フェルミオニックな開弦の境界状態の解析

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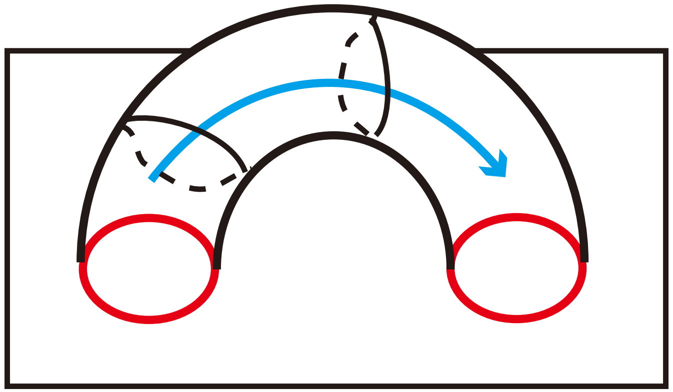
Introduction

Boundary State (closed string):

Introduction of boundaries in the closed string worldsheet representing D-branes

Boundary states describe the absorption and emission of closed strings



Boundary State in the open string sector ??

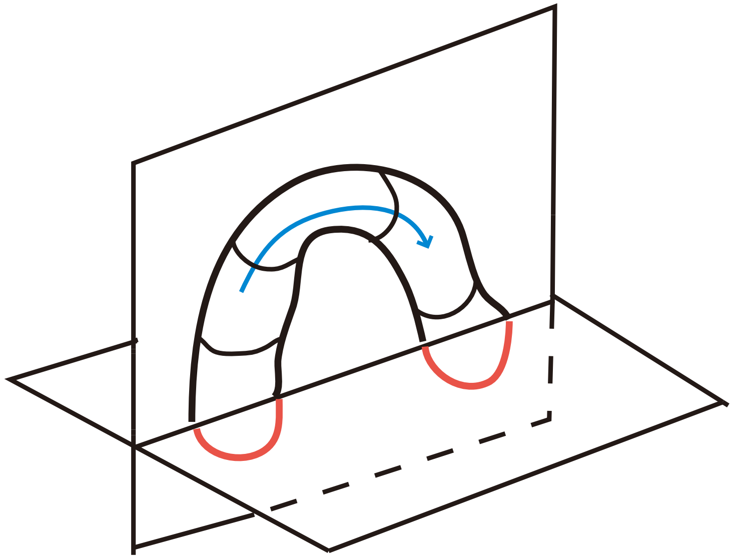
Absorption and emission of open strings

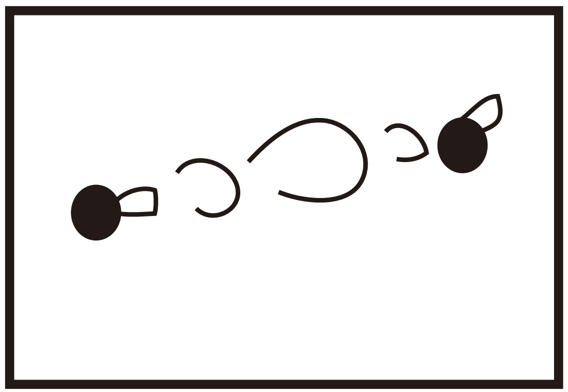
Such situation is realizable
in the configuration of multiple D-brane.

In the bosonic string case, such state is explicitly constructed.

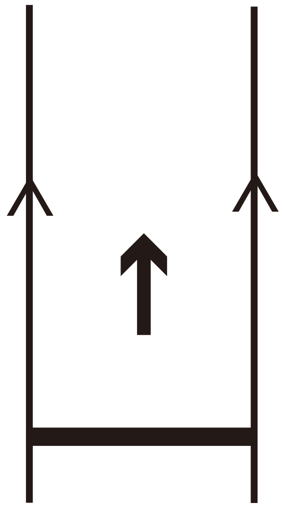
We call such a state the “Open Boundary State (OBS) ”.

[HI, Matsuo], [Imamura, HI, Matsuo]



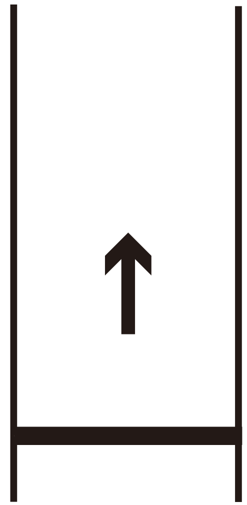


Worldsheet



closed string emission
from D-brane

Boundary does not have
deficit angle



open string emission
from D-brane

Boundary have
corner of deficit angle

two corners for one OBS

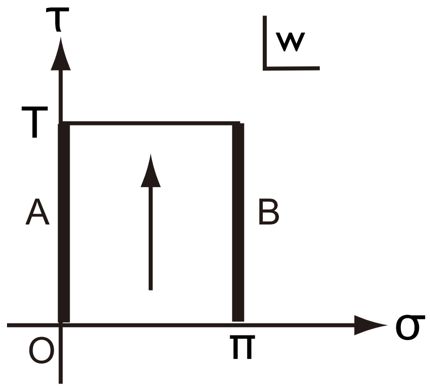
Three boundary conditions for one OBS

Explicit boundary conditions of OBS

Bosonic string case:

$$\partial_{\mathrm{n}} X^{\mu} = 0 \quad \text{Neumann b.c.}$$
$$\partial_{\mathrm{t}} X^{\mu} = 0 \quad \text{Dirichlet b.c.}$$

@ boundaries



applying these conditions, $\partial X(\sigma,\tau) \sim \sum_n \alpha_n e^{in(\sigma+i\tau)}$

b.c. for endpoints $\alpha_n + \epsilon_l \tilde{\alpha}_n = 0$

b.c. for OBS $\alpha_n - \epsilon_b \tilde{\alpha}_{-n} = 0$

$$(\alpha_n + \epsilon_l \epsilon_b \alpha_{-n})|B^o, \epsilon_l \epsilon_b\rangle = 0$$

$$|B^o\rangle \propto \exp\left(-\epsilon_l \epsilon_b \sum_{n>0} \frac{\alpha_{-n}^2}{2n}\right)|0\rangle$$

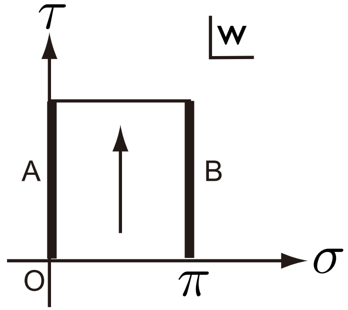
ϵ : sign for b.c.

$\epsilon = +1$ Neumann

$\epsilon = -1$ Dirichlet

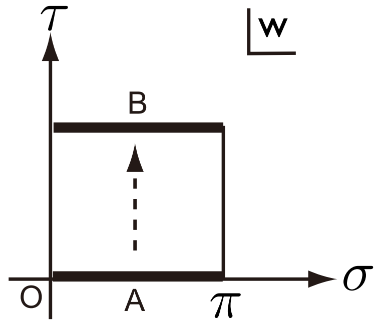
Fermionic string case:

endpoints A



$\frac{\pi}{2}$ rotation

OBS A



$$\psi(\sigma=0,\tau) = x\tilde{\psi}(\sigma=0,\tau) \qquad i^{\frac{1}{2}}\psi(\sigma,\tau=0) = x_b(-i)^{\frac{1}{2}}\tilde{\psi}(\sigma,\tau=0)$$

Combining $\quad i^{\frac{1}{2}}\psi(\sigma,\tau=0) = x_b(-i)^{\frac{1}{2}}\tilde{\psi}(\sigma,\tau=0) \qquad 0<\sigma<\pi$

and doubling trick $\quad \psi(\sigma,\tau) \equiv x_l \cdot \tilde{\psi}(-\sigma,\tau), \quad -\pi<\sigma<0$

we obtain $\quad \psi(\sigma,0) = -i\,x_b x_l \cdot \psi(-\sigma,0), \quad 0<\sigma<\pi$

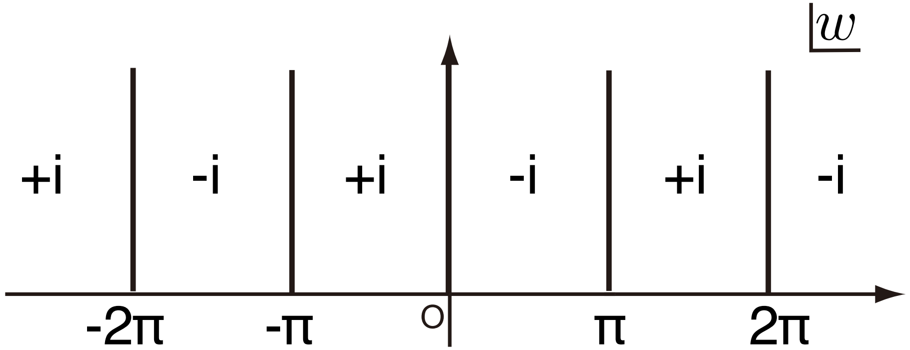
Now, ψ is defined in $-\pi<\sigma<\pi$

Boundary condition in $-\pi<\sigma<0$ is

$$\psi(\sigma,0) = i\,x_b x_l \cdot \psi(-\sigma,0), \quad -\pi<\sigma<0$$

$$\psi(\sigma,0) = -i\, x_b x_l \cdot \psi(-\sigma,0), \quad 0<\sigma<\pi$$
$$\psi(\sigma,0) = i\, x_b x_l \cdot \psi(-\sigma,0), \quad -\pi<\sigma<0$$

ψ has periodicity condition for $\sigma \sim \sigma + 2\pi$



$$[\psi(\sigma,0) + i\,\mathrm{sign}(\sin\sigma)\, x_b x_l \cdot \psi(-\sigma,0)]|B^o\rangle = 0$$

1. If conformal weight is integer, this sign function does not appear.
2. This method can be applicable to primary fields of any conformal weight.
3. the identity state $[\psi(\sigma,0) + is(\sigma)\psi(\pi-\sigma,0)]|I\rangle = 0$

$$s(\sigma) = \mathrm{sign}(\sin\sigma)$$

Oscillator representation

$$[\psi(\sigma,0) + i\,\mathrm{sign}(\sin\sigma)\eta\cdot\psi(-\sigma,0)]|B^o\rangle = 0$$

Mode expansion $\quad \psi(\sigma,0) \sim \sum_r \psi_r e^{ir\sigma}$

$$\longrightarrow \quad (\psi_r + \eta\sum_s N_{rs}\psi_{-s})|B^o\rangle = 0$$

Fourier transformation of N

$$N_{rs} = -\int \frac{d\sigma}{2\pi} e^{-i(r+s)\sigma} i\,\mathrm{sign}(\sin\sigma)$$

$$N^2 = 1, \quad N^T = N$$

$$(\psi_r + \eta \sum_s N_{rs}\psi_{-s})|B^o\rangle = 0$$

Decomposition in terms of
annihilation (mode > 0) and creation (mode < 0)

$$N = \begin{pmatrix} N_{-r,-s} & N_{-r,s} \\ N_{r,-s} & N_{r,s} \end{pmatrix} = \begin{pmatrix} n_{rs} & \tilde{n}_{rs} \\ -\tilde{n}_{rs} & -n_{rs} \end{pmatrix}, \quad \psi = \begin{pmatrix} \psi_{-r} \\ \psi_r \end{pmatrix} = \begin{pmatrix} \psi_r^\dagger \\ \psi_r \end{pmatrix}$$

$$(\psi - K \cdot \psi^\dagger)|B^o\rangle = 0$$

$$K = \eta\tilde{n}(1-\eta n)^{-1} = -\eta\tilde{n}^{-1}(1+\eta n) = -K^T$$

$$n^2 - \tilde{n}^2 = 1, \; n\tilde{n} = \tilde{n}n$$

$$\longrightarrow \quad |B^o\rangle \propto \exp\left(\sum_{r>0} \frac{\psi^\dagger K \psi^\dagger}{2}\right)|0\rangle$$

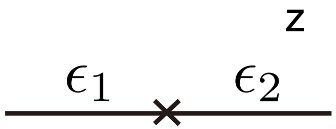
In the case where the zero modes exist,
ambiguity in decomposition arises.

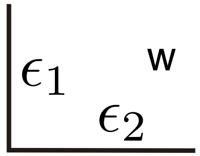
Corner anomaly and BRST invariance

<u>Insertions at the corner</u>

Bosonic string

$$z = w^2$$

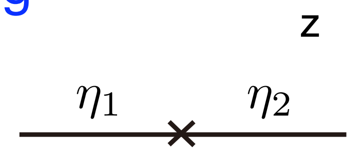


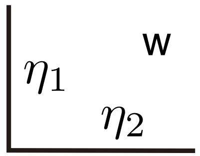


for ∂X

$\epsilon_1 = \epsilon_2$: no operators $\qquad$ $\epsilon_1 = -\epsilon_2$: twist operator σ

Fermionic string



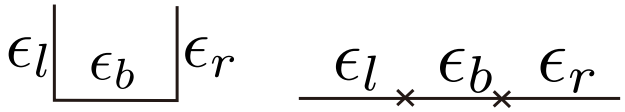


for ψ

$\eta_1 = \eta_2$: no operators $\qquad$ $\eta_1 = -\eta_2$: spin operator S

BRST invariance of OBS in 26-dim bosonic string

26 corners or pairs of boundary conditions



1. Explicit evaluation

$$Q_B|B^o\rangle \propto \sum_n n c_{-n} \left[\left(\sum \epsilon_l \epsilon_b + 6 \right) + (-1)^n \left(\sum \epsilon_r \epsilon_b + 6 \right) \right] |B^o\rangle$$

2. BRST inv. of OBS ⟷ Physical condition of vertex operator

Physical condition = (conformal weight of matter vertex =1)

From both methods #(NN,DD)=10 #(ND,DN)=16 [σ]=1/16

Vertex operator has 0 momentum.

BRST invariance gives static configurations of intersecting D-branes.

BRST invariance in 10-dim superstring

Physical condition of vertex operator

momentum of insertion = 0

⟶ static configuration of intersecting branes (no tachyon)

By using physical condition of vertex operator,

well-known fact $\#\mathrm{ND} = 4$ is realized.

This constraint should be derived from the explicit evaluation of BRST invariance of OBS in the superstring theory

In order to perform this,

more careful treatment of ∞-dim. matrix K

correct form of boundary condition

Corner weight and OBS for E-M tensor

Behavior of E-M tensor near the corner

$$z = w^2$$

Due to conformal anomaly, $T(z) \sim \dfrac{\lambda}{z^2}$ $\quad T(w) \sim \left(4\lambda - \dfrac{c}{8}\right)\dfrac{1}{w^2}$

Thus, if no insertions, there exists conformal weight of the corner.

$$\lambda_{\mathrm{corner}} = 2\lambda - \frac{c}{16}$$

Naive guess for the OBS of E-M tensor $\quad [T(\sigma,0) - T(-\sigma,0)]|B^o\rangle = 0$

From the behavior of E-M tensor near corners, the correct form is

$$[T(\sigma,0) - T(-\sigma,0) - 4\pi i(\lambda^l_{\mathrm{corner}}\delta'(\sigma) + \lambda^r_{\mathrm{corner}}\delta'(\sigma-\pi))]|B^o\rangle = 0$$

using $\quad \mathrm{disc.}\dfrac{1}{w^2} = 2\pi i\delta'(\sigma)$

In the bosonic case, this is derived by explicit calculation using OBS.

Lesson: Singularity near corners can change the boundary.

Problems and Discussion

<u>1. Change of boundary condition due to the insertions at corners</u>

for example, spin operator at corners

$$z = w^2$$

$$\psi(z) \sim z^{-1/2} \qquad \psi(w) \sim w^{-1/2}$$

possibility : singularities at corners σ=0,π in boundary conditions

$$[\psi(\sigma, 0) + is(\sigma)\eta\,\psi(-\sigma, 0)]|B^o\rangle \neq 0$$

Exact form of boundary conditions are required.

<u>2. Analytic treatment of coefficient matrix K</u>

Generating function for K can be obtained
by using some conformal mappings
Analogy to the identity state case

<u>3. OBS ⟶ solitonic operator in OSFT on D-brane ??</u>