Deformed Instantons in $\mathcal{N} = 1/2$ Super Yang-Mills Theory from The Super ADHM Construction

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Field theory on non(anti)commutative (NAC) superspace arises in string theory as low energy effective theory on D-branes in the presence of a constant graviphoton field strength background [1]. NAC deformation of 4d $\mathcal{N} = 1$ SYM theory is called $\mathcal{N} = 1/2$ SYM theory, which is realized by the following star product of $\mathcal{N} = 1$ superfields:

$$f \ast g = f \exp(P) g, \quad P = -\frac{1}{2} Q_\alpha C^{\alpha\beta} Q_\beta,$$

where $Q_\alpha$ is the (chiral) supersymmetry generator and $C^{\alpha\beta}$ is the NAC parameter. Imaanpur has argued [2] that in this theory the anti-self-dual (ASD) instanton equations should be deformed:

$$v^{\text{SD}}_{\mu\nu} + i C_{\mu\nu}\bar{\lambda} = 0, \quad \lambda = 0, \quad D_\mu \sigma^{\mu\lambda} = 0, \quad D = 0,$$

where $C_{\mu\nu} \equiv C^{\alpha\beta} \sigma_{\mu\nu} \gamma^{\alpha\beta\gamma}$.

By extending the super ADHM construction [3] (see also [4]), we formulate [5] a way to construct all the exact solutions to the deformed ASD equations. Given a connection one-form superfield $\phi$, the curvature two-form superfield $F$ can be constructed by $F = d\phi + \phi \wedge \bar{\phi}$, where $\wedge$ denotes the deformed wedge product [5]. It turns out that the deformed instanton configurations are equivalent to the curvature $F$ satisfying $F_{\mu\dot{\alpha}} = 0$ and $\ast F_{\mu\nu} = -F_{\mu\nu}$, as well as the “Yang-Mills constraints” $F_{\alpha\beta} = F_{\dot{\alpha}\dot{\beta}} = F_{\dot{\alpha}\beta} = 0$. Our deformed super ADHM construction gives such curvature superfields. We denote the bosonic and fermionic ADHM data (matrix) as $a_\alpha$ and $M$ respectively. Define the “zero dimensional Dirac operator” $\hat{\Delta}_\alpha$ as a chiral superfield extension of the one in the purely bosonic ADHM construction:

$$\hat{\Delta}_\alpha(x) = a_\alpha + x_{\alpha\beta} b^{\beta} \rightarrow \hat{\Delta}_\alpha \equiv \Delta_\alpha(y) + \theta_\alpha M.$$

With the use of the (normalized) zero mode matrix superfield $\hat{v}$ of $\hat{\Delta}_\alpha$ such that $\hat{\Delta}_\alpha \ast \hat{v} = 0$, the connection one-form for the deformed instanton is given by $\phi = -\hat{v} \ast \hat{\phi}$ as long as the condition $\hat{\Delta}_{\alpha} \ast \hat{\Delta}_{\beta} = 0$ is satisfied ($\ast$ is explained in [4]). This condition encodes the bosonic and fermionic ADHM constraints and it is found that the bosonic ADHM constraints are modified by terms proportional to $k \times k$ matrices such as $C^{12} M \ast M$. To explore the consequences of the deformation terms in the bosonic ADHM constraints would be interesting, because they may be directly contrasted with consequences of noncommutative space by NS-NS $B$ field.

References