基研研究会 in 2010/7/24

Accelerated String and Unruh effect in holographic confining gauge theory

work in progress

talker： 田港　朝貴（九州大学）

collaborators： 郷六　一生（福岡工業大学）
石原　雅文（九州大学）
久保　幸貴（九州大学）

Unruh effect

Unruh effect [Unruh '76]

The detector accelerating with a constant acceleration in the Minkowski space-time would observe a **thermal bath**.

Unruh temperature (Rindler temperature)

$$T_{Unruh} = \frac{a}{2\pi}$$

a : **constant acceleration**

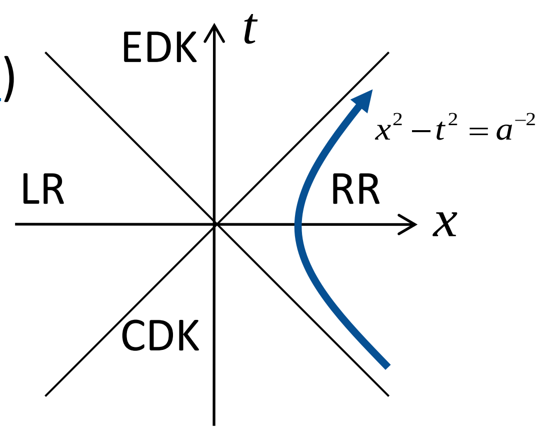
Rindler coordinates (Comoving frame)

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

$$\begin{cases} t = a^{-1} e^{a\xi} \sinh a\tau \\ x = a^{-1} e^{a\xi} \cosh a\tau \end{cases}$$

(Rindler transformation)

$$ds^2 = e^{2a\xi}\left(-d\tau^2 + d\xi^2\right) + dy^2 + dz^2$$



Unruh effect

Why Unruh effect?

■ A crucial rule in our understanding that the particle contents or vacuum of a theory depend on an observer.

■ The phenomenon of particle emission from black holes and cosmological horizons.

■ Experimentally interesting. (Extreme Light Infrastructure, ELI (Europe))

In the view of **gauge/gravity correspondence**,

■ **The properties of thermal effect in curved space-time.**

- Hawking temperature vs. Unruh temperature [Paredes, et al '09]
- Phase transition ex)chiral symmetry [Hirayama, et al '10]
  confinement-deconfinement
- The relation between thermodynamics and gravity theory

■ **Dynamics of accelerated string and radiation by Unruh effect.**

- Heavy quark energy loss in quark-gluon plasma (QGP)

Unruh effect has very fruitful physics !

Talk plan

(1)Introduction

(2)Model set-up : **Deformed** AdS (**dAdS**) for confinement

(3)Accelerating string and in curved space-time

- In the case of AdS background (review)
- In the case of **d**AdS background

(4)Unruh effect in Rindler coordinates

- Wilson loop and quark-antiquark potential
- Drag force and friction constant (~~and Einstein relation~~)

Model set-up

Deformed AdS background

[Liu and Tseytlin '99]
[Ghoroku and Yahiro '04]

A background for confinement is derived by including the R-R scalar χ

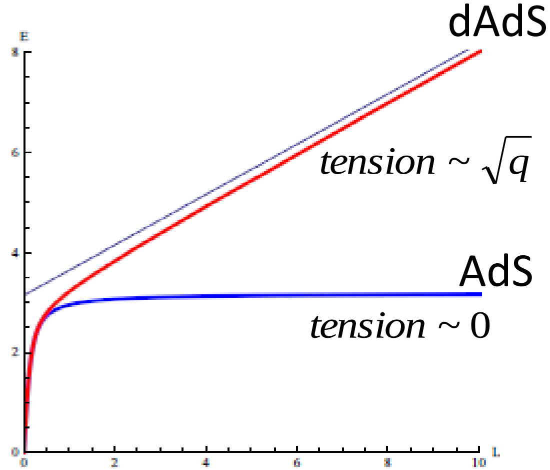
$$S = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\left(R - \frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}e^{2\Phi}(\partial\chi)^2 + \frac{1}{4\cdot 5!}F_5^{\ 2}\right)$$

The ansatz for supersymmetry : $\chi = -e^{-\Phi} + \chi_0$

The solution is given as follows

$$ds_{10}^2 = e^{\Phi/2}\left(\frac{r^2}{R^2}\left(-dt^2 + d\vec{x}^2\right) + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2\right)$$

dilaton : $e^{\Phi} = 1 + \frac{q}{r^4}$ $\quad q = \frac{\chi_0 R}{4}$



quark-antiquark potential

Properties of the gauge theory

(1) The dual gauge theory is $\mathcal{N} = 2$ $SU(N_c)$ SYM
(2) **Confinement** phase at zero temperature
(3) If **q=0**, dAdS becomes AdS

Uniformly accelerated string

In the case of AdS [Xiao '08]

AdS_5 space-time

$$ds^2_{AdS} = R^2\left[u^2\left(-dt^2 + d\vec{x}^2\right) + \frac{du^2}{u^2}\right] \qquad u = \frac{r}{R^2}$$

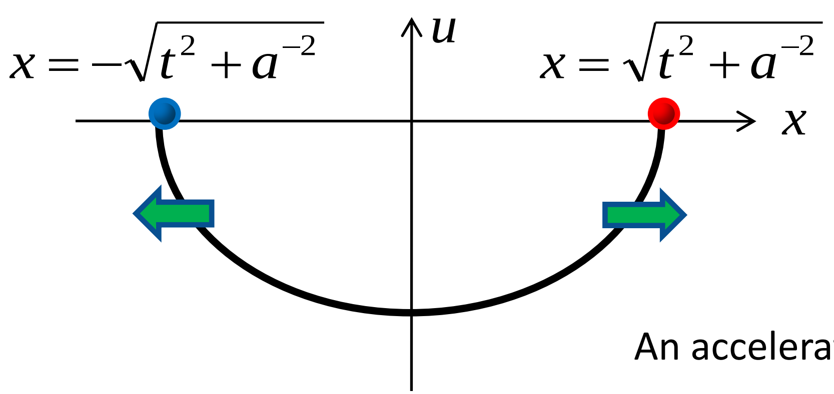
Here we define $X^\mu(\tau,\sigma)$ as a map from the string world-sheet.
We choose a static gauge by setting $(\tau,\sigma) = (t,u)$.

$$X^\mu(\tau,\sigma) = (t,u,x(t,u),0,0)$$

From Nambu-Goto action, we can find the **exact solution** of equation of motion

$$x^2 - t^2 = a^{-2} - u^{-2}$$

Since the solution becomes $x^2 - t^2 = a^{-2}$ at boundary $u \to \infty$, we can regard the constant of integration a as accelerating constant in the dual CFT.



An accelerated string

Uniformly accelerated string

Rindler transformation for AdS [Xiao '08]

$$ds^2_{AdS} = R^2\left[u^2\left(-dt^2 + d\vec{x}^2\right) + \frac{du^2}{u^2}\right]$$

$$\begin{cases} x = \sqrt{a^{-2} - u^{-2}}\, e^{a\xi} \cosh a\tau \\ t = \sqrt{a^{-2} - u^{-2}}\, e^{a\xi} \sinh a\tau \\ u = h e^{-a\xi} \end{cases}$$

(Rindler transformation)

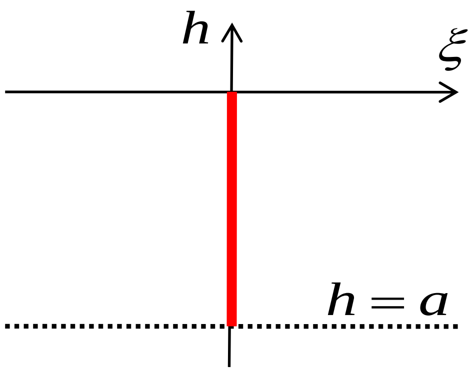
$$ds^2_{Rindler} = R^2\left[-\left(h^2 - a^2\right)d\tau^2 + h^2\left(d\xi^2 + e^{-2a\xi}\left[dy^2 + dz^2\right]\right) + \frac{dh^2}{h^2 - a^2}\right]$$

Some profiles in Rindler coordinates

- Accelerated string with a in AdS is static at $\xi = 0$ and its shape forms straight line.

  ⇒ reflection of scale independence

- The vacuum contains an event horizon at $h = a$.
- Unruh temperature $T_{Unruh} = \dfrac{a}{2\pi}$



Static string in Rindler coordinates.

Uniformly accelerated string

Rindler transformation for dAdS

$$ds^2_{dAdS} = e^{\Phi/2} R^2 \left[u^2 \left(-dt^2 + d\vec{x}^2 \right) + \frac{du^2}{u^2} \right] \qquad e^{\Phi} = 1 + \frac{q}{u^4}$$

$$\begin{cases} x = \sqrt{a^{-2} - u^{-2}} e^{a\xi} \cosh a\tau \\ t = \sqrt{a^{-2} - u^{-2}} e^{a\xi} \sinh a\tau \\ u = h e^{-a\xi} \end{cases}$$

(Rindler transformation)

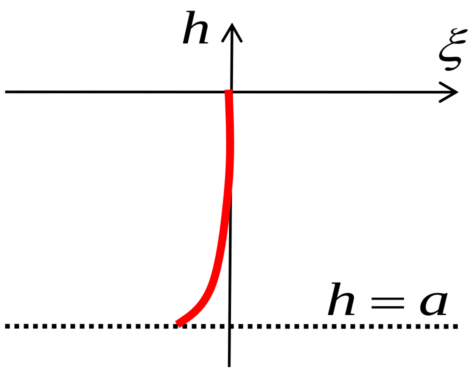
$$ds^2_{Rindler} = e^{\Phi/2} R^2 \left[-\left(h^2 - a^2\right) d\tau^2 + h^2 \left(d\xi^2 + e^{-2a\xi} \left[dy^2 + dz^2 \right] \right) + \frac{dh^2}{h^2 - a^2} \right] \qquad e^{\Phi} = 1 + \frac{q e^{4a\xi}}{h^4}$$

Some profiles in Rindler coordinates

- Accelerated string with a in this case is static at $\xi = 0$ at boundary, and its shape forms curved line.

  ⇒ **reflection of scale (dilation) dependence**

- The vacuum contains an event horizon at $h = a$.
- Unruh temperature $T_{Unruh} = \dfrac{a}{2\pi}$



Static string in Rindler coordinates.

Wilson loop

quark-antiquark potential

The quark-antiquark potential is derived from a Wilson loop in gauge theory

$$\langle W \rangle \sim e^{-V_{q\bar{q}} \int dt}$$

On the other hand, Wilson loop in the dual gravity represents as

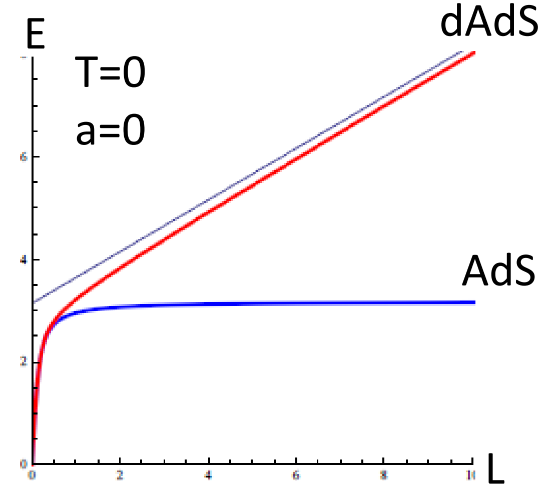
$$\langle W \rangle \sim e^{-S}$$

in terms of the Nambu-Goto action

$$S = -T_0 \int d\tau d\sigma \sqrt{-\det g_{ab}}$$

Here we calculate the quark-anti quark potential in dAdS space-time and Rindler coordinates.

We would see a **difference of thermal effect** between **Unruh temperature** and **Hawking temperature**.



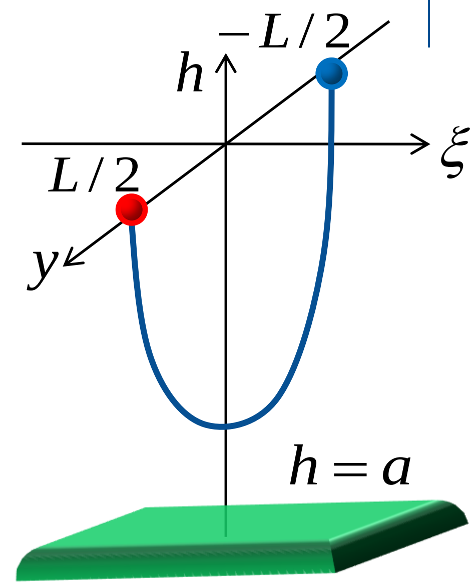
quark-antiquark potential

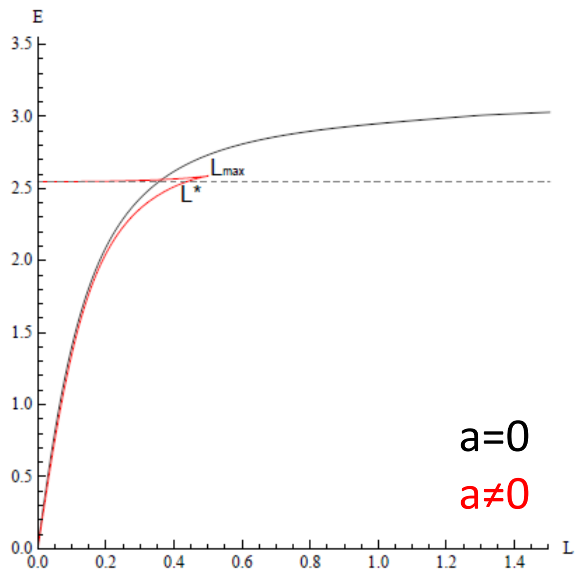
Wilson loop

String stretched to transverse direction

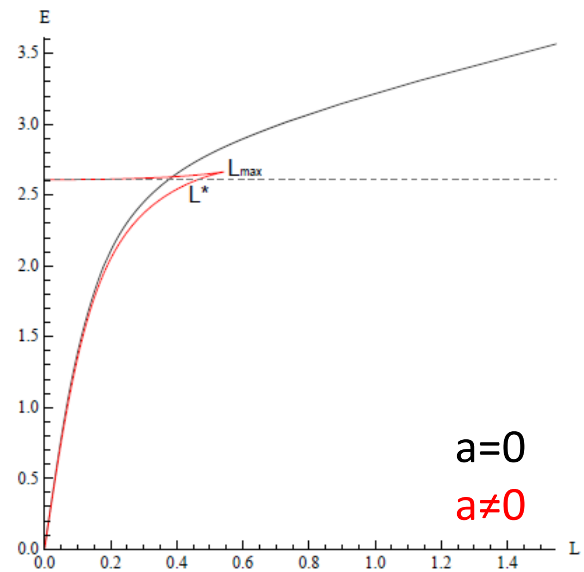
Here we show some quark-antiquark potential of U-shaped string stretched to **transverse direction to accelerated direction** ξ .

We find the maximum value of L at finite acceleration. This is reflection of **the screening of the color** force as seen in at high Hawking temperature. [Ghoroku et al '05]

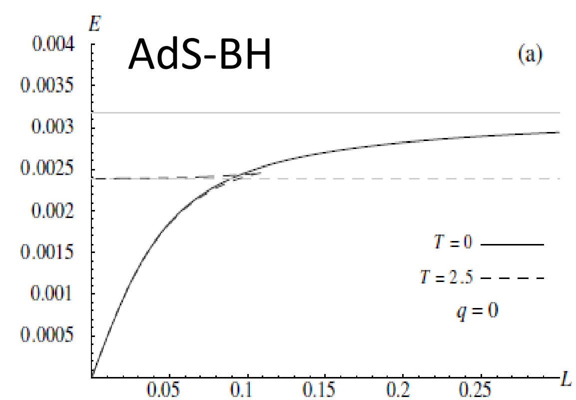




AdS background (q=0)



dAdS background (q≠0)

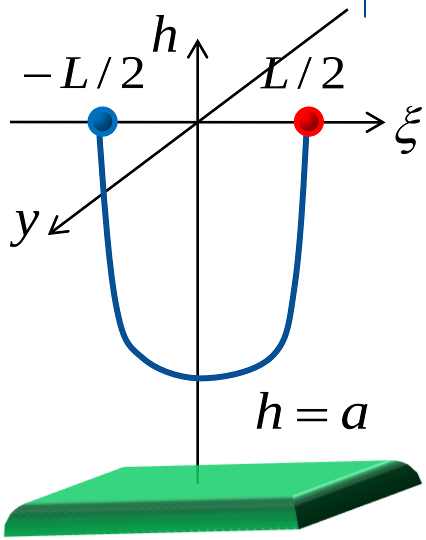


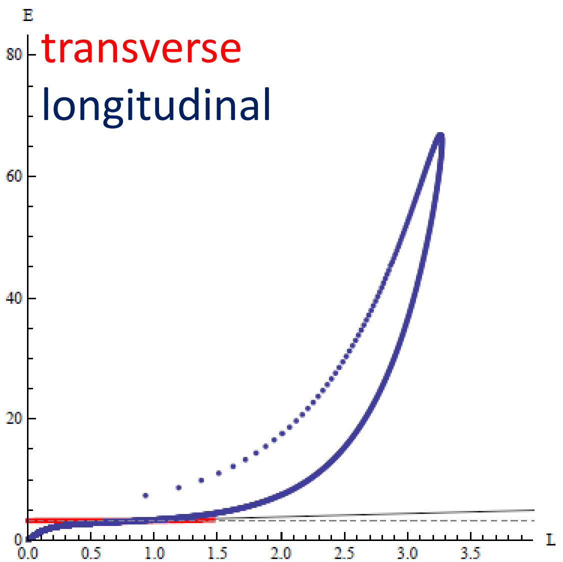
String stretched to longitudinal direction

Here we show some quark-antiquark potential of U-shaped string stretched to **longitudinal direction to ξ in dAdS**.

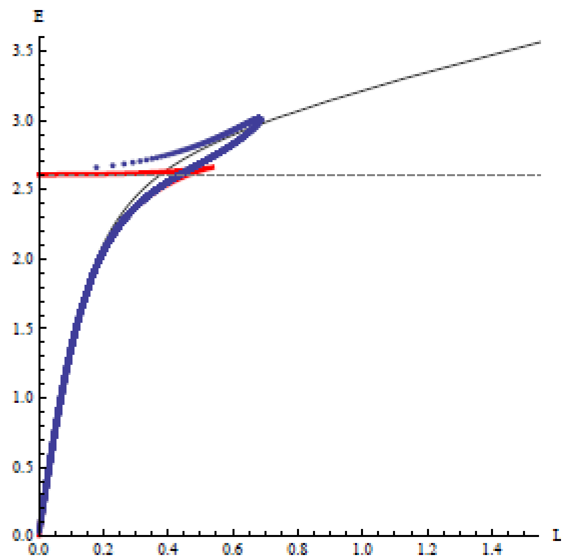
We find the maximum value of L at finite acceleration.
This is reflection of **the screening of the color**.

But, the value of energy **increases exponentially at Large L**.
This behavior is **not seen** in the both cases of
Hawking temperature and AdS space-time.





a=1



a=2

Drag force and the friction constant

We study **drag force** along transverse direction to ξ in Rindler vacuum.

A trailing string is given by

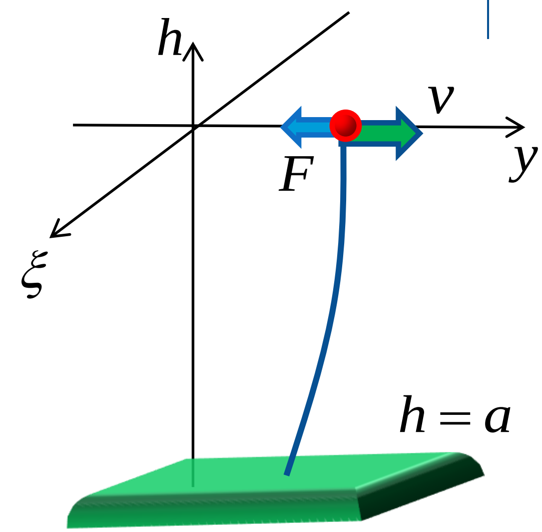
$$y = vt + \eta(h)$$

From the Nambu-Goto action in non-relativistic limit,

$$F = -T_0 \sqrt{u_*^4 R^4 + \frac{q}{R^4}}\, v = -\gamma v$$

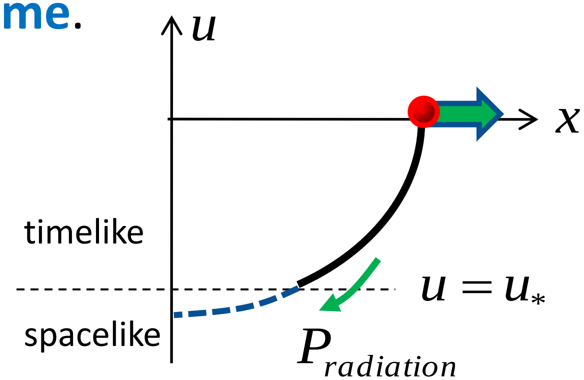
F : **drag force**

γ : **friction constant**



We also find the **friction constant in Rindler coordinates** might be equivalent to **energy loss from accelerated string in original space-time**.

$$P_{radiation} = T_0 \sqrt{u_*^4 R^4 + \frac{q}{R^4}} = \gamma$$



Summary in talk

Summary

■We discuss the **Unruh effect** in the **deformed AdS** background for the dual gauge theory that realized **confinement phase**.

■**Wilson loop in Rindler coordinates**

- It represents **deconfinement phase** and **color screening**.
- Strings stretched to the **transverse direction** to the accelerating direction
  ⇨ Its behavior is very **similar to Hawking temperature**.
- Strings stretched to the **longitudinal direction** to the acceleration direction
  ⇨ Its behavior is **different from Hawking temperature**.
  This property is **not seen in AdS case**.

■**Drag force and the friction constant**

We calculate the **drag force** and the **friction constant** on the quark moving with a constant velocity in the hot gluons. It also represents the **friction constant might be equivalent to the energy loss of accelerated string** in original background.