Inflation in Stringy Landscape

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Why do we need inflation?

Problems of the standard Big Bang theory:

- What was before the Big Bang?
- Why is our universe so homogeneous (better than 1 part in 10000) ?
- Why is it isotropic (the same in all directions)?
- Why all of its parts started expanding simultaneously?
- Why it is flat? Why parallel lines do not intersect? Why it contains so many particles?
Inflation as a theory of a harmonic oscillator

\[ V(\phi) = \frac{m^2}{2}\phi^2 \]
Equations of motion:

- Einstein:
  \[ H^2 = \left(\frac{\ddot{a}}{a}\right)^2 = \frac{m^2}{6} \phi^2 \]

- Klein-Gordon:
  \[ \ddot{\phi} + 3H \dot{\phi} = -m^2 \phi \]

Compare with equation for the harmonic oscillator with friction:

\[ \ddot{x} + \alpha x = -k x \]
Logic of Inflation:

Large $\phi$ $\rightarrow$ large $H$ $\rightarrow$ large friction

Field $\phi$ moves very slowly, so that its potential energy for a long time remains nearly constant.

$$H = \frac{\dot{a}}{a} = \frac{m\phi}{\sqrt{6}} \approx \text{const}$$

$$a \sim e^{Ht}$$

No need for false vacuum, supercooling, phase transitions, etc.
Add a constant to the inflationary potential
- obtain inflation and acceleration

\[ V = \frac{m^2}{2} \phi^2 + \Lambda \]
Predictions of Inflation:

1) The universe should be homogeneous, isotropic and flat, $\Omega = 1 + O(10^{-4}) \quad [\Omega = \rho/\rho_0]$

Observations: the universe is homogeneous, isotropic and flat, $\Omega = 1 + O(10^{-2})$

2) Inflationary perturbations should be gaussian and adiabatic, with flat spectrum, $n_s = 1 + O(10^{-1})$

Observations: perturbations are gaussian and adiabatic, with flat spectrum, $n_s = 1 + O(10^{-2})$
WMAP and cosmic microwave background anisotropy

Black dots - experimental results.

Red line - predictions of inflationary theory
Chaotic inflation in supergravity

Main problem:

\[ V(\phi) = e^K \left( K^{-1}_{\Phi \Phi} |D_\Phi W|^2 - 3|W|^2 \right) \]

Canonical Kahler potential is \( K = \Phi \bar{\Phi} \)

Therefore the potential blows up at large \(|\phi|\), and slow-roll inflation is impossible:

\[ V \sim e^{2|\phi|^2} \]

Too steep, no inflation...
A solution: shift symmetry

Kawasaki, Yamaguchi, Yanagida 2000

Equally good Kahler potential

\[ K = \frac{1}{2} (\Phi + \bar{\Phi})^2 + X \bar{X} \]

and superpotential

\[ W = m \Phi X \]

The potential is very curved with respect to \( X \) and \( \text{Re} \ \Phi \), so these fields vanish.

But Kahler potential does not depend on

\[ \phi = \sqrt{2} \ \text{Im} \ \Phi = (\Phi - \bar{\Phi}) / \sqrt{2} \]

The potential of this field has the simplest form, without any exponential terms, even including the radiative corrections:

\[ V = \frac{m^2}{2} \phi^2 \]
Inflation in String Theory

The volume stabilization problem:

A potential of the theory obtained by compactification in string theory of type IIB:

\[ V(X, Y, \phi) \sim e^{\sqrt{2}X - \sqrt{6}Y} V(\phi) \]

\(X\) and \(Y\) are canonically normalized field corresponding to the dilaton field and to the volume of the compactified space; \(\phi\) is the field driving inflation.

The potential with respect to \(X\) and \(Y\) is very steep, these fields rapidly run down, and the potential energy \(V\) vanishes. We must stabilize these fields.

Dilaton stabilization:  
Giddings, Kachru, Polchinski 2001

Volume stabilization:  
KKLT construction  
Kachru, Kallosh, A.L., Trivedi 2003  
Burgess, Kallosh, Quevedo, 2003
**Volume stabilization**

Kachru, Kallosh, A.L., Trivedi 2003

**Basic steps of the KKLT scenario:**

1) Start with a theory with runaway potential discussed above

2) Bend this potential down due to (nonperturbative) quantum effects

3) Uplift the minimum to the state with positive vacuum energy by adding a positive energy of an anti-D3 brane in warped Calabi-Yau space

\[ V_{\text{KKLT}} = V_{\text{AdS}} + \frac{D}{\sigma^2} \]
The results:

- It is possible to stabilize internal dimensions, and to obtain an accelerating universe. Eventually, our part of the universe will decay and become ten-dimensional, but it will only happen in $10^{10^{120}}$ years.

- Apparently, vacuum stabilization can be achieved in $10^{100} - 10^{1000}$ different ways. This means that the potential energy $V$ of string theory may have $10^{100} - 10^{1000}$ minima where we (or somebody else) can enjoy life...
All moduli on K3 x K3 can be stabilized
Self-reproducing Inflationary Universe
String Theory Landscape

Perhaps $10^{100} - 10^{1000}$ different minima

Lerche, Lust, Schellekens 1987

Bousso, Polchinski; Susskind; Douglas, Denef...
Anthropic principle in combination with inflationary cosmology and string theory implies, in particular, that if inflationary 4D space-time is possible in the context of string theory, then we should live in a 4D space even if other compactifications are much more probable.

Such arguments allow one to concentrate on those problems which cannot be solved by using anthropic reasoning.
We must understand how to introduce a proper probability measure in stringy landscape, taking into account volume of the inflating universe. What is important, however, is a gradual change of the attitude towards anthropic reasoning:

Previously anthropic arguments were considered as an “alternative science”. Now one can often hear an opposite question: Is there any alternative to the anthropic considerations combined with the counting of possible vacuum states? What is the role of dynamics in the world governed by chance?

Here we will give an example of the “natural selection” mechanism, which may help to understand the origin of symmetries.
Quantum effects lead to particle production, which results in moduli trapping near enhanced symmetry points.

These effects are stronger near the points with greater symmetry, where many particles become massless.

This may explain why we live in a state with a large number of light particles and (spontaneously broken) symmetries.
Basic Idea

is related to the theory of preheating after inflation
Kofman, A.L., Starobinsky 1997

Consider two interacting moduli with potential

$$V = \frac{g^2}{2} \phi^2 \chi^2$$

It can be represented by two intersecting valleys

Suppose the field $\Phi$ moves to the right with velocity $v$. Can it create particles $\chi$?

**Nonadiabaticity condition:**

$$\dot{m}_\chi > m^2$$

$$g \dot{\phi} \equiv g v > g^2 \phi^2$$

$$\phi < \sqrt{v/g}$$
When the field $\phi$ passes the (red) nonadiabaticity region near the point of enhanced symmetry, it created particles $\chi$ with energy density proportional to $\phi$. Therefore the rolling field slows down and stops at the point when

$$\rho_\chi = g|\phi| n_\chi = \frac{v^2}{2}$$

Then the field falls down and reaches the nonadiabaticity region again…

![Diagram of field and particles](image_url)
When the field passes the nonadiabaticity region again, the number of particles $\chi$ (approximately) doubles, and the potential becomes two times more steep. As a result, the field becomes trapped at a distance that is two times smaller than before.
Trapping of the scalar field

\[ \phi \]

\[ t \]
Thus anthropic and statistical considerations are supplemented by a dynamical selection mechanism, which may help us to understand the origin of symmetries in our world.
Anthropic principle says that we can live only in those parts of the universe where we can **survive**

Moduli trapping is a dynamical mechanism which may help us to find places where we can **live well**
Two types of string inflation models:

- **Moduli Inflation.** The simplest class of models. They use only the fields that are already present in the KKLT model.

- **Brane inflation.** The inflaton field corresponds to the distance between branes in Calabi-Yau space. Historically, this was the first class of string inflation models.
Inflation in string theory

- KKLMMT brane-anti-brane inflation
- D3/D7 brane inflation
- Racetrack modular inflation
- DBI inflation (non-minimal kinetic terms)
Racetrack Inflation

the first working model of the moduli inflation

Blanco-Pilado, Burgess, Cline, Escoda, Gomes-Reino, Kallosh, Linde, Quevedo

Superpotential: \[ W = W_0 + A e^{-aT} + B e^{-bT} \]

Kahler potential: \[ K = -3 \log(T + T^*) \]

Effective potential for the field \( T = X + i Y \)

\[
V = \frac{E}{X^\alpha} + \frac{e^{-aX}}{6X^2} \left[ aA^2 (aX + 3) e^{-aX} + 3W_0 aA \cos(aY) \right] + \\
+ \frac{e^{-bX}}{6X^2} \left[ bB^2 (bX + 3) e^{-bX} + 3W_0 bB \cos(bY) \right] + \\
+ \frac{e^{-(a+b)X}}{6X^2} \left[ AB (2abX + 3a + 3b) \cos((a - b)Y) \right]
\]
Parameters and Potential

\[ A = \frac{1}{50}, \quad B = -\frac{35}{1000}, \quad a = \frac{2\pi}{100}, \quad b = \frac{2\pi}{90}, \quad W_0 = -\frac{1}{25000} \]
Meanwhile for inflation with a flat spectrum of perturbations one needs

\[ K = -3 \log(\rho + \bar{\rho} - \phi \phi) \]

\[ W = W_0 + e^{-a\rho} \]

\[ m_\phi^2 = 2H^2 \]

This can be achieved by taking \( W \) depending on \( \phi \) and by fine-tuning it at the level \( O(1\%) \)
This model is complicated and requires fine-tuning, but it is based on some well-established concepts of string theory. Its advantage is that the smallness of inflationary parameters has a natural explanation in terms of warping of the Klebanov-Strassler throat.

Fine-tuning may not be a problem in the string theory landscape paradigm.

Further developed by: Burgess, Cline, Stoica, Quevedo; DeWolfe, Kachru, Verlinde; Iisuka, Trivedi; Berg, Haack, Kors; Buchel, Ghodsi
This is a stringy version of D-term Inflation
Binetruy, Dvali; Halyo
String inflation and shift symmetry

\[ K = -3 \log \left( \rho + \bar{\rho} - \frac{1}{2} \left( s + \bar{s} \right)^2 \right) \]

Hsu, Kallosh, Prokushkin 2003

Shift symmetry protects flatness of the inflaton potential in the \( S - \bar{S} \) direction. This is not just a requirement which is desirable for inflation, but, in a certain class of string theory models, it may be a consequence of a classical symmetry slightly broken by quantum corrections.

Hsu, Kallosh, 2004
and work in progress
Double Uplifting

\[ V_{\text{tot}} = V_{\text{KKLT}}(\sigma) + V_2(\sigma, s, \phi_{\pm}) \]

First uplifting: KKLT

\[ V_{\text{KKLT}} = V_{\text{AdS}} + \frac{D}{\sigma^2} \]

Kallosh, A.L., in progress
Second uplifting in D3/D7 model

\[ V(s, \sigma, \phi_{\pm} = 0) = V_{\text{AdS}} + \frac{D}{\sigma^2} + \frac{C}{\sigma^3} \]

D3 contribution

effective FI
Inflationary potential at $\phi_{\pm} = 0$ as a function of $S$ and $\sigma$

Shift symmetry is broken only by quantum effects
Potential of D3/D7 inflation with a stabilized volume modulus

\[ V_2(\phi_+, \phi_-, s, \sigma_{cr}) = \frac{g^2}{2} |\phi_+\phi_-|^2 + \frac{g^2}{2} s^2 [|\phi_+|^2 + |\phi_-|^2] + \frac{g^2}{8} [|\phi_+|^2 - |\phi_-|^2 - 2\xi]^2 \]

\[ V_{1\text{-}loop} = \frac{g^2 \xi^2}{2} \left( 1 + \frac{g^2}{8\pi^2} \ln \frac{|S^2|}{|S_c^2|} \right) \]

Unlike in the brane-antibrane scenario, inflation in D3/D7 model does not require fine-tuning because of the shift symmetry.
The height of the KKLT barrier is smaller than $|V_{AdS}| = m^{2_{3/2}}$. The inflationary potential $V_{infl}$ cannot be much higher than the height of the barrier. Inflationary Hubble constant is given by $H^2 = V_{infl}/3 < m^{2_{3/2}}$.

Constraint on the Hubble constant in this class of models:

$$H < m^{3/2}$$
\[ V(\phi) = e^K \left( K_{\Phi \Phi}^{-1} |D_{\Phi} W|^2 - 3|W|^2 \right) \]

In the AdS minimum in the KKLT construction \( D_{\Phi} W = 0 \)

Therefore

\[ V_{\text{AdS}} = -3 e^K |W^2| = -3 m_{3/2}^2 \]

\[ 3H^2 = V_{\text{inflation}} \leq V_{\text{barrier}} \sim |V_{\text{AdS}}| = 3m_{3/2}^2 \]

\[ H \leq m_{3/2} \]
A new class of KKLT models

Kallosh, A.L. hep-th/0411011

Using racetrack superpotential with two exponents, one can obtain a supersymmetric Minkowski vacuum without any uplifting of the potential

Inflation in the new class of KKLT models can occur at $H >> m_{3/2}$

Small mass of gravitino, no correlation with the height of the barrier and with the Hubble constant during inflation

Even in these models inflation occurs only at $V << 1$. This may lead to a problem with initial conditions for inflation, which we are going to address now.
In all versions of string inflation, the process of inflation begins at $V<<<1$. However, a hot closed universe collapses within the time $t = S^{2/3}$, in Planck units. It can survive until the beginning of inflation at $t = H^{-1} = V^{-1/2}$ only if $S > V^{-3/4}$

For $V=10^{-16}$ (typical for string inflation) the initial entropy (the number of particles) must be $S > 10^{12}$. Such a universe at the Planck time consisted of $10^{12}$ causally independent domains. Thus, in order to explain why the universe is so large and homogeneous one should assume that it was large and homogeneous from the very beginning...
Thus it is difficult to start expansion of the universe with a low-scale inflation in any of the standard Friedmann models (closed universe or infinite flat or open universe).

Can we create a finite flat universe?

Yes we can!

Take a box (a part of a flat universe) and glue its opposite sides to each other. What we obtain is a torus, which is a topologically nontrivial flat universe.

The size of the torus (our universe) grows as $t^{1/2}$, whereas the mean free path of a relativistic particle grows much faster, as $t$. Therefore until the beginning of inflation the universe remains smaller than the size of the horizon $t$. 
If the universe initially had a Planckian size (the smallest possible size), then within the cosmological time $t \gg 1$ (in Planck units) particles run around the torus many times and appear in all parts of the universe with equal probability, which makes the universe homogeneous and keeps it homogeneous until the beginning of inflation.
Creation of a closed inflationary universe, and of an infinite flat or open universe is exponentially less probable than creation of a compact topologically nontrivial flat or open universe.
This does not necessarily mean that our universe looks like a torus, and that one should look for circles in the sky. Inflation in string theory is always eternal, due to large number of metastable dS vacua (string theory landscape).

The new-born universe typically looks like a bagel, but the grown-up universe looks like an eternally growing fractal.
Eternal inflation is a general property of all landscape-based models: The fields eternally jump from one minimum to another, and the universe continues to expand exponentially.

After a very long stage of cosmological evolution, the probability that the energy density at a given point is equal to $V$ becomes given by the following “thermodynamic” expression:

$$ P \sim \exp S(\phi) \sim \exp \left( \frac{24\pi^2}{V(\phi)} \right) $$

Here $S$ is the Gibbons-Hawking entropy for dS space.
Let $10^{500}$ flowers blossom

$\Lambda > 0$

$\Lambda = 0$

$\Lambda < 0$
However, at some point the fields must stop jumping, as in old inflation, and start rolling, as in new or chaotic inflation: the last stage of inflation must be of the slow-roll type. Otherwise we would live in an empty open universe with $\Omega << 1$.

How can we create initial conditions for a slow-roll inflation after the tunneling?
Initial Conditions for D3/D7 Inflation

In D3/D7 scenario flatness of the inflaton direction does not depend on fluxes.

The field drifts in the upper valley due to quantum fluctuations and then tunneling occurs due to change of fluxes inside a bubble.

Eternal inflation in a valley with different fluxes:

\[ H \ggg m \]

Slow roll inflation:

\[ H \gg m \]
The resulting scenario:

1) The universe eternally jumps from one dS vacuum to another due to formation of bubbles. Each bubble contains a new dS vacuum. The bubbles contain no particles unless this process ends by a stage of a slow-roll inflation. Here is how:

2) At some stage the universe appears in dS state with a large potential but with a flat inflaton direction, as in D3/D7 model. Quantum fluctuations during eternal inflation in this state push the inflaton field S in all directions along the inflaton valley.

3) Eventually this state decays, and bubbles are produced. Each of these bubbles may contain any possible value of the inflaton field S, prepared by the previous stage. A slow-roll inflation begins and makes the universe flat. It produces particles, galaxies, and the participants of this conference:)