Study of QCD critical end-point using Wilson-type fermions

N_f=3	PRD91,014508(2015)
	PRD96,034523(2017)
	PRD101,054509(2020)
N_f=3,+μ	PRD92,114511(2015)
N_f=2+1	PRD94,114507(2016)
	arXiv:1909.05441
N_f=4	arXiv:1812.01318

Shinji Takeda

(Kanazawa University)

KANAZAWA UNIVERSITY 1862—1949 金沢大学

in collaboration with

X-Y. Jin(RIKEN->ANL), Y. Kuramashi(Tsukuba U.),

Y. Nakamura(RIKEN) , H. Ohno(Tsukuba U.) & A. Ukawa(RIKEN->JSPS)

QCD phase diagram and lattice QCD@YITP

October 27, 2021

Plan

- Columbia plot
- Strategy to determine the order of phase transition
- Our Wilson-type fermion studies
  - $N_f = 3$
  - $N_f = 2 + 1$
  - $N_f = 4$
- Summary

Nature of QCD phase transition

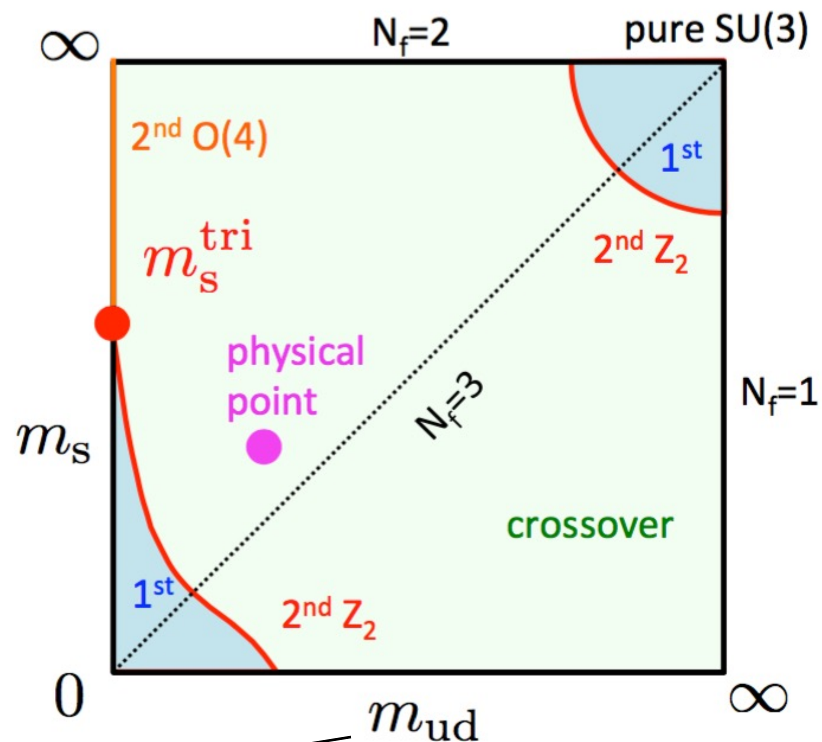
Depends on

- Quark masses
- # flavor
- Chemical potential
- …

Brown et al ‘90

Columbia plot

at zero density $\mu = 0$



Heavy quarks (c,b,t) are ignored

$$m_u = m_d = m_{ud}$$

Nature of QCD phase transition

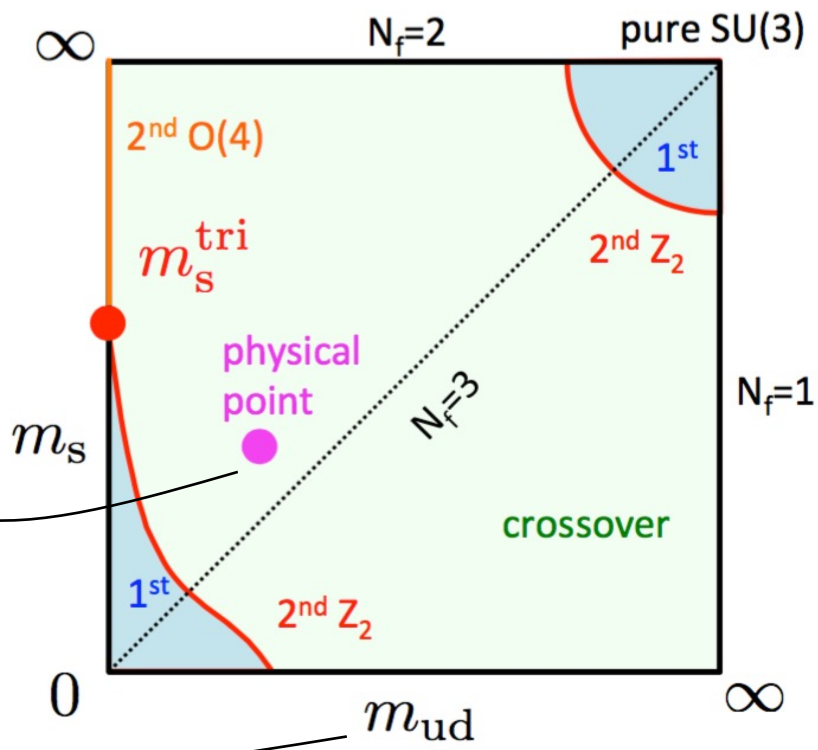
Depends on

- Quark masses
- # flavor
- Chemical potential
- …

Brown et al '90

Columbia plot

at zero density $\mu = 0$



Continuum limit is taken

Aoki et al '06
Bazavov et al '12
Bhattacharya et al '14

$$m_u = m_d = m_{ud}$$

Columbia plot

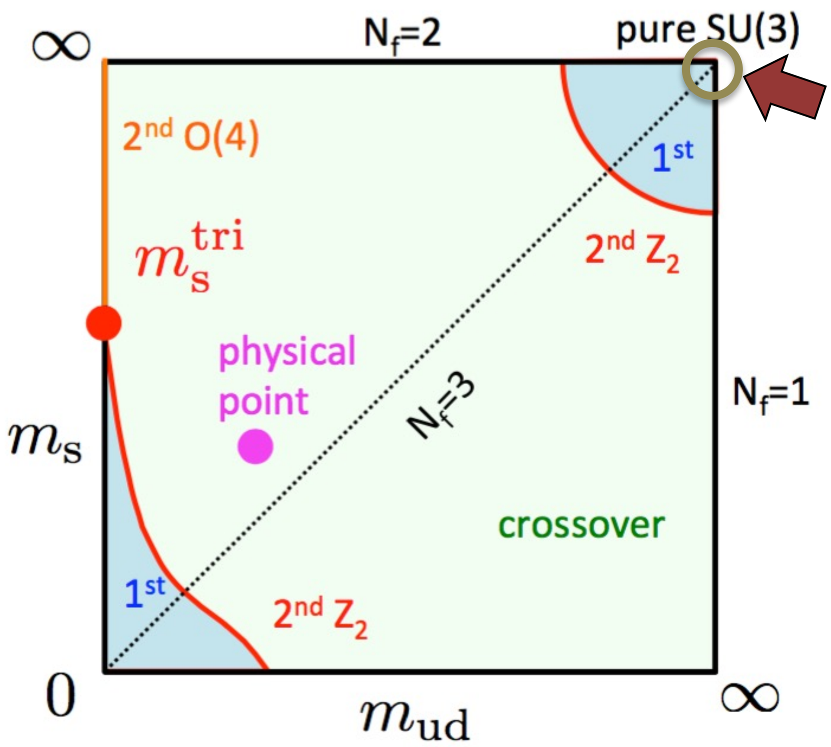
Heavy mass region

- Ginzburg-Landau analysis
- Renormalization group analysis
- Monte Carlo/Finite size scaling predict/support

Brown '88, Fukugita et al '90

1st order PT in static limit

OK!

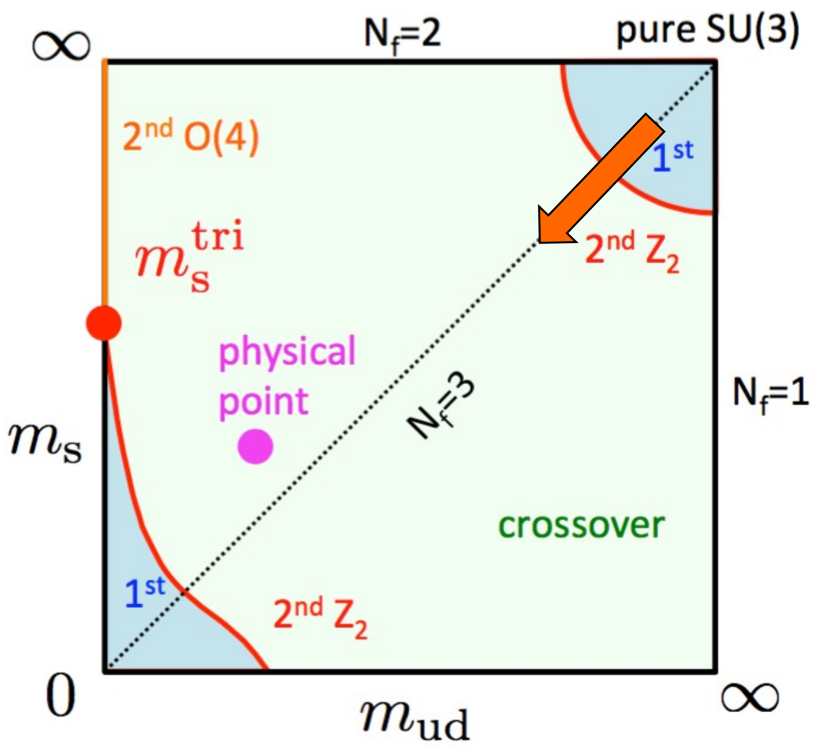


Columbia plot

Heavy mass region

m < ∞

- weaken 1st order PT
- 1st -> 2nd -> crossover



Columbia plot

Heavy mass region

$m < \infty$

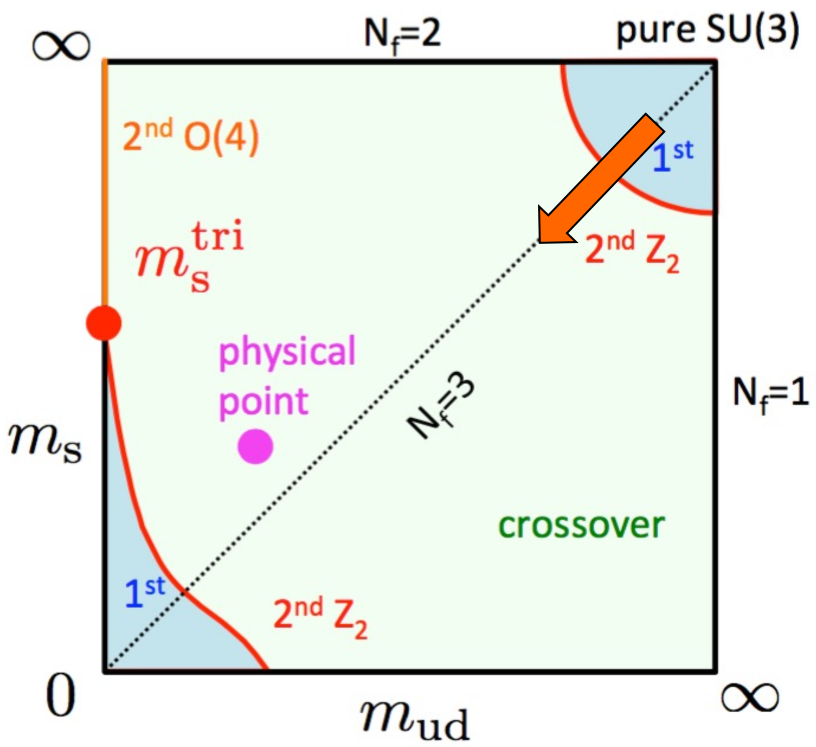
- weaken 1st order PT
- 1st -> 2nd -> crossover

Saito et al '12
Ejiri et al '19
Philipsen et al '21

Lattice simulation

- m_π^c = O(3-5) GeV
- Z_2 universality class
- Continuum limit ?

$$am_\pi > 1$$



Columbia plot

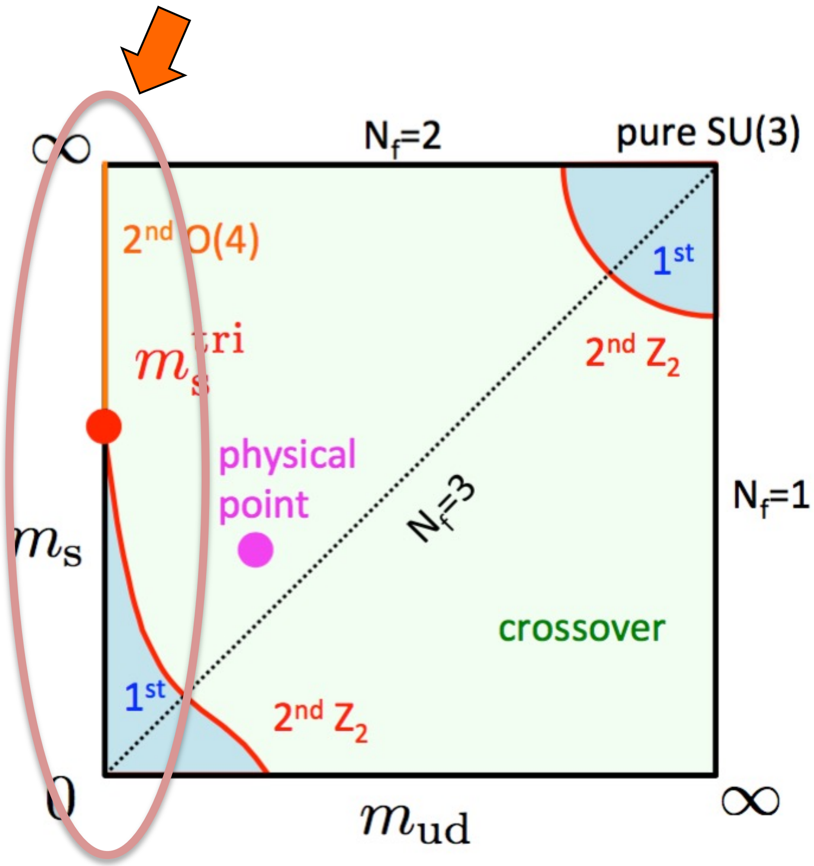
Chiral region ⇒ Chiral symmetry is important

Sigma model

meson field: $M_{ij} = \langle \bar{q}_L^i q_R^j \rangle$
(order parameter)

$$\mathcal{L}_{\text{eff}} = \text{tr}\left(\partial_\mu M^\dagger \partial_\mu M\right) + m_0^2 \text{tr}\left(M^\dagger M\right) + g_1 \left(\text{tr}\left(M^\dagger M\right)\right)^2 + g_2 \text{tr}\left(\left(M^\dagger M\right)^2\right) + c_{\text{A}}\left(\det M + \det M^\dagger\right)$$

$U_A(1)$ anomaly



Columbia plot

Chiral region

Sigma model

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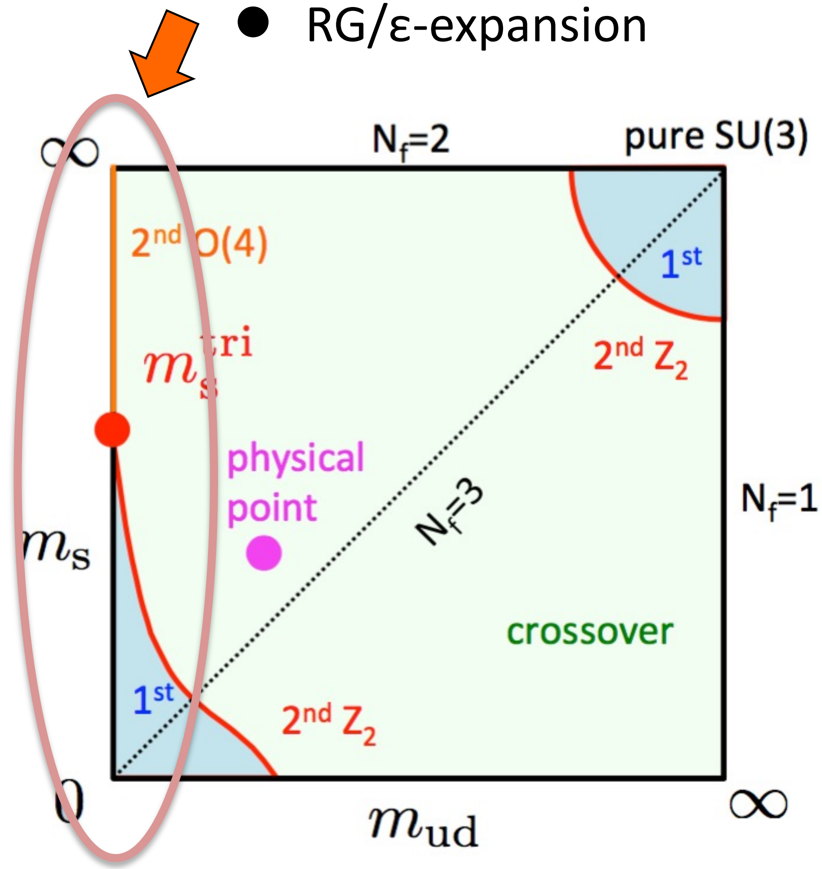
→ $U_A(1)$ anomaly

analysis by

Goldberg '83
Pisarski & Wilczek '84
Wilczek '92
Rajagopal & Wilczek '93

- Ginzburg-Landau
- RG/ε-expansion



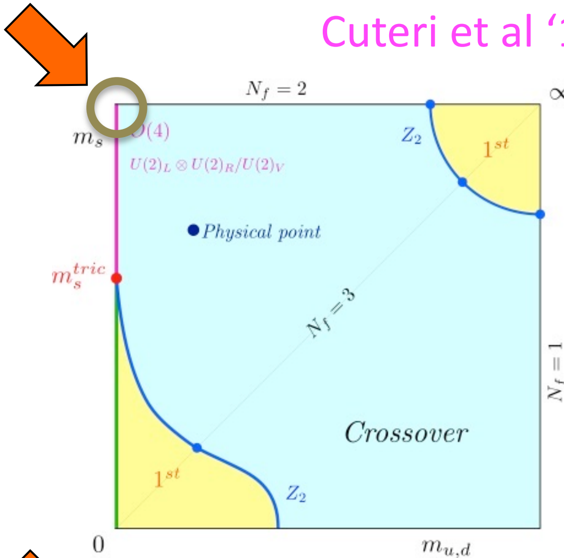
Columbia plot

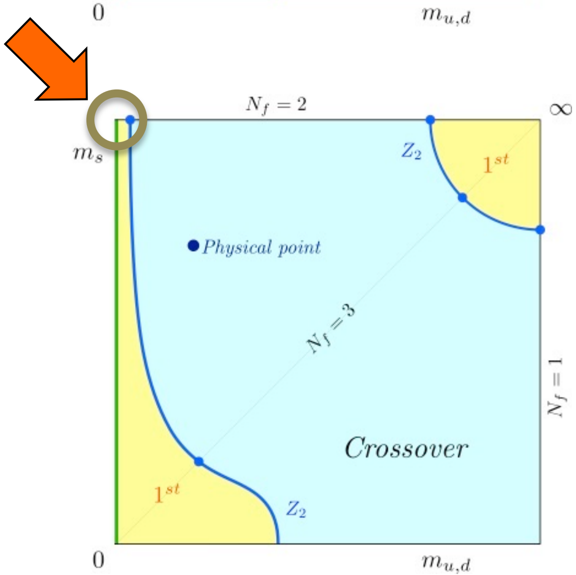
Chiral limit N_f=2

Sigma model (not predictive)   Pisarski & Wilczek '84

$U_A(1)$	order of PT
broken (anomaly)	2^{nd} O(4)
restore	1^{st} or 2^{nd} $U_L(2) \times U_R(2)/U_V(1)$

Cuteri et al '18





Columbia plot

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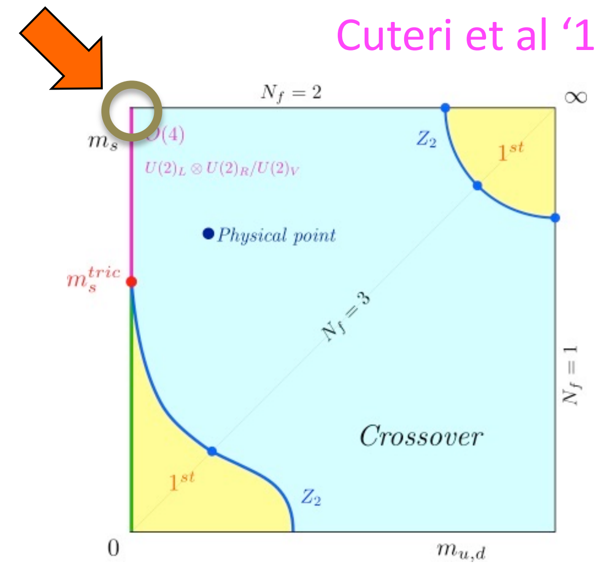


Lattice QCD with N_f=2

Finite lattice spacing → Continuum limit?

Controversial !!

Ref.	N_t	fermion	method	order
Bonati et al' 14	4	staggered	imaginary μ	1^{st}
Philipsen et al '16	4	Wilson	imaginary μ	1^{st}
Umeda et al '16	4	improved Wilson	scaling of order parameter	2^{nd} O(4)
Philipsen et al '18	4	staggered	non-integer N_f, tri-critical	1^{st}

For update see Philipsen's talk tomorrow

Columbia plot

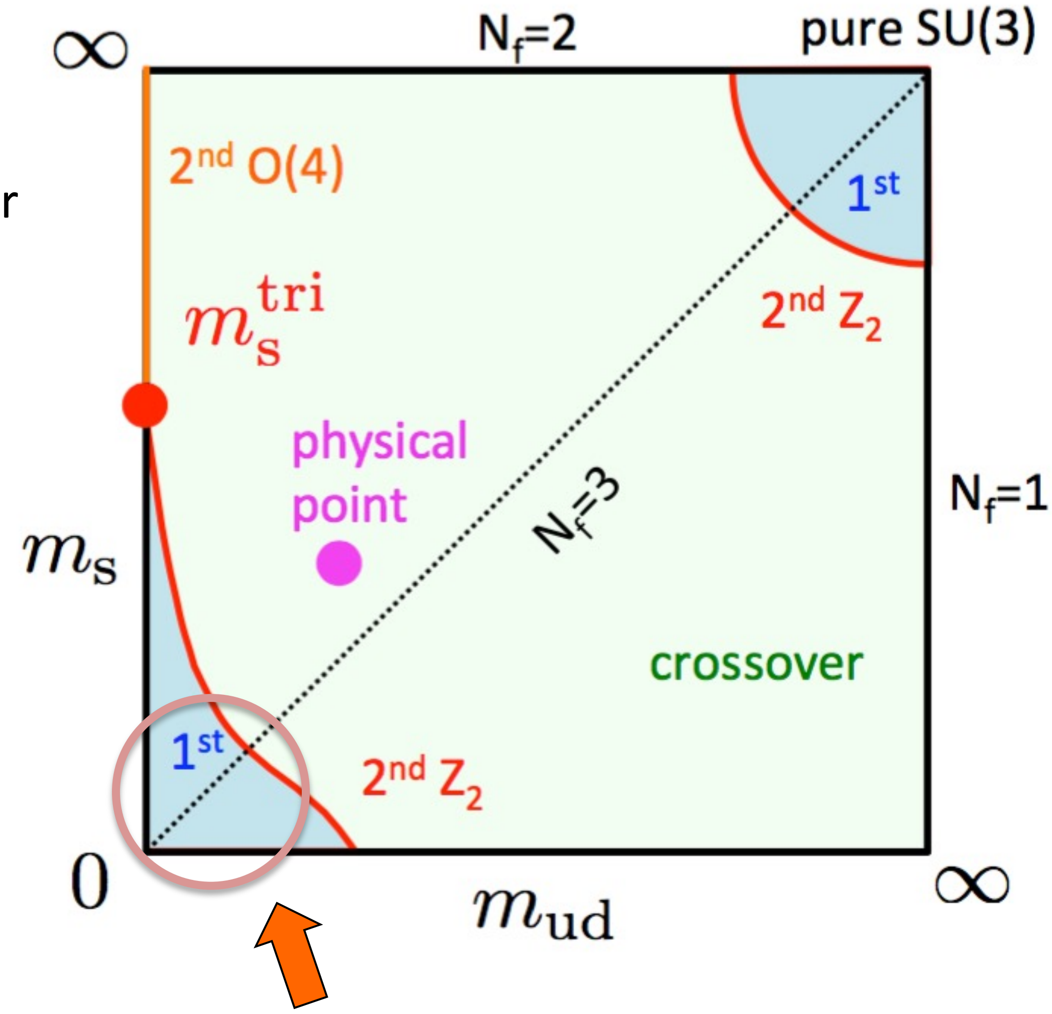
Chiral region N_f=3

Sigma model predicts 1st order in the chiral limit

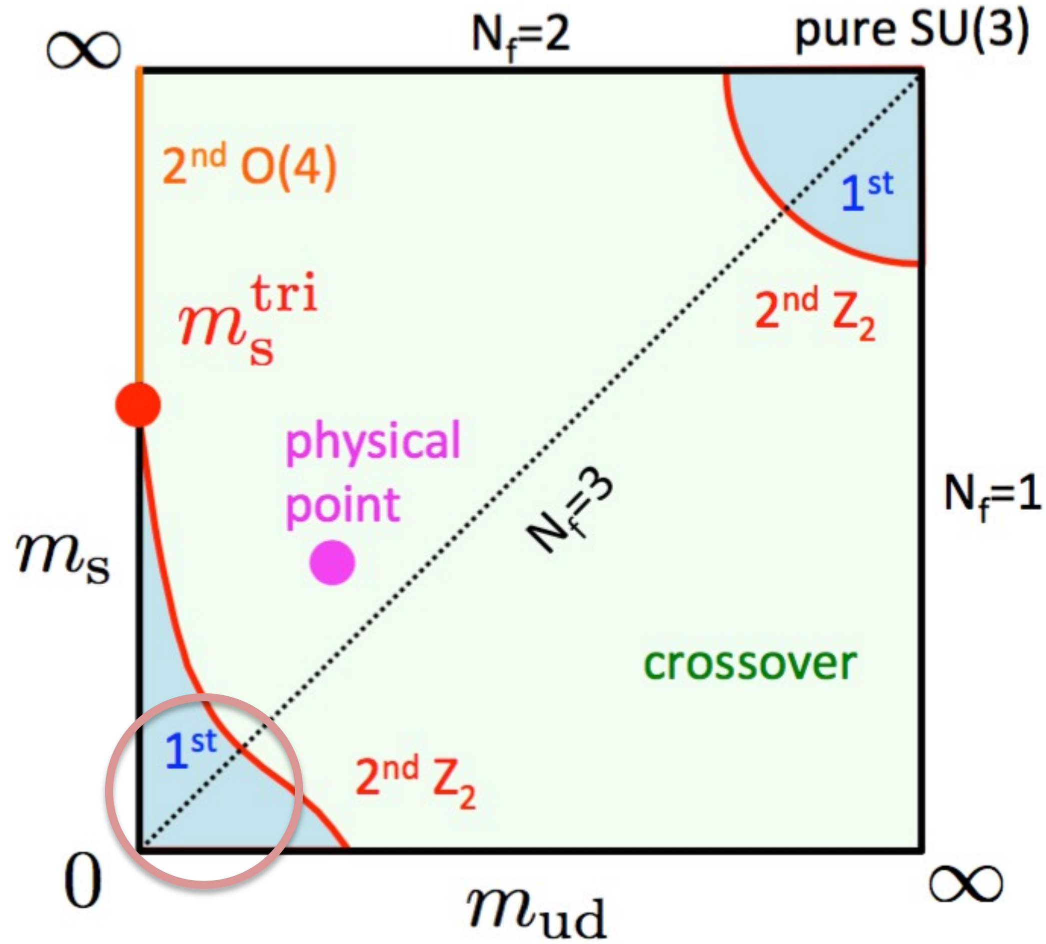
Goldberg ’83
Pisarski & Wilczek ’84

seems robust?

How to explore by lattice QCD?



Strategy for lattice study

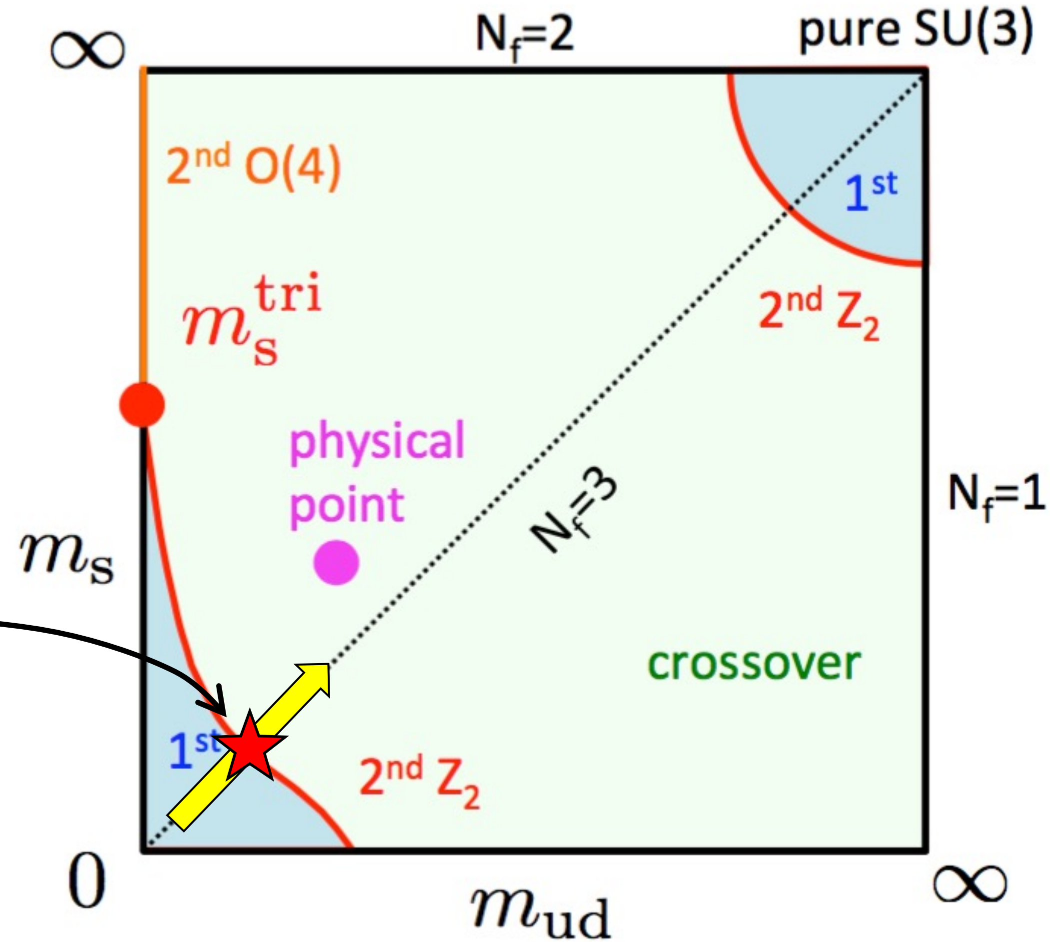


identify critical point
on N_f=3 symmetric line

how to identify critical point?

Strategy for lattice study

identify critical point
on N_f=3 symmetric line



how to identify critical point?

Kurtosis

Karsch et al '01

Binder cumulant

$$K \equiv \frac{\langle (\mathcal{O} - \langle \mathcal{O} \rangle)^4 \rangle}{\langle (\mathcal{O} - \langle \mathcal{O} \rangle)^2 \rangle^2} - 3$$

order parameter $\mathcal{O} \sim \bar{\psi}\psi$

Kurtosis

Karsch et al '01

Binder cumulant

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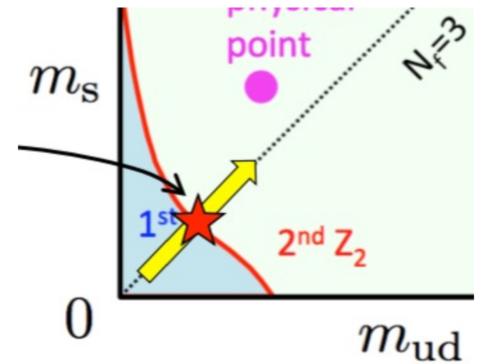
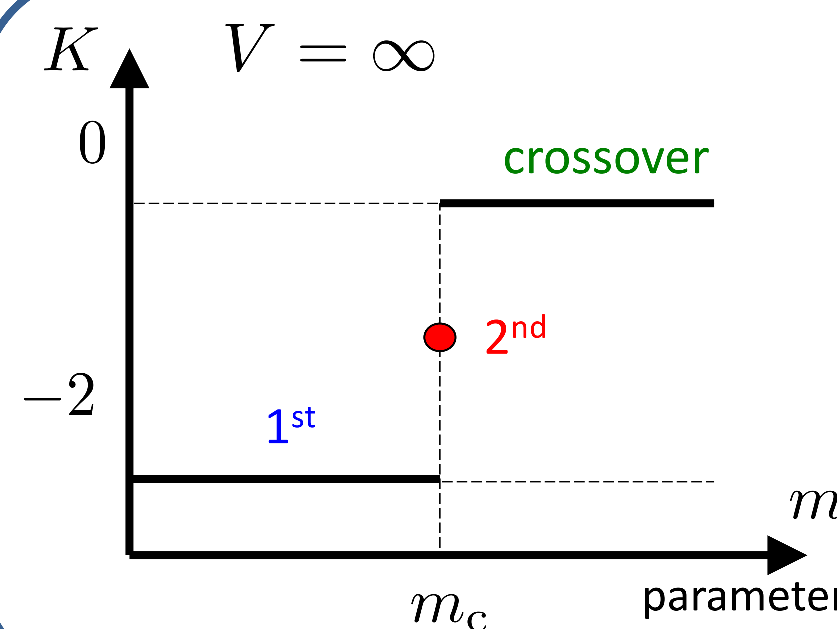


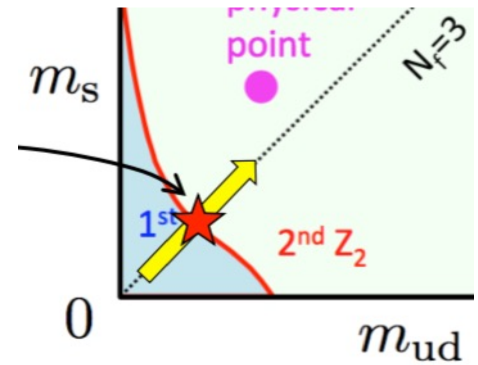
$$K = \begin{cases} -2 & \text{1st order} \\ -2 < c < 0 & \text{2nd order} \\ 0 & \text{crossover} \end{cases}$$

in the thermodynamic limit



Kurtosis

Karsch et al '01



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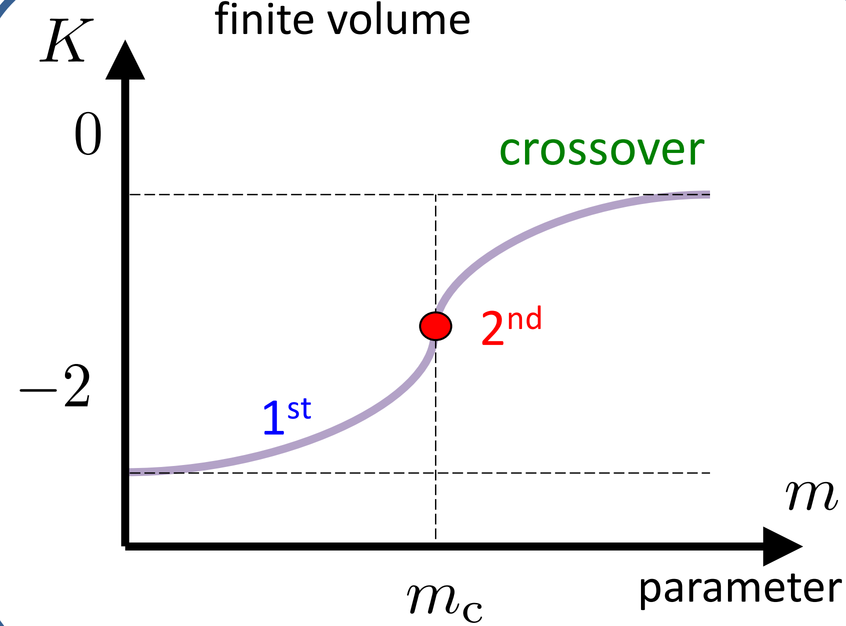
in the thermodynamic limit

finite volume



Kurtosis

Karsch et al ‘01

Binder cumulant

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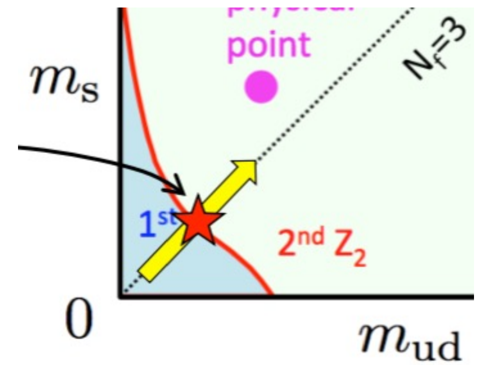

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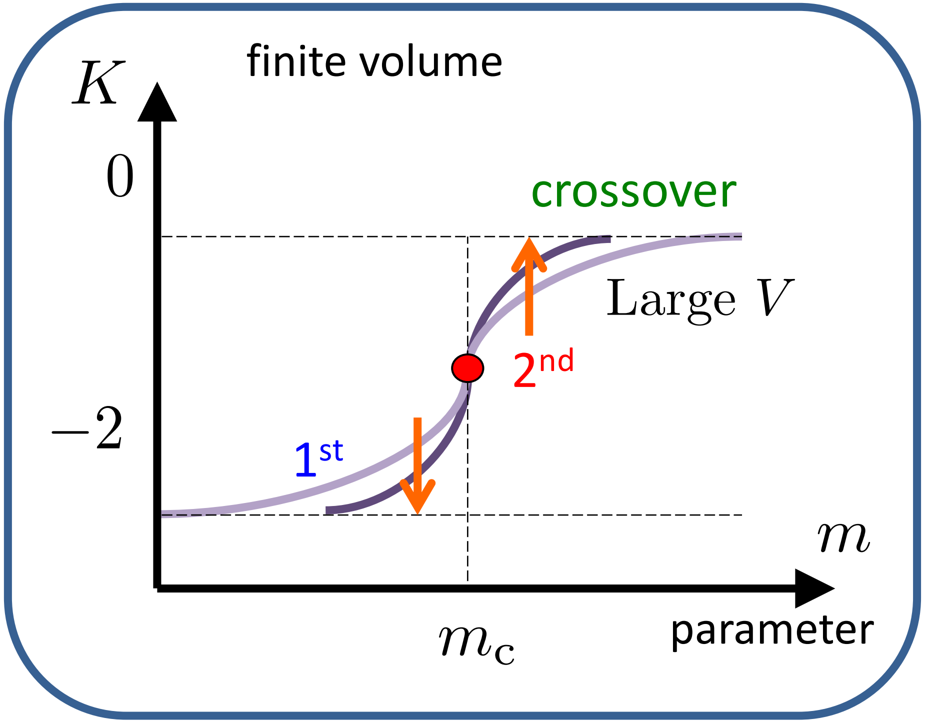
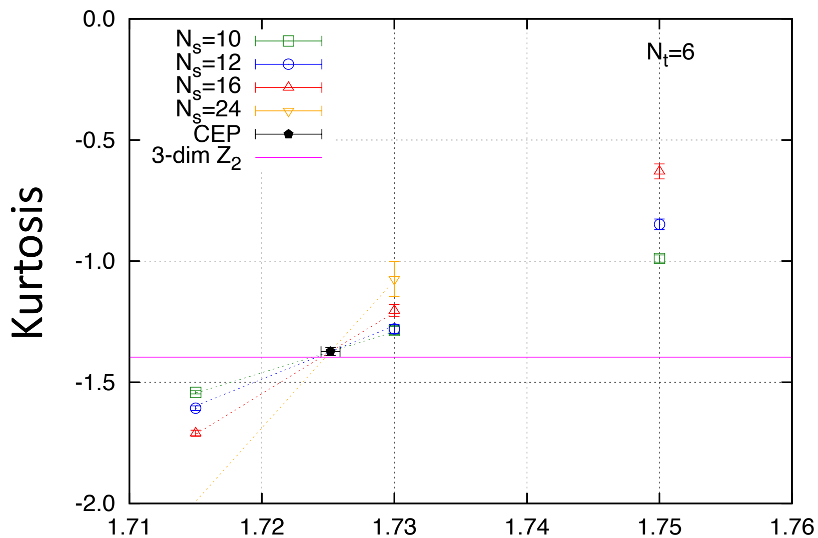


Wilson fermion study for N_f=3

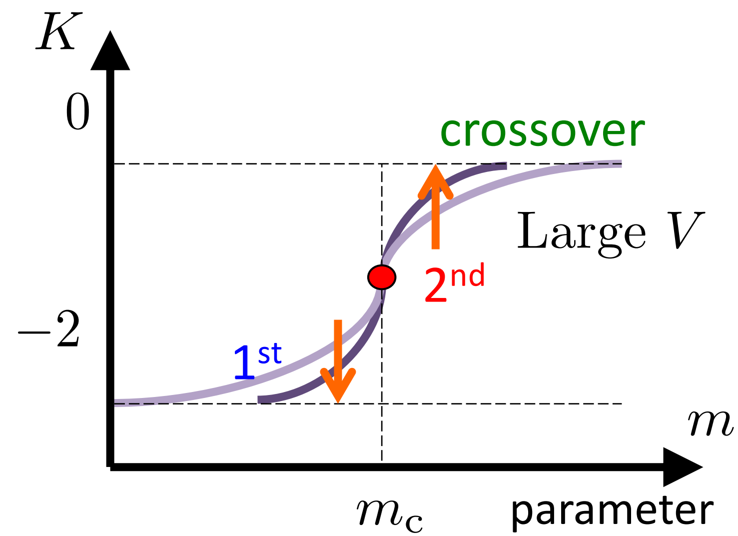
- Iwasaki gauge
- NP O(a) improved Clover-Wilson fermions

Kurtosis intersection

$N_t = 6$



$\beta = 6/g^2$

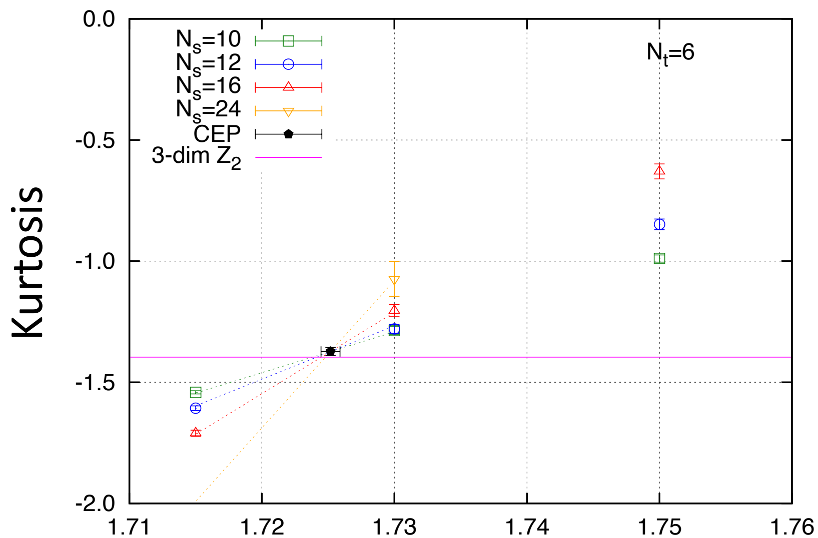


$$K = K_E + A N_s^{1/\nu}(\beta - \beta_E)$$

$$V = N_s^3$$

Kurtosis intersection

$N_t = 6$



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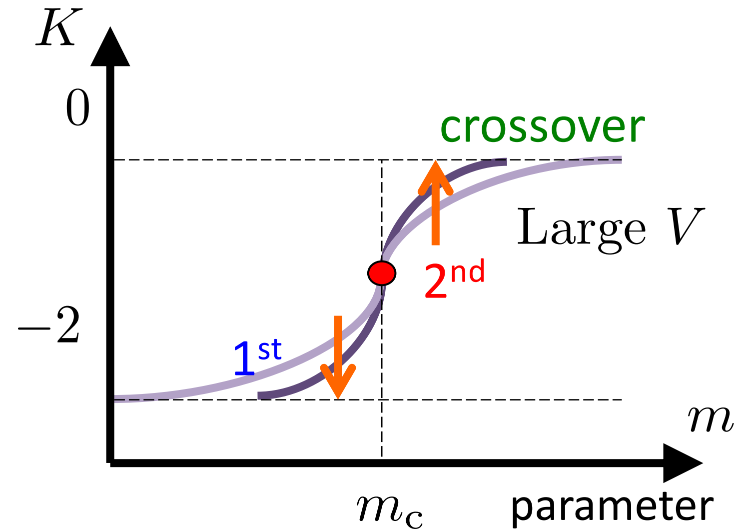


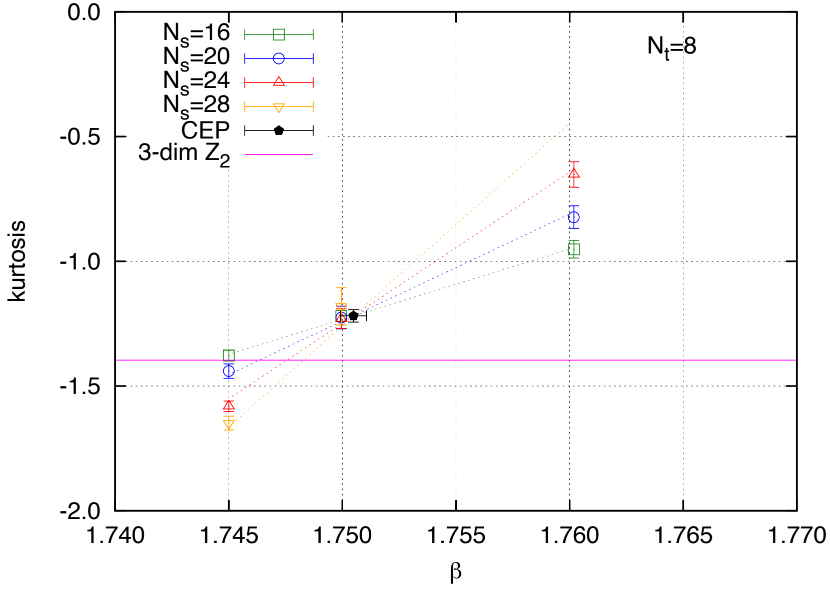
$$K = K_E + A N_s^{1/\nu}(\beta - \beta_E)$$

$V = N_s^3$

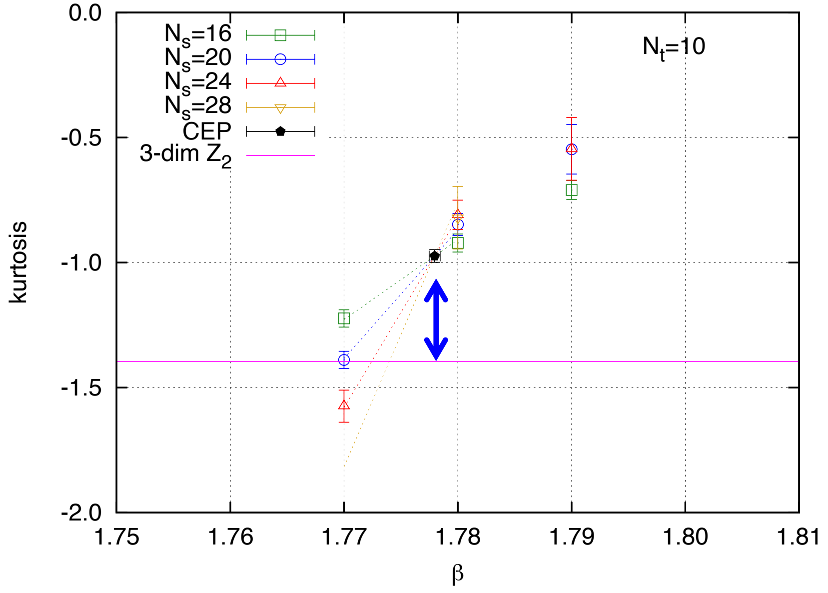
3D Z_2 universality class is confirmed

Kurtosis intersection

$N_t = 8$



$N_t = 10$



Due to additional correction term originated from energy-like observable?

Karsch et al '01

$$\mathcal{O} = \mathcal{M} + \mathcal{E}$$

Including correction term

$$K = \left[K_{\rm E} + A N_{\rm s}^{1/\nu}(\beta - \beta_{\rm E})\right]\left(1 + B N_{\rm s}^{y_{\rm t} - y_{\rm h}}\right)$$

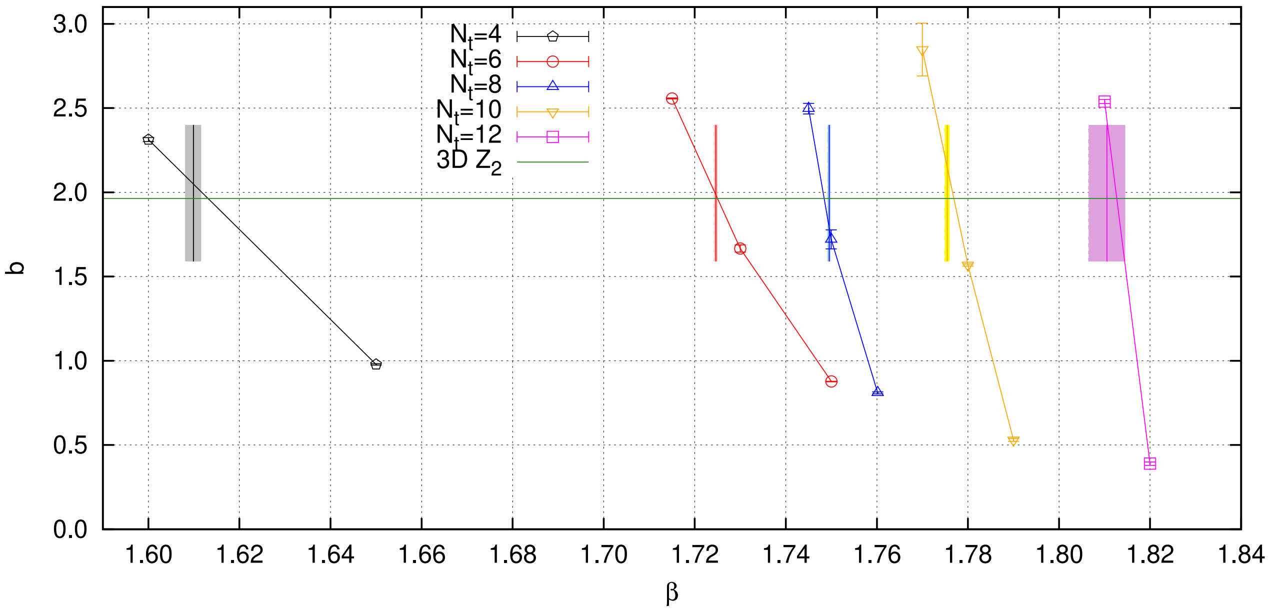
Correction term originated from energy-like observable

N_t	Fit	β_E	κ_E	K_E	ν	A	B	$y_t - y_h$	χ^2/d.o.f.
4	1	1.6115(26)	0.1429337(13)	−1.383(48)	0.84(13)	0.88(42)	×	×	1.75
	2	1.61065(61)	0.1429713(13)	−1.396	0.63	0.313(12)	×	×	3.05
	3	1.6099(17)	0.1430048(13)	−1.396	0.63	0.311(14)	0.10(21)	−0.894	3.77
6	1	1.72518(71)	0.1406129(14)	−1.373(17)	0.683(54)	0.58(17)	×	×	0.68
	2	1.72431(24)	0.1406451(14)	−1.396	0.63	0.418(11)	×	×	0.70
	3	1.72462(40)	0.1406334(14)	−1.396	0.63	0.422(12)	−0.052(52)	−0.894	0.70
8	1	1.75049(57)	0.1402234(11)	−1.219(25)	0.527(55)	0.146(88)	×	×	0.73
	2	1.74721(42)	0.14031921(76)	−1.396	0.63	0.404(36)	×	×	5.99
	3	1.74953(33)	0.1402512(10)	−1.396	0.63	0.414(13)	−1.33(15)	−0.894	0.73
10	1	1.77796(48)	0.1396661(17)	−0.974(25)	0.466(45)	0.084(52)	×	×	0.22
	2	1.7694(16)	0.1398724(22)	−1.396	0.63	0.421(95)	×	×	10.03
	3	1.77545(53)	0.1397274(17)	−1.396	0.63	0.559(29)	−2.97(25)	−0.894	0.43

Critical point is determined with assuming Z_2 universality class for cross check

Exponent of susceptibility

$$\chi_{\rm max} \propto (N_{\rm s})^b \qquad b = \gamma/\nu \quad \text{for 2nd PT}$$



Critical point estimated by kurtosis intersection is consistent with that of susceptibility

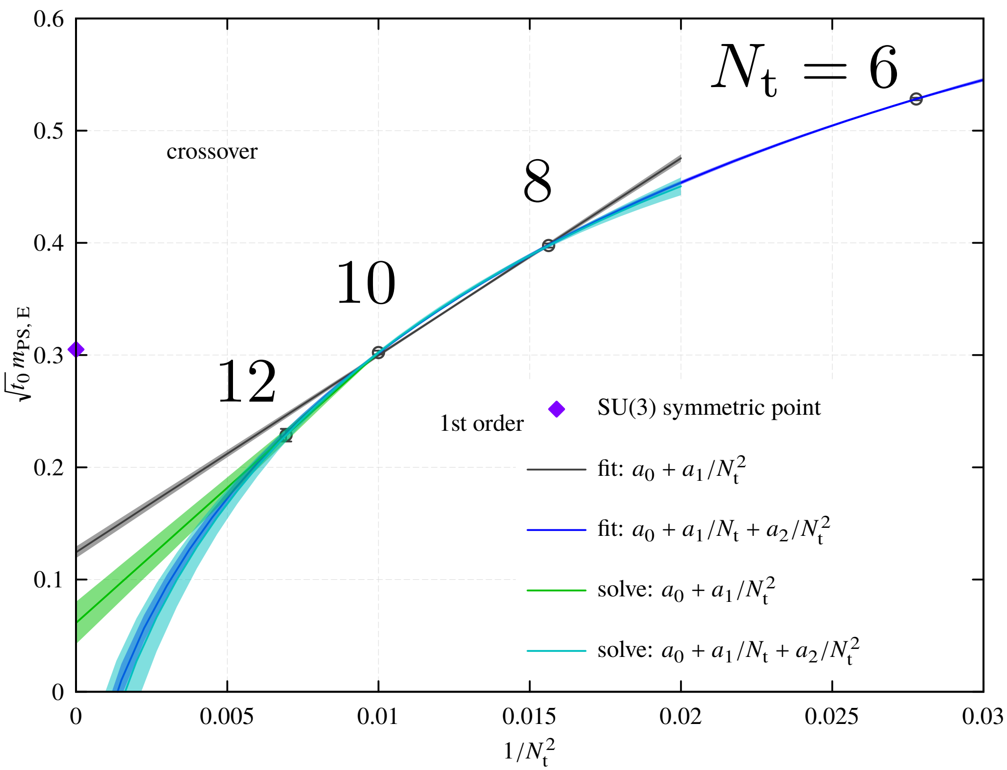
Continuum extrapolation

bare parameter space (β_E,m_E) → physical (hadron mass) parameter space

$m_{\mathrm{PS,E}}$

$1/\sqrt{t_0} = 1347(30)\mathrm{MeV}$

Borsanyi et al, 2012



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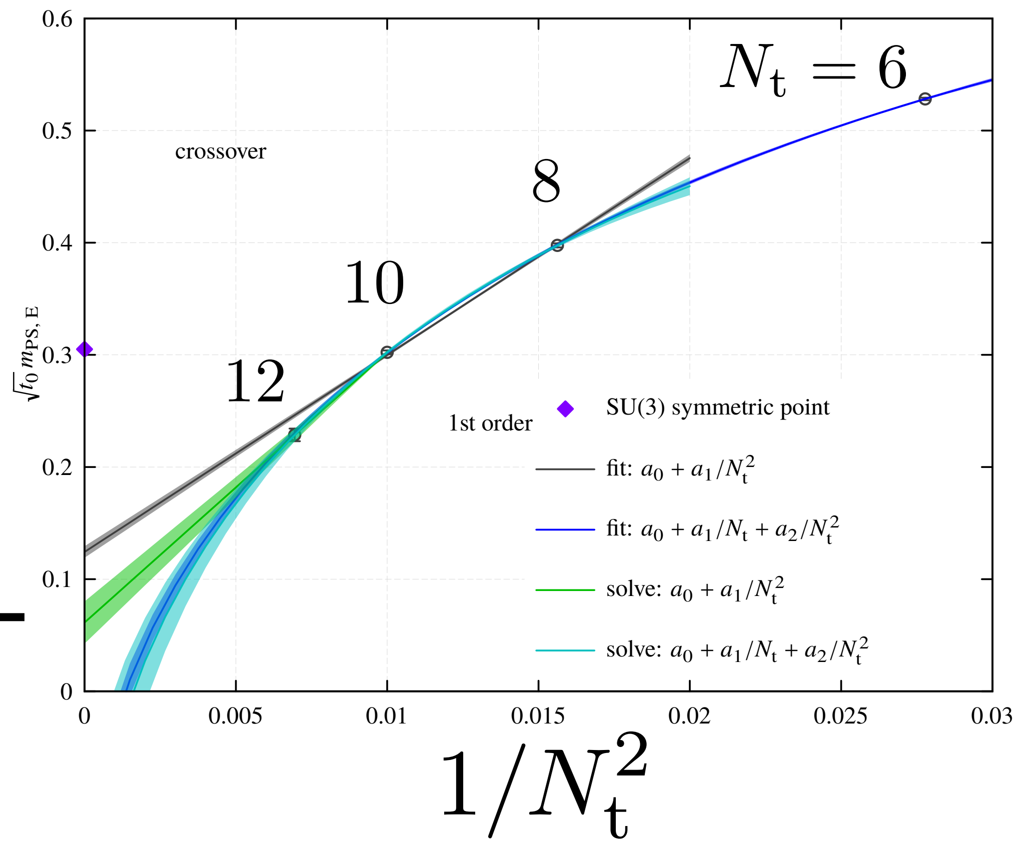


upper bound

$$m_{\mathrm{PS,E}} \lesssim 110\mathrm{MeV}$$

Summary for N_f=3 QCD critical point

Action	N_t	$m_{PS,E}$	Ref.
Staggered, standard	4	290MeV	Karsch et al ’01, Liao ‘01
Staggered, p4	4	67MeV	Karsch et al ‘04
Staggered, standard	6	150MeV	de Forcrand et al ‘07
Staggered, HISQ	6	$\lesssim$ 50MeV	Ding et al ‘17
Staggered, stout	4-6	could be 0	Varnhorst ‘14

Wilson, standard	4	$\lesssim$ 670MeV	Iwasaki et al, ‘96
Wilson-Clover	6-10	$\lesssim$ 170MeV	ST et al, ‘17
Wilson-Clover	6-12	$\lesssim$ 110MeV	ST et al, ‘20

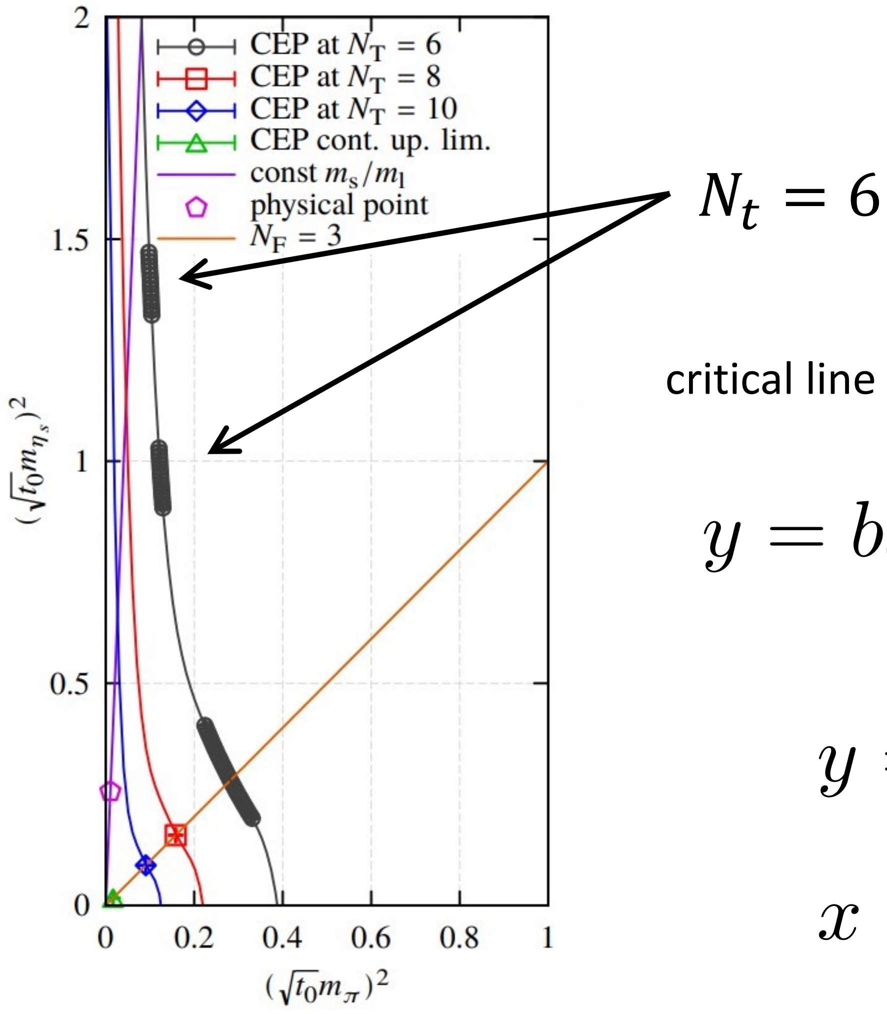
Consensus: 1st order region gets smaller for both fermions

If critical mass is zero (2nd) or no CP (crossover), then contradict Pisarski & Wilczek ?

Wilson fermion study for N_f=2+1

N_t=6 study

Y. Nakamura *et al.*, PoS LATTICE 2019 (2019) 053



critical line

$$y = b_0 + a_0 x^{2/5} + \sum_{i=1}^{5} a_i x^i$$

$$y = \left(\sqrt{t_0} m_{\eta_s}\right)^2$$

$$x = \left(\sqrt{t_0} m_\pi\right)^2$$

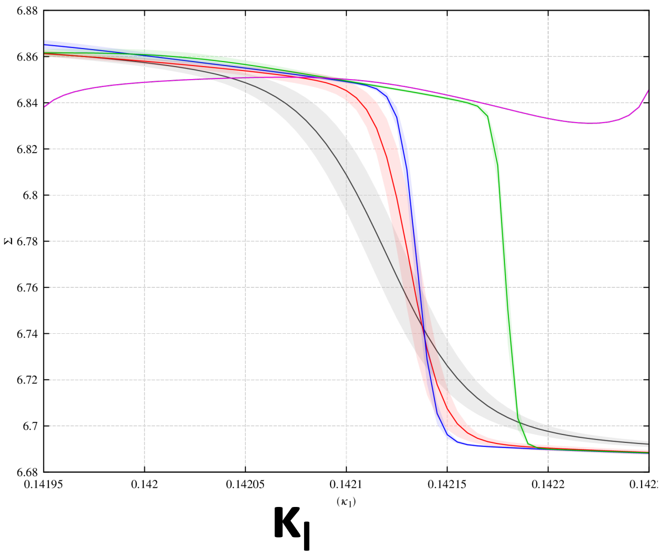
N_t=8 study is on-going

preliminary

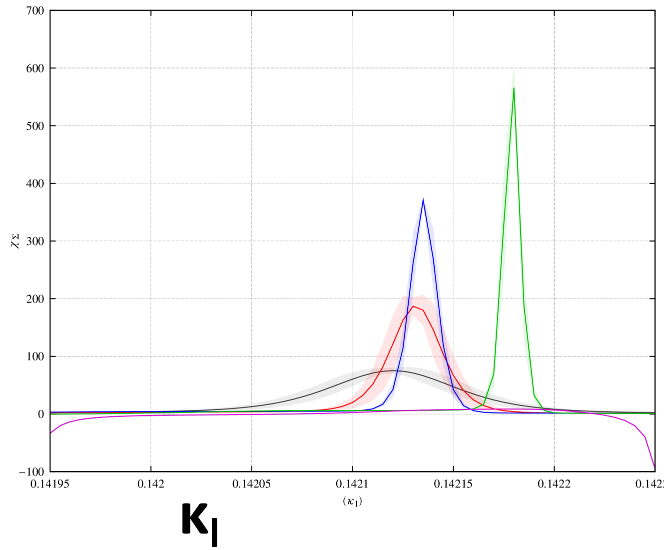
Lattice 2021, H. Ohno's talk

$\beta = 1.75$, $\kappa_s = 0.1327$

Chiral condensate



Chiral susceptibility



line: reweighting

N_s = 12

N_s = 16

N_s = 20

N_s = 24

N_s = 28

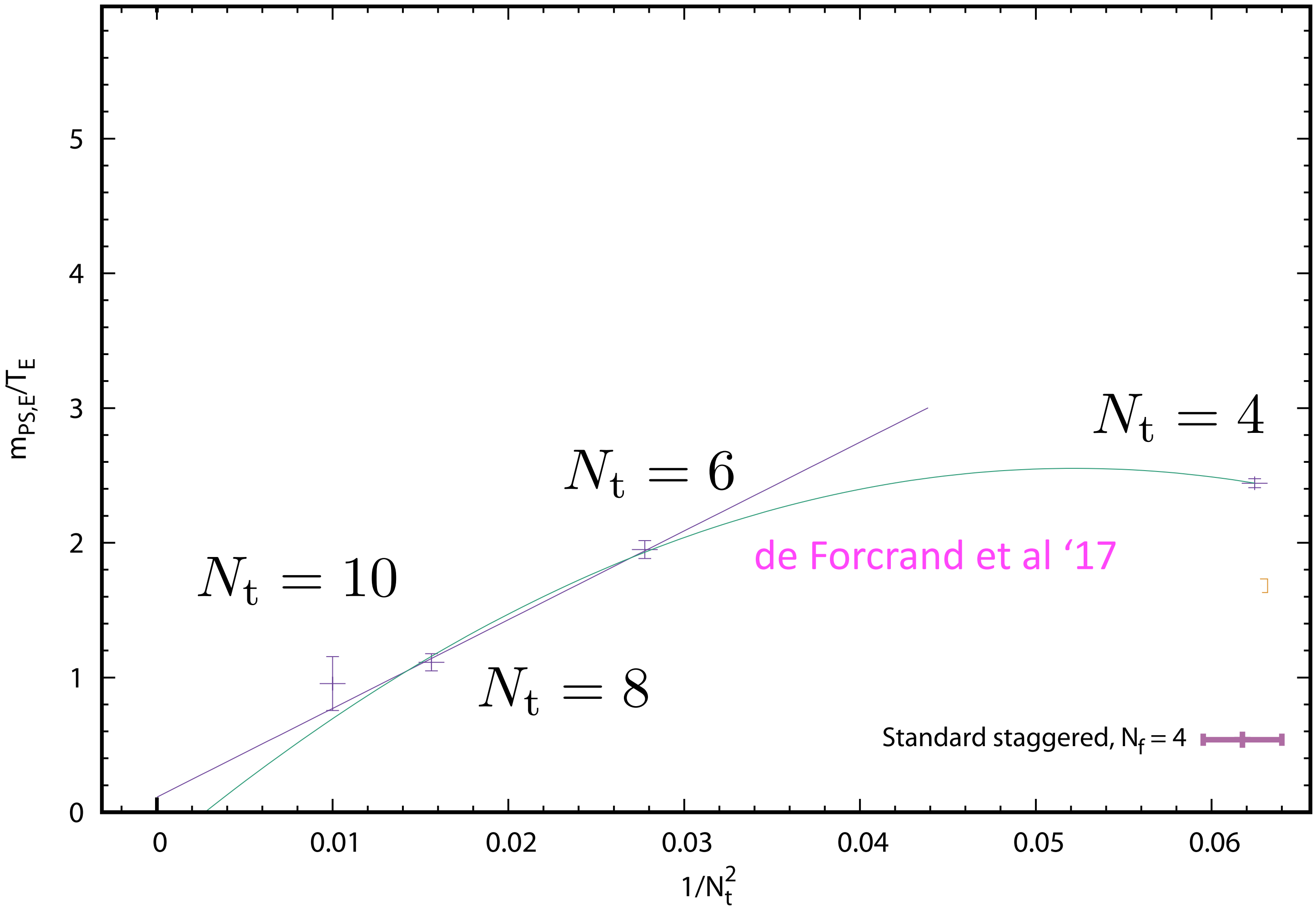
There might be a first order phase transition around κ_l = 0.14218

Wilson fermion study for N_f=4

Why N_f=4?

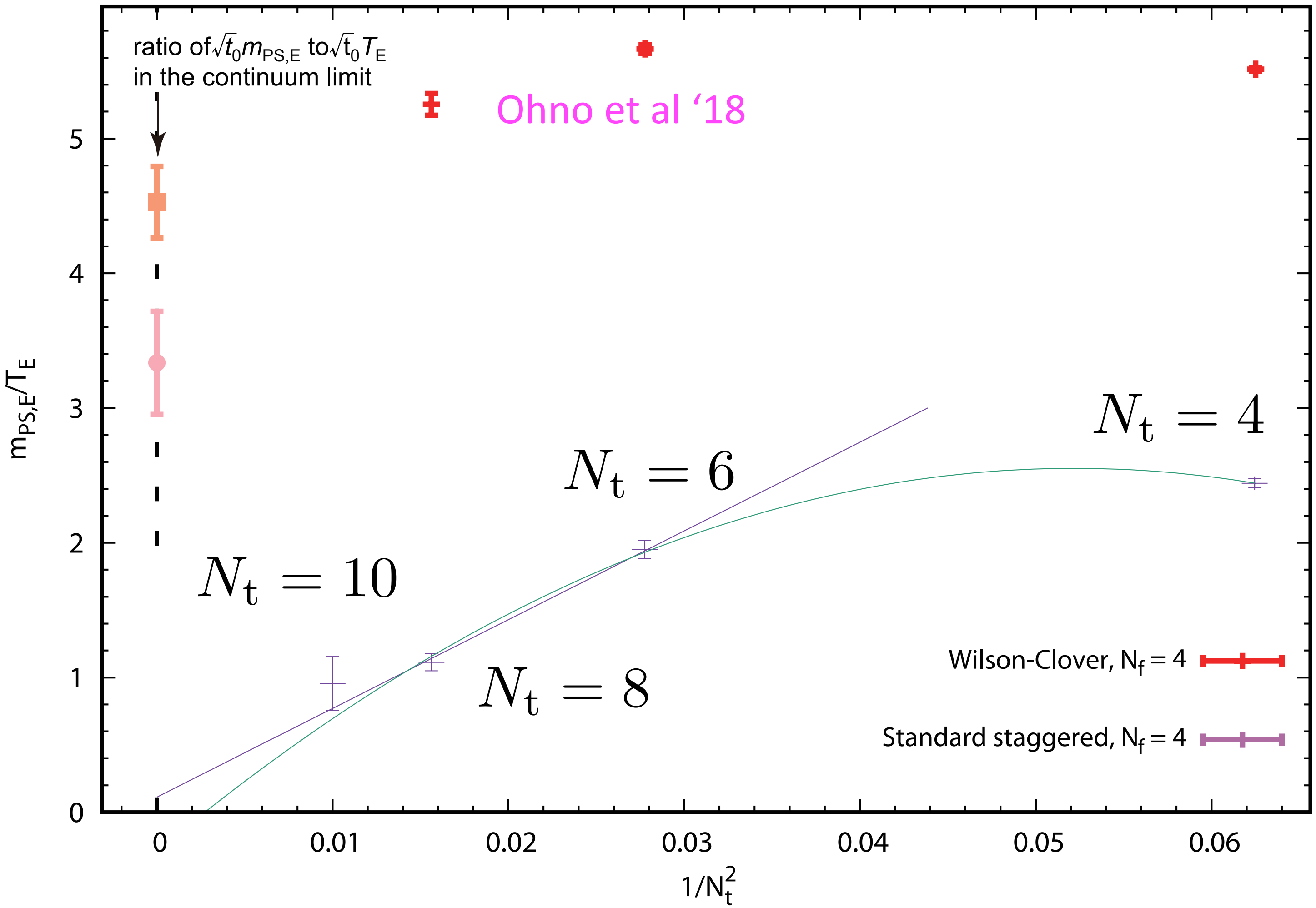
- No rooting issue
- Check Universality: staggered and Wilson should meet in continuum limit
- likely 1^{st} order PT by sigma model
- critical mass is heavier → cheap cost

Staggered vs. Wilson

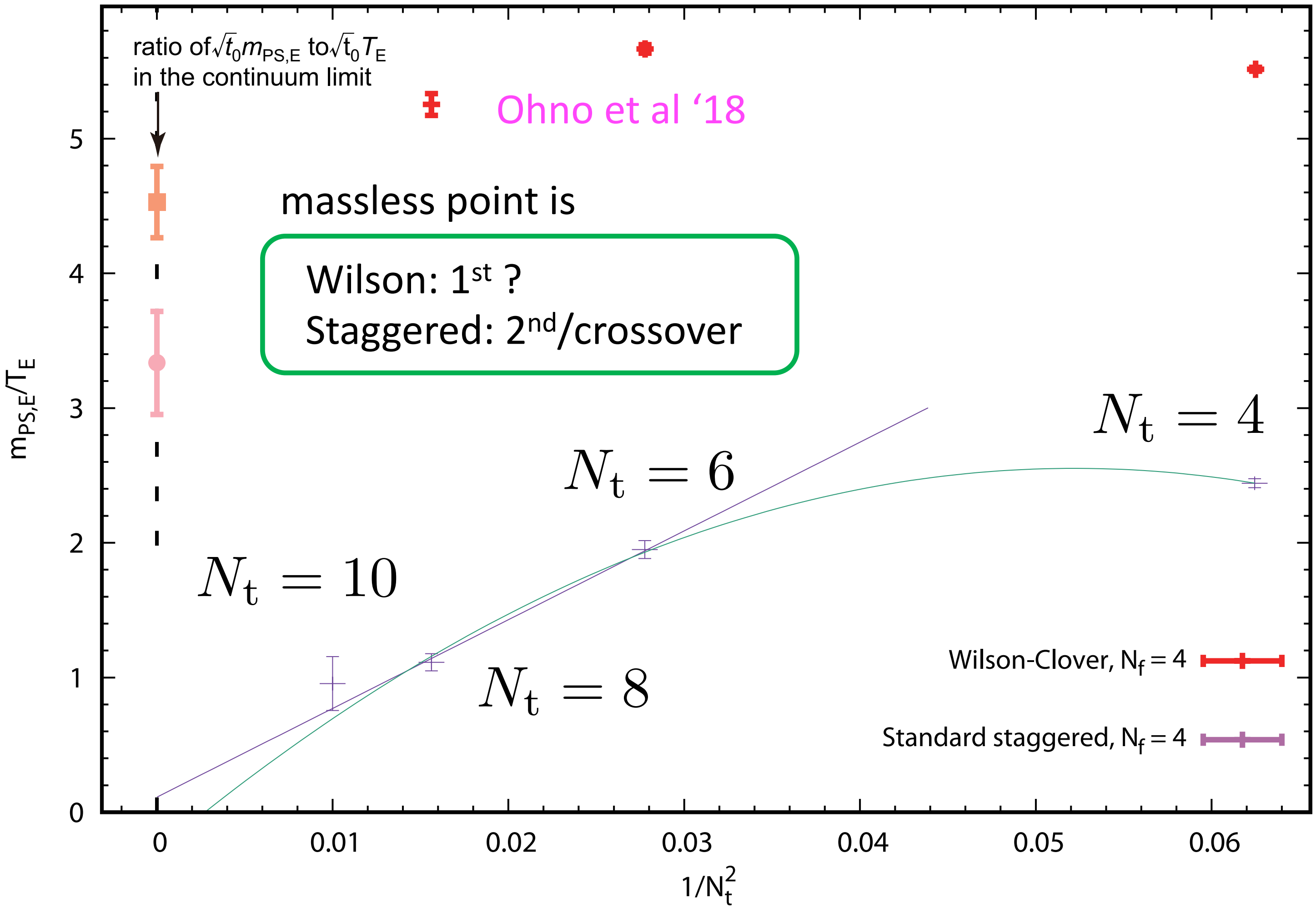


m_c=0?
or
no CP

Staggered vs. Wilson



Staggered vs. Wilson

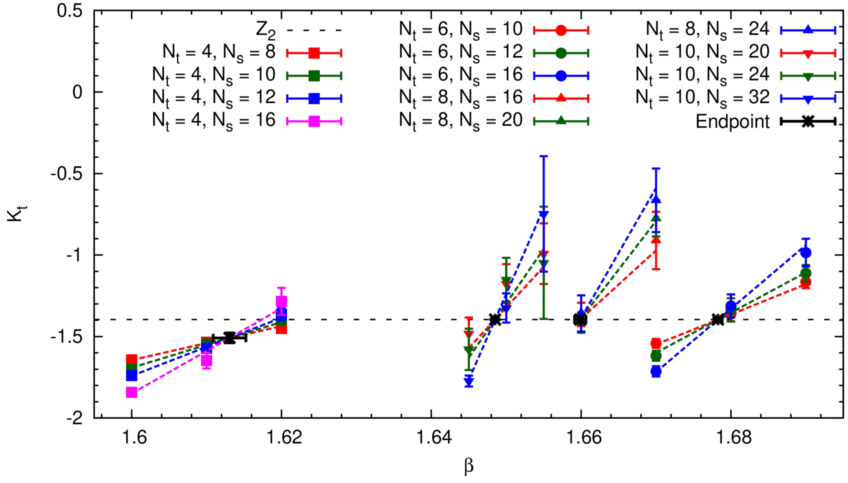


N_t=10 study is on-going

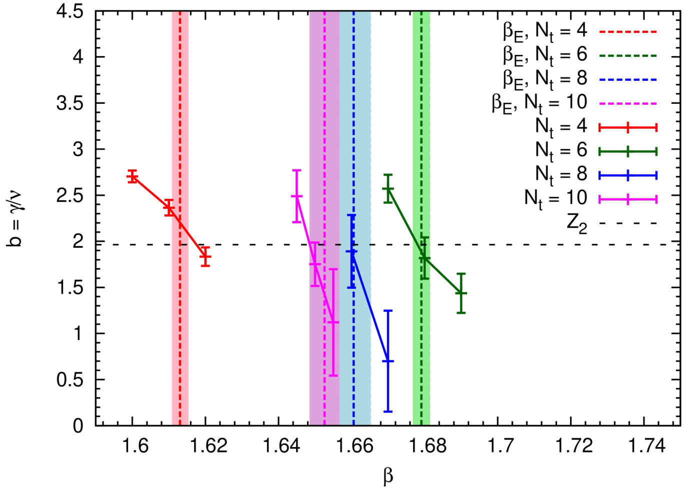
preliminary

Lattice 2021, H. Ohno's talk

kurtosis intersection



exponent of susceptibility



Continuum limit is left for future work

Summary

- N_f=2: Controversial (actions, methods)
- N_f=3: Critical mass is nearly zero (for both Staggered & Wilson in continuum limit)

If critical mass is zero
→ Pisarski & Wilczek ?

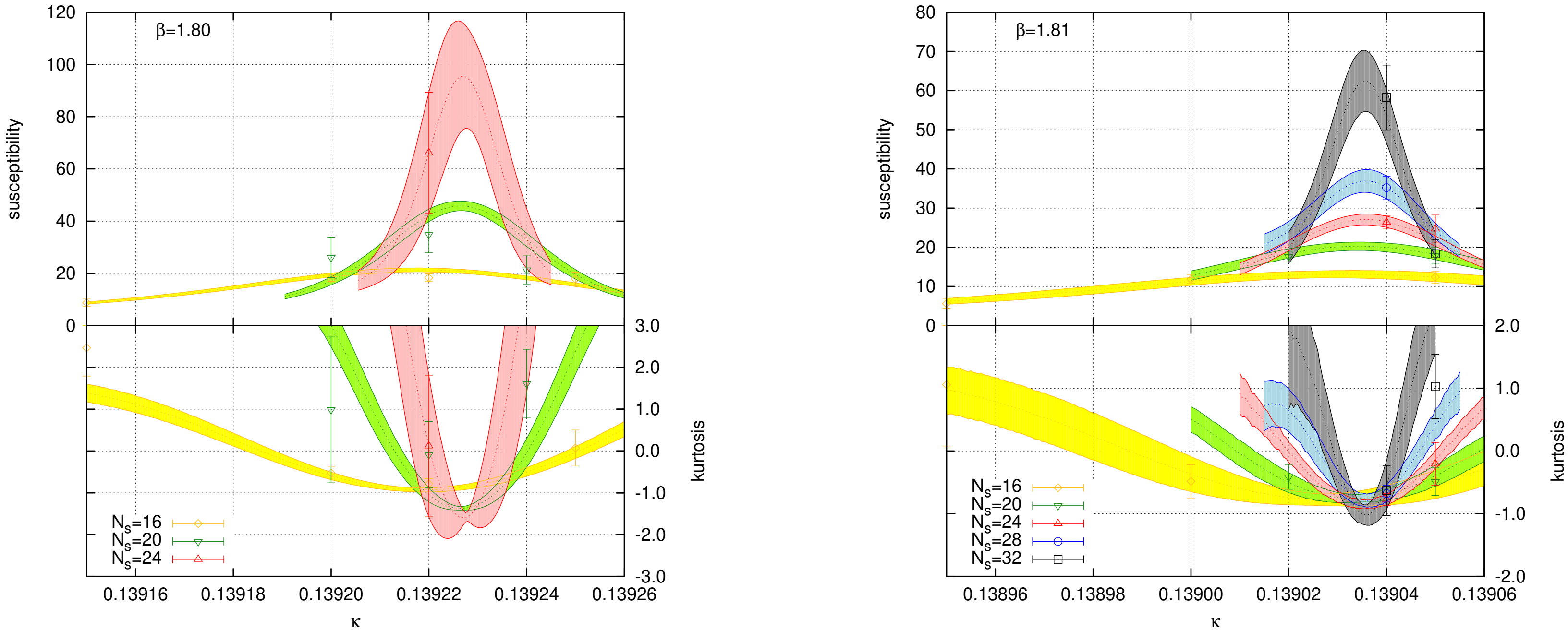
- N_f=4 (no rooting issue) :
  - Staggered: critical mass is nearly zero
  - Wilson: finite critical mass? (large N_t study is on-going)
- Caution
  - Staggered: Rooting issue
  - Wilson: Large cutoff error

Continuum limit should be taken

- Chiral fermions(DWF/Overlap) may help us!?

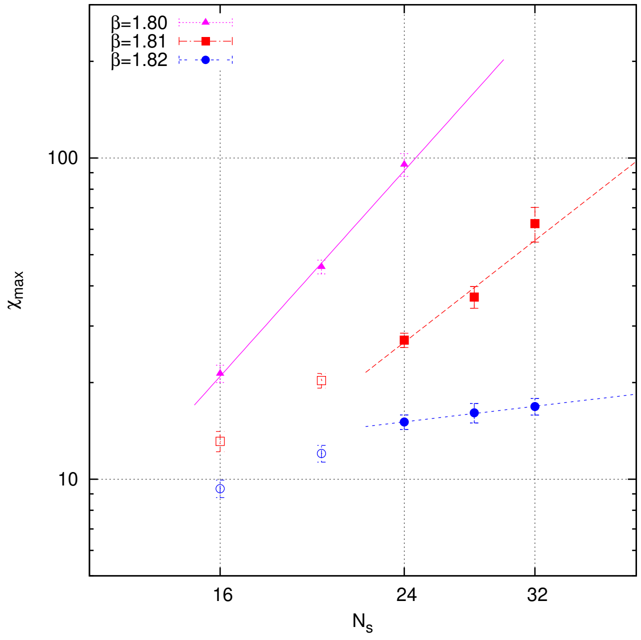
Back up

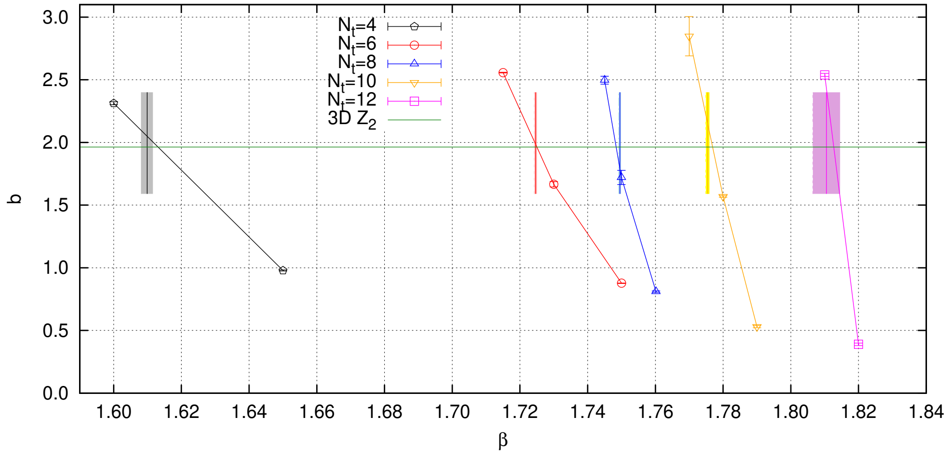
Nf=3, susceptibility and kurtosis



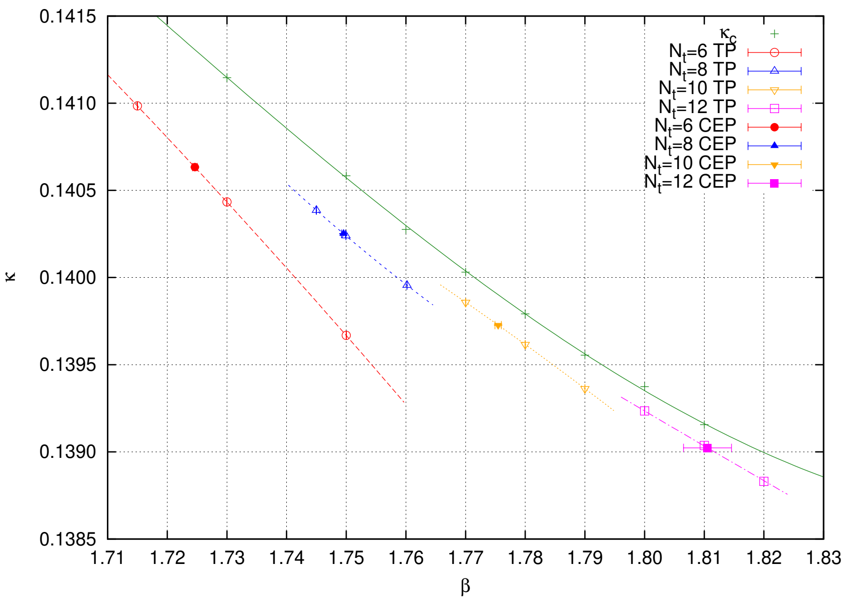
Nf=3, Exponent of susceptibility

$$\chi_{\max} \propto (N_{\mathrm{s}})^{b}$$

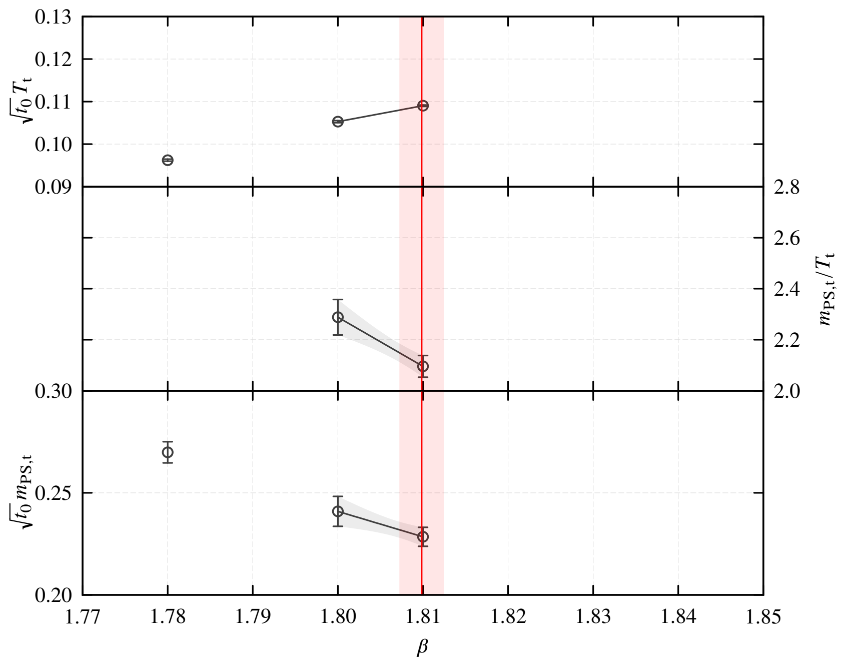




Nf=3

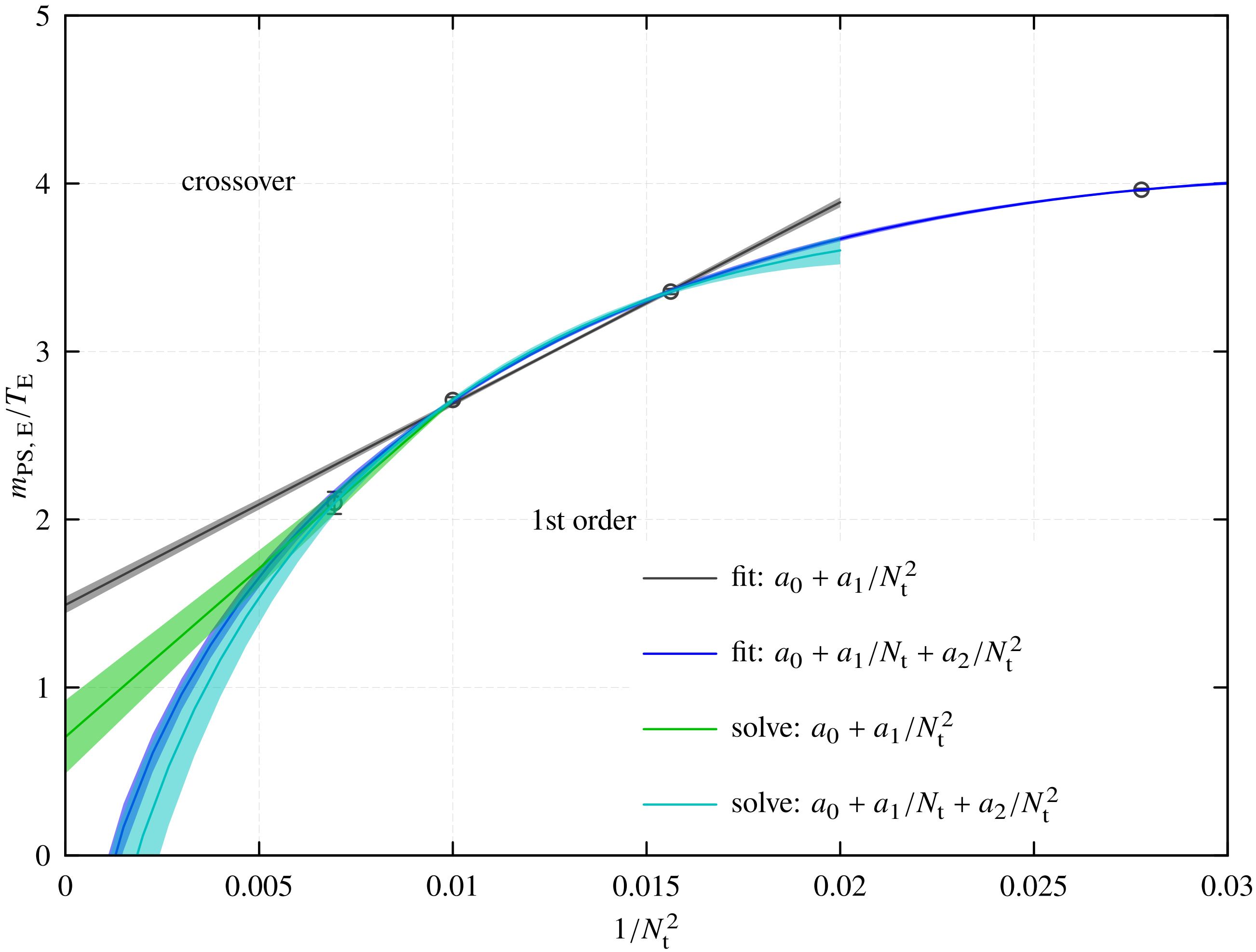


κc
Nt=6 TP
Nt=8 TP
Nt=10 TP
Nt=12 TP
Nt=6 CEP
Nt=8 CEP
Nt=10 CEP
Nt=12 CEP
0.1415
0.1410
0.1405
0.1400
0.1395
0.1390
0.1385
κ
1.71 1.72 1.73 1.74 1.75 1.76 1.77 1.78 1.79 1.80 1.81 1.82 1.83
β



$\sqrt{t_0}T_t$
0.13
0.12
0.11
0.10
0.09
2.8
2.6
2.4
2.2
2.0
$m_{PS,t}/T_t$
0.30
0.25
0.20
$\sqrt{t_0}m_{PS,t}$
1.77 1.78 1.79 1.80 1.81 1.82 1.83 1.84 1.85
β

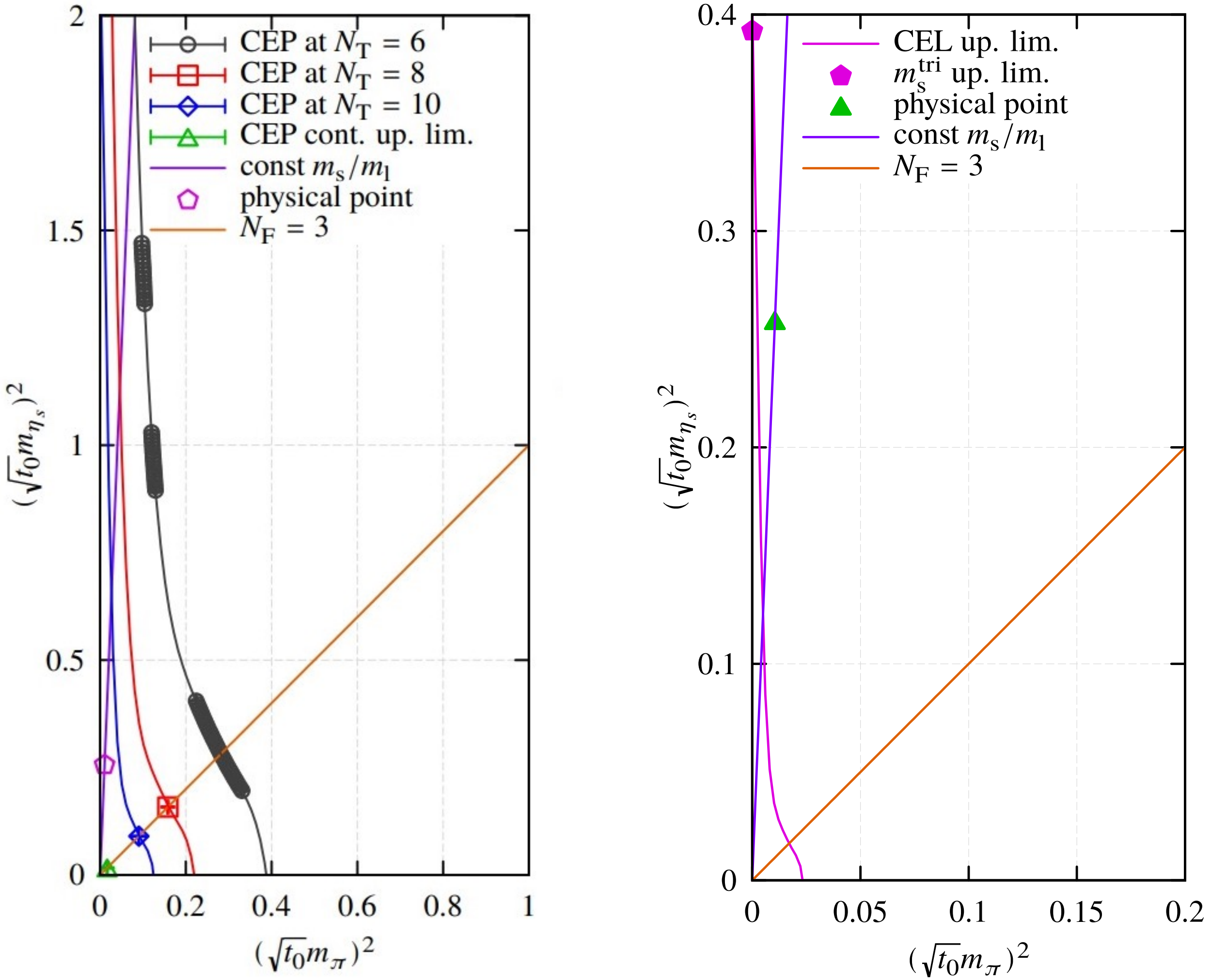
Nf=3



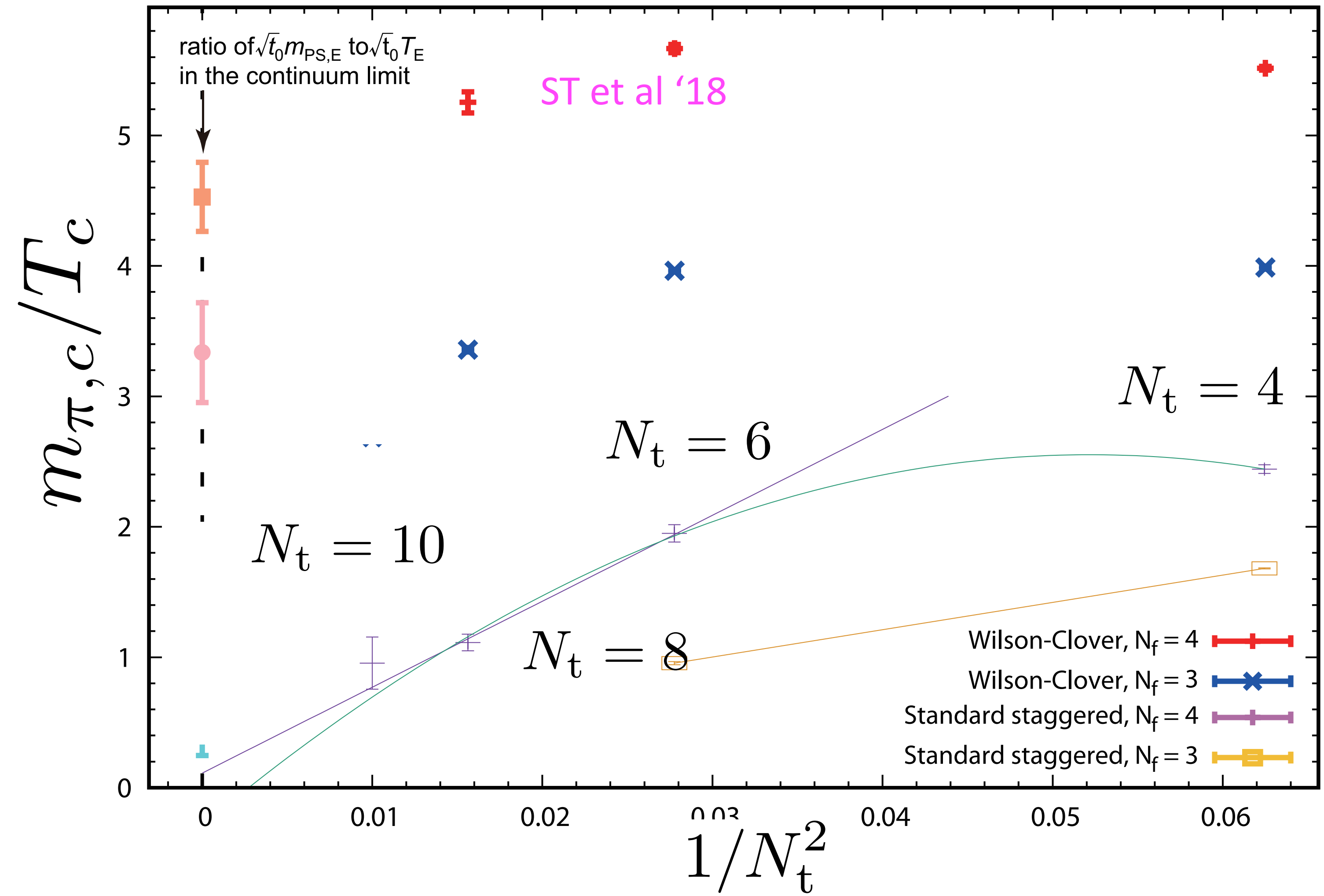
crossover
1st order
fit: $a_0 + a_1/N_t^2$
fit: $a_0 + a_1/N_t + a_2/N_t^2$
solve: $a_0 + a_1/N_t^2$
solve: $a_0 + a_1/N_t + a_2/N_t^2$
$m_{\mathrm{PS,E}}/T_{\mathrm{E}}$
$1/N_t^2$
0
1
2
3
4
5
0.005
0.01
0.015
0.02
0.025
0.03

Nf=2+1, N_t=6

Y. Nakamura *et al.*, PoS LATTICE 2019 (2019) 053



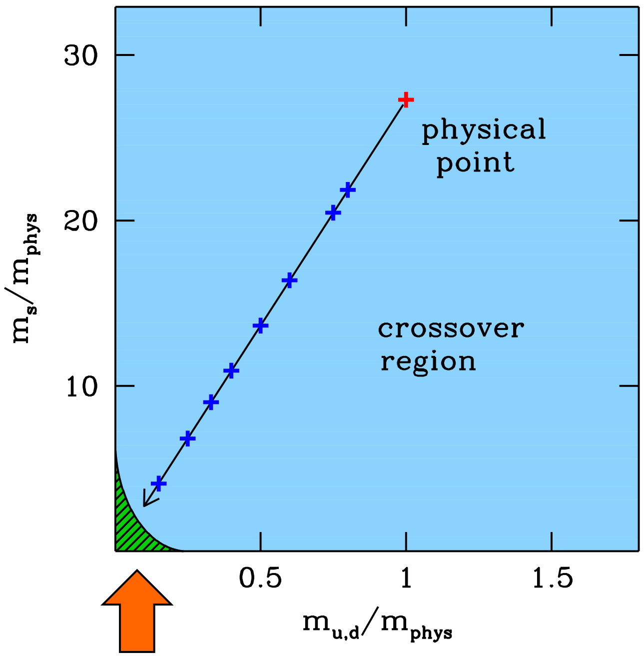
Staggered vs. Wilson for Nf=4



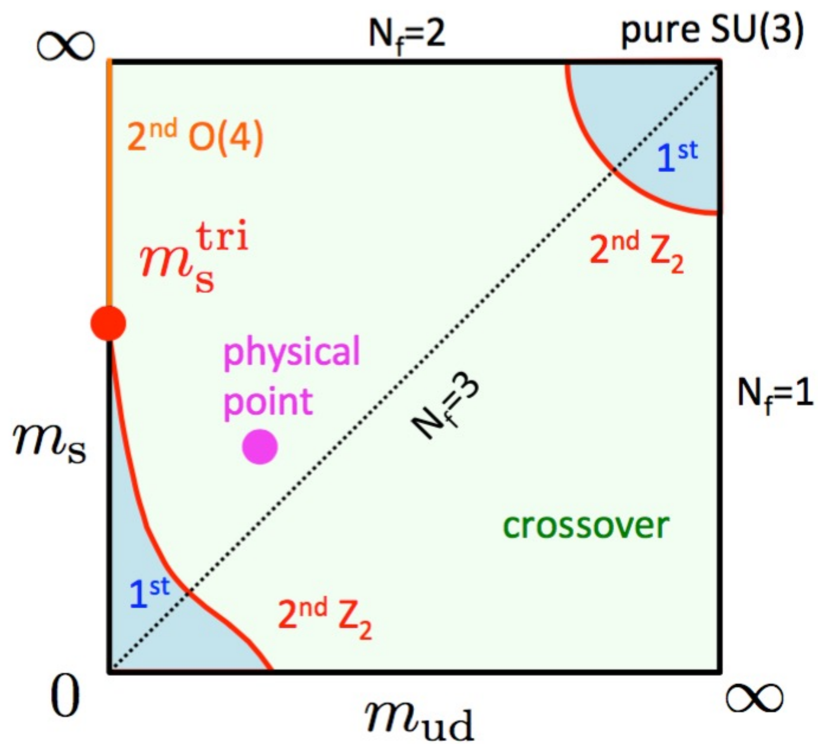
Columbia plot

Chiral region N_f=3

Lattice study Endrodi et al '07
stout Staggered, N_t=4,6



Very small 1st order region!

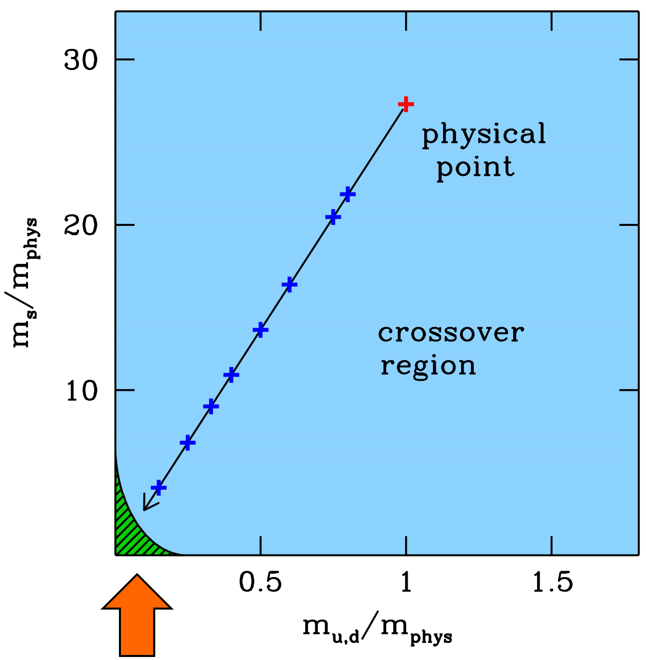


Columbia plot

Chiral region N_f=3

Lattice study Endrodi et al '07

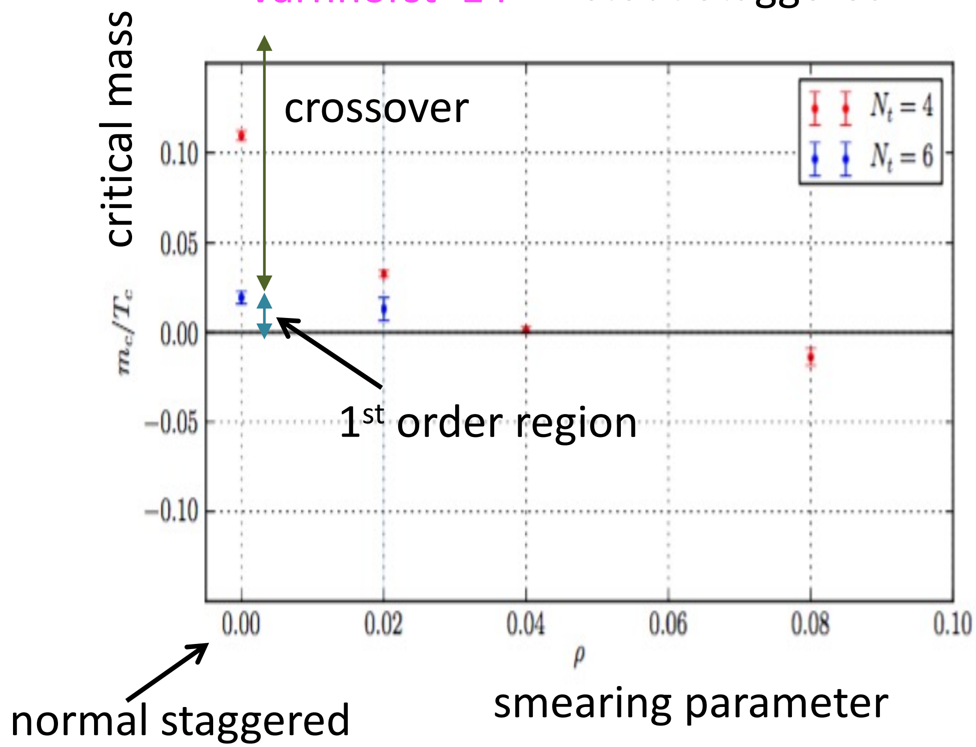
stout Staggered, N_t=4,6



Very small 1st order region!

N_f=3

Varnhorst '14 stout Staggered



No 1st order region!

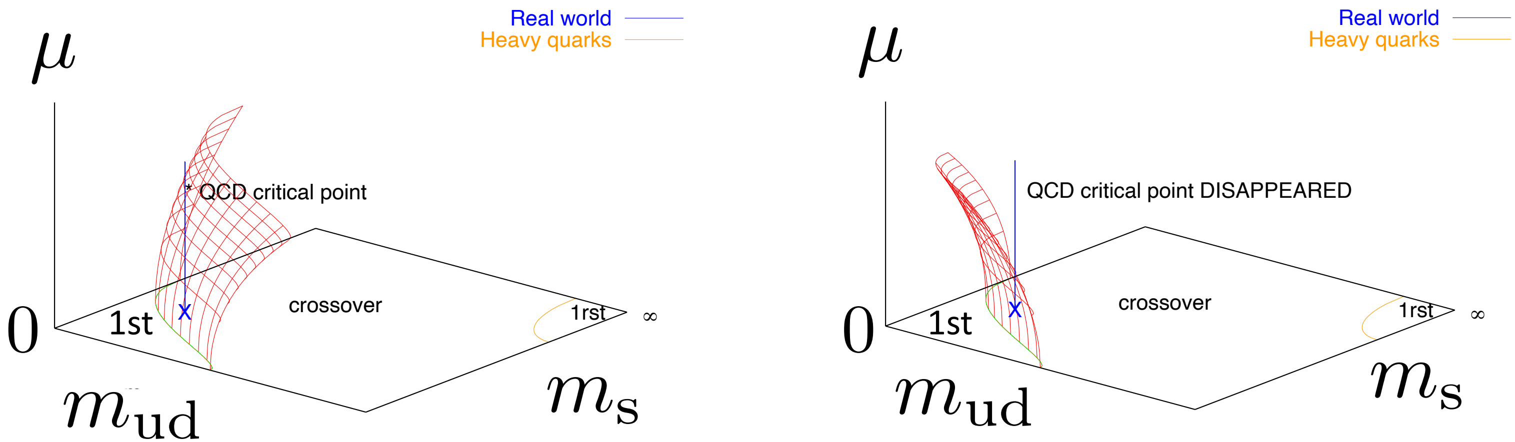
$N_f=3, \quad \mu \neq 0$

N_f=3, $\mu \neq 0$

Is curvature of critical surface positive or negative?

Positive: normal scenario

negative: exotic scenario

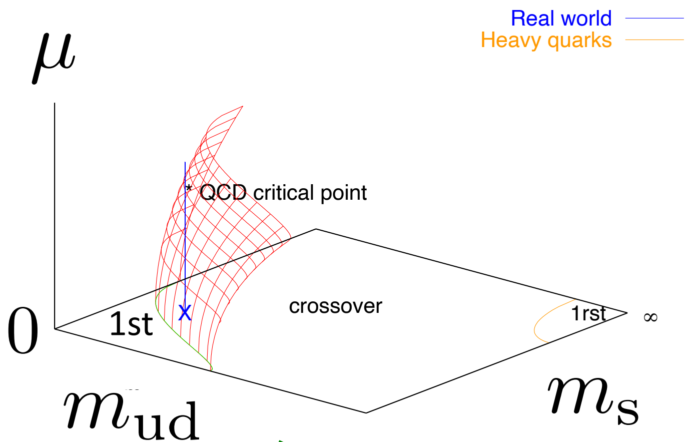


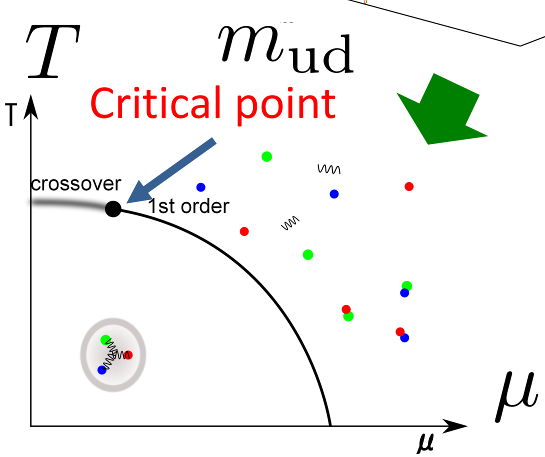
de Forcrand et al '07

N_f=3, $\mu\neq0$

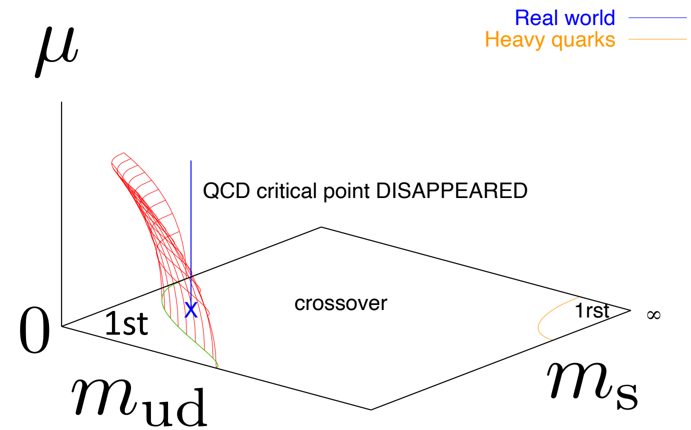
Is curvature of critical surface positive or negative?

Positive: normal scenario





negative: exotic scenario

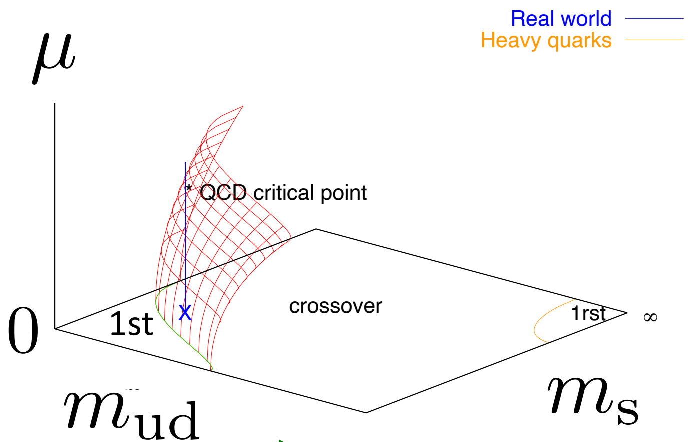


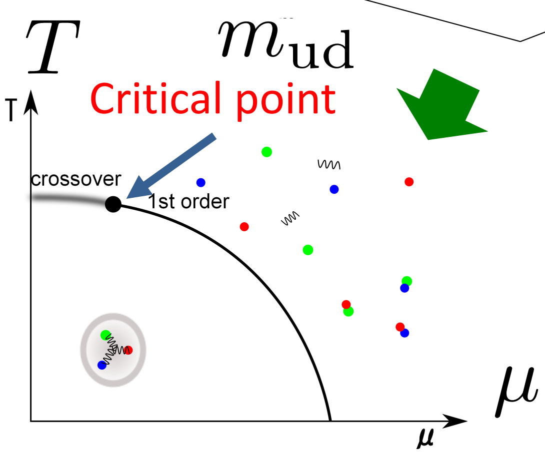
de Forcrand et al '07

N_f=3, $\mu \neq 0$

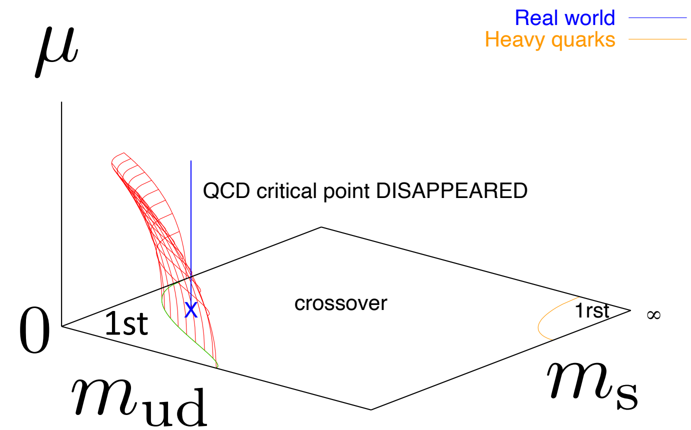
Is curvature of critical surface positive or negative?

Positive: normal scenario





negative: exotic scenario



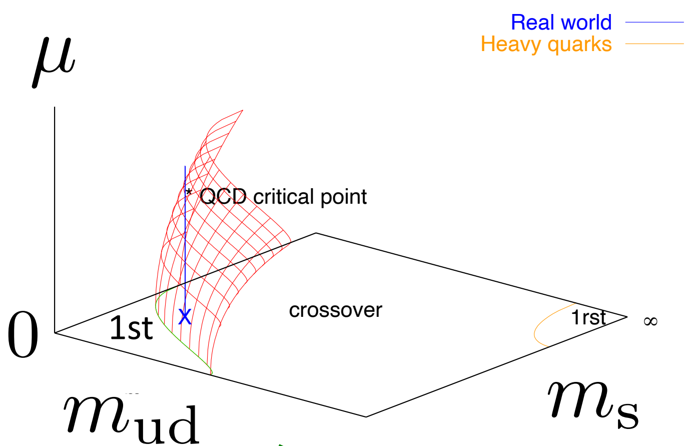
No critical point!!!

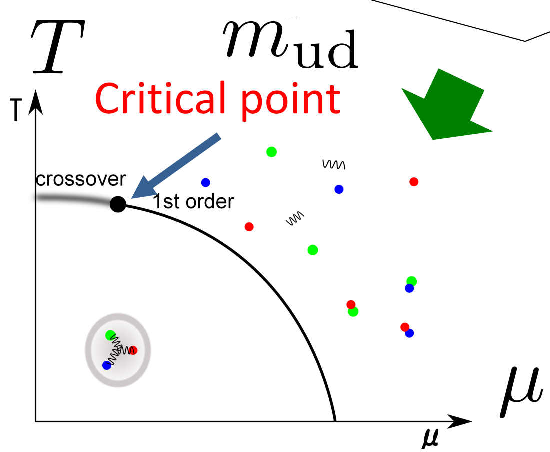
de Forcrand et al '07

N_f=3, $\mu\neq 0$

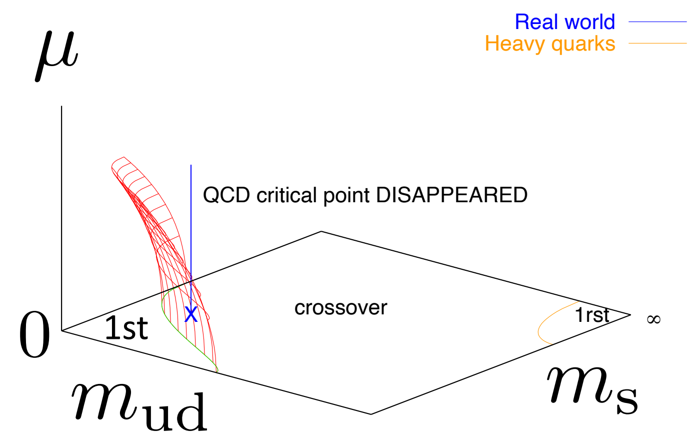
Is curvature of critical surface positive or negative?

Positive: normal scenario





negative: exotic scenario



No critical point!!!

$$\frac{m_c(\mu)}{m_c(0)} = 1-0.7(4)\left(\frac{\mu}{\pi T}\right)^2$$

de Forcrand & Philipsen '07

N_f =3

N_t =4

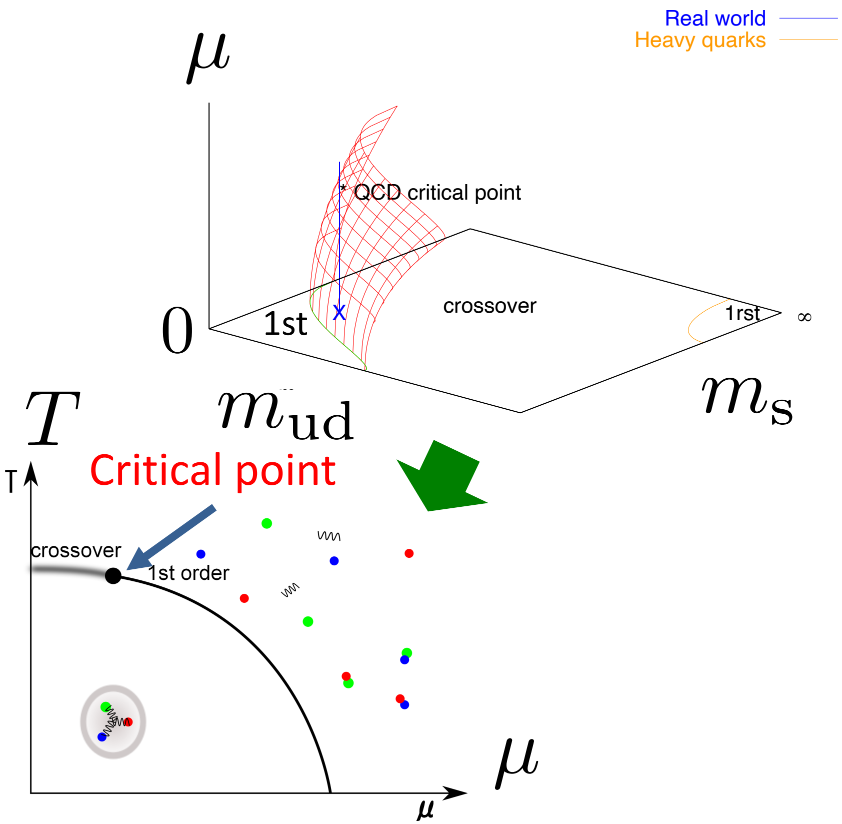
Staggered

imaginary μ

N_f=3, $\mu \neq 0$

Is curvature of critical surface positive or negative?

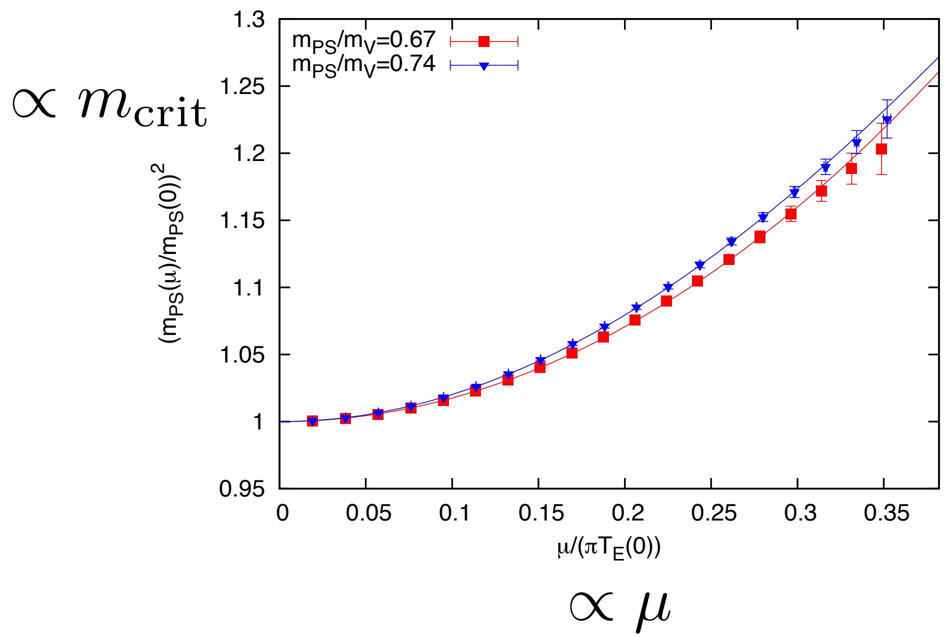
Positive: normal scenario



ST et al '15

Wilson-clover, N_f=3, N_t=6, phase RW

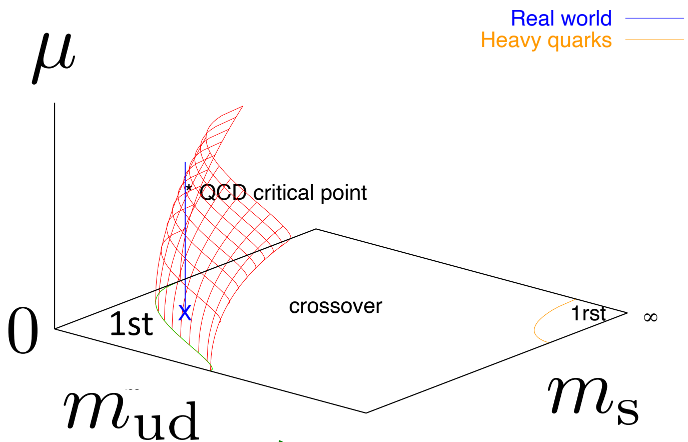
Positive!

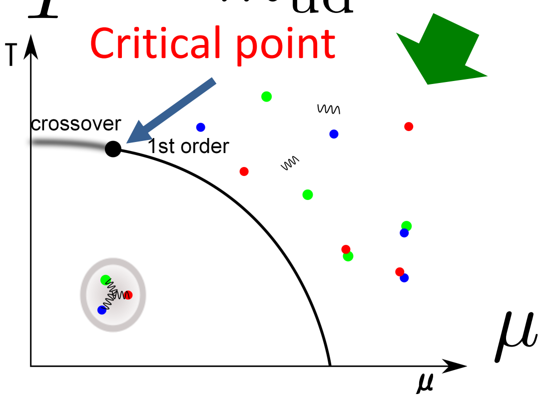


N_f=3, $\mu \neq 0$

Is curvature of critical surface positive or negative?

Positive: normal scenario





ST et al '15

Wilson-clover, N_f=3, N_t=6, phase RW