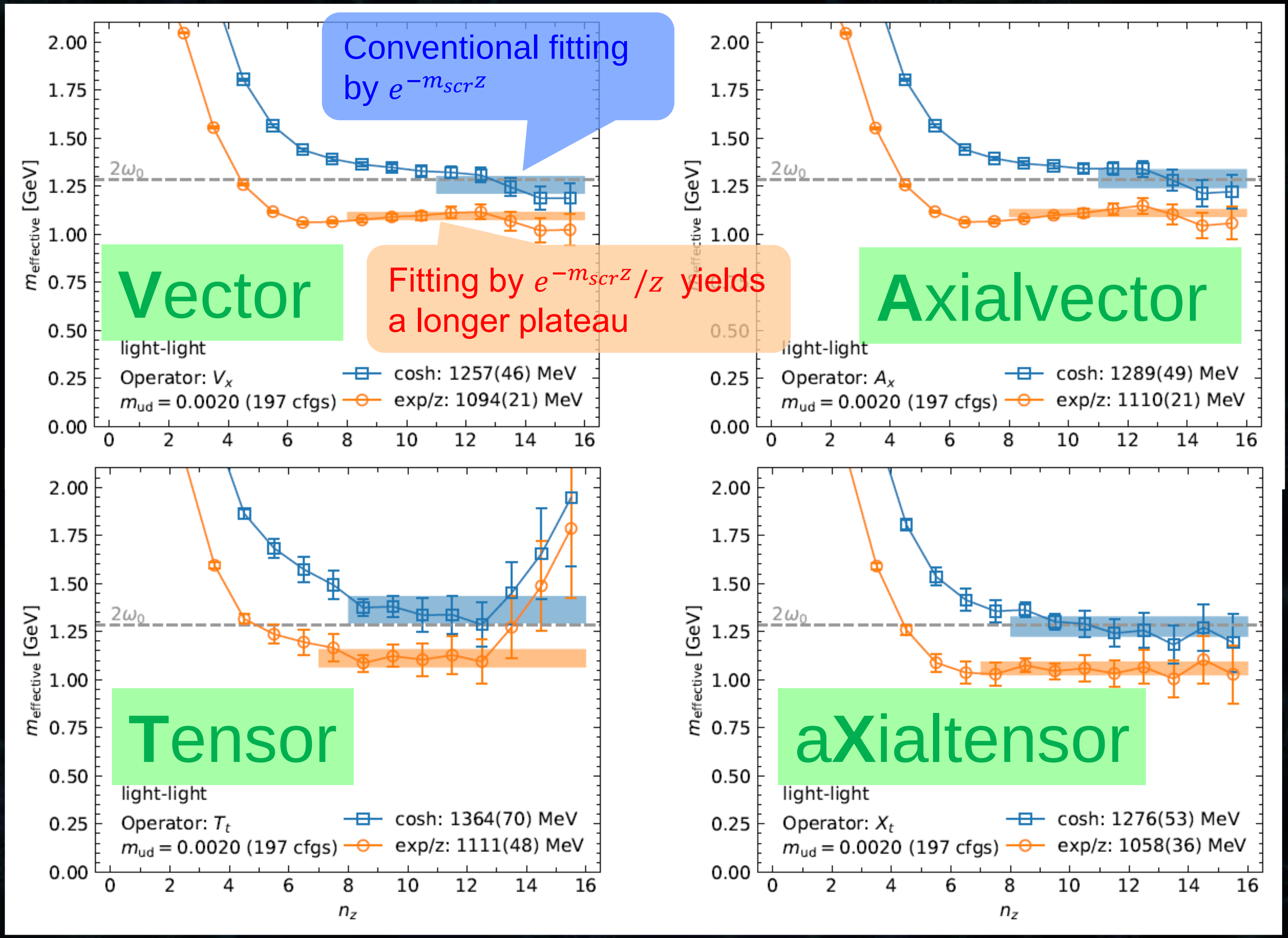
Ansatz 2 (2-quark threshold):

$$\rho_\Gamma(\omega) \equiv \theta(\omega - m_{scr})(c_0 + c_1\omega + \cdots)$$

$$C_\Gamma(z) \sim e^{-m_{scr} z}$$

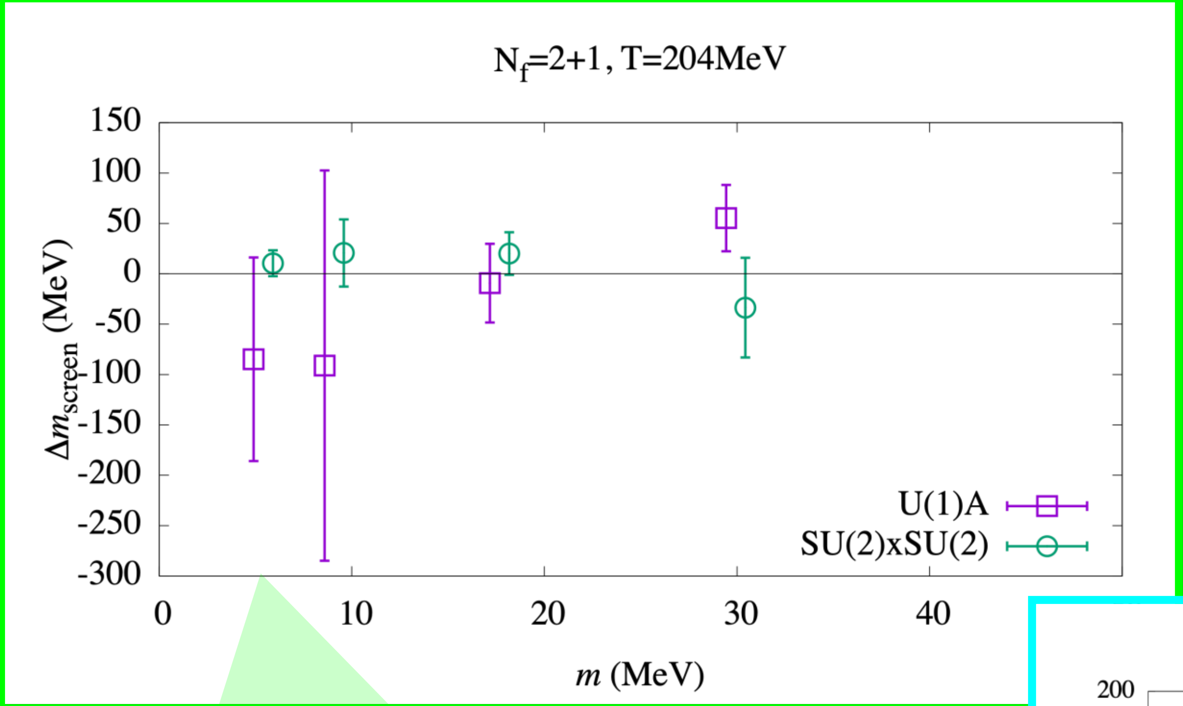
$$C_\Gamma(z) \sim e^{-m_{scr} z}(1/z + O(1/z^2))$$

Examples of effective screening masses



JLQCD, preliminary (2021)

Screening mass difference

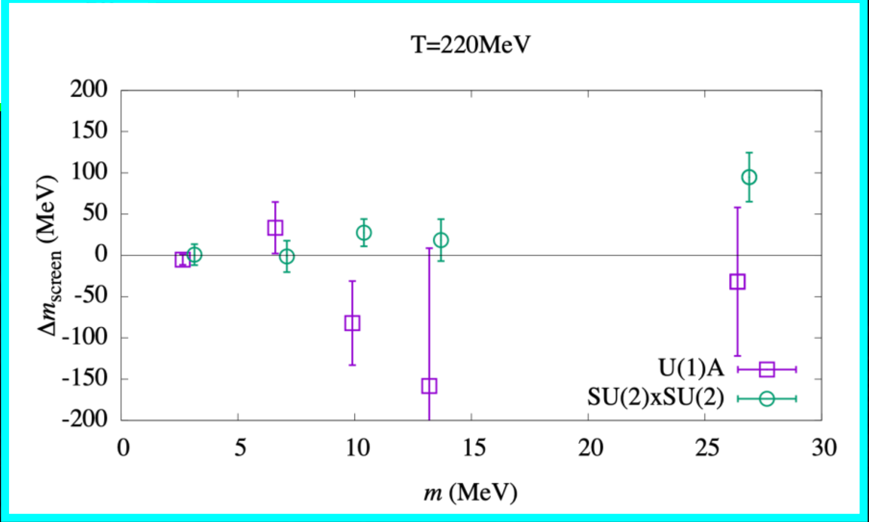


$$\Delta m_{scr}^{SU(2)} = m_{scr}^{V} - m_{scr}^{A}$$
$$\Delta m_{scr}^{U(1)_A} = m_{scr}^{T} - m_{scr}^{X}$$

At physical m_q,

- SU(2) × SU(2): consistent with zero toward the chiral limit
- $U(1)_A$: consistent with zero within errors ($\Delta m_{scr}/m_{scr}$ ~ 100MeV/1GeV)

Nf=2, JLQCD, 2011.01499



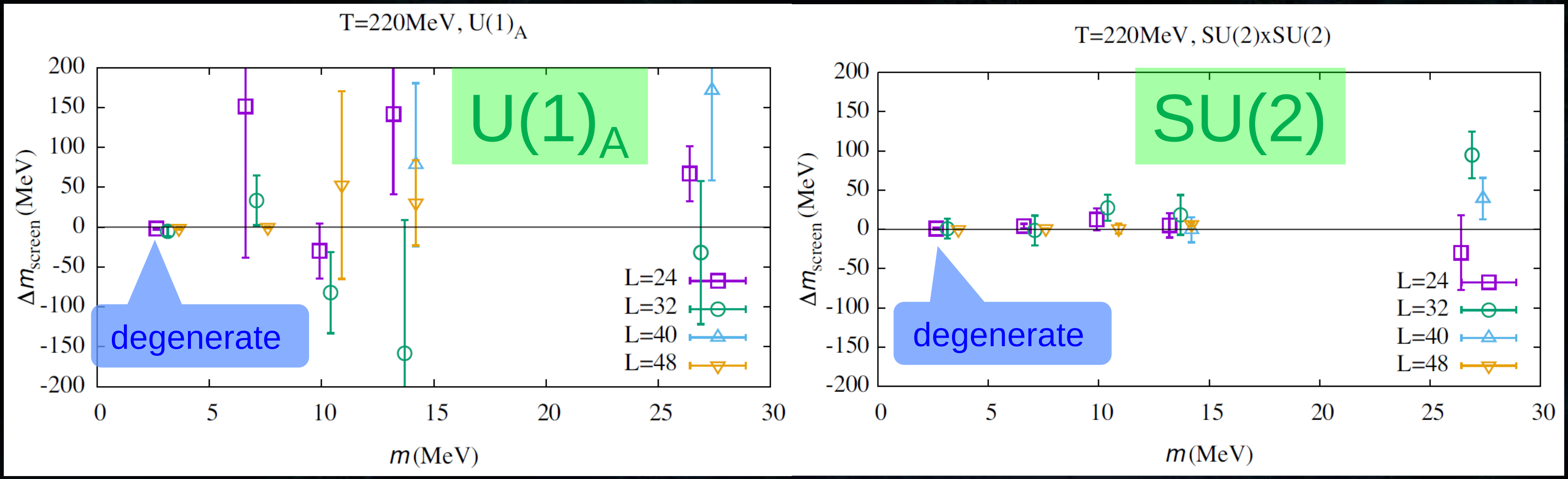
Summary

- We study high-temperature phase (1.1T_c and 1.3T_c) with N_f=2 and N_f=2+1 chiral fermions
- Top. susceptibility drops to be consistent with zero at physical m_q
- $U(1)_A$ susceptibility is also strongly suppressed in the chiral limit
- From screening masses, we find degenerate chiral and $U(1)_A$ partners
- In progress: T=165MeV (<T_c at N_f=2), T=153MeV (~1.0T_c at N_f=2+1), and V-dependence

Backup

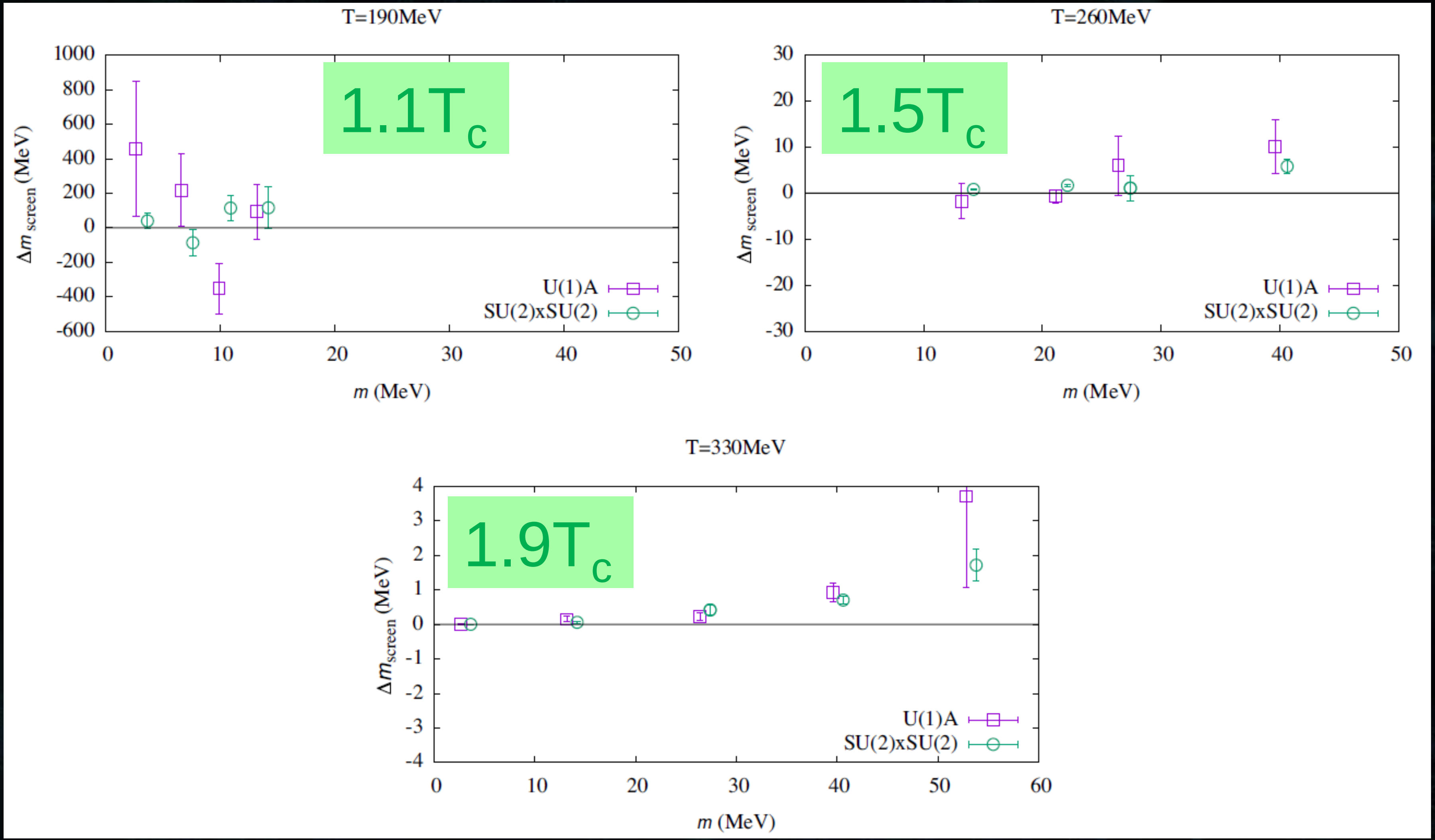
Screening mass difference (mesons)

T=220MeV~1.25T_c



Nf=2: S. Aoki et al. (JLQCD), PRD103, 074506 (2021) [arXiv:2011.01499]

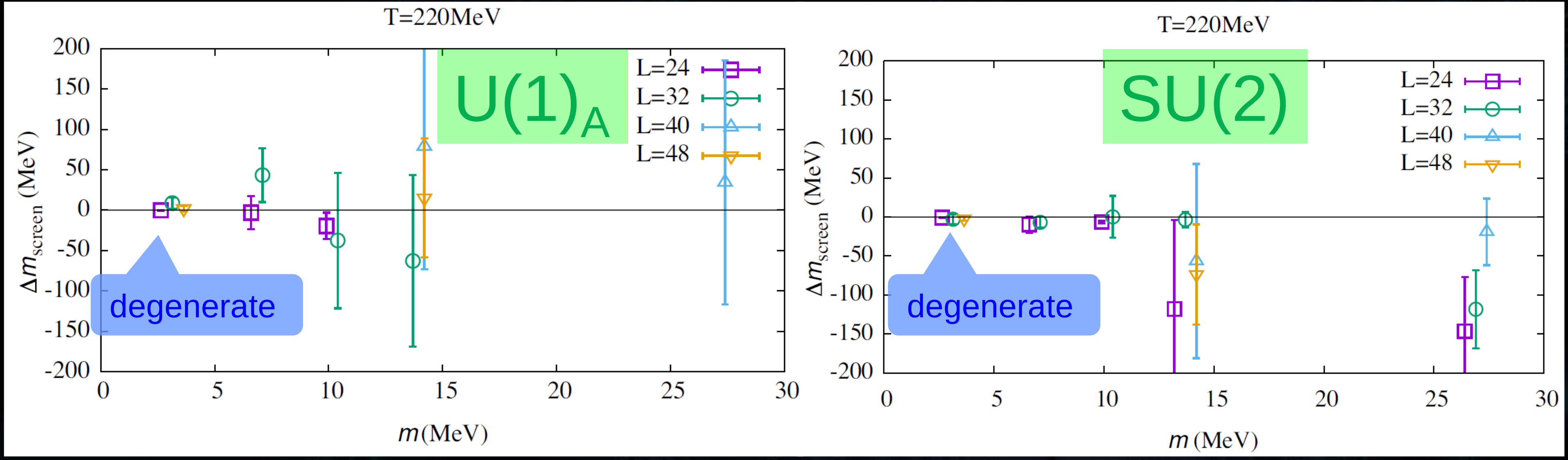
Screening mass difference (mesons)



Nf=2: S. Aoki et al. (JLQCD), PRD103, 074506 (2021) [arXiv:2011.01499]

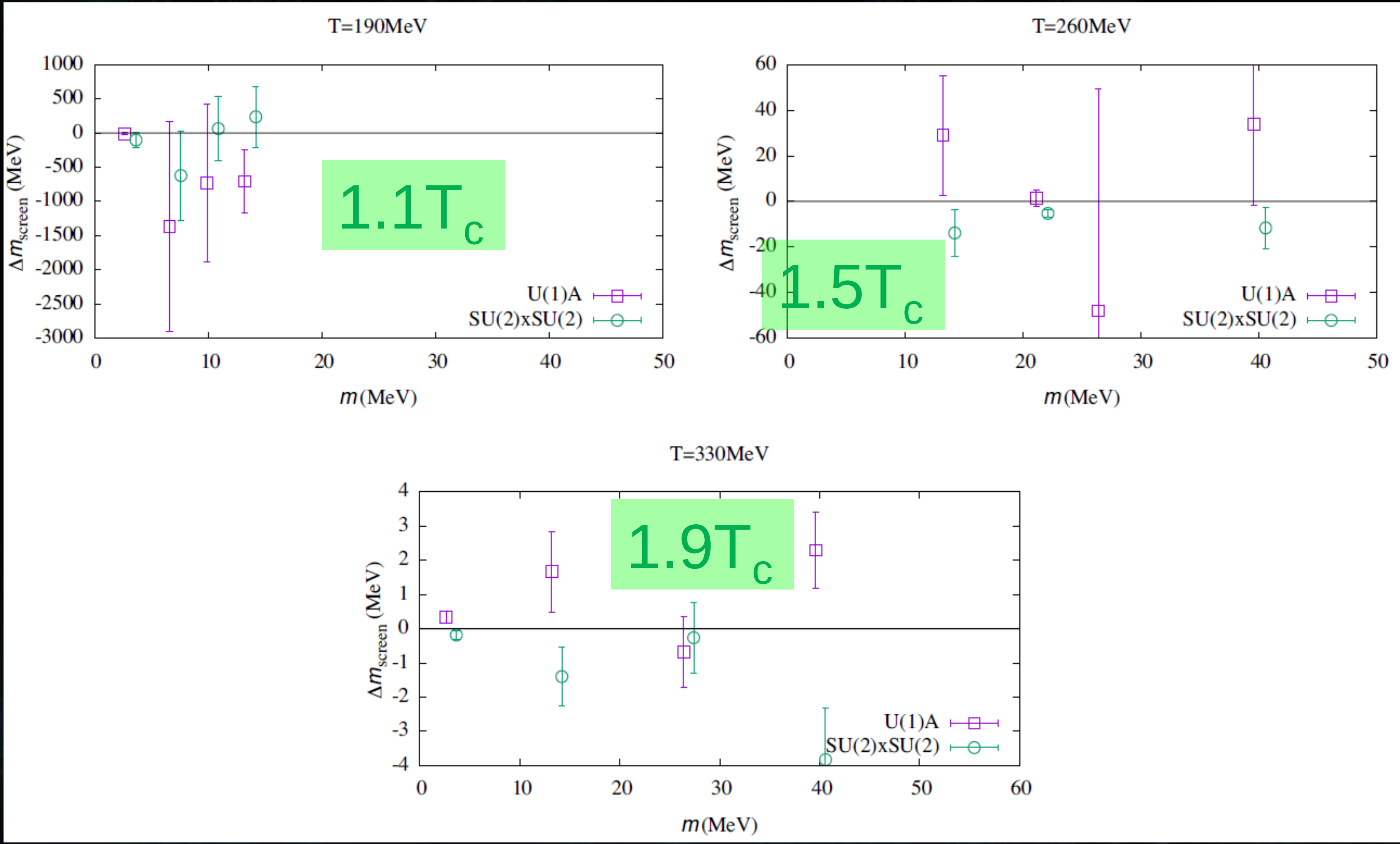
Screening mass difference (baryons)

T=220MeV~1.25T_c



Nf=2: S. Aoki et al. (JLQCD), PRD103, 074506 (2021) [arXiv:2011.01499]

Screening mass difference (baryons)

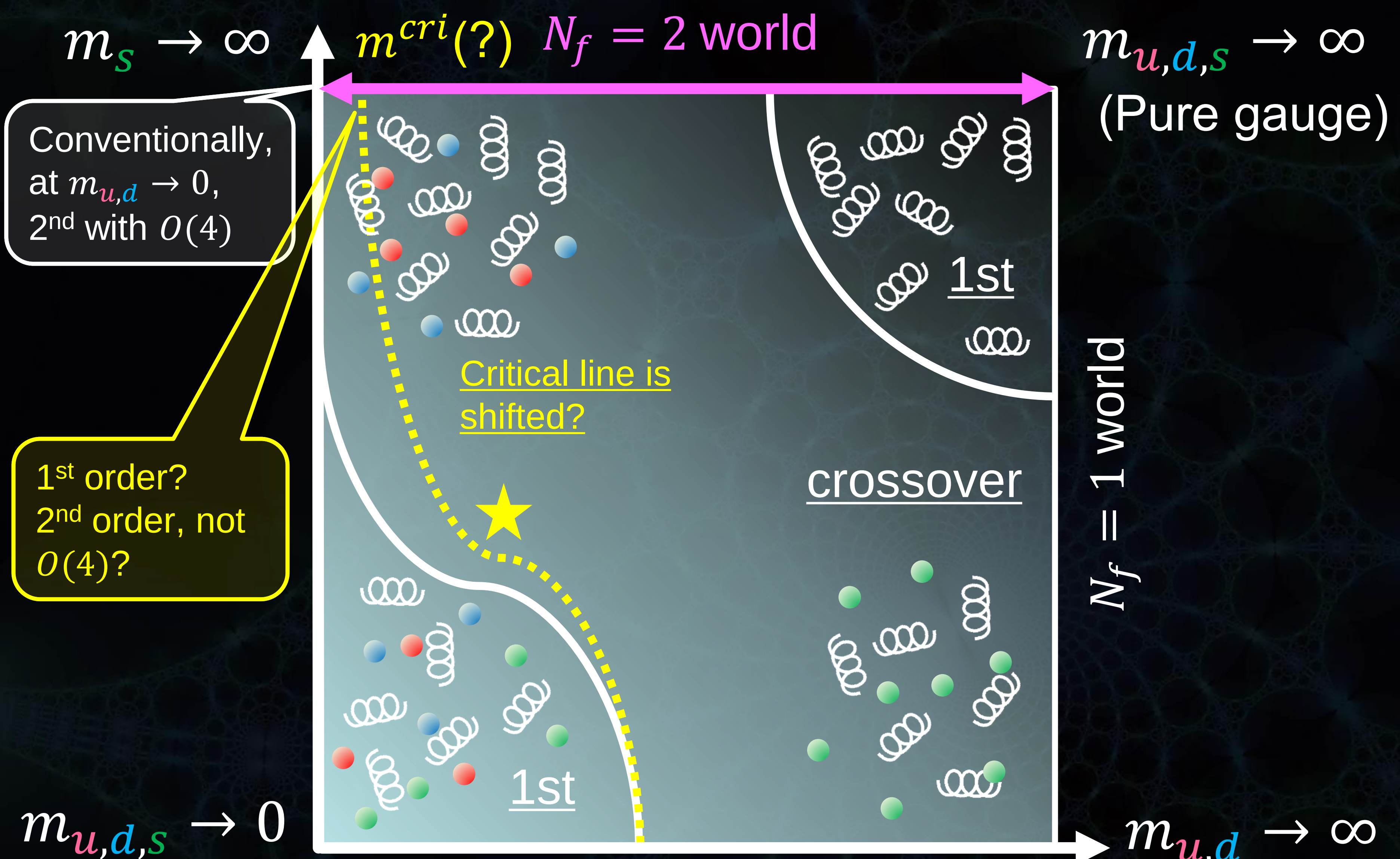


Nf=2: S. Aoki et al. (JLQCD), PRD103, 074506 (2021) [arXiv:2011.01499]

Cf.) S. Aoki, H. Fukaya, and Y. Taniguchi, PRD86

If $U(1)_A$ is restored…

Colombia plot is modified?



S. Aoki, H. Fukaya, and Y. Taniguchi, PRD86, 114512 (2012)
A. Tomiya et al. (JLQCD), PRD96, 034509 (2017)

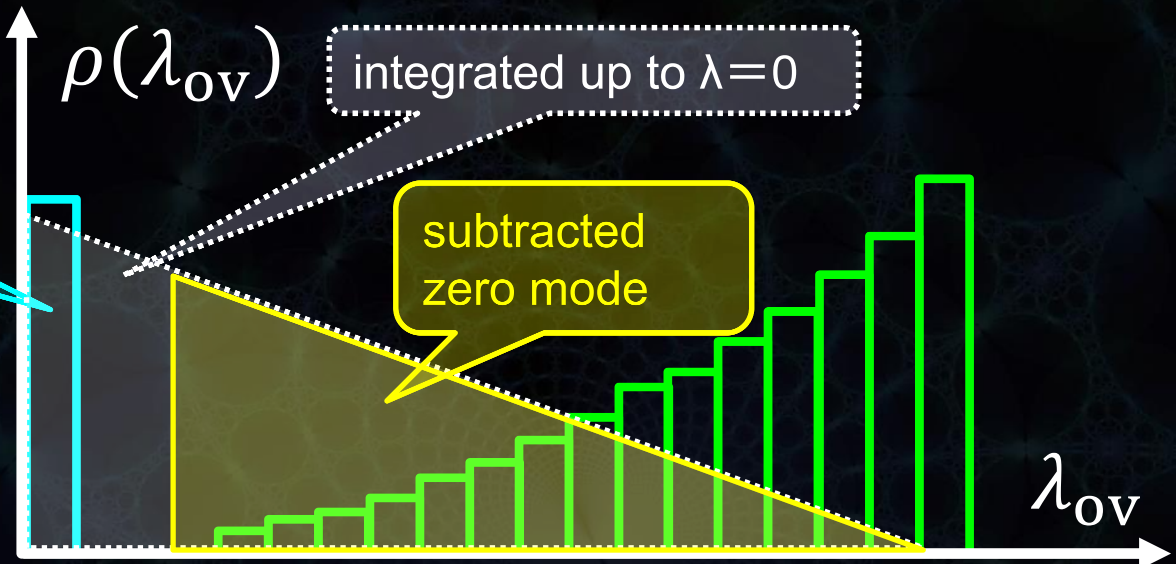
Note 1:

$U(1)_A$ susc. = Low modes + Zero mode?

$$\Delta_{\pi-\delta} = \int_0^\infty d\lambda\, \rho(\lambda) \frac{2m^2}{(\lambda^2 + m^2)^2}$$

$$\Delta^{\rm ov}_{\pi-\delta} \equiv \frac{1}{V(1-m^2)^2} \sum_i \frac{2m^2(1-\lambda^{(i)2}_{\rm ov})^2}{\lambda^{(i)4}_{\rm ov}}$$

The factor of $1/\lambda^4$ enhances zero-mode contribution?



In $V \to \infty$ limit, we know zero-mode contribution is suppressed:

$$\Delta^{\rm ov}_{0-mode} = \frac{2N_0}{Vm^2} (\propto 1/\sqrt{V})$$

New order parameter: we subtract zero mode

$$\bar{\Delta}^{\rm ov}_{\pi-\delta} \equiv \Delta^{\rm ov}_{\pi-\delta} - \frac{2N_0}{Vm^2}$$

S. Aoki, H. Fukaya, and Y. Taniguchi, PRD86, 114512 (2012)
A. Tomiya et al. (JLQCD), PRD96, 034509 (2017)

Note 1:

$U(1)_A$ susc. = Low modes + Zero mode ?

$$\Delta_{\pi-\delta} \equiv \int_0^\infty d\lambda\, \rho(\lambda) \frac{2m^2}{(\lambda^2+m^2)^2}$$

$$\rho_{0-mode}(\lambda) = \frac{1}{V} \sum_{0-mode} \delta(\lambda)$$

$$\Delta_{\text{zero}} = \int_0^\infty d\lambda \frac{1}{V} \sum_{0-mode} \delta(\lambda) \frac{2m^2}{(\lambda^2+m^2)^2}$$

$$= \frac{1}{V} \sum_{0-mode} \frac{2m^2}{m^4}$$

$$= \frac{1}{V} \sum_{0-mode} \frac{2}{m^2} = \frac{2N_0}{Vm^2}$$

$$\langle N_{L+R}^2 \rangle = \mathcal{O}(V)$$
$$\langle N_{L+R} \rangle = \mathcal{O}\left(\sqrt{V}\right)$$

$$\lim_{V\to\infty} \Delta_{\text{zero}} = 0$$

Zero mode contributions in $\Delta_{\pi-\delta}$ will be suppressed in $V \to \infty$ limit

S. Aoki et al. (JLQCD), PRD103, 074506 (2021)

Note 2:

$U(1)_A$ susc.＝Physics＋Ultraviolet divergence？

$$\Delta_{\pi-\delta} = \int_0^\infty d\lambda\, \rho(\lambda) \frac{2m^2}{(\lambda^2+m^2)^2}$$

$\rho(\lambda) \sim \lambda^3$　　$\sim 1/\lambda^4$

$$\Delta^{\mathrm{ov}}_{\pi-\delta} \propto m^2 \ln\Lambda + \cdots$$

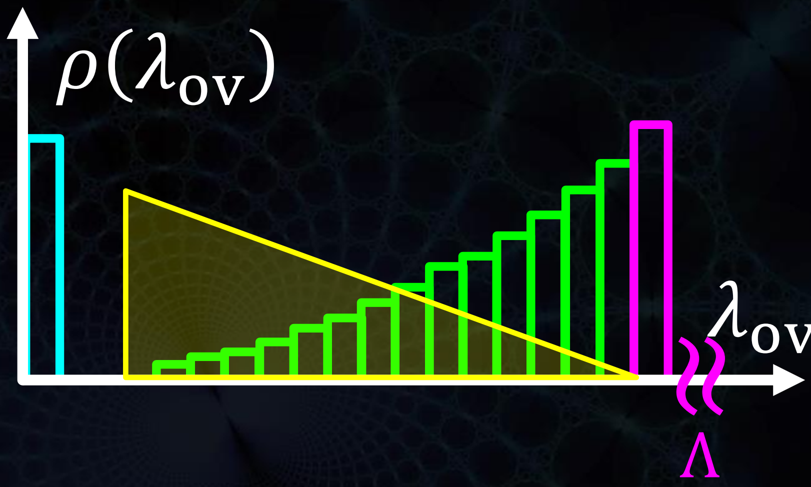
The term depends on cutoff Λ and valence quark mass m

We assume valence quark mass dependence of $\Delta_{\pi-\delta}$ (for small m):

$$\Delta_{\pi-\delta}(m) = \frac{a}{m^2} + b + cm^2 + O(m^4)$$

Zero-mode
(disappears in $V \to \infty$)

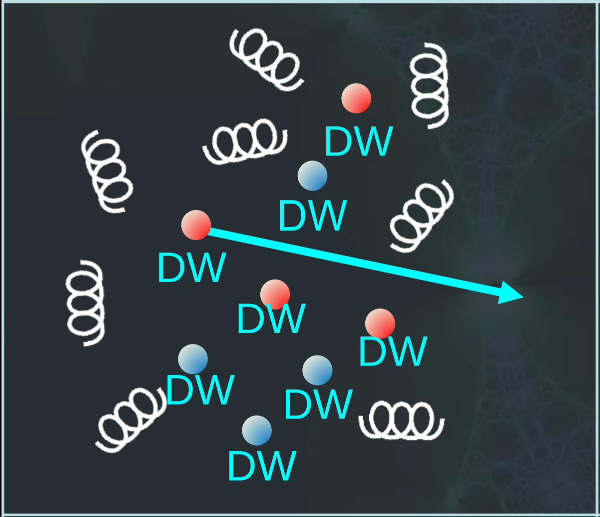
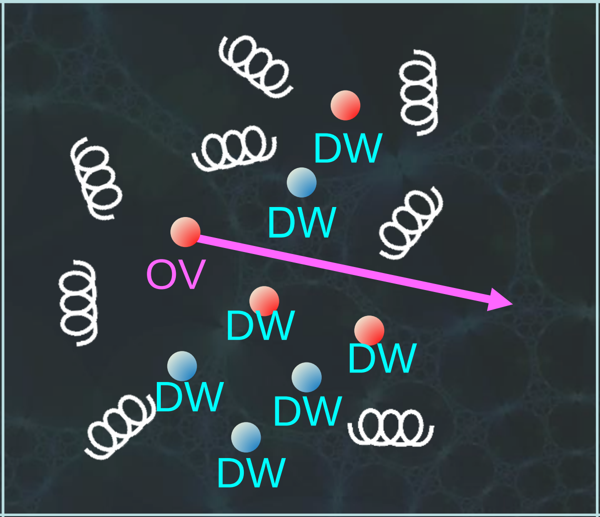
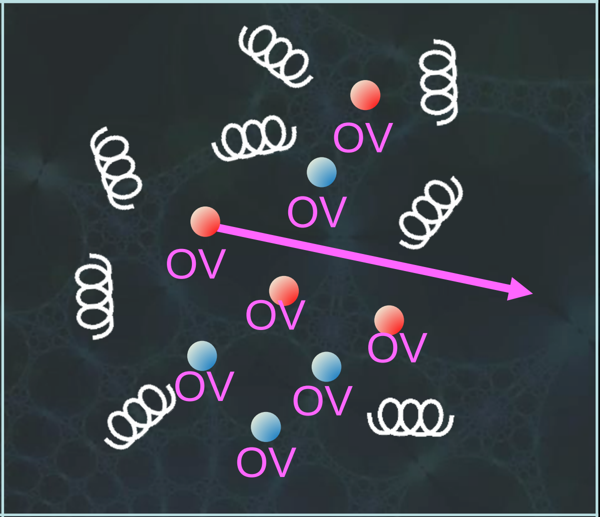
$m^2 \ln\Lambda$
(disappears in $m \to 0$)

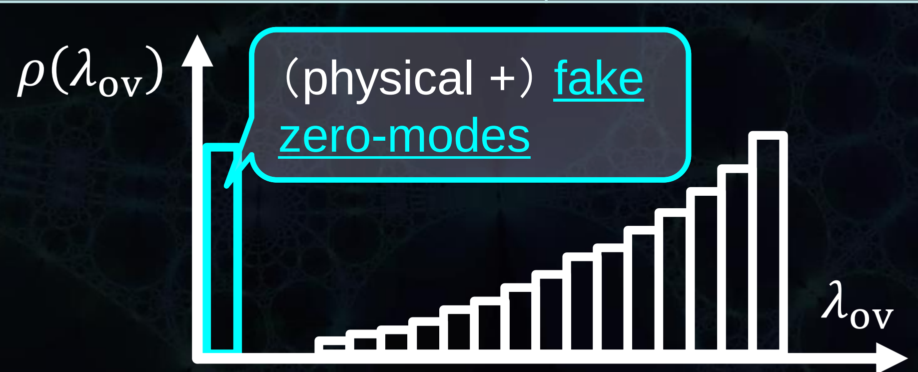


⇒ From 3 eqs. for $\Delta_{\pi-\delta}(m_1)$, $\Delta_{\pi-\delta}(m_2)$, $\Delta_{\pi-\delta}(m_3)$, a and c are eliminated

⇒ $\Delta_{\pi-\delta} \sim b + O(m^4)$ (, that depends on sea quark mass)

Valence quark and Sea quark

DW on DW	**OV on DW**	**OV on OV**
		
Almost good chiral symmetry	Fake zero-mode appears as an artifact	Exact chiral symmetry, but, very high cost



DW / OV reweighting
⇒can remove fake zero mode

A. Tomiya et al. (JLQCD) PRD96 (2017) 034509

DW/OV reweighting removes fake zero-modes

OV on DW:
Fake zero-modes by partially quenched

OV on OV:
removed fake zero-modes
⇒ Only physical zero-modes survive!