- *Phase structure of Topologically massive gauge theory with fermion by spectral function*
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1 An alternative to Dyson-Schwinger eq

1 Old attemt by Bloch-Nordsieck near $p^2 = m^2$,

$$S_F(p) \simeq \frac{\gamma \cdot p + m}{m^2(1 - p^2/m^2)^{1-D}}, D = \frac{\alpha(d-3)}{2\pi}, \alpha = \frac{e^2}{4\pi}.$$

2 3-dim-th suggests validity in whole region.

3 QED:severe infrared divergences with massless photon-$>$ cured by vacuum polarization,$1/N$ approximation,

4 TMG:Chern-Simon and Instanton effects modify aomalous dimension.

5 C-S- dependence of Fermion mass is assumed to be small.

6 $\left\langle \overline{\psi}\psi \right\rangle_{\pm} = finite$ or ∞ for 2 and 4-spinor.

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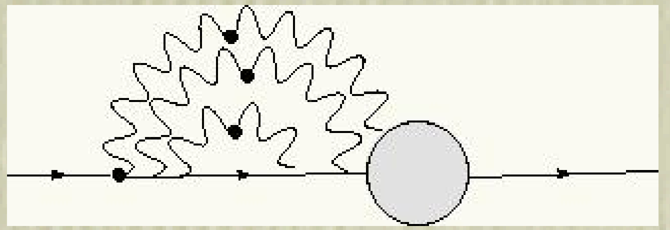


2 Soft-photon summation

Photon attached with external line is most singular by low-energy theorem

$$T_1 = -ie\frac{(r+k)\cdot\gamma + m}{(r+k)^2 - m^2}\gamma_\mu \epsilon^\mu(k,\lambda) \times \mathrm{exp}(i(k+r)\cdot x)U(r,s). \quad (1)$$

$O(e^2)$ spectral function F is given

$$F = \int \frac{d^3k}{(2\pi)^2}\delta(k^2)\theta(k_0)\,\mathrm{exp}(ik\cdot x)\sum_{\lambda,S} T_1\overline{T_1}. \quad (2)$$

In this case the spectral function ρ is given

$$\rho(p) = \int d^3x\,\mathrm{exp}(-ip\cdot x)\frac{\mathrm{exp}(-m\,|x|)}{4\pi|x|}\,\mathrm{exp}(F). \quad (3)$$

Model independent form

$$\sum_{\lambda,s} T_1\overline{T}_1 = -e^2(\frac{\gamma\cdot r}{m}+1)[\frac{m^2}{(r\cdot k)^2}+\frac{1}{(r\cdot k)}+\frac{d-1}{k^2}].$$

2.1 Evaluation of F

μ :infrared cut-off.

$$D_F^{(0)}(x)_+ = \int \frac{d^3k}{i(2\pi)^2}\delta(k^2-\mu^2)\theta(k^0)\exp(ik\cdot x) = \frac{\exp(-\mu\,|x|)}{8\pi i\,|x|}, \tag{4}$$

$$F = ie^2m^2\int_0^\infty \alpha d\alpha D_F(x+\alpha r) - e^2\int_0^\infty d\alpha D_F(x+\alpha r) - ie^2(d-1)\frac{\partial}{\partial\mu^2}D_F(x). \tag{5}$$

In quenced case for finite μ, F is written as

$$F = -\frac{e^2}{8\pi}(\frac{\exp(-\mu\,|x|)}{\mu}-|x|\,\mathrm{Ei}(\mu\,|x|))-\frac{e^2}{8\pi\sqrt{r^2}}\mathrm{Ei}(\mu\,|x|)$$

$$-(d-1)\frac{e^2}{16\pi\mu}\exp(-\mu\,|x|), r^2 = m^2, \qquad (6)$$

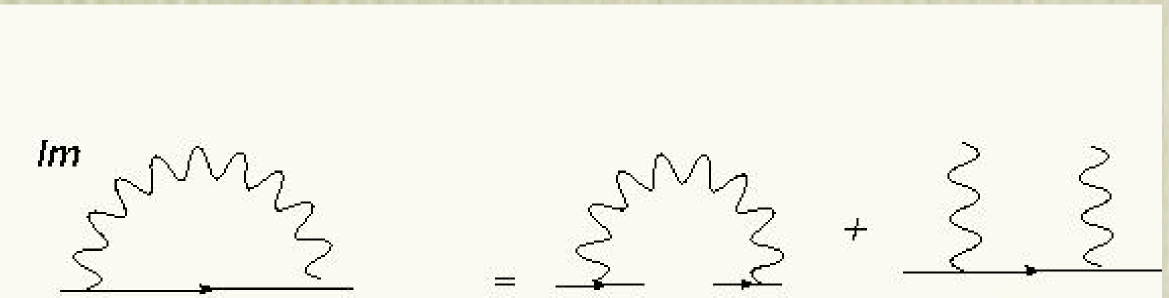
where

$$\mathrm{Ei}(z) = \int_1^\infty \frac{\exp(-zt)}{t}dt, \qquad (7)$$

$$\mathrm{Ei}(\mu\,|x|) = -\gamma - \ln(\mu\,|x|) + (\mu\,|x|) + O(\mu^2). \qquad (8)$$

For the leading order in μ we have at short distance

$$F = \frac{(1+d)e^2}{16\pi\mu} + \frac{e^2\gamma}{8\pi m} + \frac{e^2}{8\pi m}\ln(\mu\,|x|) + \frac{e^2}{8\pi}\,|x|\ln(\mu\,|x|) - \frac{e^2}{16\pi}\,|x|\,(d+1-2\gamma). \qquad (9)$$



where γ is Euler's constant and m is a physical mass.

$$m\overline{\rho}(x) = \frac{m\exp(-m\,|x|)}{4\pi\,|x|}\exp(F) \qquad (10)$$

There is mass shift and its log correction

$$\Delta m|x| = \frac{e^2}{8\pi}|x|\ln(\mu\,|x|) - \frac{e^2}{16\pi}|x|(d+1-2\gamma). \quad (11)$$

$\exp(F)$ is parametrized in the following form

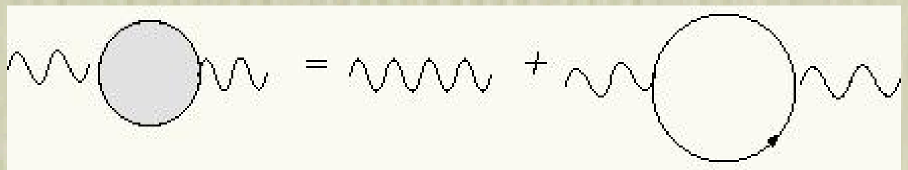
$$\exp(F) = A(\mu\,|x|)^{D+C|x|}, \quad (12)$$

$$A = \exp(\frac{\gamma e^2}{8\pi m} + \frac{e^2}{16\pi\mu}(d+1)),$$

$$D = \frac{e^2}{8\pi m}, C = \frac{e^2}{8\pi}.\text{Gauge invariant.} \quad (13)$$

F acts to change power of $|x|$ and mass.If $D = 1, S_F(0) = finite. \left\langle \overline{\psi}\psi \right\rangle \propto \mu.$

3 Screening effects



$$\rho^{(0)}(s) = \delta(s-\mu^2) \to \rho_\gamma^F, c = \frac{e^2 N}{8}.$$

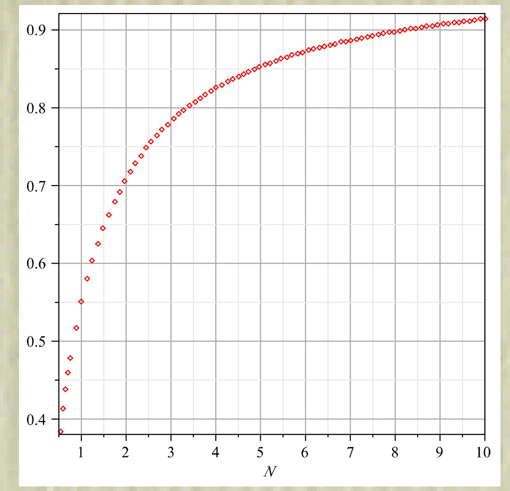
$$D_F(p) = \int_0^\infty \frac{\rho_\gamma^F(s)ds}{p^2 - s + i\epsilon} \tag{14}$$

$$\rho_\gamma^F(k) = \delta(k) + \frac{\mathrm{Im}\,\Pi(k)\theta(k^2-4m^2)}{\pi(-k^2+\mathrm{Re}\,\Pi(k))^2 + (\mathrm{Im}\,\Pi(k))^2}. \tag{15}$$

$$\begin{aligned}\Pi(k) &= -\frac{e^2N}{8\pi}[(\sqrt{-k^2} + \frac{4m^2}{\sqrt{-k^2}})\ln(\frac{2m+\sqrt{-k^2}}{2m-\sqrt{-k^2}}) - 4m], \\ &= -\frac{e^2N}{8} i\sqrt{-k^2}(-k^2>0, m=0).\end{aligned} \tag{16}$$

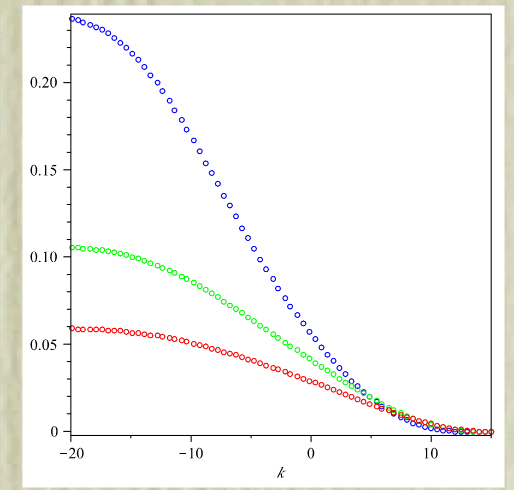
Assumption $\mathbf{m}_{phy} = \mathbf{c}/\mathbf{N}\boldsymbol{\pi}$.

$$Z_3^{-1} = \int_0^\infty \rho_\gamma^F(s)ds. \tag{17}$$



Z_3^{-1} for $N = 1/2..10$ in unit of $e^2 = 1$.

$$\exp(\widetilde{F}(x)) = \int d\mu^2 \rho(\mu^2)\, \exp(F(x,\mu))$$

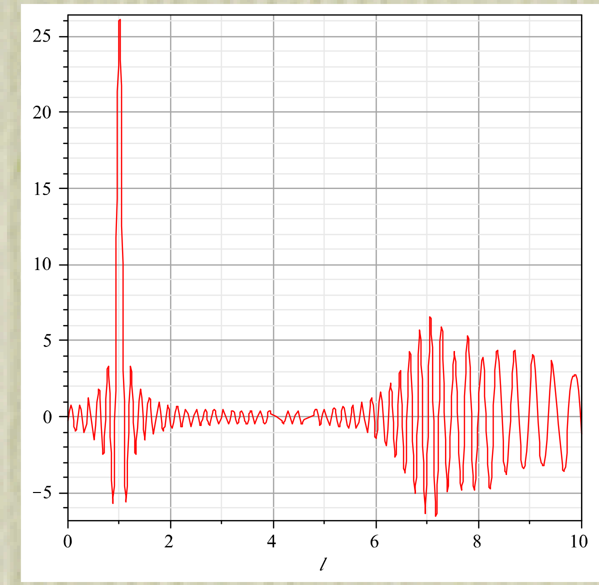


$4S_F(x)$ for $N = 1\tilde{}3, D = 1$ in unit of e^2.

4 Minkowski region

$p^2/m^2 = s$

$$\rho(s) = \frac{1}{2\pi m^2}\int_{-\infty}^{\infty} dx e^{-i(s-1)x} \exp(\widetilde{F}(\frac{x}{m^2})) \quad (18)$$

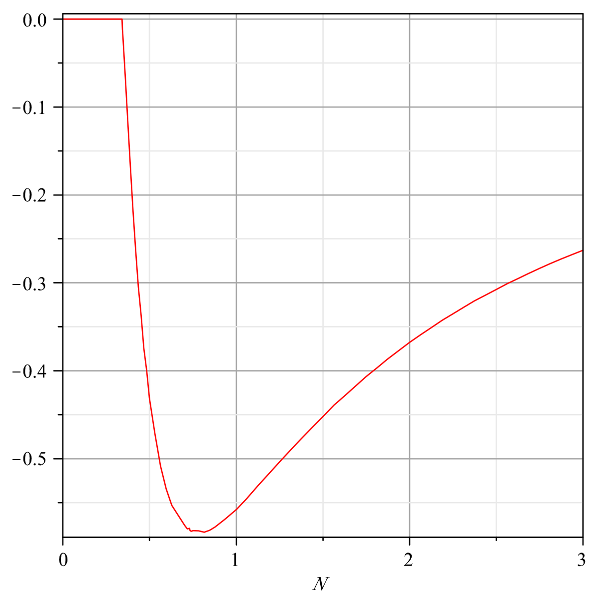


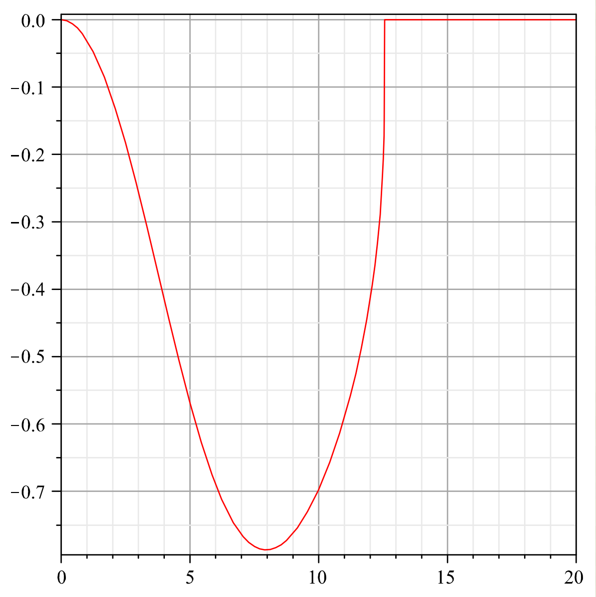
$\rho(l)$ for $c = 1, N = 1$.

5 $\langle\overline{\psi}\psi\rangle$

$$\langle\overline{\psi}\psi\rangle = -4 \lim_{x\to 0} m\overline{\rho}(x).$$

We have $\langle\overline{\psi}\psi\rangle$ as a function of N and c in $1/N$.For small N or large c,fermion mass becomes large.In this case the photon spectral function has a quenched $\delta(s)$ which yields vanishment of $\langle\overline{\psi}\psi\rangle$.





$\langle\overline{\psi}\psi\rangle$ as a function of N for $c = 1$.

$\langle\overline{\psi}\psi\rangle$ as afunction of c for $N = 1$.Critical behavior.

For D=1, $m = c/N\pi$ we have $\left\langle \overline{\psi}\psi \right\rangle \approx -O(10^{-3})e^2$ which agrees Schwinger-Dyson analysis for small $N.m_{phys} > m(p = 0)$.Chiral symmetry $U(2n)$, breaks dynamically into $SU(n) \times SU(n) \times U(1) \times U(1)$ as in QCD.

6 Summary

1 if we input mass, we have massshift,log correction and anomalous dimension which are gauge invariant.
2 $d = -1$,Including vacuum polarization we can avoid infrared divergences.
3 There may be a critical coupling for $\left\langle \overline{\psi}\psi \right\rangle \neq 0$.

7 Chern-Simon QED,QCD

$$L = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4}\theta\epsilon^{\mu\nu\rho}F_{\mu\nu}A_\rho + \overline{\psi}(i\gamma\cdot(\partial - ieA) - m)\psi + \frac{1}{2d}(\partial\cdot A)^2, \quad (19)$$

$$L = \frac{1}{4g^2}tr(F_{\mu\nu}F^{\mu\nu}) - \frac{\theta}{4g^2}\epsilon^{\mu\nu\rho}tr(F_{\mu\nu}A_\rho - \frac{2}{3}A_\mu A_\nu A\rho) + \overline{\psi}(i\gamma\cdot(\partial - ieA) - m)\psi + i\partial^\mu\overline{C}\cdot D_\mu C + \frac{1}{2g^2d}(\partial\cdot A)^2 \quad (20)$$

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}, g_{\mu\nu} = diag(1, -1, -1)$$

$$\begin{aligned}
\gamma_0 &\equiv \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}, \gamma_{1,2} \equiv -i\begin{pmatrix} \sigma_{1,2} & 0 \\ 0 & -\sigma_{1,2} \end{pmatrix}, \\
\gamma_4 &\equiv \gamma^4 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \gamma_5 \equiv \gamma_0\gamma_1\gamma_2\gamma_3 = \begin{pmatrix} 0 & -iI \\ iI & 0 \end{pmatrix}, \\
\gamma_{45} &= \gamma^{45} = -i\gamma_4\gamma_5, \gamma_{\mu 4} = i\gamma_\mu\gamma_4, \gamma_{\mu 5} = i\gamma_\mu\gamma_5. \quad (21)
\end{aligned}$$

There are two redundant matrices which anticommutes with other three γ matrices.

There exists two kinds of chiral transformation $\psi \rightarrow \exp(i\alpha\gamma_4)\psi, \psi \rightarrow \exp(\alpha\gamma_5)\psi,$

for massless theory invariant.$U(2)$ symmetry is generated by $\{I_4, \gamma_4, \gamma_5, \gamma_{45}\}$.

Mass term breaks $\{\gamma_4, \gamma_5\}$ symmetry down to

$U(1) \times U(1)$ generated by $\{I_4, \gamma_{45}\}$.

$$\tau \equiv \gamma_{45} = \frac{i}{2}[\gamma_4, \gamma_5] = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \tau_\pm = \frac{1 \pm \tau}{2}. \tag{22}$$

Ordinary mass $m_e\overline{\psi}\psi$ breaks chiral symmetry.Parity violating mass $m_o\overline{\psi}\tau\psi$ is parity odd but singlet under chiral transformation.Parity transform is $x' = (x^0, -x^1, x^2)$.Here $\overline{\psi}\tau\psi$ is a spin density.

$$\psi^+\frac{i}{2}[\gamma_1, \gamma_2]\psi = \overline{\psi}\tau\psi = n_\uparrow(x) - n_\downarrow(x). \tag{23}$$

$$\Pi = \gamma_{14}\exp(i\phi_p\gamma_{45}), \Pi\overline{\psi}\tau\psi\Pi^{-1} = -\overline{\psi}\tau\psi.$$

Chiral representation with mass $m_\pm = m_e \pm m_o$

$$\begin{aligned} S_F(p) &= \frac{1}{m_\epsilon I + m_O\tau - \gamma\cdot p} \\ &= \frac{(\gamma\cdot p + m_+)\tau_+}{p^2 - m_+^2 + i\epsilon} + \frac{(\gamma\cdot p + m_-)\tau_-}{p^2 - m_-^2 + i\epsilon} \end{aligned} \quad (24)$$

7.0.1 two-component spinor (τ_+)

quenched case θ is an intrinsic cut-off and we have no

infrared divergences.θ is assumed to modify the anomalous dimension of fermion.

$$D_F^0(k) = -i\left(\frac{g_{\mu\nu} - k_\mu k_\nu/k^2 - i\theta\epsilon_{\mu\nu\rho}/k^2}{k^2 - \theta^2 + i\epsilon}\right) - id\frac{k_\mu k_\nu}{k^4}, \quad (25)$$

$$\rho_0^e(s) = \delta(s-\theta^2), \rho_0^O(s) = \frac{1}{\theta}[(\delta(s-\theta^2) - \delta(s)]. \quad (26)$$

$$\sum_{\lambda,S} T_1\overline{T}_1 = \frac{-e^2(\gamma\cdot p+m)}{2m}[\frac{m^2}{(p\cdot k)^2}+\frac{1}{p\cdot k}+\frac{(d-1)}{k^2}]$$
$$-\frac{\gamma\cdot p}{m}\frac{e^2}{4\theta}\frac{m\tau}{p\cdot k}. \quad (27)$$

In $O(e^2)$ we have $1/p\cdot k$

$$.-\left\langle\frac{1}{p\cdot k}\right\rangle \sim \frac{\gamma+\ln(\theta|x|)}{8\pi m}(\theta|x|\ll 1). \quad (28)$$

As in QED_3 we set anomalous dimension $D\geq 1$ for finite vacuum expectation value

$$D=\frac{e^2}{8\pi m}+\frac{e^2}{32\pi\theta}\geq 1, m=e^2/8\pi/(1-\frac{e^2}{32\pi\theta}), \quad (29)$$

which may be consistent with the Laddar Schwinger-Dyson eq.by T.Matsuyama & H.Nagahiro.It has been shown that $\left\langle\overline{\psi}\psi\right\rangle \propto \theta$ in [6].θ dependence of mass m may be small.For infinitesimal θ linear approximation holds,we have $m=e^2/8\pi$ as in QED.

In QCD_3 instanton effects is

$$\begin{aligned} \theta &= \theta = ng^2/4\pi, n(0, \pm 1, \pm 2, ...), \\ D &= \frac{e^2}{8\pi m} + \frac{1}{8n} \geq 1, m = e^2/8\pi/(1 - \frac{1}{8n}), \\ n &\neq 0, \left\langle \overline{\psi}\psi \right\rangle = finite. \end{aligned} \tag{30}$$

unquenched case This approximation holds for $\theta \geq 2m$.Including vacuum polarization function that is written by parity even and odd piece

$$\Pi_{\mu\nu}(k) = (g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})\Pi^e(k) + i\theta\epsilon_{\mu\nu\rho}k_\rho \Pi^O(k), \tag{31}$$

The exact propagator is given by

$$D_{\mu\nu}(k) = (g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} - i\mathcal{M}(k)\frac{\epsilon_{\mu\nu\rho}k_\rho}{k^2})\frac{-i}{k^2 - \Pi(k)}. \tag{32}$$

We have a spectral function for unquenched case

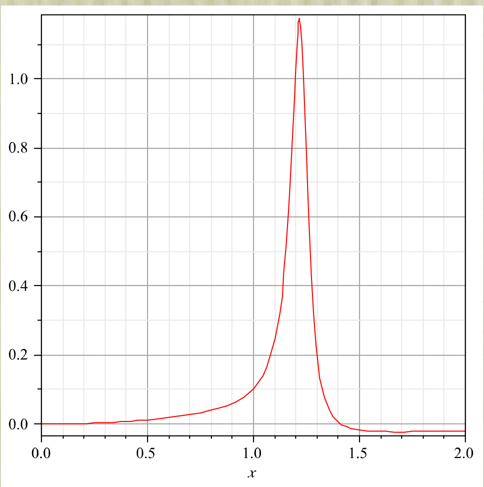
$$(g_{\mu\nu} - k_\mu k_\nu/k^2)\rho^e(k) - i\epsilon_{\mu\nu\rho}k_\rho/\theta\rho^O(k).$$

$$\widetilde{F}(x) = \int ds(\rho^e(s)F^e(\sqrt{s},x) + \rho^O(s)F^O(\sqrt{s},x)$$
$$= D^e \ln(\overline{\theta^e}|x|) + D^O \ln(\overline{\theta^O}|x|), \quad (33)$$

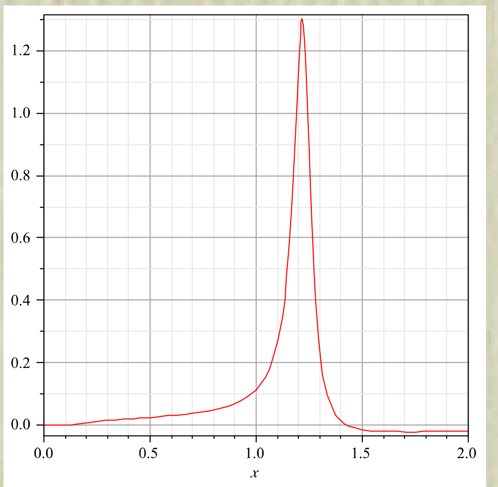
$$\exp(\widetilde{F}(x)) \sim (\overline{\theta^e})^{D^e}(\overline{\theta^O})^{D^O}|x|^D. \quad (34)$$

Thus the phase structures are the same with that of quenched case. C-S QED:large N,only broken phase.

Topologically massive QCD: large N_c,only broken phase.



$\rho_g^e(x)$ for $c = 1, n = 1, N_f = 3, SU(3)$.



$\rho_g^O(x), c = 1, n = 1, N_f = 3, SU(3)$.

Gauge dependedepent terms of vacuum polarization Π has a cut

$$\Pi^e(\Pi^O) \propto \frac{d}{\sqrt{-p^2}} \tag{35}$$

for $d \neq 0$ gauge.Spectral function from this term vanishes in the infrared.Thus it has no effects on the infrared behaviour of gluon propagator.

R.Pisarsky& S.Rao discussed the infrared behaviour of Π at higher order and point out the above results in peruturbative sense.

7.0.2 4-component spinor

It is easy to derive the anomalous dimension in the 2-component spinor case.

$$m_\pm = m_e \pm m_O \tag{36}$$

There is no extra constraints for

$$D_{\pm} = \frac{e^2}{8\pi m_{\pm}} \pm \frac{e^2}{32\pi\theta}. \tag{37}$$

8 Summary

QED, $U(2) - > U(1) \times U(1)$

C-S QED can be understood by our mehod.

2-spinor: $\left\langle \overline{\psi}\psi \right\rangle_{+} = finite.$

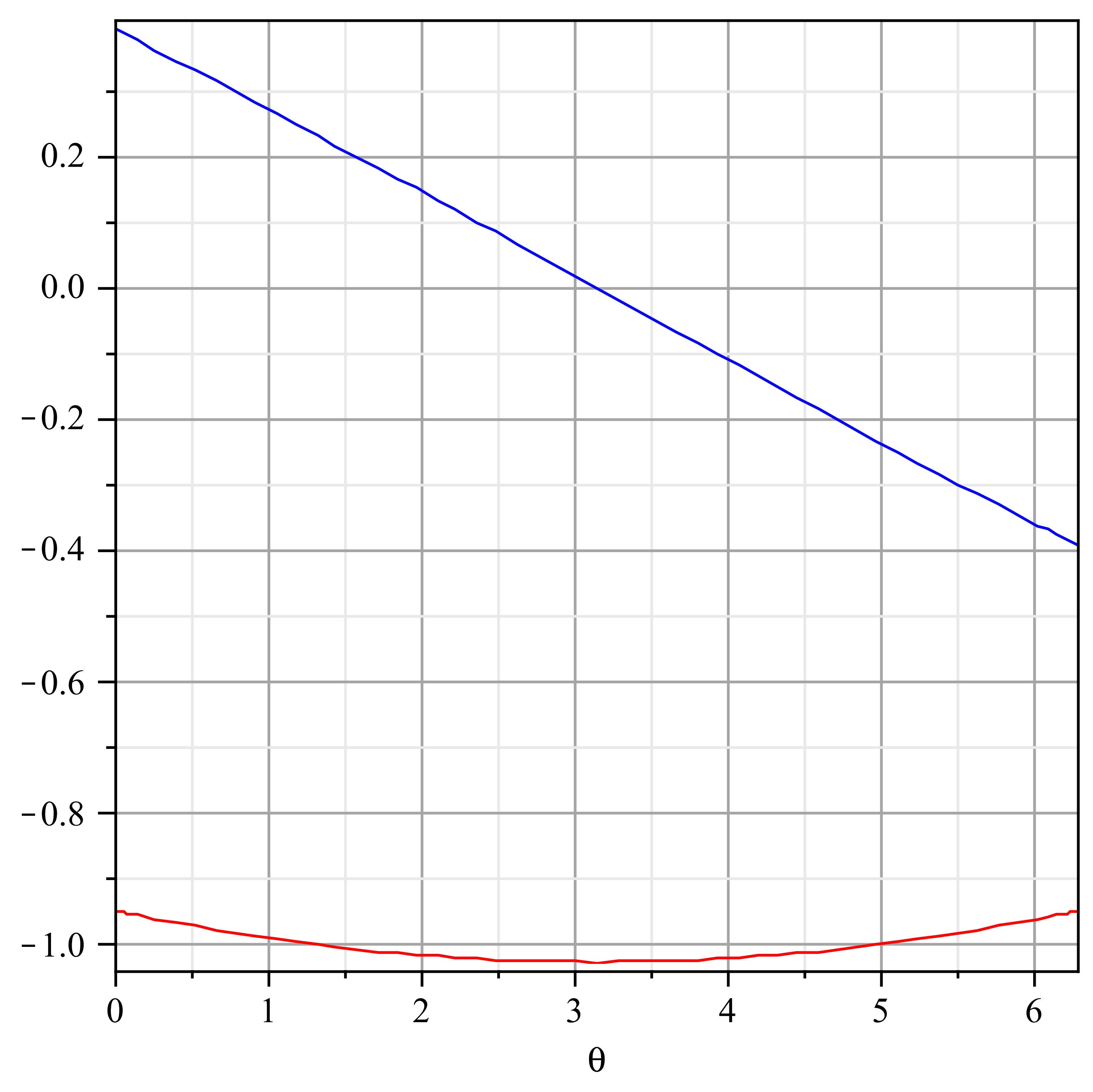
4-spinor: Both of $\left\langle \overline{\psi}\psi \right\rangle_{\pm}$ can be finite ?.

What can be done for Topologically massive QCD ?

Summation for n.

$$\sum_{n=-\infty}^{\infty} \exp(in\theta)$$

mass



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