Quantum Entanglement of Excited States by Heavy Local Operators in Large-c 2d CFT at Finite Temperature

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Motivations

- Entanglement Entropy (EE) for Excited States by local operators

\[ S_A = -T \rho_A \log \rho_A \rho = O(x) \langle \Psi | \rho | O(x) \rangle \]

\[ \rho_A = T \rho \]

- “Can we characterize the local operators from entanglement measures?”

Free scalar

- RCFT [He-Numasawa-Takayanagi, Kawamura 14]

- Large-[Caputa-Numasawa-Takayanagi 14]

- Large-c [Asplund-Bernamonti-Galli-Hartman 14]

- Finite T [Caputa-Simon-Stikonas-Takayanagi, etc.,

Next! Large-c & Finite T!

- Mutual Information (MI)

\[ I_{A:B} = S_A + S_B - S_{A:B} = \frac{1}{2} \sum_{k} \min \{ \rho_{A(k)}, \rho_{B(k)} \} \]

Upper bound for the connected 2-pt functions!

When does MI vanish? \[ I_{A:B} = 0 \]

No entanglement (and correlation) between A and B!

Set-up

- Large-c 2d CFT \( c \to \infty \)

- Finite T

- Thermo-field double (TFD) state

\[ \text{TFD}_t = \sum_{\xi} e^{-\frac{\xi t}{T}} \xi | \Omega \rangle_{\text{DL}} \otimes | \mathcal{F} \rangle \]

\[ T \rho_{\text{TFD}} = \sum_{\xi} e^{-\frac{\xi t}{T}} \langle \xi | \rho | \xi \rangle_{\text{DL}} \otimes | \mathcal{F} \rangle \]

- Heavy local operator

\[ \rho_{L} = \rho - \psi_L (L)_{\text{TFD}} = \rho - \psi_L (L)_{\text{TFD}} \]

\[ \rho \sim \psi_L (L)_{\text{TFD}} \]

- Time evolution

\[ \psi_L (L)_{\text{TFD}} \psi_L (L)_{\text{TFD}} \]

Consider the time evolution by

\[ e^{-\frac{\xi t}{T}} \rho e^{-\frac{\xi t}{T}} \]

\[ \rho_{L} = \rho - \psi_L (L)_{\text{TFD}} = \rho - \psi_L (L)_{\text{TFD}} \]

\[ \text{TFD}_t \rightarrow \text{TFD}_s \]

\[ \psi_L (L)_{\text{TFD}} \psi_L (L)_{\text{TFD}} \]

\[ \text{CTFD} \]

\[ \text{CFT}_B \]

Results

- \( \Delta S_A \)

\[ \frac{\Delta S_A}{c \cdot \ell} = \left\{ \begin{array}{ll}
0 & \text{if } y < \frac{L}{2} + \frac{\ell}{2} \\
\psi_L (\frac{\ell}{2} + y) & \text{if } y < \frac{L}{2} \leq \frac{\ell}{2}
\end{array} \right. 
\]

\[ \frac{\Delta S_A}{c \cdot \ell} = \left\{ \begin{array}{ll}
\psi_L (y) & \text{if } \frac{L}{2} < y < L \\
0 & \text{if } y > L
\end{array} \right. 
\]

Scrambling time \( T_{\text{sc}} \)

\[ T_{\text{sc}} = \left\{ \begin{array}{ll}
0 & \text{for large } \ell \text{ and } c \to \infty
\end{array} \right. 
\]

\[ T_{\text{sc}} = \frac{L}{2} - \frac{1}{2} \frac{c}{\ell} \log \left( \frac{\sin \frac{\ell}{2} \pi}{\sin \frac{\ell}{2} \pi} \right) + \frac{1}{2} \frac{c}{\ell} \log \left( \frac{8 \sin \frac{\ell}{2} \pi}{4 \ell} \right) 
\]

\[ \sigma_{\infty} = \frac{\pi c}{3 \ell} E_0 = \frac{\pi c}{4 \ell} 
\]

Holographic computations

- By using Ryu-Takayanagi formula, we can also compute in the holographic model

\[ \text{Perfect matching to the Large-c 2d CFT (leading results)!} \]

The dual geometry:

Free falling particle in external BTZ BH (including the back-reaction)

\[ \text{[Caputa-Numasawa-Takayanagi, etc.]} \]

- Especially, for large \( \ell \)

Almost null energy localized at the horizon

\[ \approx \text{Shock wave geometry} \]

\[ \text{Holographic model} \]

Some more physics?

1/c corrections, Recurrence

Complexity of states

Bounds on chaos

Other approaches based on quantum information?

e.g., Quantum information metric [Numasawa-Shiba-Takayanagi-15]

[Lashkari-Stikonas-15]

Detail: Large-c computations

By using Conformal map \( w = e^{-\frac{\xi t}{T}} \) from cylinder to plane,

\[ S_A^{(n)} = \frac{c}{6} (n + 1) \log \frac{c}{2 \pi L} \sin \left( \frac{\pi L}{2} \right) + \frac{c}{2} \frac{n c}{\ell} \log \left( \frac{\sin \frac{\ell}{2} \pi}{\sin \frac{\ell}{2} \pi} \right) + \frac{c}{2} \frac{n c}{\ell} \log \left( \frac{8 \sin \frac{\ell}{2} \pi}{4 \ell} \right) \]

\[ G_{\ell}(z, \bar{z}) \]

\[ G_{\ell}(z, \bar{z}) \approx \exp \left[ \frac{nc}{6} \left( \int \frac{H_{\ell}(z)}{c} \sigma_0 \right) \right] + c \epsilon \]

[ Zamolodchikov 87]

2 Heavy-2 Light

Expanding \( f \) around \( z \sim 1 \) & \( n \to \infty \)

\[ \Delta S_B = \frac{c}{6} \log \left( \frac{2^{1/4} \pi}{2^{1/4} \pi} \right) + \frac{c}{2} \frac{n c}{\ell} \log \left( \frac{8 \sin \frac{\ell}{2} \pi}{4 \ell} \right) \]

\[ \approx - \frac{1}{4} \frac{2 \sqrt{c}}{c} \frac{c}{\ell} \frac{1/4 \pi}{2^{1/4} \pi} \]

Similarly,

\[ S_B = \frac{c}{6} \log \left( \frac{2^{1/4} \pi}{2^{1/4} \pi} \right) + \frac{c}{2} \frac{n c}{\ell} \log \left( \frac{8 \sin \frac{\ell}{2} \pi}{4 \ell} \right) \]

\[ \Delta S_B = 0 \]

(For any \( \ell \))

The 6-pt function can be approximated by 2 dominant contributions:

- (choose bigger one)

\[ S_{A:B} = S_A + S_B \]

\[ I_{A:B} = 0 \]

\[ S_{A:B} \neq S_A + S_B \]

\[ I_{A:B} \neq 0 \]