Black-brane Solutions in Heterotic String Theroy

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Introduction

Cobordism conjecture:
In a consistent quantum gravity, it is said that for every possible charge, there exists an object carrying that charge. Recently, this hypothesis has been refined by the cobordism conjecture [1] and made applicable to more general charges.

Charges:
In heterotic string theory, there are some charges classified by the homotopy groups of the gauge groups $(E_8 \times E_8) \rtimes \mathbb{Z}_2$ or $\mathrm{Spin}(32)/\mathbb{Z}_2$, especially we focus on

$$\pi_{7-4}((E_8 \times E_8) \rtimes \mathbb{Z}_2) = \mathbb{Z} \times \mathbb{Z}, \qquad \pi_{7-0}(\mathrm{Spin}(32)/\mathbb{Z}_2) = \mathbb{Z}. \tag{1}$$

According to the conjecture, branes carrying such charges must exist in this theory [2]. Analogous to the magnetic monopole in pure 4d $U(1)$ Maxwell theory, these branes are codimension 5 and 9 objects respectively (table 1 and figure 1).

Table.1:

theory	codimension	surrounding sphere	equator	non-triviality
4d $U(1)$ Maxwell	3	S^2	S^1	$\pi_1(U(1))$
10d heterotic G	$9-p$	S^{8-p}	S^{7-p}	$\pi_{7-p}(G)$

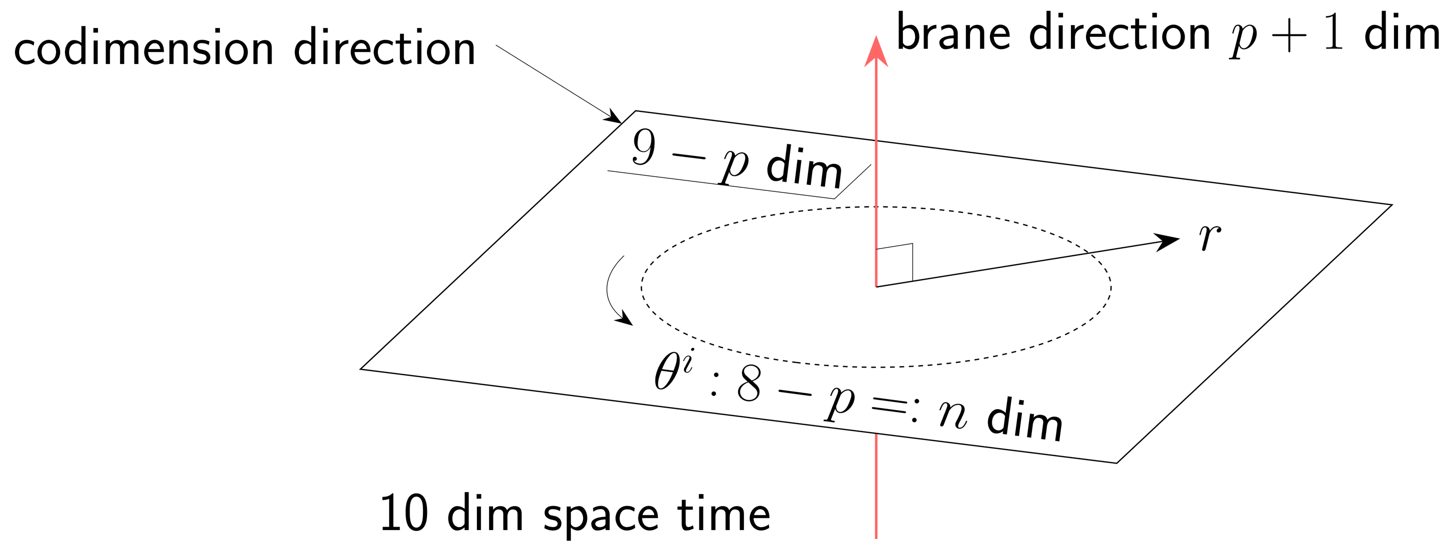

Figure.1:

Purpose:
As long as quantum corrections are small, if non-trivial classical solutions exist, the corresponding object should also exist in the quantum theory.
The purpose of this research is to provide evidence for the existence of branes predicted by the cobordism conjecture [2] by giving black brane solutions.

Ansatz and Equations of Motion

Action:
Assume that the quantum corrections are small and the supergravity action is reliable. For our purpose, we have to solve the equations of motion derived from

$$S_{\text{het}} = \int \mathrm{d}^{10}x \sqrt{G} e^{-2\Phi} \left(\mathcal{R} + 4(\partial\Phi)^2 - \frac{\alpha'}{4} \mathrm{tr}'(F_{\mu\nu}F^{\mu\nu}) \right) + \dots \tag{2}$$

Especially, we want a configuration carrying the charge (1) measured by

$$\int_{S^n} \mathrm{tr}(F^{\frac{n}{2}}), \tag{3}$$

i.e., the gauge field is a non-trivial configuration on S^n classified by $\pi_{n-1}(G)$.

Ansatz:
For simplicity, we assume that the solution has following properties:

- static
- spherical
- flatness along the brane
- extremal

Under these ansatzes, the fields can be taken in the following form

$$\mathrm{d}s^2 = \mathrm{d}r^2 + R(r)^2 \mathrm{d}\theta^2_{n-\text{sphere}} + (\mathrm{d}x^\alpha \mathrm{d}x_\alpha)_{\text{brane direction}} \tag{4}$$
$$\Phi = \Phi(r) \tag{5}$$
$$A = A_{\theta^i}(\theta)\mathrm{d}\theta^i. \tag{6}$$

Since A does not have the r and x^α component like the $U(1)$ magnetic monopole, it is only important that the kinetic term of A in (2) takes the following form:

$$\frac{1}{2}\mathrm{tr}(F_{\theta_1\theta_2}F^{\theta_1\theta_2}) = \frac{\mathsf{C}}{R^4} \quad \longleftarrow \text{constant related to the charge} \tag{7}$$

Equations of motion:
Under these ansatzes, the equation of motion becomes

$$0 = \sigma'' - 2\left(\Phi - \frac{n}{2}\sigma\right)'\sigma' - \frac{n-1}{l_0^2}(e^{-2\sigma} - e^{-4\sigma}), \tag{8}$$
$$0 = \Phi'' - 2\left(\Phi - \frac{n}{2}\sigma\right)'\Phi' + \frac{n(n-1)}{4l_0^2}e^{-4\sigma}, \tag{9}$$

Asymptotic Behavior

Throat region:
The equation (8) has a trivial solution

$$\sigma = 0 \Leftrightarrow R(r) = l_0. \tag{11}$$

Also equation (8) and (9) can be solved for Φ

$$\Phi = -\frac{n(n-1)}{\sqrt{8\alpha'\mathsf{C}}} r + (\text{const}). \tag{12}$$

Thus, the brane solution has a throat region of radius l_0 as shown in figure 2 and linear dilaton near the horizon.

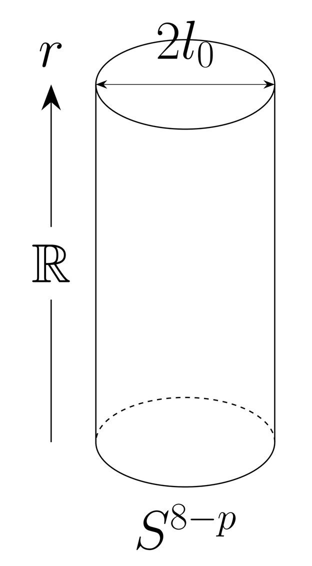

Figure.2: Throat solution

Asymptotic flat region:
On the other hand, since σ is large in the region far from the brane, we can ignore $e^{-4\sigma}$ term in the equation (8) and (9). Then, we can find the following solutions:

$$R = r \tag{13}$$
$$\Phi = (\text{const}) \tag{14}$$

Thus, the brane solution has an asymptotically flat region far from the brane.

Whole:

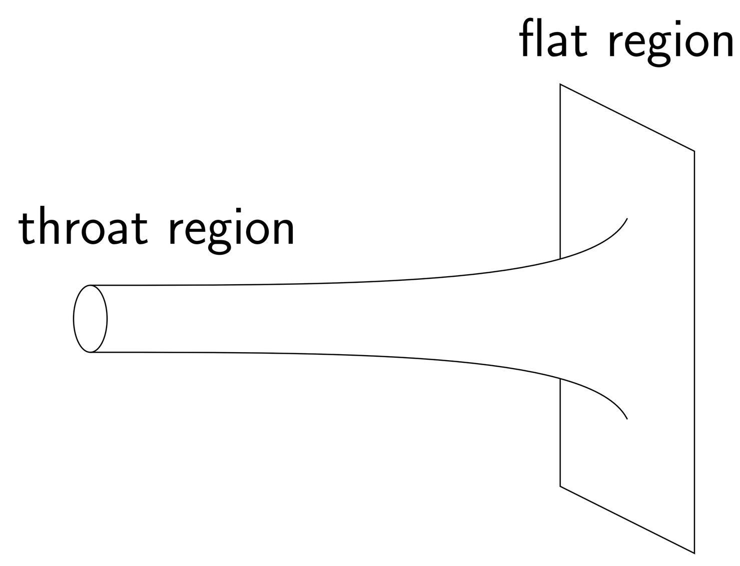

Figure.3: Codimension direction of the black-brane solution.

Therefore, we can deduce that the solution which smoothly connects the throat region and the asymptotic region is the desired solution (see figure 3). If we find such solutions, the purpose is achieved.

Results

For $n = 4, 8$, we have not yet obtained analytical solutions yet, but we constructed solutions using numerical calculations. We used the initial conditions that slightly perturb the throat solution. For example, the case of $n = 4$ solution is shown in figure 4 and 5.

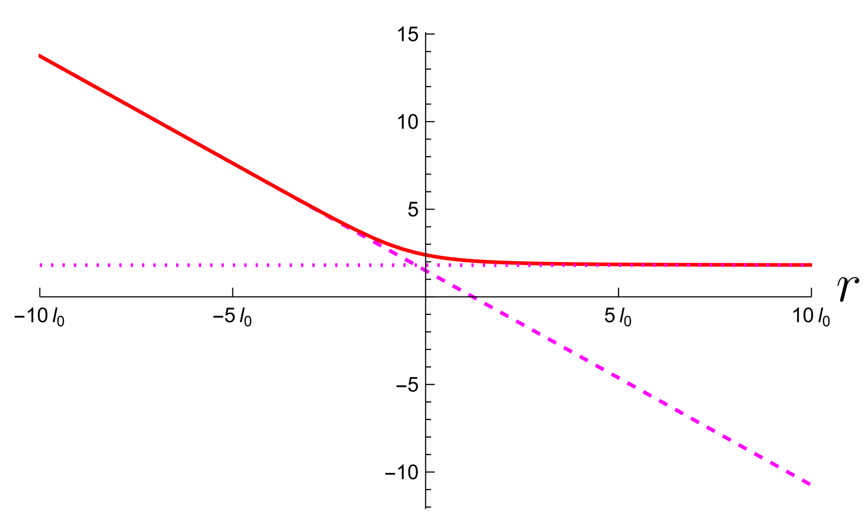

Figure.4: Dilaton $\Phi(r)$

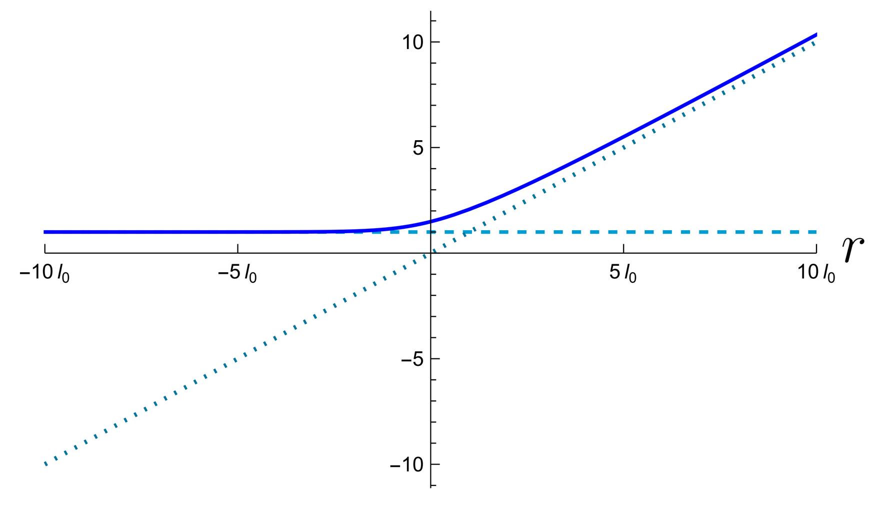

Figure.5: Radius $\frac{R(r)}{l_0}$

- Dilaton configuration $\Phi(r)$.
- Linear dilaton $-\sqrt{\frac{n(n-1)}{8}}(r/l_0) + \Phi_0$.
- $\Phi_\infty \sim \Phi(40 l_0)$.
- Radius function $R(r)/l_0$.
- Throat radius 1.
- $R(r)/l_0 = r/l_0$.

We checked that for the case of $n = 2$, which has an analytical solution, our numerical calculations reproduce the analytical solution. Therefore we conclude that the black brane solutions with the desired properties have been obtained.

References

[1] Jacob McNamara and Cumrun Vafa. Cobordism Classes and the Swampland. 9 2019.
[2] Justin Kaidi, Kantaro Ohmori, Yuji Tachikawa, and Kazuya Yonekura. Nonsupersymmetric Heterotic Branes. *Phys. Rev. Lett.*, Vol 131, No 12, p. 121601, 2023.

← where,

$$l_0 = \sqrt{\alpha'\mathsf{C}/n(n-1)}, \quad \sigma = \log(R/l_0). \tag{10}$$