4D N=1 gauge theories from M5 branes on $A_k$ singularity with orientifold

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1. Introduction

6D N=(2,0) SCFT on Riemann surface

4D N=2 theory

This procedure is very successful for understanding duality network of 4D N=2 theories.

We want to generalized to 4D N=1

6D N=(1,0) SCFT on Riemann surface

4D N=1 theory

So many 6D N=(1,0) SCFT [Heckman et al.] [Gaiotto Razamat] M5 on $C^2/Z_k$ orbifold “class $S_k$.”

In this talk, we further impose $H^4/Z_k$ orientifold action

In addition, we restrict the case for odd $k$, $k=2m+1$

2. Type IIA brane system set up

Orientifold

3. Orbifold and Orientifold projection

Original theory : N=2 SU(N) conformal quiver

$N-2$ SU(N)$_a$ vector multiplet $(Q^{(a)},\bar{Q}^{(a)})$ ($a=1,\ldots,n$)

SU(N)$_{1+}$ SU(N)$_{1-}$ bifundamental hypermultiplet $(Q^{(+)}(a),\bar{Q}^{(a)})$ ($a=1,\ldots,n-1$)

+ fund. (antifund.) hyper at end nodes

**Orientifold projection**

(a) spatial rotation

spatial rotation charge = R-charge

\[ \Phi : 1 \rightarrow Q: -1/2 \rightarrow Q: 1/2 \] (charge of scalar component)

(b) action on gauge index

for charge (i,j) sector filed

\[ Q \rightarrow Q \frac{\xi-i\eta}{\xi-i\eta} \]

assign (half-)integer R-charge

4. Quiver diagram representation of the projection

N=2 SU(N) conformal quiver

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N=2 Usp-SO conformal quiver

4D N=1 quiver for the brane system

**Fixed plane**

antisymmetric rep. chiral

symmetric rep. chiral

Thus, we found

\[ (y^{(a)},\bar{y}^{(a)}) \rightarrow N=1 \text{ SU(N)}^k \text{ necklace quiver} \]

\[ (Q^{(a)},\bar{Q}^{(a)}) \rightarrow \text{ bifundamental chiral which zig-zag} \]

\[ \text{identify fields with their orientifold image} \]

\[ \text{If image of a field is itself, then} \]

\[ \text{gauge node : SU} \rightarrow \text{SO (O4$^-$) /Usp (O4$^+$)} \]

\[ \text{bifundamental} \Phi \rightarrow \text{antisymmetric rep. (O4$^-$)} \]

\[ \text{symmetric rep. (O4$^+$)} \]

For simplicity, we assign O4$^+$ plane to gauge node with integer $Z_k$ charge

Since orientifold action causes $Q \rightarrow \bar{Q}$

such situation does not occur for $Q$ and $\bar{Q}$

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We impose two conditions:

1. Anomaly free
2. Obtain N=2 Usp-SO conformal quiver for m=0

Gauge group

\[ \prod_{i=0}^{m} \left( \text{Usp}(2N) \times \prod_{i=0}^{m} \left( \text{SU}(2N+4i+4) \times \text{SU}(2N+4i+4) \right) \right) \]

Chiral fields for even a

- \(Q^{(a,i)}\) \(\text{SU}(2N+4i+4) \times \text{SU}(2N+4i+4)\) \(i = 1, 2, \ldots, m\)
- \(X^{(a)}\) \(\text{SU}(2N)\)

Chiral fields for odd a

- \(Q^{(a,i)}\) \(\text{SU}(2N+4i+4) \times \text{SU}(2N+4i+4)\) \(i = 0, 1, \ldots, m\)
- \(Y^{(a)}\) \(\text{SU}(2N)\)

Superpotential coupling

For even a

- \(W^{(a,i)} = \text{Tr} \bar{Q}^{(a,i)} \Phi^{(a,i)} Q^{(a,i+1)}\) \(i = 0, 1, \ldots, m-1\)
- \(W^{(a,m)} = \text{Tr} Q^{(a,m)} X^{(a)} Q^{(a,m)}\)

For odd a

- \(V^{(a,i)} = \text{Tr} Q^{(a,i)} \Phi^{(a,i-1)} Q^{(a,i)}\) \(i = 1, 2, \ldots, m\)
- \(V^{(a,0)} = \text{Tr} Q^{(a,0)} Y^{(a)} Q^{(a,0)}\)

Exactly marginal deformation

To identify the 4D theory with compactification of 6D theory on Riemann surface, it must be hold

\[# \text{of exact marginal deformation} = # \text{of complex structure moduli parameter of Riemann surface} \]

Global symmetry

Intrinsic symmetry \((U(1)_a)^{1/2} \times (U(1)_b)^{1/2}\) \(\times Q \times X \times Y\)

Flavor symmetry

- \(U(1)\) rotation of fields crossing with each orange arrow \((k-1)\) independent ones
- Sign of charge determined by orientation

Associated Riemann surface

We identify the quiver theory with a sphere with \((n+1)\) punctures

\((n-1)\) puncture associated with \(U(1)_a\)

Two punctures associated with ends of the quiver

We can glue punctures (gauging the flavor symmetry) and obtain more general theories in the same way as for N=2 class S theories.

Conjecture

Exchange of same type of punctures = duality of the theory

Future work

Check the conjecture in terms of index

Closing puncture (giving VEV to meson or baryon op.)

How about even k (need additional flavor for anomaly cancellation)