Localization on D-brane and Gauge theory/Matrix model

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In this talk, I talked about the relation between the instanton counting of four-dimensional \( \mathcal{N} = 2 \) supersymmetric gauge theories [1] and two-dimensional (bosonic) gauge theories based on the work [2].

Let us consider Type IIB superstring theory on \( \mathbb{R}^{1,3} \times \mathbb{C} \times M_4 \), where \( M_4 \) is a four-dimensional ALE space. If \( N_c \) D5-branes are wrapped on a 2-cycle \( \Sigma \) in \( M_4 \), 4D \( SU(N_c) \) \( \mathcal{N} = 2 \) SYM theory appears on the extra \( \mathbb{R}^{1,3} \) space on the D5-branes except for the compactified 2-cycle. We also introduce a noncommutativity to the \( \mathbb{R}^{1,3} \) space-time, which does not affect the instanton contribution to the prepotential of the \( \mathcal{N} = 2 \) SYM theory. According to the concept of the large \( N \) reduction and the noncommutativity, the gauge theory on the D5-branes reduces to a two-dimensional large \( N \) topological field theory on the internal 2-cycle.

By evaluating the vacuum expectation value of an operator which couples to D-instanton on the D-strings and taking into account the moduli parameters of the 4D SYM, we found that we obtain the partition function of the two-dimensional generalized Yang-Mills theory [2],

\[
Z = \sum_{\{n_i\}} \prod_{1 \leq i < j \leq N} (g_s n_i - g_s n_j)^2 e^{- \frac{4}{\pi} \sum_{i} W(g_s n_i)}. \tag{1}
\]

If we regard \( \{n_i\} \) as “positions” of free fermions, any configuration of \( \{n_i\} \) can be thought to be an excitation state from the “vacuum state” that is defined by the densest state around the critical points of the potential \( W(x) \). As a result, there appear two fermi surfaces in this system. In addition, we take a large \( N \) limit by fixing the combination, \( \left( \frac{g_s N}{2N_c} \right)^{2N_c} e^{-A \mu (\frac{g_s N}{2N_c})^{N_c}} \equiv \Lambda^{2N_c} \).

As a result, we can rewrite (1) as \( Z_{\text{YM}_2} = Z_{\text{Nek}}(\{a\}; \Lambda)^2 \), where \( Z_{\text{Nek}}(\{a\}; \Lambda) \) is so-called Nekrasov’s partition function for 4D \( \mathcal{N} = 2 \) SYM [1].

We also showed that the similar partition functions for \( cN = 2 \) theory with hypermultiplets can be obtained from suitably deformed two-dimensional theories. As an important by-product, we derived the instanton partition function for \( \mathcal{N} = 2 \) A_2-type quiver gauge theory.

References
