\(A_\infty\) structure from the Berkovits formulation of open superstring field theory

Tomoyuki Takezaki, The University of Tokyo, Komaba

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See also arXiv: 1505.07065 and 1510.00364 by Theodore Erler

1. Introduction

For the Neveu-Schwarz sector of open superstring field theory, we now have two formulations: the Berkovits formulation [1] and the \(A_\infty\) formulation [2].

<table>
<thead>
<tr>
<th>Berkovits</th>
<th>(A_\infty)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hilbert space</td>
<td>large HS small HS</td>
</tr>
<tr>
<td>gauge fixing</td>
<td>difficult straightforward</td>
</tr>
<tr>
<td>closed form expression</td>
<td>yes</td>
</tr>
</tbody>
</table>

We find the same structure in these two formulations.

2. The Berkovits formulation

The Berkovits formulation [1] is based on the large Hilbert space of the superconformal ghosts. The equation of motion and the gauge transformation in the free theory are

\[
Q\eta\Phi_B = 0, \quad \delta\Phi_B = QA + \eta\Omega
\]

where \(Q\) is the BRST operator of the superstring, and \(\eta\) is the zero mode of the superconformal ghost \(\eta(z)\). The full action takes the Wess-Zumino-Witten-like form

\[
S_{\text{WZW}}[\Phi_B] = -\int_0^1 dt \left< B(t), Q B_B(t) \right>
\]

with

\[
B_B(t) = (\eta\Phi_B(t)e^{-\Phi_B(t)}, \quad B(t) = (\delta\Phi_B(t))e^{-\Phi_B(t)}
\]

where \(B_B(0) = 0\) and \(\Phi_B(1) = \Phi_B\). The \(t\) dependence of the action is topological, and the action is a functional of \(\Phi_B\).

The gauge invariance and the topological \(t\) dependence follow from

\[
\eta B_B(t) = B_B(t)B_B(t), \quad \delta B_B(t) = \eta B_B(t) - B_B(t)B_B(t) + B(t)B_B(t).
\]

A regular formulation in small Hilbert space (reduced Berkovits formulation) can be obtained by fixing gauge freedom generated by \(\eta\). The point is that we can use \(\xi\); a line integral of superconformal ghost \(\xi(z)\) [3].

3. The \(A_\infty\) formulation

The \(A_\infty\) formulation [2] is based on the small Hilbert space of the superconformal ghosts. The action and the gauge transformation are given by the same set of multi-string products \(\{M_n\}_{n \in \mathbb{N}}\):

\[
S_{\infty}[\Psi_A] = \frac{1}{2} \omega(\Psi_A, Q \Psi_A) + \frac{1}{3} \omega(\Psi_A, M_2(\Psi_A, \Psi_A)) + \frac{1}{4} \omega(\Psi_A, M_3(\Psi_A, \Psi_A, \Psi_A)) + \cdots
\]

where \(M_1 = Q\). The gauge invariance of the action follows from \(A_\infty\) relations

\[
0 = Q^2 \alpha
\]

where \(a, b, c, d, \ldots\) are arbitrary string fields. If \(M_2\) is associative, we can set \(M_{n \geq 3} = 0\). However, such a choice leads to divergent results (Witten’s original theory).

In the \(A_\infty\) formulation, the singularity in \(A_\infty\) takes the Wess-Zumino-Witten-like form takes the Wess-Zumino-Witten-like form

\[
S_{\infty}[\Psi_A] = -\int_0^1 dt \left< A_1(t), Q A_B(t) \right>
\]

where \(A_1(t)\) and \(A_2(t)\) satisfies

\[
\eta A_1(t) = A_1(t)A_1(t), \quad \delta A_1(t) = \eta A_1(t) - A_1(t)A_1(t) + A_1(t)A_1(t).
\]

The expression of \(A_2(t)\) and \(A_1(t)\) to the second order is

\[
A_2(t) = \Psi_A(t) + \frac{1}{3} \xi(\Psi_A(t))\Psi_A(t)
\]

where \(\Psi_A(0) = 0\) and \(\Psi_A(1) = \Psi_A\). Equating \(A_2\) and \(B_2\), we obtain a field redefinition between the reduced Berkovits formulation and the \(A_\infty\) formulation.

We also uplift the \(A_\infty\) formulation to the large Hilbert space, and find its relation to the Berkovits formulation.

4. Main results of our paper

In our paper, we transform the action of the \(A_\infty\) formulation into the same form as the Berkovits formulation.

\[
S_{\infty}[\Psi_A] = -\int_0^1 dt \left< A_1(t), Q A_B(t) \right>
\]

where \(A_1(t)\) and \(A_2(t)\) satisfies

\[
\eta A_1(t) = A_1(t)A_1(t), \quad \delta A_1(t) = \eta A_1(t) - A_1(t)A_1(t) + A_1(t)A_1(t).
\]

Equating \(A_2\) and \(B_2\), we obtain a field redefinition between the reduced Berkovits formulation and the \(A_\infty\) formulation.

5. Conclusion

We can understand the Berkovits formulation and the \(A_\infty\) formulation as different parametrizations of \(A_1, A_2\).

6. Future directions

Recently, an action of open superstring field theory including the Ramond sector is constructed [4]. They started with the Berkovits action, and coupled it to the Ramond string field. In our next paper [5], we construct an \(A_\infty\) action with Ramond sector, and show its equivalence with [4].


