Double dualization of twisted chiral and another GLSM for five-branes of codimension two

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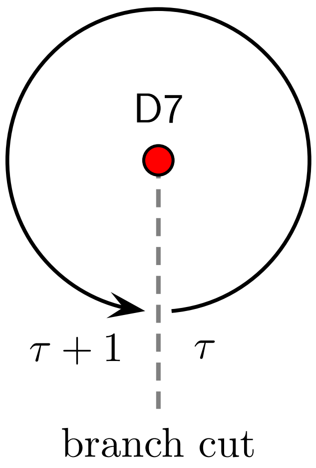
1. 5-branes of codim-2

D7-brane in 10D type IIB string is coupled to

$$\tau(z) \equiv C + \mathrm{i}e^{-\phi} = \frac{\vartheta}{2\pi} + \frac{\mathrm{i}}{2\pi} \log \frac{\Lambda}{\rho}$$

magnetic “charge” (monodromy) : $\tau \to \tau + 1$



feature of τ $\rightarrow$ F-theory

Vafa (1996)

Explore branes of codim-2 :

codim-2 branes	D7	defect NS5	KK-vortex
D	10B	8 (+ T^2)	8 (+ T^2)
coupled to	$\tau = C + \mathrm{i}e^{-\phi}$	$\tau_K =$ Kähler str. of T^2	$\tau_C =$ complex str. of T^2
monodromy	$SL(2,\mathbb{Z})_S$	$SL(2,\mathbb{Z})_K$	$SL(2,\mathbb{Z})_C$
pair	NS7 ($[0,1]$7 or 7_3)	5^2_2	KK-vortex′

Approaches : many works of various kinds

- Mass / tension
- Supergravity
- Worldvolume
- Double field theory (DFT)
- Group theory
- String sigma model ✔

Investigate quantum corrections to background geometry in the UV level

$\rightarrow$ Gauged linear sigma model (GLSM)

In our previous works, we have already understood the classical background geometry.

We have also investigated the worldsheet instanton corrections along S^1 in T^2.

How can we obtained the full corrections along T^2 ?

$\rightarrow$ Go back to the constituents of the corrections in the GLSM.

2. Formulation

2D $\mathcal{N} = (2, 2)$ superfield formalism

$$e^{\int \vartheta F}$$

dynamical theta-angle in 2D

$$e^{\int \vartheta F}$$ is provided by twisted superpotential $$e^{\int \Theta \Sigma}$$

$\Theta, \Sigma = \overline{D}_+ D_- V$: twisted chiral

ϑ, as imaginary part of Θ, receives corrections by topological Chern class.

This is applied to worldsheet instanton corrections of 5-branes of codim-3.

ϑ describes $\begin{cases} \boldsymbol{S}^1 \text{ transverse to H-monopole (NS5)} \\ \text{dual } \boldsymbol{S}^1 \text{ of Taub-NUT (KK-monopole)} \end{cases}$,

and topological Chern class provides quantum corrections as $\begin{cases} \text{KK-modes} \\ \text{winding modes} \end{cases}$.

Tong (2002); Harvey, Jensen (2005); Okuyama (2005)

Apply this to gauge theory for 5-brane of codim-2 with $\underset{\vartheta}{\boldsymbol{S}^1} \times \underset{\varphi}{\boldsymbol{S}^1} = \boldsymbol{T}^2$.

We study $U(1) \times U(1)$ gauge theory $$\mathrm{e}^{\int \vartheta F + \int \varphi \widehat{F}}$$

This is provided by the same twisted chiral (to modify our previous model)

$$\mathrm{e}^{\int \Theta \Sigma + \mathrm{i} \int \Theta \widehat{\Sigma}}$$

Duality transformation :

$$\mathscr{L} = -\frac{1}{2g^2}(\partial_m \vartheta)^2 + \epsilon^{mn}(\partial_m \vartheta)(\partial_n \gamma)$$

- integrate out γ : $\mathscr{L} = -\frac{1}{2g^2}(\partial_m \vartheta)^2$
- integrate out ϑ : $\mathscr{L} = -\frac{g^2}{2}(\partial_m \gamma)^2$

It is well known if only ϑ in Θ is dualized.

Roček, Verlinde (1991); Hori, Vafa (2000)

However, how to dualize both ϑ and φ in Θ ?

Duality transformation by reducible superfield :

$$\mathscr{L} = \int d^4\theta \Big(-\frac{1}{g^2}|R|^2 - 2(R+\overline{R})V - 2i(R-\overline{R})\widehat{V} - R\,L - \overline{R}\,\overline{L} \Big)$$

- integrate out L : $\mathscr{L} = -\frac{1}{g^2}\int d^4\theta\,|\Theta|^2 - \int d^2\widetilde{\theta}\,\Theta(\Sigma + i\widehat{\Sigma}) + \text{h.c.}$
- integrate out R : $\mathscr{L} = +g^2 \int d^4\theta\,\big|L + 2(V + i\widehat{V})\big|^2$

L : twisted linear supefield $0 = \overline{D}_+ D_- L$

$L = X + \overline{W} + Y$

X, W : chiral, Y : twisted chiral

Grisaru, Massar, Sevrin, Troost (1998); TK (2015)

Role of X, W, Y :

$$\mathrm{Im}\,(X + \overline{W}) = \mathrm{Im}\,\Theta \quad \xleftrightarrow{\text{dual}} \quad \mathrm{Im}\,(X - \overline{W})$$

$$\mathrm{Re}\,(X + \overline{W}) = \mathrm{Re}\,\Theta \quad \xleftrightarrow{\text{dual}} \quad \mathrm{Re}\,(X - \overline{W})$$

Y : link LHS with RHS

Different from the duality transformation by irreducible superfields,

we can dualize real/imaginary part of Θ without any disturbance !

$L = X + \overline{W} + Y$ carries both original and dual fields $\rightarrow$ "doubled" GLSM

“Doubled” GLSM for 5-branes of codim-2 $\mathscr{L} = \mathscr{L}_{\text{gauge}} + \mathscr{L}_{\text{CHM}} + \mathscr{L}_{\text{NHM}}$:

$$\mathscr{L}_{\text{gauge}} = \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e^2}(-|\Sigma|^2 + |\Phi|^2) + \frac{1}{\widehat{e}^2}(-|\widehat{\Sigma}|^2 + |\widehat{\Phi}|^2) \Big\}$$

$$\mathscr{L}_{\text{CHM}} = \int \mathrm{d}^4\theta \, \Big\{ |Q|^2 \mathrm{e}^{+2V} + |\widetilde{Q}|^2 \mathrm{e}^{-2V} \Big\} - \int \mathrm{d}^2\theta \, \widetilde{Q}\Phi Q + \text{h.c.}$$
$$+ \int \mathrm{d}^4\theta \, \Big\{ |P|^2 \mathrm{e}^{+2\widehat{V}} + |\widetilde{P}|^2 \mathrm{e}^{-2\widehat{V}} \Big\} - \int \mathrm{d}^2\theta \, \widetilde{P}\widehat{\Phi} P + \text{h.c.}$$

$$\mathscr{L}_{\text{NHM}} = \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{g^2}|\Psi|^2 + g^2 \big| L + 2(V + \mathrm{i}\widehat{V}) \big|^2 \Big\}$$
$$+ \Big\{ \int \mathrm{d}^2\theta \, (s - \Psi)\Phi + \int \mathrm{d}^2\widetilde{\theta} \, t\Sigma + \text{h.c.} \Big\} + \epsilon^{mn}\partial_m(\vartheta A_n)$$
$$+ \Big\{ \int \mathrm{d}^2\theta \, (\widehat{s} - \Psi)\widehat{\Phi} + \int \mathrm{d}^2\widetilde{\theta} \, \widehat{t}\widehat{\Sigma} + \text{h.c.} \Big\} + \epsilon^{mn}\partial_m(\varphi \widehat{A}_n)$$

Our previous GLSM has a prepotential C of chiral Ψ (i.e., $\Psi = \overline{D}_+ \overline{D}_- C$) TK, Sasaki (2013)

$\rightarrow$ quite complicated to analyze

3. Worldsheet instantons

Intrinsic part :

$$S = \int d^2\sigma \Big\{ \frac{1}{2e^2} F_{12}^2 + |D_m q|^2 + \frac{e^2}{2}(|q|^2 - t)^2 + i\vartheta F_{12} + \frac{1}{2\widehat{e}^2} \widehat{F}_{12}^2 + |D_m p|^2 + \frac{\widehat{e}^2}{2}(|p|^2 - \widehat{t})^2 + i\varphi \widehat{F}_{12} \Big\} + \ldots$$

$$\geq |\vec{t}|^2 \sqrt{|\vec{Q}|^2} - i\vec{\vartheta} \cdot \vec{Q}$$

$$\vec{\vartheta} = (\vartheta, \varphi)\,, \qquad \vec{t} = (t, \widehat{t})\,, \qquad \vec{Q} = \Big(-\int d^2\sigma\, F_{12}\,, -\int d^2\sigma\, \widehat{F}_{12} \Big)$$

Chern numbers

BPS eqs are given as Abrikosov-Nielsen-Olesen (ANO) vortex eq.

$$F_{12} \mp e^2(|q|^2 - t) = 0\,, \qquad (D_1 \pm iD_2)q = 0$$

- Doubled NLSM

$$\mathscr{L} = -\frac{H}{2}\big\{(\partial_m x)^2 + (\partial_m y)^2 + (\partial_m \varphi)^2 + (\partial_m \vartheta)^2\big\} + \Omega\epsilon^{mn}(\partial_m \varphi)(\partial_n \vartheta)$$
$$+\epsilon^{mn}(\partial_m \varphi)(\partial_n \widetilde{\gamma}) + \epsilon^{mn}(\partial_m \vartheta)(\partial_m \gamma)$$

provides string sigma model for defect NS5, KK-vortex, and 5^2_2 (skip)

with $H = \log \frac{\Lambda}{\rho}, \quad \Omega = \arctan\left(\frac{y}{x}\right), \quad \rho = \sqrt{x^2 + y^2}$

- Worldsheet instantons originate from ANO vortices

$$H \;\to\; H + \sum_{(n,m)\neq(0,0)} \mathrm{e}^{in\vartheta}\mathrm{e}^{im\varphi}\mathrm{e}^{-\rho\sqrt{n^2+m^2}}$$

Witten (1993); Morrison, Plesser (1994); Schroers (1996)

also derived in DFT : Shiozawa's poster

Worldsheet instantons on

defect NS5 : point-like instantons, i.e., small instantons arrayed along T^2

$\to$ deforms background geometry

$\to$ KK-modes along T^2

Witten (1996); Tong (2002)

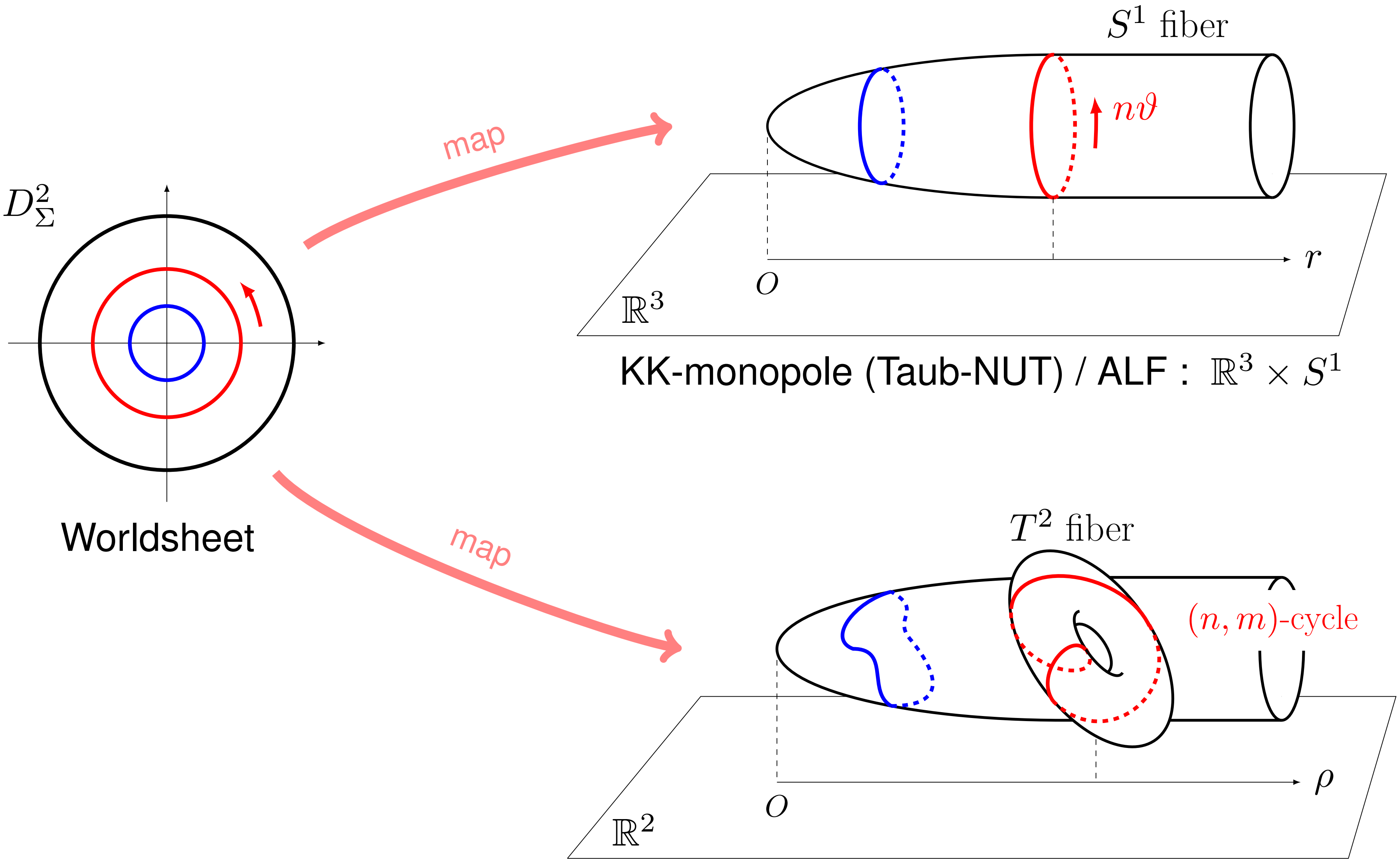
KK / 5^2_2 : disk instantons

$\to$ does not appear as deformation of background geometry

$\to$ Winding modes along T^2

Gregory, Harvey, Moore (1997); Tong (2002); Harvey, Jensen (2005); Okuyama (2005); TK, Sasaki (2013)

Worldsheet instantons as disk instantons



KK-monopole (Taub-NUT) / ALF : $\mathbb{R}^3 \times S^1$

KK-vortex / ALG : $\mathbb{R}^2 \times T^2$

4. Summary

- We captured string worldsheet instanton corrections

  via ANO vortices in GLSM.

- These instanton corrections are understood as

  corrections by KK-modes and/or winding modes along T^2.

To realize above, we developed duality transformations by reducible superfield.

Thanks

Appendix

A. Duality transformations

$$e^{\int \Theta \Sigma + \mathrm{i} \int \Theta \widehat{\Sigma}}$$

technically disturbs the standard T-duality (Hori-Vafa) transformation :

$$\begin{aligned} & \Theta + \overline{\Theta} \quad \Leftrightarrow \quad -(\Gamma + \overline{\Gamma}) - 2V \\ \text{and/or} \quad & \Theta - \overline{\Theta} \quad \Leftrightarrow \quad -(\Gamma - \overline{\Gamma}) + 2\mathrm{i}\widehat{V} \end{aligned}$$

by parity transform along x^{-}-direction

Duality transformation by irreducible superfields :

Roček, Verlinde (1991); Hori, Vafa (2000)

$$\mathscr{L} \ = \ \int \mathrm{d}^4\theta \, \Big(- \frac{1}{2g^2} B^2 - 2BV - 2(\Gamma + \overline{\Gamma}) B \Big)$$

- integrate out Γ : $\quad \mathscr{L} \ = \ - \dfrac{1}{g^2} \displaystyle\int \mathrm{d}^4\theta \, |\Theta|^2 - \int \mathrm{d}^2\widetilde{\theta} \, \Theta \Sigma + \mathrm{h.c.}$
- integrate out B : $\quad \mathscr{L} \ = \ + \dfrac{g^2}{2} \displaystyle\int \mathrm{d}^4\theta \, (\Gamma + \overline{\Gamma} + 2V)^2$

How to generate $\Theta(\Sigma + \mathrm{i}\widehat{\Sigma})$?

Available duality transformation w/ irreducible superfields ?

Roček, Verlinde (1991); Hori, Vafa (2000)

$$-\frac{1}{g^2}\int \mathrm{d}^4\theta\, |\Theta|^2 - \int \mathrm{d}^2\widetilde{\theta}\, \Theta(\Sigma + \mathrm{i}\widehat{\Sigma}) + \mathrm{h.c.}$$

dualize imaginary part $\downarrow$ twisted chiral $\Theta \to$ chiral Γ

$$+\frac{g^2}{2}\int \mathrm{d}^4\theta\, (\Gamma + \overline{\Gamma} + 2(V + \mathrm{i}\widehat{V}))^2$$

dualize real part $\downarrow$ chiral $\Gamma \to$ twisted chiral

??

use reducible superfields

Grisaru, Massar, Sevrin, Troost (1998); TK (2015)

B. **Another GLSM for 5-branes of codim-2**

Doubled NLSM

$$\mathscr{L} = -\frac{H}{2}\big\{(\partial_m x)^2 + (\partial_m y)^2 + (\partial_m \varphi)^2 + (\partial_m \vartheta)^2\big\} + \Omega \epsilon^{mn}(\partial_m \varphi)(\partial_n \vartheta) + \epsilon^{mn}(\partial_m \varphi)(\partial_n \widetilde{\gamma}) + \epsilon^{mn}(\partial_m \vartheta)(\partial_m \gamma)$$

provides string sigma model for defect NS5, KK-vortex, and 5^2_2.

dNS5 : $$\mathscr{L} = -\frac{H}{2}\big\{(\partial_m x)^2 + (\partial_m y)^2 + (\partial_m \varphi)^2 + (\partial_m \vartheta)^2\big\} + \Omega \epsilon^{mn}(\partial_m \varphi)(\partial_n \vartheta)$$

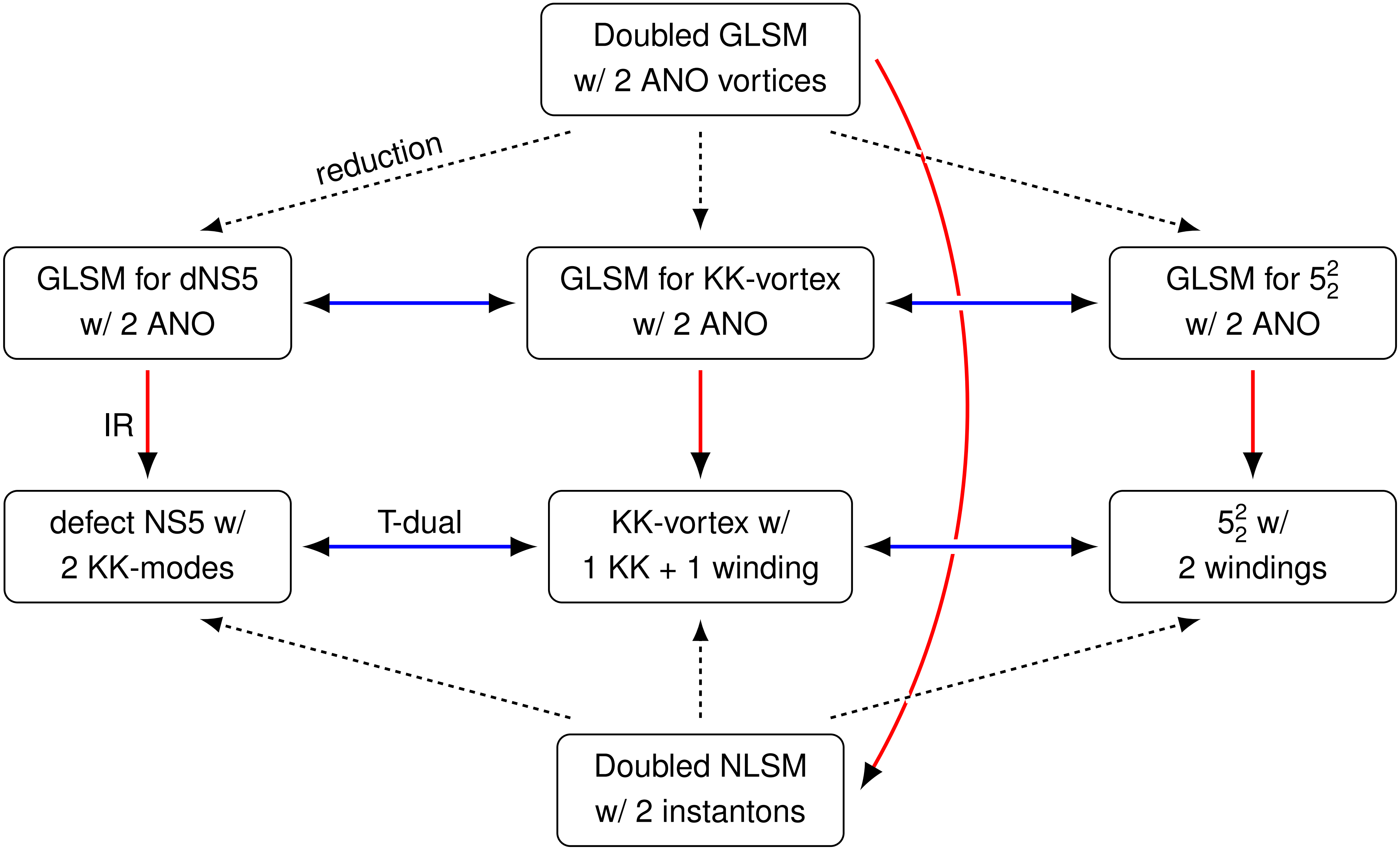
KK-v : $$\mathscr{L} = -\frac{H}{2}\big\{(\partial_m x)^2 + (\partial_m y)^2 + (\partial_m \varphi)^2\big\} - \frac{1}{2H}(\partial_m \gamma - \Omega \partial_m \varphi)^2$$

5^2_2 : $$\mathscr{L} = -\frac{H}{2}\big\{(\partial_m x)^2 + (\partial_m y)^2\big\} - \frac{H}{K}\big\{(\partial_m \widetilde{\gamma})^2 + (\partial_m \gamma)^2\big\} - \frac{\Omega}{K}\epsilon^{mn}(\partial_m \widetilde{\gamma})(\partial_n \gamma)$$

$$H = \log \frac{\Lambda}{\rho}, \quad \Omega = \arctan\Big(\frac{y}{x}\Big), \quad \rho = \sqrt{x^2 + y^2}, \quad K = H^2 + \Omega^2$$

instanton corrections : $$H \to H + \sum_{(n,m)\neq(0,0)} \mathrm{e}^{\mathrm{i}n\vartheta}\mathrm{e}^{\mathrm{i}m\varphi} K_0\big(\rho\sqrt{n^2+m^2}\big)$$

modified Bessel


Doubled GLSM
w/ 2 ANO vortices
reduction
GLSM for dNS5
w/ 2 ANO
GLSM for KK-vortex
w/ 2 ANO
GLSM for 5^2_2
w/ 2 ANO
IR
defect NS5 w/
2 KK-modes
T-dual
KK-vortex w/
1 KK + 1 winding
5^2_2 w/
2 windings
Doubled NLSM
w/ 2 instantons