Stability of Fuzzy $CP^2$ in IIB Matrix Model.

$1^{)}$ KEK, $2^{)}$ Grad. Univ. of Advanced Studies, $3^{)}$ National Taiwan Normal Univ.
Hiromichi Kaneko$^{2)}$, Yoshihisa Kitazawa$^{1)}$,$^{2)}$, Dan Tomino$^{3)}$

E-mail: kanekoh@post.kek.jp, kitazawa@post.kek.jp, dan@home.phy.ntnu.edu.tw

IIB matrix model is a candidate of a non-perturbative formulation of string theory, and we can observe the spacetime dimension dynamically with investigating the effective action. Up to now, it has been found that 4 dimensional spacetime tends to minimize the effective action. Therefore, to examine the background’s dynamics in IIB matrix model may be of help to explain 4 dimensional spacetime in string theory.

In previous work, we have found that the fuzzy $S^2 \times S^2$ background is not stable at most symmetric point (Two fuzzy spheres is equal.). We also have found more symmetric manifolds is stable. Therefore $CP^2$ which is a more symmetric manifold is interesting and will be stable. It is also interesting that the effective action of $CP^2$ is compared with $S^2 \times S^2$’s one.

IIB matrix model is

$$ S_{IIB} = -\frac{1}{4} Tr [A_{\mu}, A_{\nu}]^2 - \frac{1}{2} Tr \bar{\psi} \Gamma_{\mu} [A_{\mu}, \psi], \quad (1) $$

where $A_\mu$ and $\psi$ are $N \times N$ Hermitian matrices. We can separate $A_{\mu}$ and $\psi$ into background fields and quantum fluctuations. Embedding fuzzy $CP^2$, we take the bosonic background fields as $SU(3)$ algebra. The irreducible representations of $SU(3)$ can be classified by the Young Tableaux ($p, q$) $\equiv \begin{array}{c|c|c|c|c} 1 & \cdot & \cdot & \cdot & q+p \end{array}$. When the representation is ($p, 0$), the background becomes fuzzy $CP^2$. In ($p, p$) rep., it becomes 6 dim. geometry. In ($p, q$) where $p \neq q$ and $q$ is fixed, we can find that the background acts like the $U(q+1)$ gauge theory of the fuzzy $CP^2$. Then we have to observe the effective action which is on the fuzzy $CP^2$ and the related manifolds.

We calculate the effective action up to 2-loop level and observe the minimum of it. The results are

- Fuzzy $CP^2$ is a solution of IIB matrix model up to 2-loop level.
- Fuzzy $CP^2$ is stable as far as $SU(3)$ symmetry is not broken.
- The minimum of the effective action of $CP^2$ is comparable to $S^2 \times S^2$ one.
- The 4 dim. manifold is preferable to the 6 dim. one. This is the same as the fuzzy spheres case.
- The scaling behavior of the effective action on the fuzzy $CP^2$ is $O(N)$. This is the same as the fuzzy $S^2 \times S^2$ and $T^4$ cases.

Note: See hep-th/0506033, the references of my talk are found.