Quantum Gravity as an Average

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Overview

Emergence of Geometry

Question: Does gravity emerge from *averaging* of quantum theories?

This could mean either:

- Average over many configurations of the same theory, or
- Ensemble average over many different theories.

The first is well established:

- A typical heavy state is a black hole microstate (complicated)
- Coarse grained state is a black hole geometry (simple).

What about the second?

Gravity as an Average

Evidence: A 2d theory, JT gravity, is a theory of random matrices:

$$Z_{JT}(\beta) \equiv \int_{\partial M = S^1(\beta)} Dg\ e^{-S} \approx \int dH\ p(H)\ \mathrm{Tr}\,(e^{-\beta H})$$

What about more realistic theories of gravity?

An intermediate step would be 3D gravity:

- AdS_3 gravity should be dual to a CFT_2 with $c = \frac{3\ell}{2G} \sim \frac{1}{\hbar}$.

Is it actually an ensemble average of many CFTs?

- Two pieces of evidence support this...

The Spectrum of 3D gravity

First, AdS_3 gravity should have a discrete spectrum.

But the partition function

$$Z_{GR}(T^2) \equiv \int_{\partial M = T^2(\tau)} Dg\ e^{-S} = \sum_{g_o} e^{-cS(g_o)+S^{(1)}(g_0)+\ldots}$$
$$\approx \sum_J \int d\Delta\ \rho_J(\Delta)\ q^{\Delta+J}\bar{q}^{\Delta-J}$$

appears to have a continuous spectrum $\rho_J(\Delta)$.

Perhaps this is because

$$Z_{GR}(T^2) = \langle Z_{\mathcal{C}}(T^2)\rangle \equiv \sum_{\text{CFTs } \mathcal{C}} Z_{\mathcal{C}}(T^2)p(\mathcal{C})?$$

A.M. & Witten, ...

Euclidean Wormholes

Second, a CFT partition function on $\Sigma_1 \cup \Sigma_2$ will factorize.

But AdS_3 gravity has Euclidean wormholes

- Connected solutions M with $\partial M = \Sigma_1 \cup \Sigma_2$.

So it appears that

$$Z_{GR}(\Sigma_1 \cup \Sigma_2) \neq Z_{GR}(\Sigma_1) Z_{GR}(\Sigma_2)$$

Perhaps this is because

$$Z_{GR}(\Sigma_1 \cup \Sigma_2) = \langle Z_{\mathcal{C}}(\Sigma_1) Z_{\mathcal{C}}(\Sigma_2) \rangle \neq \langle Z_{\mathcal{C}}(\Sigma_1) \rangle \langle Z_{\mathcal{C}}(\Sigma_2) \rangle ?$$

Maldacena & Maoz, . . .

Random CFT

Goal: Develop a theory of random CFTs

The data that define a CFT are

- A list of operator dimensions Δ_i and spins J_i
- A list of OPE coefficients C_{ijk}

for primary operators.

To average over this data we must

1. Determine the allowed values of $\{C_{ijk}, \Delta_i, J_i\}$.
2. Fix a probability distribution over these allowed values.
3. Compute ensemble averages and compare to gravity in AdS_3.

This is hard!

A Simpler Case

Instead, we will first consider a simpler class of CFTs

- $c = N$ and $U(1)^N \times U(1)^N$ current algebra

where these can be accomplished.

We will conclude that

$$\langle Z(\Sigma) \rangle = \sum_{g_o} e^{-cS(g_o)+S^{(1)}(g_o)}$$

of an exotic theory of 3D gravity, "$U(1)$ Gravity."

With certain assumptions, we can enumerate all saddles and compute all loop corrections in this theory of gravity.

Plan for Today

- U(1) Gravity and an Average over CFTs
- Wormholes and Statistics of CFTs
- Speculations & Confusions on Pure Gravity

Averaging over CFTs

Warmup: $N = 1$

A single free boson $X(z, \bar{z}) \sim X(z, \bar{z}) + 2\pi R$, with

$$I = \int d^2 z \, \partial X \, \bar{\partial} X$$

has $c = 1$ and $U(1) \times U(1)$ current algebra.

R is a coordinate on the moduli space $\mathcal{M}_1$ of such theories.

The 2-pt function of the marginal operator $\partial X \bar{\partial} X$ determines a metric on $\mathcal{M}_1$:

$$ds^2 = \frac{dR^2}{R^2}$$

We can average over $\mathcal{M}_1$ with this measure

$$\langle \cdot \rangle = \int_1^\infty \frac{dR}{R} (\cdot)$$

N Free Bosons

For N free bosons

$$I = \int d^2z \ \left(G_{pq}\, \partial X^p\, \bar{\partial} X^q + B_{pq}\, dX^p \wedge dX^q \right)$$

the $\{G_{pq}, B_{pq}\}$ are coordinates on Narain's moduli space

$$\mathcal{M}_N = O(N,N,\mathbb{Z}) \backslash O(N,N) / O(N) \times O(N)$$

The metric is the homogeneous one:

$$ds^2 = G^{mn} G^{pq} \left(dG_{mp} dG_{nq} + dB_{mp} dB_{nq} \right)$$

We can average over $m \in \mathcal{M}_N$ with this measure:

$$\langle \cdot \rangle = \frac{1}{Vol(\mathcal{M}_N)} \int_{\mathcal{M}_N} d\mu \, (\cdot)$$

Siegel-Weil Formula

The average partition function on a genus g surface Σ is

$$\langle Z(m,\Omega)\rangle = \frac{1}{(\det \Delta_0)^{N/2}} E_{N/2}(\Omega)$$

where Ω is the period matrix, and the Eisenstein series

$$E_{N/2}(\Omega) = \sum_{\gamma \in Sp(2g,\mathbb{Z})/P} (\det \operatorname{Im} \gamma\Omega)^{N/2} = \sum_{(C,D)=1} \left(\frac{\det \operatorname{Im} \Omega}{|C\Omega + D|^2} \right)^{N/2}$$

is a sum over the modular group $Sp(2g,\mathbb{Z})$ of Σ

$$\gamma\Omega = (A\Omega + B)(C\Omega + D)^{-1}, \quad \text{with} \quad \gamma = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in Sp(2g,\mathbb{Z})$$

Bulk Gravity Interpretation

An Exotic Theory of Gravity

So what is the bulk gravity theory?

The boundary theory has $U(1)^{2N}$ global symmetry, so our bulk theory should have a corresponding gauge symmetry.

The simplest possibility is $U(1)^{2N}$ Chern-Simons theory:

$$S_{bulk} = \sum_{p=1}^{N} \int A^p \wedge dB^p$$

This will lead to a $U(1)^{2N}$ current algebra.

Our boundary theory does not have an independent stress tensor, so we will not include a separate Einstein-Hilbert term.

- A "composite" graviton built from gauge fields.

Perturbative Expansion

We view $U(1)^{2N}$ Chern-Simons theory as the "perturbative part" of the bulk gravity theory.

The parameter $N \sim \hbar^{-1}$ controls quantum corrections.

So our exotic theory has $\mathcal{O}(1/\hbar)$ perturbative fields

- No distinction between "classical" and "loop" effects.

But $U(1)$ Chern-Simons theory is a free theory.

- We can just compute it on any 3-manifold.

Perturbative Expansion 2

The simplest bulk geometry Y with $\partial Y = \Sigma$ is a handlebody:



The contribution from $U(1)^{2N}$ Chern-Simons fields fluctuating around the trivial connection on Y is

$$Z^{CS}(Y) = \sqrt{\frac{\det \operatorname{Im} \Omega}{\det \Delta_0}}$$

one of the terms in $\langle Z(m, \Omega) \rangle$!

It is a $U(1) \times U(1)$ current algebra block on Σ.

Non-Perturbative Corrections

We must augment this with a "sum over geometries."

This is the Eisenstein series:

$$E_{N/2}(\Omega) = \sum_{\gamma \in Sp(2g,\mathbb{Z})/P} (\det \operatorname{Im} \gamma\Omega)^{N/2} \sim \sum_{g_o} e^{-\frac{1}{\hbar} S(g_o)}$$

The coset $Sp(2g,\mathbb{Z})/P$ labels the handlebodies Y which fill in Σ:



The choice of γ labels which cycles are contractible in the bulk.

Sum over Topologies

So our theory of gravity includes a sum over handlebodies:

- An all-genus version of the "black hole Farey tail."

The difference is that the sum over geometries is computable and convergent.

It is one loop exact, and exactly equals an average over a space of CFTs:

$$\langle Z(\Sigma) \rangle = \sum_{g_o} e^{-cS(g_o)+S^{(1)}(g_o)}$$

This works at all genus.

In a sense, it is an exact holographic duality.

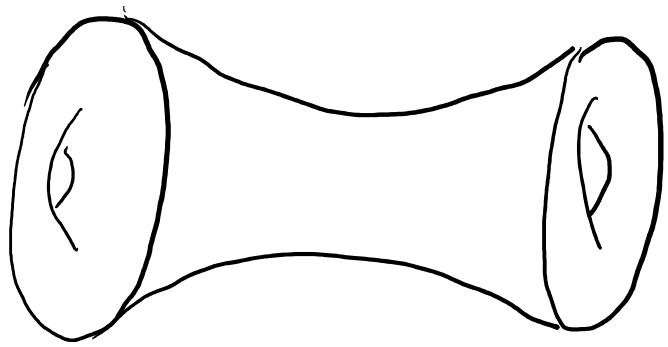
Disconnected Geometries

We can also compute correlation functions:

$$\langle Z(\Sigma \cup \Sigma') \rangle \neq \langle Z(\Sigma) \rangle \langle Z(\Sigma') \rangle$$

which do not factorize.

This is interpreted as coming from wormholes:



These give (exponentially) small but non-zero contributions to statistics.

Spectral Form Factor

The sum over $T^2 \cup T^2$ wormholes gives a two point function:

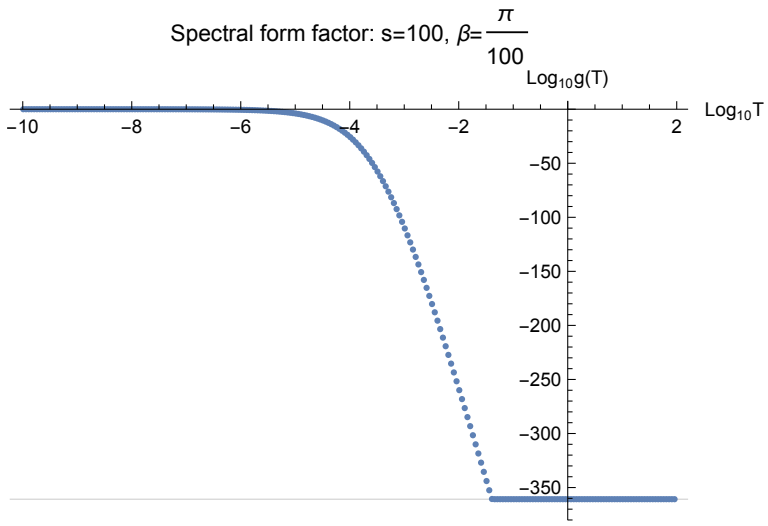
$$\langle \rho^p(\Delta_1, j_1)\rho^p(\Delta_2, j_2)\rangle$$

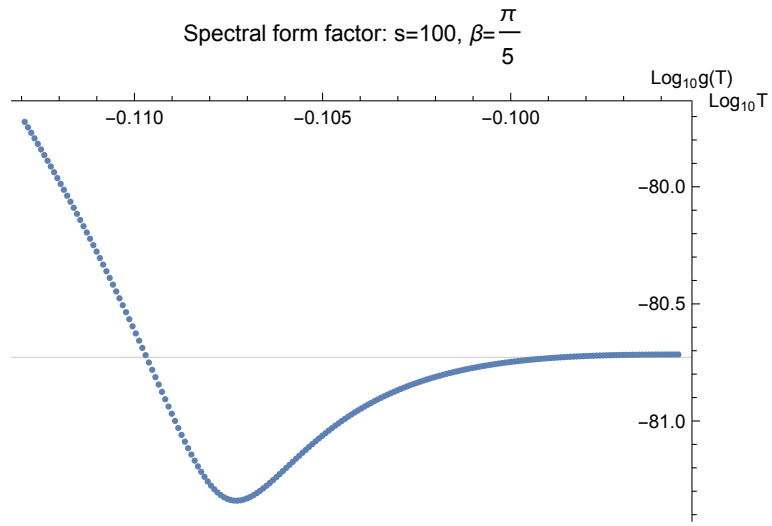
which exhibits neither Poisson nor random matrix statistics.

The spectral form factor

$$\langle Z(\beta + it)Z(\beta - it)\rangle \to Z(2\beta)$$

has a plateau (reflecting discreteness) but no linear ramp:





w/ S. Collier

Other Ensembles of CFTs?

The real drawback of the Narain theories is that they are too integrable, since $c = c_{currents}$.

Ensemble averaging is harmless only in self-averaging theories.

Can we find other, more chaotic examples?

Optimistically, pure gravity should be an average over all CFTs:

$$Z_{GR}(\Sigma) \equiv \int_{\partial M=\Sigma} Dg\ e^{-cS} \stackrel{?}{=} \sum\!\!\!\!\!\!\int_{CFTs\ \mathcal{C}} \frac{1}{|Aut(\mathcal{C})|} Z_{\mathcal{C}}(\Sigma)$$

This is difficult to understand precisely.

It would imply that a typical large c CFT is sparse (or extremal):

- i.e. that $\Delta_1 = \mathcal{O}(c)$?

The CAD$83,000 Question

Where does ensemble averaging come from?

Recall: coarse graining over (extremal) black hole microstates gives an emergent $AdS \times \mathcal{M}$ near horizon region.

- Coarse graining over states in a single higher dimensional, UV complete theory.

Low dimensional AdS theories describe near-horizon dynamics of black holes/branes.

Perhaps ensemble averaging in low-dimensional AdS is a remnant of this coarse graining over BH microstates in the parent theory.

Thank you!