Walls in 3-d supersymmetric Kähler nonlinear sigma models on SO(2N)/U(N) and Sp(N)/U(N)

Masato Arai
Czech Technical University in Prague

Based on collaboration with Sunyoung Shin

Introduction – progress on domain wall

- Soliton - Playing an important role in physics
  - Domain wall solution ⇨ Brane world scenario

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- Soliton - Playing an important role in physics
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- Massive hyper-Kähler nonlinear sigma model on $T^*G_{N,M}$
  - D=5 N=1 SUSY U(N) gauge theory coupled to M massive flavors including the Fayet-Iliopoulos term with infinite gauge coupling limits
  - ${}_N C_M$ number of discrete vacua

MA, Nitta, Sakai, 2003

  - Domain wall solutions interpolating them are derived based on the moduli matrix approach

Isozumi, Nitta, Ohashi, Sakai, 2004

    - The moduli space of the wall: Grassmannian $G_{N,M} = \dfrac{U(N)}{U(N-M)\times U(M)}$

Introduction

- $G_{N,M}$: one of compact Hermitian symmetric spaces (HSS)

$$G_{N,M} = \frac{U(N)}{U(N-M)\times U(M)} \quad Q^N = \frac{SO(N+2)}{SO(N)\times U(1)} \quad \frac{SO(2N)}{U(N)} \quad \frac{Sp(N)}{U(N)}$$

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- Question: How about a NLSM on $T^*\mathcal{M}$ ($\mathcal{M}$ is the HSS except $G_{N,M}$)?
  - The moduli matrix approach can be used in SUSY gauge theory.
  - Need to describe a NLSM on $T^*\mathcal{M}$ as a SUSY gauge theory (with infinite gauge coupling limit)
  - Difficult to construct a SUSY gauge theory corresponding to a NLSM on $T^*\mathcal{M}$ other than $T^*G_{N,M}$, for example, T^*Q^N .

(cf. MA, Nitta, 2006, MA, Lindström, Kuzenko, 2007)

Introduction

- Simplifying setup
  - Observation: Cotangent part is irrelevant for vacua and domain walls in $T^*G_{N,M}$

    Isozumi, Nitta, Ohashi, Sakai, 2004

$$\mathcal{L} = \int d^4\theta K(\Phi, \Phi^\dagger, \overbrace{\Psi, \Psi^\dagger}) + \text{mass term}$$

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  - The cotangent part can be dropped for investigation of vacua and walls.

  - The result is respected as one of the massive Kähler NLSM on $G_{N,M}$.

$$\mathcal{L} = \int d^4\theta K(\underbrace{\Phi, \Phi^\dagger}_{\text{Base manifold part}}) + \text{mass term}$$

  - The situation would be the same for other NLSMs on $T^*\mathcal{M}$ ($\mathcal{M}$ is the HSS).

Purpose of our work

- Domain walls in massive kähler NLSM on the complex quadric surface $Q^N = \dfrac{SO(N+2)}{SO(N) \times U(1)}$ in 3-dimensional space-time.

MA, Lee, Shin, 2009

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- We consider domain walls for massive Kähler NLSM on

$$\frac{SO(2N)}{U(N)} \qquad \frac{Sp(N)}{U(N)}$$

formulated by gauge theory.

Setup

- Starting with the massless NLSM on $\frac{SO(2N)}{U(N)}, \frac{Sp(N)}{U(N)}$ in 4 dimensions.

  Higashijima, Nitta, 1999

  - 4D N=1 U(N) gauge theory coupled to 2N flavors including FI term (with $g \to \infty$)

$$\mathcal{L} = \int d^4\theta (\phi_a{}^i \bar{\phi}_i{}^b (e^V)_b{}^a - r^2 V_a{}^a) + \left(\int d^2\theta \, \phi_0^{ab} (\phi_b{}^i J_{ij} \phi^{\mathrm{T}j}{}_a) + \text{c.c.} \right)$$

$$(i = 1, \cdots, 2N, \quad a = 1, \cdots, N)$$

$$J = \mathbf{1} \otimes \begin{pmatrix} 0 & 1 \\ \epsilon & 0 \end{pmatrix}, \quad \epsilon = \begin{cases} +1 & SO(2N)/U(N) \\ -1 & Sp(N)/U(N) \end{cases}$$

  - ϕ : vector rep. of SO(2N) or Sp(N)
  - $\phi_0^T = \epsilon\phi_0$: symmetric (SO(2N))/anti-symmetric (Sp(N)) rep.
  - No kinetic term for gauge part and ϕ_0 , whose eqs. of motion give constraints

$$\phi_a{}^i \bar{\phi}_i{}^b - \delta_a{}^b = 0 \quad \text{(D-term)} \qquad \phi_a{}^i J_{ij} \phi^{\mathrm{T}j}{}_b = 0 \quad \text{(F-term)}$$

Setup

- Dimensional reduction
  - Getting a non-trivial scalar potential

$$\mathcal{L}_{\text{bos}} = -D_\mu \phi^i D^\mu \bar{\phi}^i - 4|(\phi_0)^{ab}\phi_b{}^i|^2 + \cdots$$

Cartan subalgebra of $SO(2N), Sp(N)$

$$\frac{\partial \phi_a{}^i}{\partial x^3} = i\phi_a{}^j M_j{}^i \qquad M_j{}^i = \text{diag}(m_1, m_2, \cdots, m_N) \otimes \sigma_3$$

$$\mathcal{L}_{\text{bos}} = -D_m \phi^i D^m \bar{\phi}^i \underbrace{- |i\phi_a{}^j M_j{}^i - i\Sigma_a{}^b \phi_b{}^i|^2 - 4|(\phi_0)^{ab}\phi_b{}^i|^2}_{\Rightarrow\, -V} + \cdots$$

$(\Sigma = v_3)$

Giving rise to discrete vacua

Vacua

- Vacuum condition

$$0 = V = |i\phi_a{}^j M_j{}^i - i\Sigma_a{}^b \phi_b{}^i|^2 + 4|(\phi_0)^{ab}\phi_b{}^i|^2$$

$$\Rightarrow \quad \phi_0 = 0, \quad \phi_a^i \neq 0 \qquad (m_k - \Sigma_a)\phi_a{}^{2k-1} = 0,\ (m_k + \Sigma_a)\phi_a{}^{2k} = 0$$

$(\Sigma_a{}^b \to \mathrm{diag}(\Sigma_1, \Sigma_2, \cdots \Sigma_N)$ by gauge trans.)

with constraints $\phi_a{}^i \bar{\phi}_i{}^b - \delta_a{}^b = 0, \quad \phi_a{}^i J_{ij} \phi^{\mathrm{T}j}{}_b = 0$

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- Ex. SO(4)/U(2) & Sp(2)/U(2) case

$$\Sigma_{\langle 1\rangle} = (m_1, m_2) \qquad \Sigma_{\langle 2\rangle} = (m_1, -m_2)$$

$$\phi_{\langle 1\rangle} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad \phi_{\langle 2\rangle} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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BPS equation

- Bogomol'nyi completion
  - Supposing a non-trivial configuration along $x = x_1$ direction, $v_0 = v_2 = 0$

$$E = \int dx \left(|D_1 \phi_a{}^i \mp (\phi^i_a M_j{}^i - \Sigma_c{}^b \phi_b{}^i)|^2 + 4|(\phi_0)^{ab} \phi_b{}^i|^2 \right) \pm T \geq \pm T$$

$$T = \int dx \partial_1 (\phi_a{}^i M_i{}^j \bar{\phi}_j{}^a)$$

with constraints $\phi_a{}^i \bar{\phi}_i{}^b - \delta_a{}^b = 0, \quad \phi_a{}^i J_{ij} \phi^{\mathrm{T}j}{}_b = 0$

- BPS equation

$$\left. \begin{aligned} &(D_1 \phi)_a{}^i \mp (\phi_a{}^j M_j{}^i - \Sigma_a{}^b \phi_b{}^i) = 0 \\ &(\phi_0)^{ab} \phi_b{}^i = 0 \end{aligned} \right\} \Longrightarrow \phi_0 = 0, \quad \phi \neq 0$$

Solving the BPS equation – moduli matrix approach

- Rewriting the equations

$$(D_1\phi)_a{}^i - (\phi_a{}^j M_j{}^i - \Sigma_a{}^b \phi_b{}^i) = 0 \quad \Longrightarrow \quad \partial_1 f_a{}^i = f_a{}^j M_j{}^i$$

$$\boxed{\phi_a{}^i = (S^{-1})_a{}^b f_b{}^i, \quad \Sigma - i v_1 = S^{-1}\partial_1 S} \quad S \in \mathbf{C}$$

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- Solution

$$\phi_a{}^i = (S^{-1})_a{}^b H_{0b}{}^j (e^{Mx})_j{}^i$$

$H_{0a}{}^i$: integration constants called the moduli matrix

- Constraints

$$H_0 e^{2Mx} H_0^\dagger = S S^\dagger \qquad H_0 J H_0^{\mathrm{T}} = 0$$

Solving the BPS equation – moduli matrix approach

- Moduli matrix ${H_0}_a{}^i$ – key issue in solving the equation
  - It includes information of vacua, boundary conditions and positions of walls.
  - The rest of work is to choose them so that they satisfy

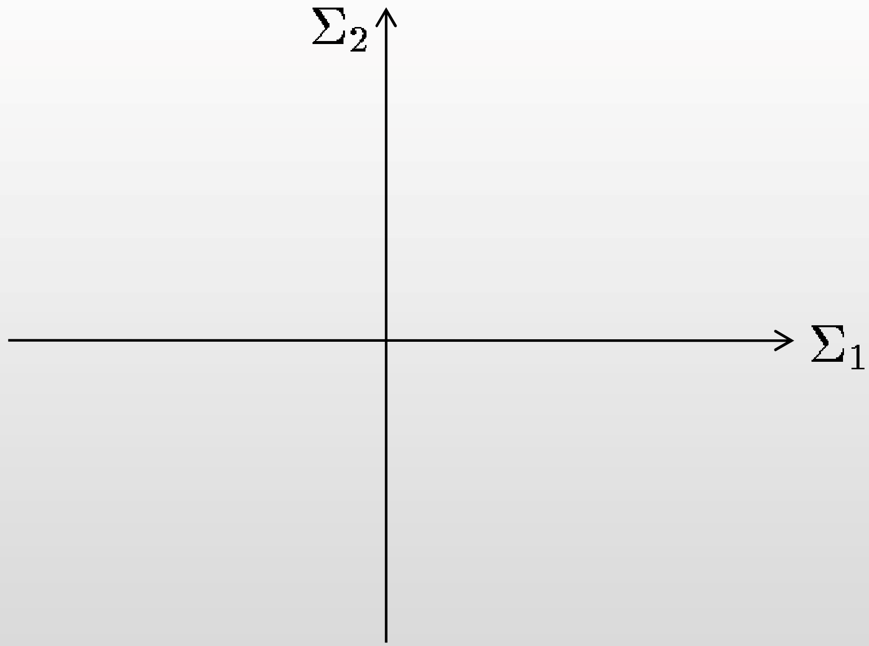
$$H_0 J H_0^{\mathrm{T}} = 0$$

  and appropriate boundary conditions at $x = \pm\infty$.
  - $H_0 e^{2Mx} H_0^\dagger = S S^\dagger$ determines S . ⇒ $\phi_a{}^i = (S^{-1})_a{}^b {H_0}_b{}^j (e^{Mx})_j{}^i$

Solution - N=2 case

- SO(4)/U(2) case
  - 4 discrete vacua

$$(m_k - \Sigma_a)\phi_a{}^{2k-1} = 0,\ \ (m_k + \Sigma_a)\phi_a{}^{2k} = 0$$



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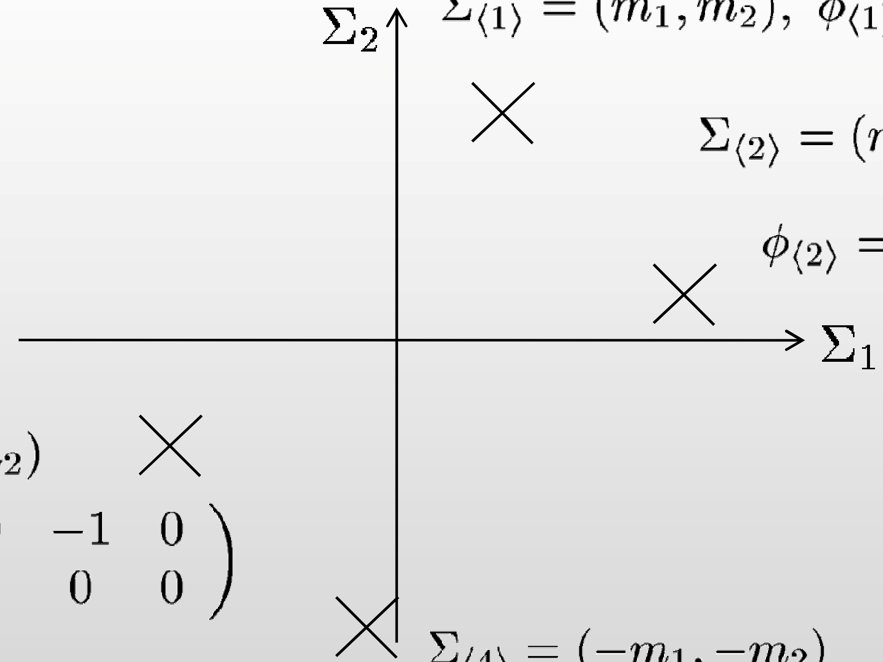

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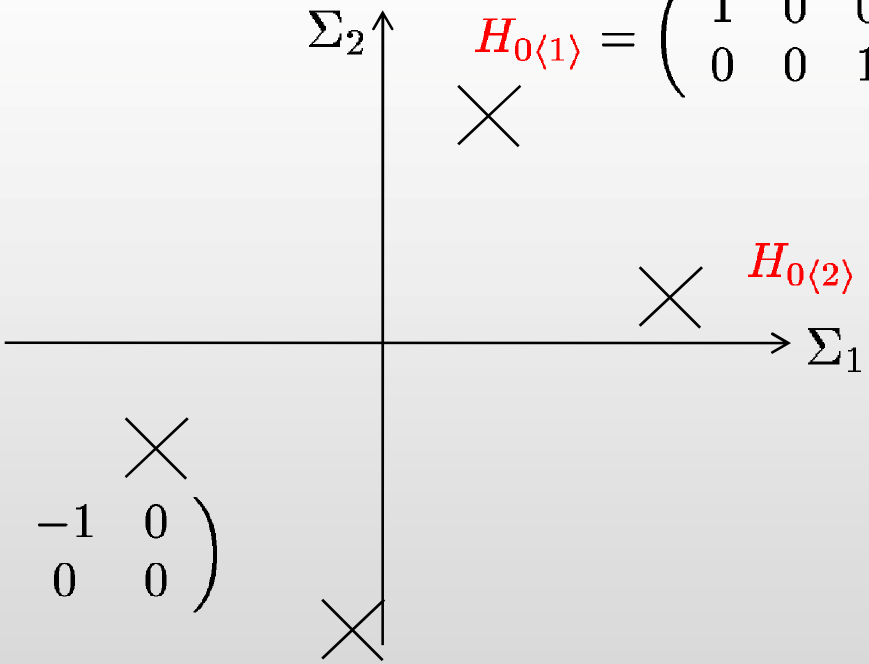

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  - Single walls

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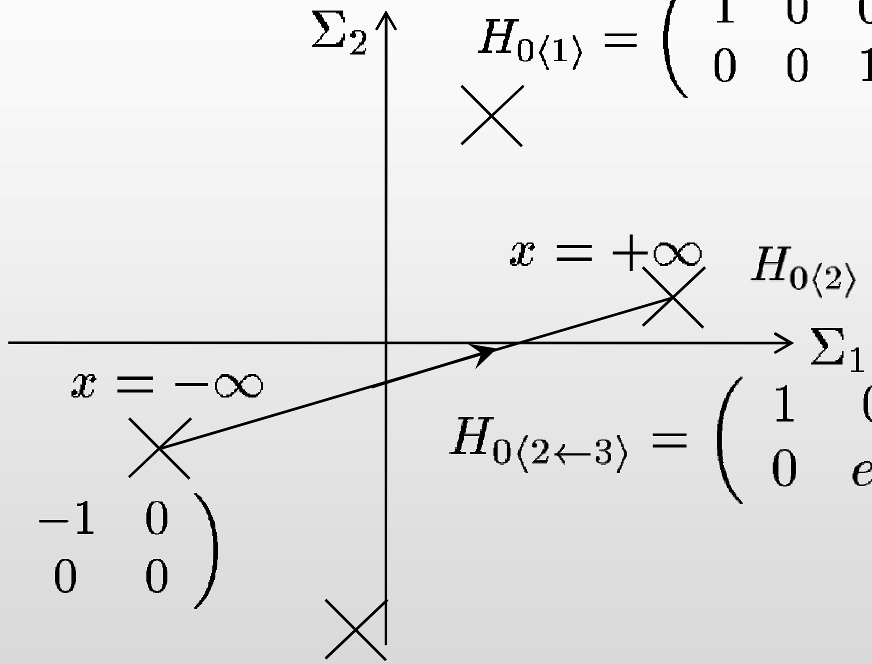

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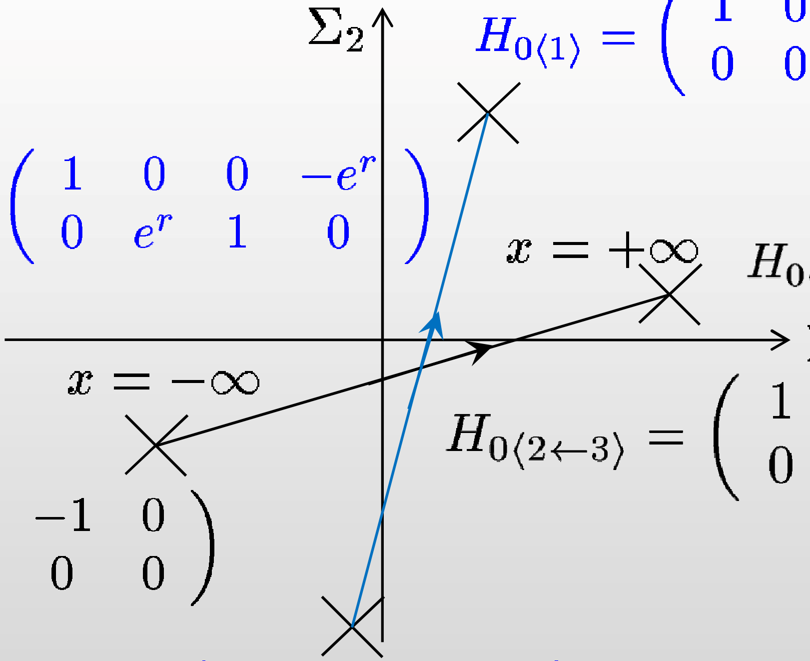

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Solution - N=2 case

- Expected result

$$\frac{SO(4)}{U(2)} \simeq \frac{SU(2)\times SU(2)}{SU(2)\times U(1)} \simeq \frac{SU(2)}{U(1)} \simeq \mathbf{C}P^1$$

  - Massive nonlinear sigma model on $\mathbf{C}P^1$ ➡ 2 discrete vacua and 1 domain wall
  - Our result : 4 discrete vacua and 2 domain walls, double structure
    - $H_0 J H_0^{\mathrm{T}} = 0$ (O(4) constraint) ➡ $(H_0, H_0 P)$ $\quad \det P = -1$

$$H_{0\langle 1\leftarrow 4\rangle} = H_{0\langle 2\leftarrow 3\rangle} P \qquad P = \begin{pmatrix} \mathbf{1}_2 & 0 \\ 0 & \sigma_1 \end{pmatrix}$$

    - Massless limit: $H_{0\langle 1\leftarrow 4\rangle}, H_{0\langle 2\leftarrow 3\rangle}$ (vacuum moduli)

$$\begin{array}{ccc} \langle 1\rangle \bullet & \xrightarrow{\quad H_{0\langle 1\leftarrow 4\rangle} \quad} & \bullet \langle 4\rangle \\ a=0 & & a=\infty \\ \langle 2\rangle \bullet & \xrightarrow{\quad H_{0\langle 2\leftarrow 3\rangle} \quad} & \bullet \langle 3\rangle \end{array} \qquad \phi = \frac{1}{\sqrt{1+|a|^2}} \begin{pmatrix} 1 & 0 & 0 & -a \\ 0 & a & 1 & 0 \end{pmatrix}$$

    - $H_{0\langle 1\leftarrow 4\rangle}, H_{0\langle 2\leftarrow 3\rangle}$ are identified since P is a symmetry of the theory.

Solution - N=2 case

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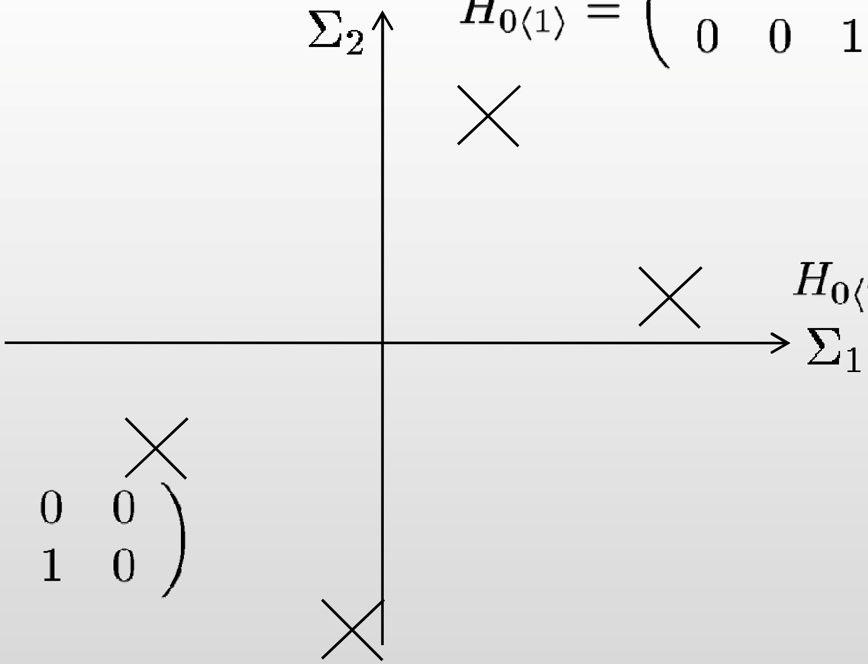
    - $H_{0\langle 1\leftarrow 4\rangle}, H_{0\langle 2\leftarrow 3\rangle}$ are identified since P is a symmetry of the theory.
    - Massive case: P is not a symmetry ⬅ mass term (Cartan subalgebra of SO(2N))

Solution - N=2 case

- Sp(2)/U(2) case
  - 4 discrete vacua

$$H_{0\langle 1\rangle} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



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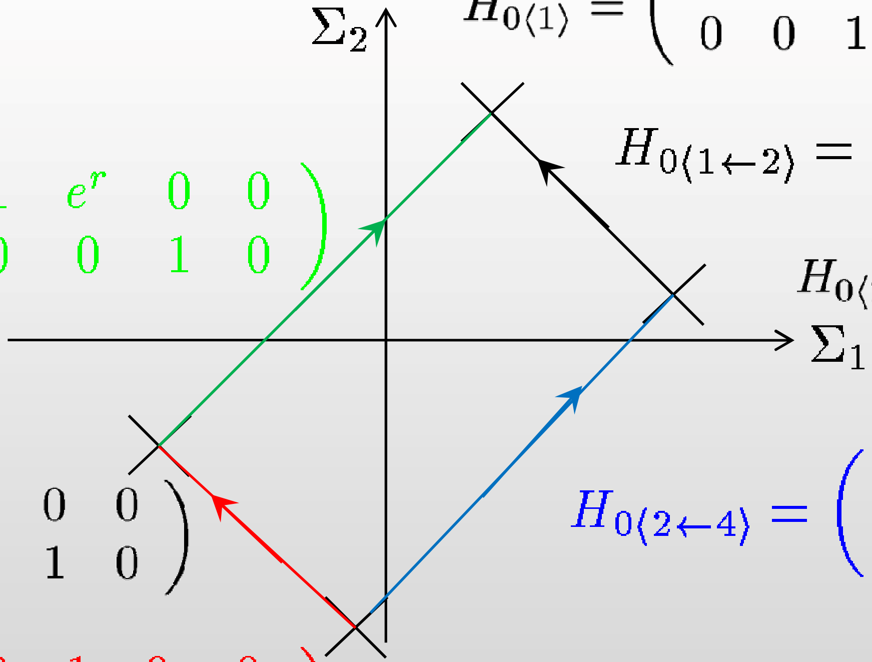
Solution - N=2 case

- Sp(2)/U(2) case
  - Single wall



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  - Double walls

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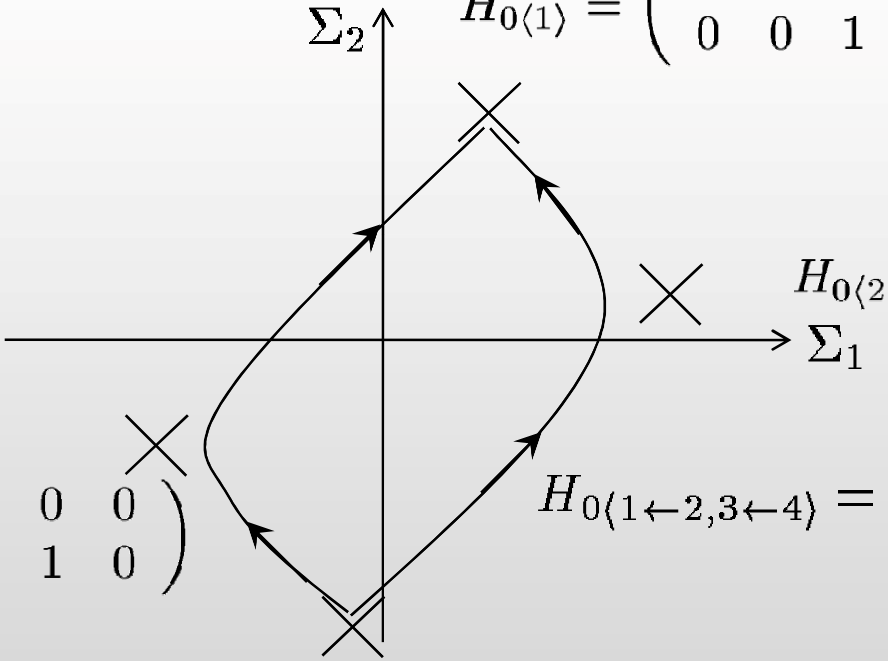

$$H_{0\langle 2\rangle} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$H_{0\langle 1\leftarrow 2,3\leftarrow 4\rangle} = \begin{pmatrix} 1 & e^{r_2} & 0 & 0 \\ 0 & 0 & 1 & e^{r_1} \end{pmatrix}$$

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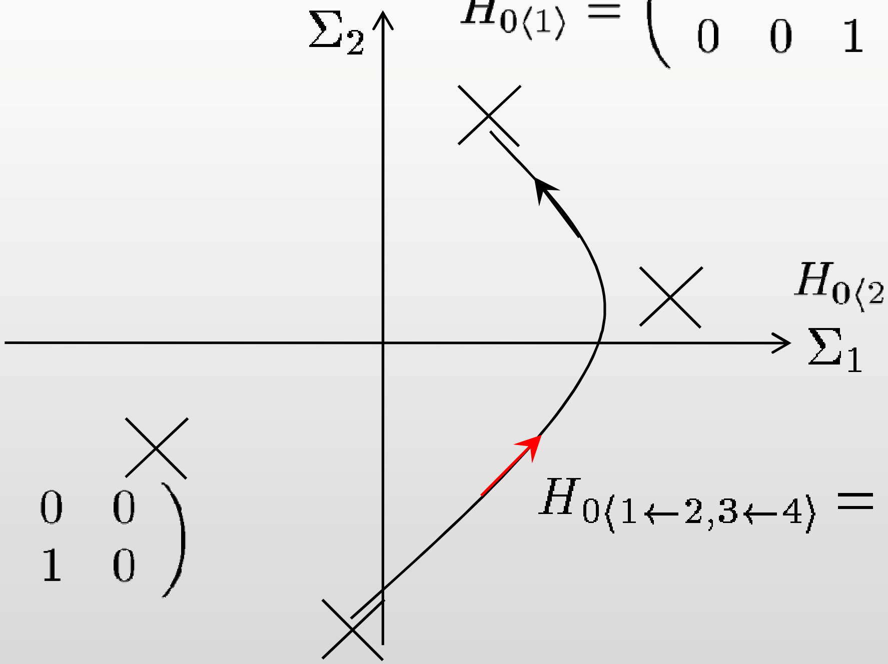
Solution - N=2 case

- Sp(2)/U(2) case
  - Double walls

$$H_{0\langle 1\rangle} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



$$H_{0\langle 2\rangle} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

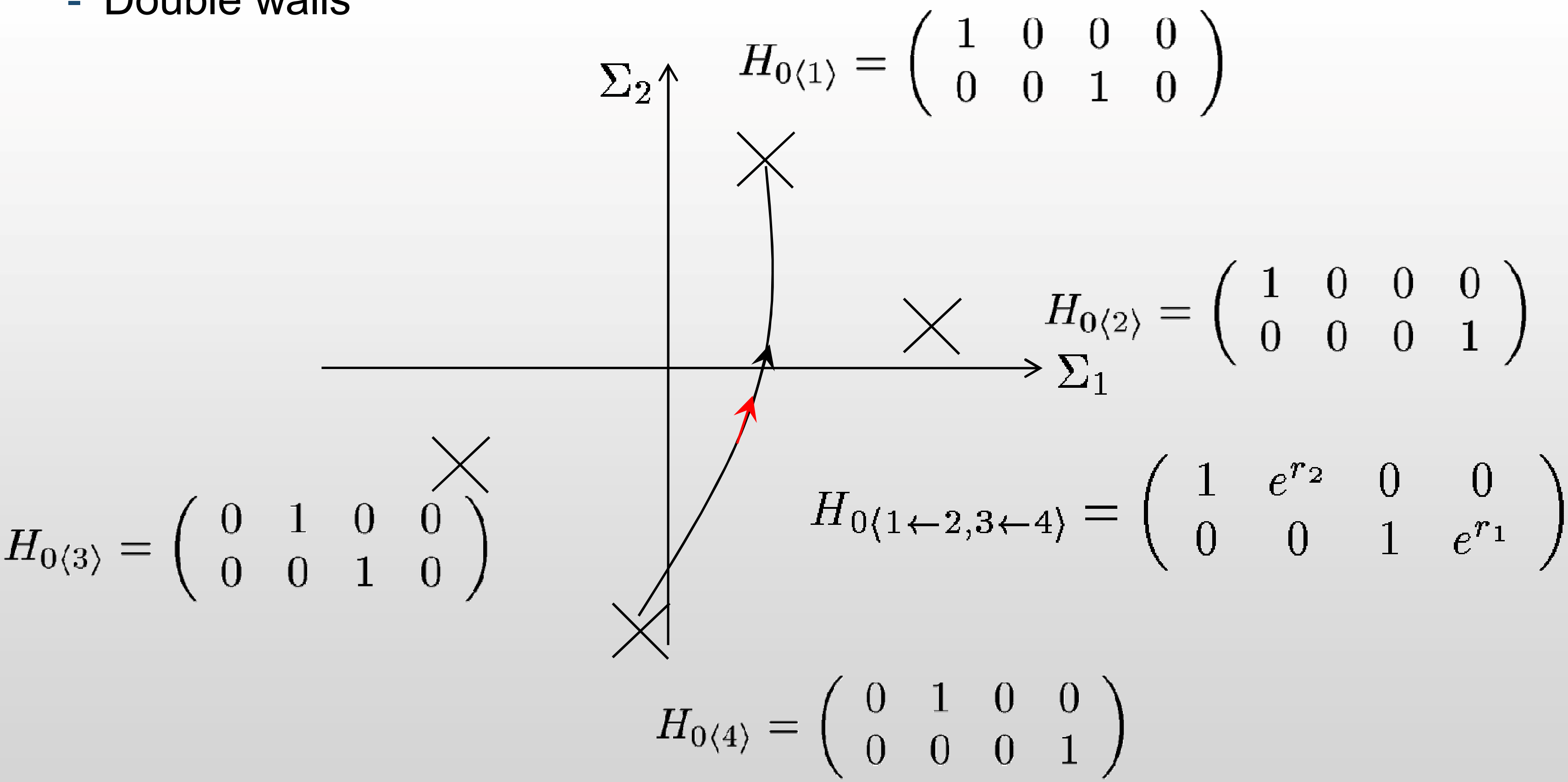
$$H_{0\langle 1\leftarrow 2,3\leftarrow 4\rangle} = \begin{pmatrix} 1 & e^{r_2} & 0 & 0 \\ 0 & 0 & 1 & e^{r_1} \end{pmatrix}$$

$$H_{0\langle 3\rangle} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$H_{0\langle 4\rangle} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Solution - N=2 case

- Sp(2)/U(2) case
  - Double walls



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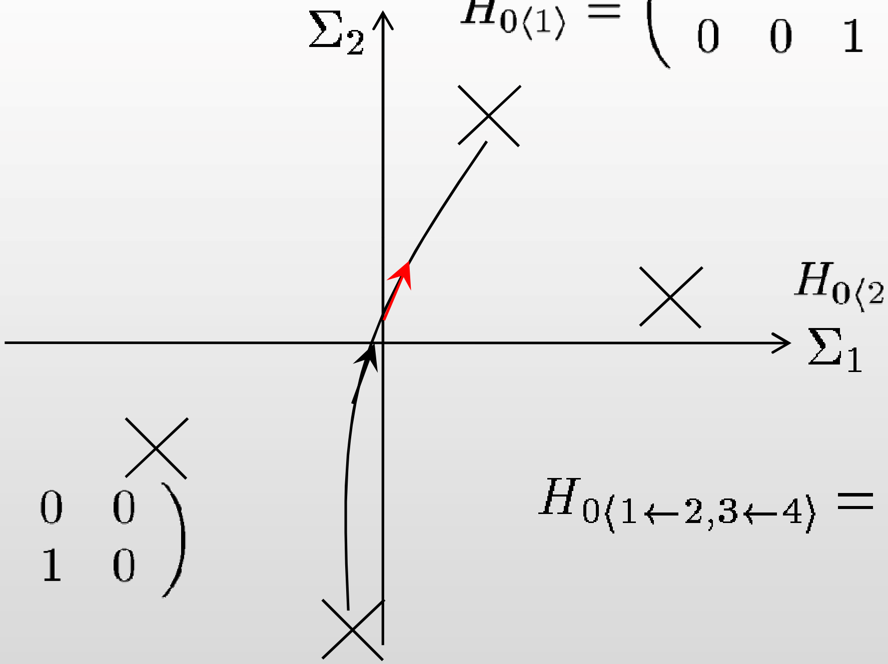

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- Sp(2)/U(2) case
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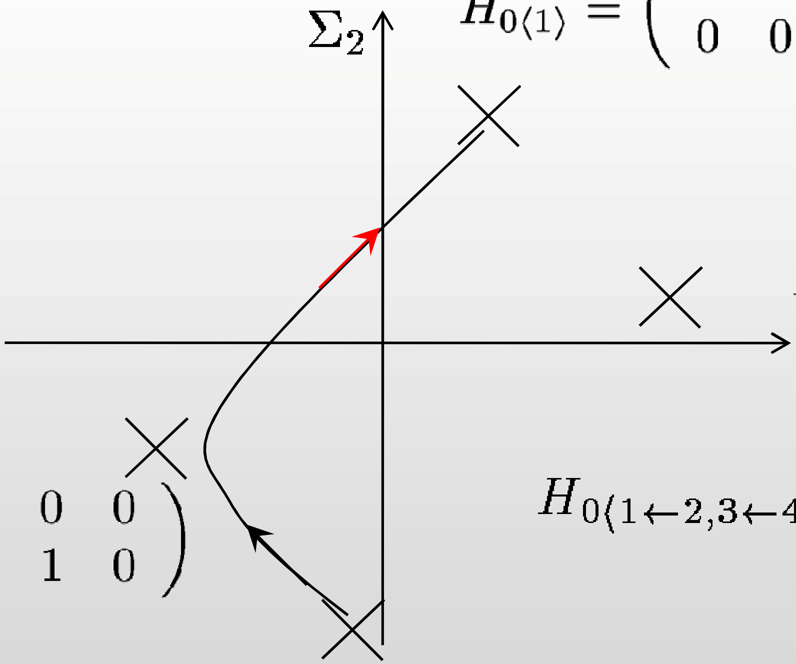

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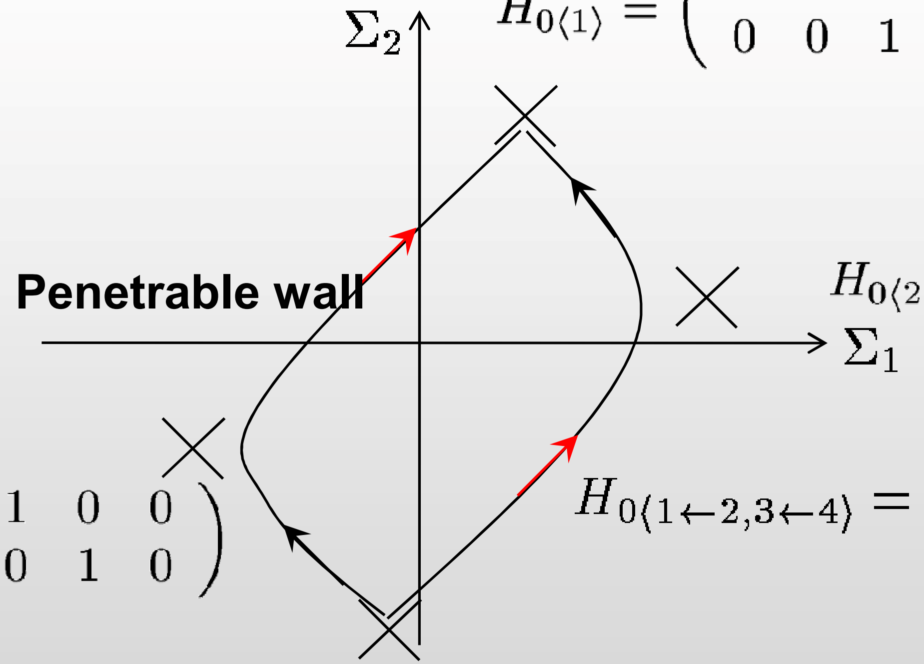

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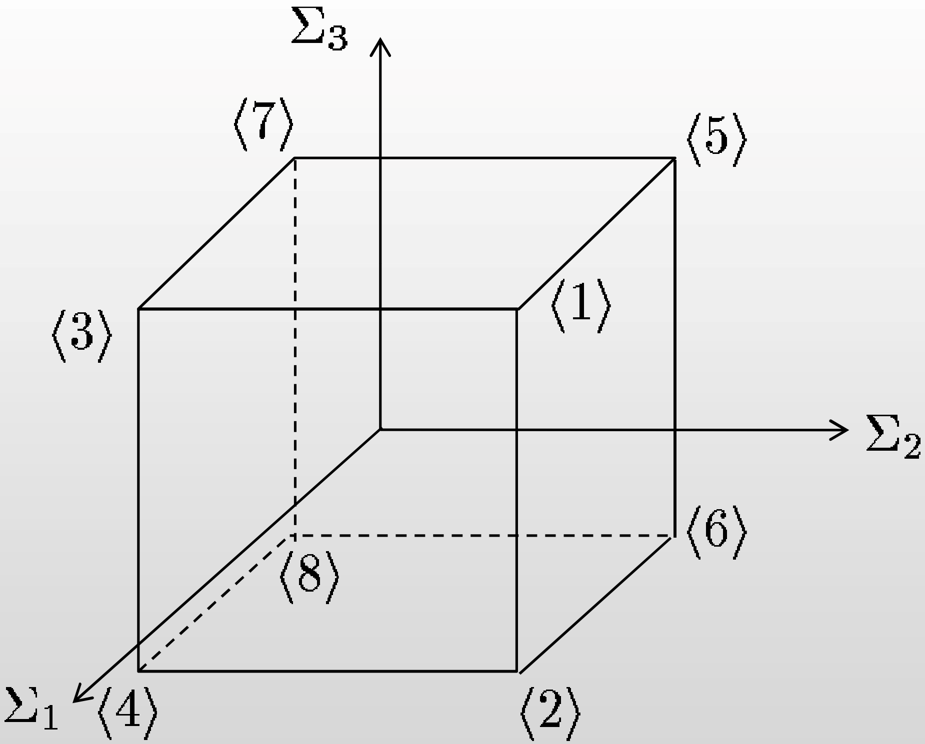
Result is consistent with $Sp(2)/U(2) \simeq Q^3$ **case (our previous work)**

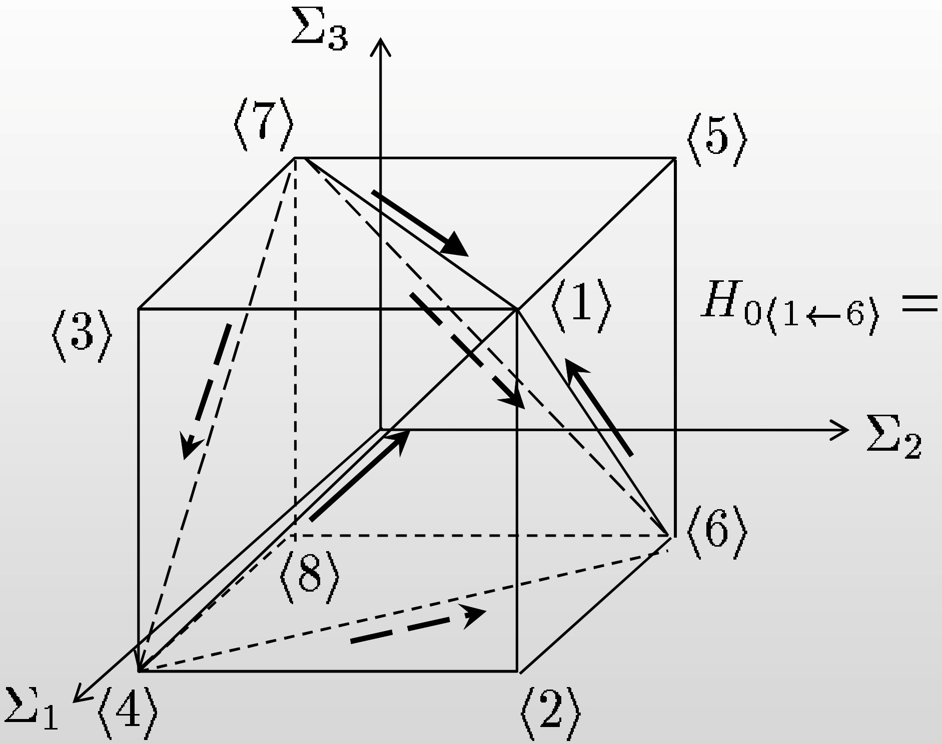
Solution - N=3 case

- SO(6)/U(3) case
  - $SO(6)/U(3) \simeq \mathbf{C}P^3$: Expected 4 discrete vacua ➡ 8 discrete vacua



Solution - N=3 case

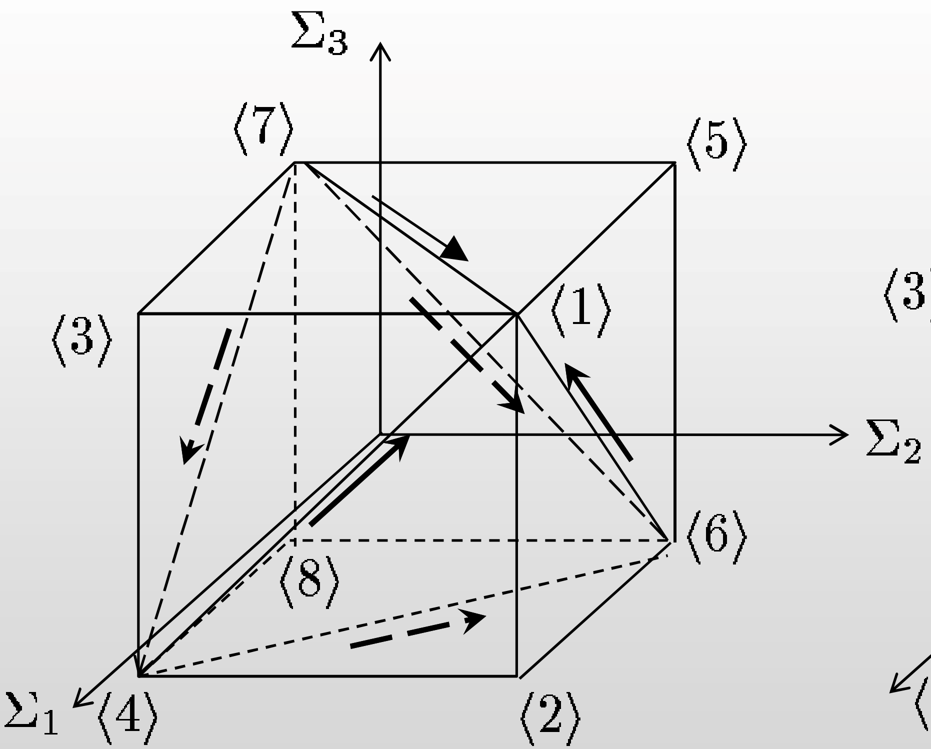
- SO(6)/U(3) case
  - single walls

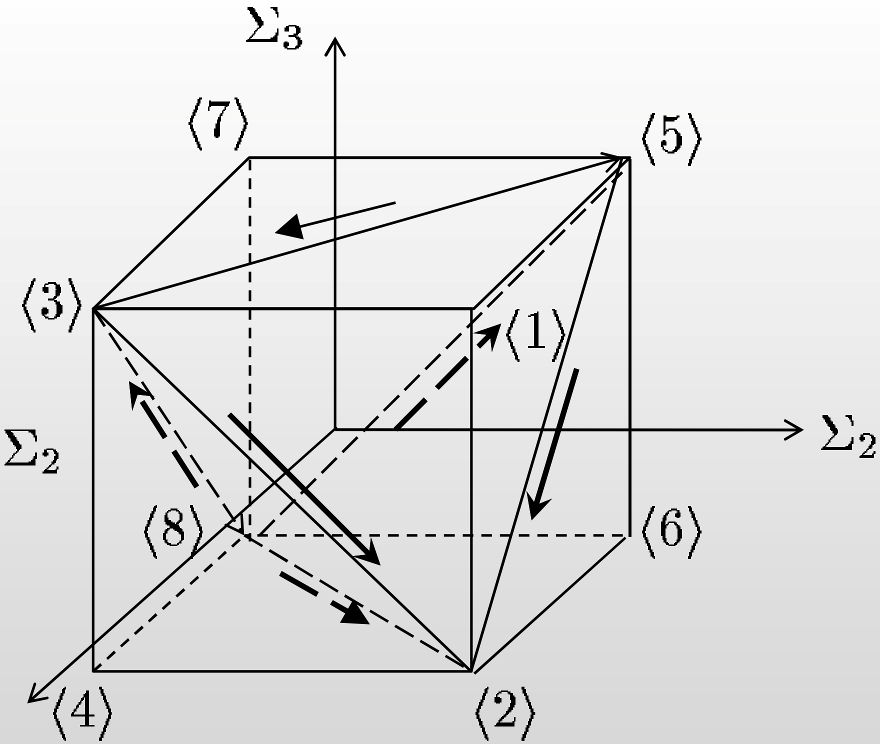


$$H_{0\langle 1 \leftarrow 6 \rangle} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -e^r \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & e^r & 0 & 0 & 1 & 0 \end{pmatrix}$$

Solution - N=3 case

- SO(6)/U(3) case
  - single walls

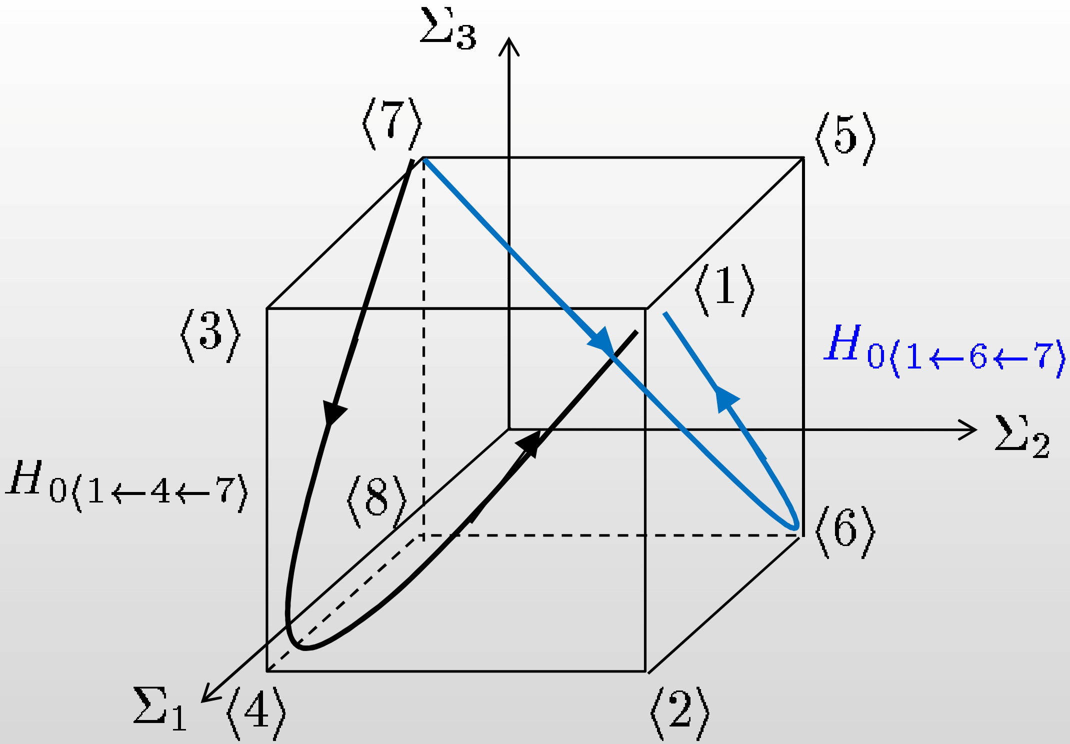




Double structure: $H_0 \leftrightarrow H_0 P, \det P = -1$

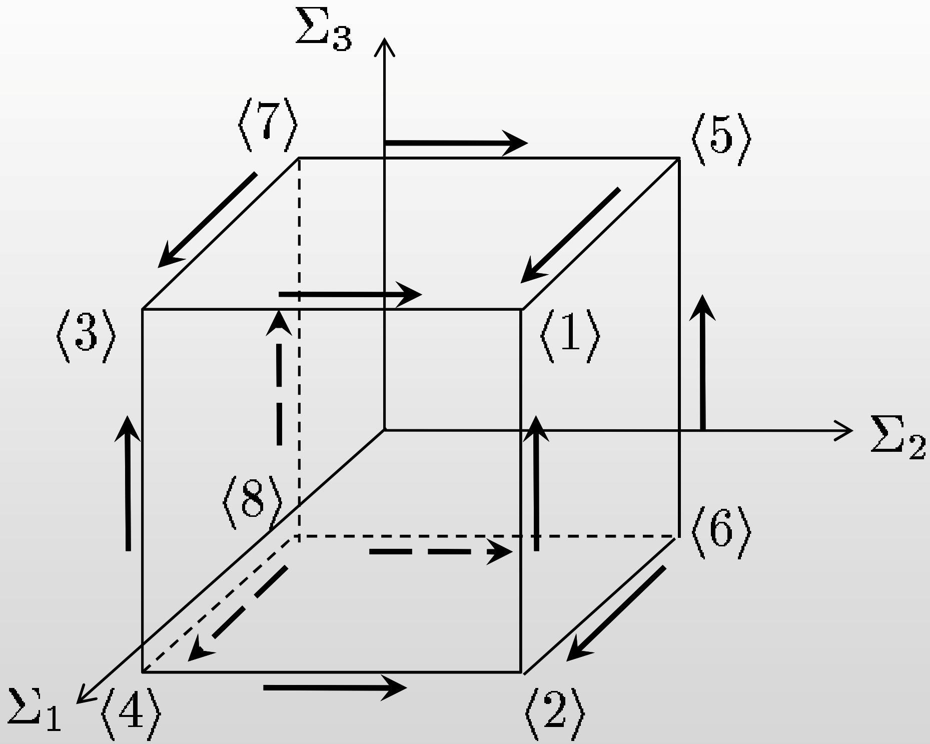
Solution - N=3 case

- SO(6)/U(3) case
  - Double walls



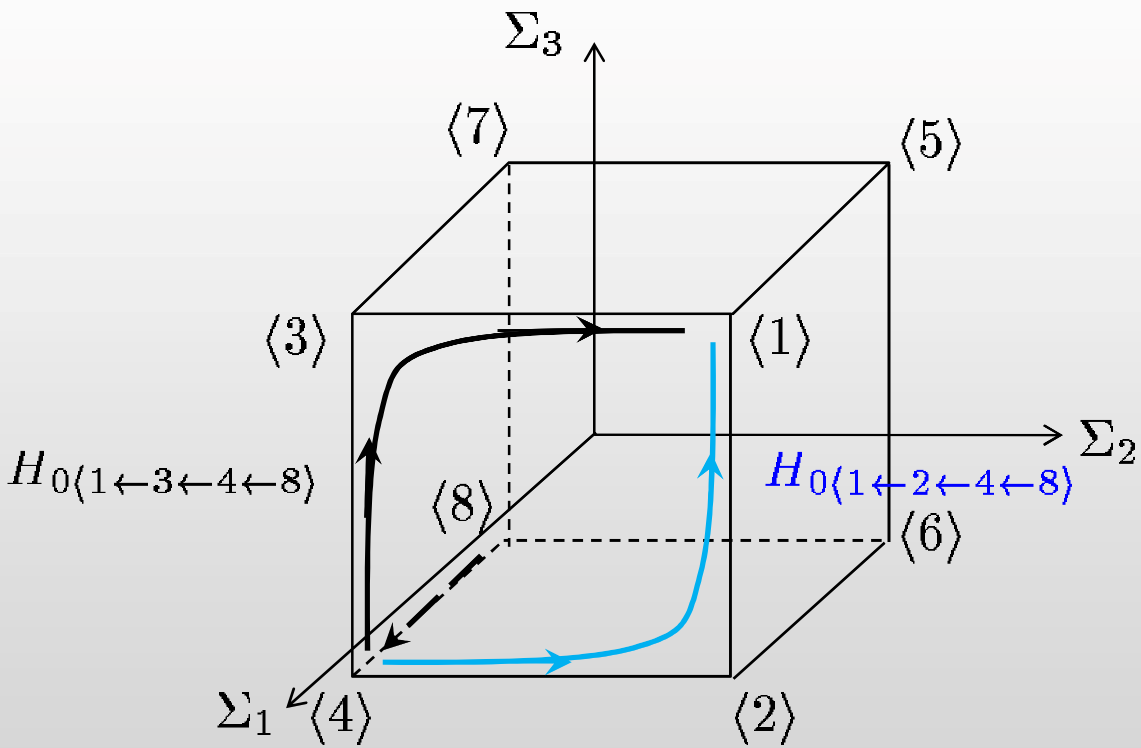
Solution - N=3 case

- Sp(3)/U(3) case
  - 8 discrete vacua, 12 single walls



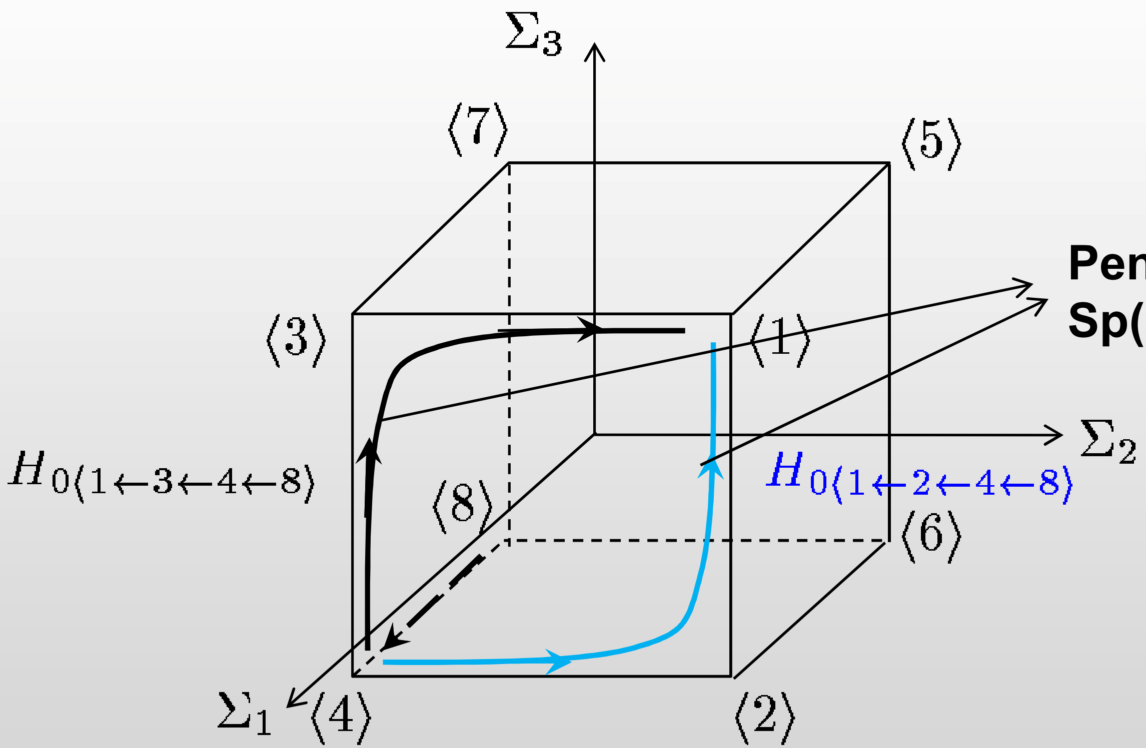
Solution - N=3 case

- Sp(3)/U(3) case
  - Double, triple walls



Solution - N=3 case

- Sp(3)/U(3) case
  - Double, triple walls



Penetrable like Sp(2)/U(2) case

Summary & Discussion

- Investigating domain walls in massive Kähler NLSM on SO(2N)/U(N) and Sp(N)/U(N) in 3-dimensional space-time.
- Showing 2^N discrete vacua in both models.
- BPS wall solutions
  - Deriving BPS domain wall solutions up to N=3 case