

Bridging two semiclassical confinement mechanisms: monopole and center vortex

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弦の理論と場の理論2024

August 8, 2024

[arXiv:2405.12402](https://arxiv.org/abs/2405.12402) in collaboration with Yuya Tanizaki (YITP, Kyoto U.)

Confinement mechanism(s)

Two promising scenarios for quark confinement: monopole and center vortex

Dual superconductor picture (monopole condensation)

[Nambu '74, 't Hooft '75, Mandelstam '76,...]

monopole condensation

⇒ dual Meissner effect

⇒ linear $q\bar{q}$ -potential



Center-vortex proliferation ['t Hooft '78, ...]

A diagram showing a blue Wilson loop (a horizontal ellipse) intersecting a vertical red line representing a center vortex. The text "Wilson loop" is written in blue above the loop, and "center vortex" is written in brown below the red line.

$$= e^{\frac{2\pi i}{N}}$$

Center vortex: rotating Wilson loop by $e^{\frac{2\pi i}{N}}$.
Proliferation ⇒ $\langle W(C) \rangle \sim e^{-\sigma (\text{Area})}$

cf.) restoration of $\mathbb{Z}_N^{[1]}$: proliferation of co-dim-2 defects

Connection between them? [Ambjørn-Giedt-Greensite '99, Engelhardt-Reinhard '99, Cornwall '99]
“monopole as junction of center vortices”

Summary

Quark confiners: monopole and center vortex

Weak-coupling semiclassical realizations:

3d monopole semiclassics

[Ünsal '07, Ünsal-Yaffe '08,...]

SU(N) Yang-Mills on $\mathbb{R}^3 \times S^1$ with
“center-stabilizing deformation”

$\Rightarrow$ **confinement by 3d monopole gas**

2d center-vortex semiclassics

[Tanizaki-Ünsal '22, ...]

SU(N) Yang-Mills on $\mathbb{R}^2 \times T^2$ with ‘t
Hooft flux

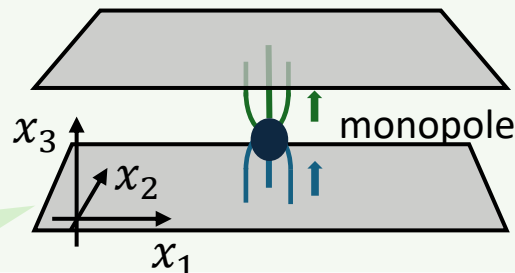
$\Rightarrow$ **confinement by 2d center-vortex gas**



This work: Consider an interpolating setup on $(\mathbb{R}^2 \times S^1) \times S^1$

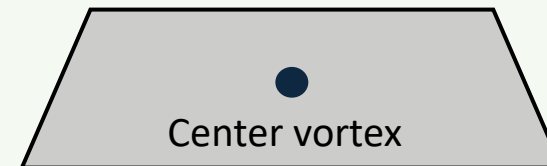
Monopole in $\mathbb{R}^2 \times S^1$

“monopole as junction of
center vortices”



=

Center vortex in 2d



Semiclassical approaches to confinement

Motto: deforming $SU(N)$ YM to **weakly-coupled** theory with **keeping confinement**.

compactification

center-stabilizing deformation
(to avoid deconfinement transition)

3d monopole semiclassics

[Ünsal '07, Ünsal-Yaffe '08,...]

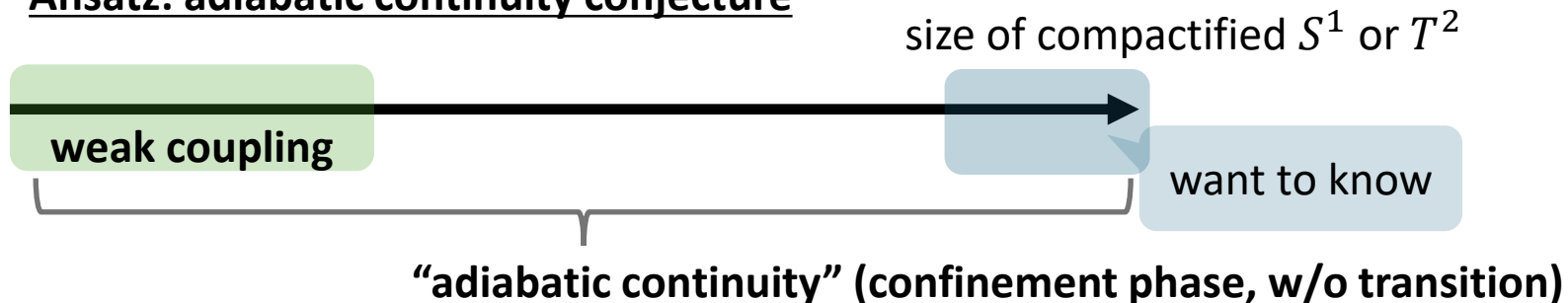
$SU(N)$ Yang-Mills on $\mathbb{R}^3 \times S^1$ with
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2d center-vortex semiclassics

[Tanizaki-Ünsal '22, ...]

$SU(N)$ Yang-Mills on $\mathbb{R}^2 \times T^2$ with ‘t
Hooft flux
 $\Rightarrow$ **confinement by 2d center-vortex gas**

Ansatz: adiabatic continuity conjecture



3d Monopole semiclassics

[Ünsal '07, Ünsal-Yaffe '08,...] (cf. [Davies-Hollowood-Khoze-Mattis '99,...])

- **SU(N) Yang-Mills on $\mathbb{R}^3 \times S^1$ with “center-stabilizing deformation”** [Ünsal-Yaffe '08]:

$$S = S_{YM} + \int d^3x \sum_{n=1}^{[N/2]} a_n |\text{tr}(P^n)|^2$$

Add a potential for Polyakov loop (by hand) to keep center symmetry

⇒ Center symmetry is kept for **small S^1** (, realizing weak-coupling confinement)

- **3d effective theory on $\mathbb{R}^3$**

The Polyakov loop behaves as an adjoint scalar field.

At the center symmetric vacuum, “ $\langle P \rangle \sim C$ ” (up to gauge)

⇒ adjoint higgsing $SU(N) \rightarrow U(1)^{N-1}$

e.g.) clock matrix for $N = 3$

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{2\pi i}{3}} & 0 \\ 0 & 0 & e^{\frac{4\pi i}{3}} \end{pmatrix}$$

3d effective theory = 3d $U(1)^{N-1}$ gauge theory + monopoles

- Polyakov confinement by dilute gas of monopoles (in 3d Abelian gauge theory) [Polyakov '77]

Magnetic Debye screening ⇒ magnetic fluctuations enhanced ⇒ area law $\langle W(C) \rangle \sim e^{-\sigma (\text{Area})}$

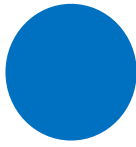
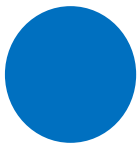
3d Monopole semiclassics (some details)

- **N kinds of monopoles:** $Q_{top} = 1/N$ fractional instantons

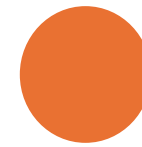
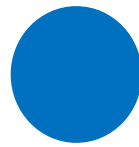
“compactness of adjoint higgs”

[Kraan-van Baal '98] [Lee-Lu '98]

(N-1) BPS monopoles (for $U(1)^{N-1}$) + KK monopole



.....



Magnetic charge: $\vec{\alpha}_1$ $\vec{\alpha}_2$ $\vec{\alpha}_{N-1}$ $\vec{\alpha}_N (= -\vec{\alpha}_1 - \dots - \vec{\alpha}_{N-1})$

- **3d effective theory**

$\vec{\alpha}_1, \dots, \vec{\alpha}_{N-1}$: simple roots

$\vec{\alpha}_N (= -\vec{\alpha}_1 - \dots - \vec{\alpha}_{N-1})$: affine root

3d abelian duality: $U(1)^{N-1}$ gauge field $\rightarrow U(1)^{N-1}$ -valued compact boson $\vec{\sigma}$ ($d\vec{\sigma} = * \vec{f}$)

In terms of $\vec{\sigma}$ (dual photon/magnetic potential), the 3d effective theory is,

$$S = \int d^3x \left[\frac{\# g^2}{L} |d\vec{\sigma}|^2 - \# e^{-\frac{8\pi^2}{Ng^2}} \sum_{i=1, \dots, N} \cos(\vec{\alpha}_i \cdot \vec{\sigma} + \theta/N) \right]$$

Monopole amplitude

$$[\mathcal{M}_i] \sim e^{-\frac{8\pi^2}{Ng^2}} e^{i\vec{\alpha}_i \cdot \vec{\sigma} + i\theta/N}$$

2d center-vortex semiclassics

[Tanizaki-Ünsal '22,] (cf. [Yamazaki-Yonekura '17])

Setup: $SU(N)$ Yang-Mills on $\mathbb{R}^2 \times T^2$ with 't Hooft flux

- **'t Hooft flux for T^2 (or $\mathbb{Z}_N^{[1]}$ background)**

A unit 't Hooft flux $\Leftrightarrow$ choose $g_3(0)g_4(L)g_3^\dagger(L)g_4^\dagger(0) = e^{\frac{2\pi i}{N}}$.

$(g_3(x_4), g_4(x_3))$: transition functions on T^2

Up to gauge, we can take $g_3 = S$, $g_4 = C$ (shift and clock matrices of $SU(N)$).

- **Consequences from 't Hooft-twisted compactification**

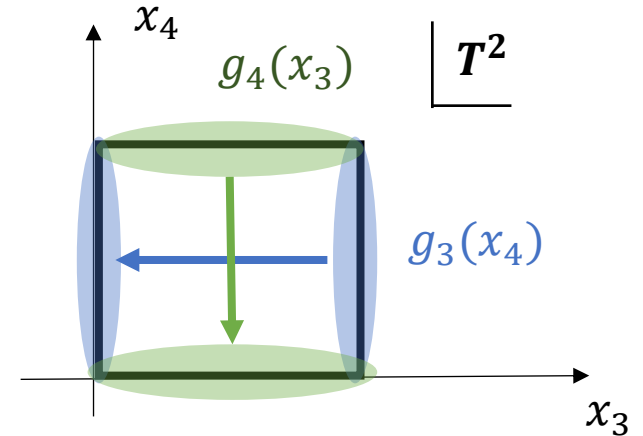
✓ **Center symmetry is kept at small T^2**

Classically, $P_3 = S$ and $P_4 = C \Rightarrow \langle \text{tr } P_3 \rangle = \langle \text{tr } P_4 \rangle = 0$.

✓ **Perturbatively gapped gluons: $O(1/NL)$ KK mass**

✓ **Numerical evidence for center vortex/fractional instantons (as a local solution)**

[Gonzalez-Arroyo-Montero '98, Montero '99,].



$$\begin{cases} a(\vec{x}, x_3 + L, x_4) = g_3^\dagger a g_3 - i g_3^\dagger d g_3 \\ a(\vec{x}, x_3, x_4 + L) = g_4^\dagger a g_4 - i g_4^\dagger d g_4 \end{cases}$$

e.g.) $N = 3$

$$S = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{2\pi i}{3}} & 0 \\ 0 & 0 & e^{\frac{4\pi i}{3}} \end{pmatrix}$$

exists locally,
(not globally if 'regularity' at infinity is imposed)

2d center-vortex semiclassics [Tanizaki-Ünsal '22]

• Dilute gas of center vortices

The center-vortex and anti-center-vortex vertices are:

$$[\mathcal{V}] = K e^{-\frac{8\pi^2}{Ng^2} + i\frac{\theta}{N}}, \quad [\bar{\mathcal{V}}] = K e^{-\frac{8\pi^2}{Ng^2} - i\frac{\theta}{N}}$$

with a dimensionful constant K .

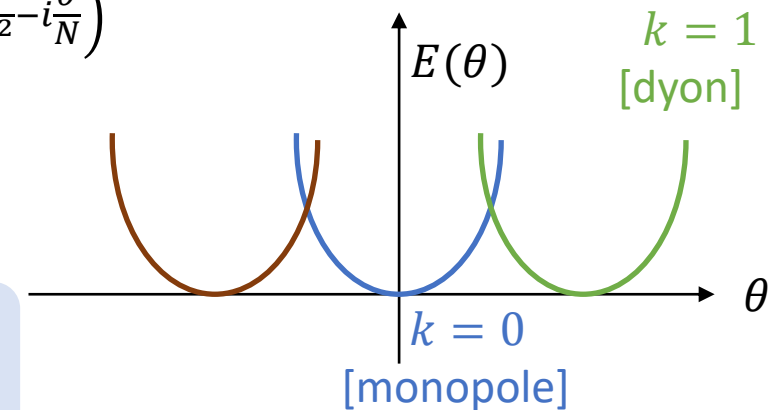
For calculating partition function, we compactify $\mathbb{R}^2$ without 't Hooft flux.
 $\Rightarrow$ total topological charge is constrained $Q_{top} \in \mathbb{Z}$

Then, the dilute gas approximation yields, (only configurations with $Q_{top} \in \mathbb{Z}$ are admitted)

$$\begin{aligned} Z_{2d} &= \sum_{n, \bar{n} \geq 0} \frac{1}{n! \bar{n}!} \delta_{n - \bar{n} \in N\mathbb{Z}} \left(V K e^{-\frac{8\pi^2}{Ng^2} + i\frac{\theta}{N}} \right)^n \left(V K e^{-\frac{8\pi^2}{Ng^2} - i\frac{\theta}{N}} \right)^{\bar{n}} \\ &= \sum_{k \in \mathbb{Z}_N} \exp \left[-V \left(-2K e^{-\frac{8\pi^2}{Ng^2}} \cos \left(\frac{\theta - 2\pi k}{N} \right) \right) \right] \end{aligned}$$

N semiclassical vacua

Energy density of k-th vacuum
 $\rightarrow$ multibranch structure!



✓ One can also derive area-law falloff of the Wilson loop from the dilute gas of center vortices.

Summary of Backgrounds / Question

Motto: deforming $SU(N)$ YM to **weakly-coupled** one with **keeping confinement**.

3d monopole semiclassics

[Ünsal '07, Ünsal-Yaffe '08,...]

$SU(N)$ Yang-Mills on $\mathbb{R}^3 \times S^1$ with

“center-stabilizing deformation”

$\Rightarrow$ **3d $U(1)^{N-1}$ gauge theory**

+ monopole gas

2d center vortex semiclassics

[Tanizaki-Ünsal '22, ...]

$SU(N)$ Yang-Mills on $\mathbb{R}^2 \times T^2$ with 't Hooft flux

$\Rightarrow$ **confinement by 2d center-vortex gas**



Question: Relation between them?
How monopole transmutes to center vortex?

Interpolating setup

Interpolating setup: SU(N) Yang-Mills on $\mathbb{R}^2 \times \overbrace{(S^1)_3 \times (S^1)_4}^{\text{'t Hooft flux}}$
(L_4 : always small)
center-stabilizing deformation

$L_3 \rightarrow \infty$

3d monopole semiclassics

SU(N) Yang-Mills on $\mathbb{R}^3 \times S^1$ with
center-stabilizing deformation

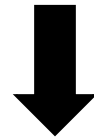
$L_3 \rightarrow L_4$

2d center vortex semiclassics

SU(N) Yang-Mills on $\mathbb{R}^2 \times T^2$ with
't Hooft flux

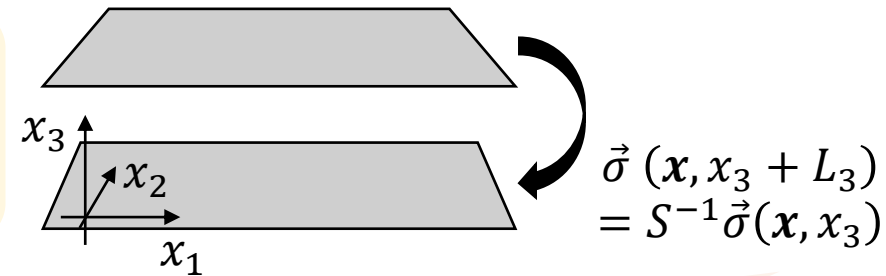
3d effective theory on $\mathbb{R}^2 \times (S^1)_3$

Interpolating setup: SU(N) Yang-Mills on $\mathbb{R}^2 \times \overbrace{(S^1)_3 \times (S^1)_4}^{\text{'t Hooft flux}}$
 (L_4 : always small) center-stabilizing deformation



small L_4 , adjoint higgsing by P_4

3d $U(1)^{N-1}$ gauge theory + monopoles on $\mathbb{R}^2 \times (S^1)_3$
 with “shift-twisted” boundary conditions



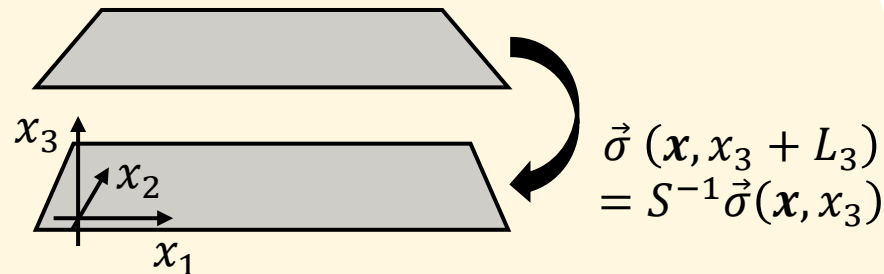
In the gauge $\langle P_4 \rangle = C$ (clock matrix),
 the transition function for $(S^1)_3$ is $g_3 = S$ (shift matrix),
 ($\sim$ Weyl-permutation for dual photon $\vec{\sigma}(x, x_3)$)

For SU(2) case:
 $\sigma(x, x_3 + L_3) = -\sigma(x, x_3)$

From 3d monopole gas to 2d center-vortex gas

3d $U(1)^{N-1}$ gauge theory + monopoles on $\mathbb{R}^2 \times (S^1)_3$ with “shift-twisted” boundary conditions

$$S_{3d}[\vec{\sigma}] = \int d^3x \left[\frac{\# g^2}{L_4} |d\vec{\sigma}|^2 - \# e^{-\frac{8\pi^2}{Ng^2}} \sum_{i=1, \dots, N} \cos(\vec{\alpha}_i \cdot \vec{\sigma} + \theta/N) \right]$$



Monopole amplitude

$$[\mathcal{M}_i] \sim e^{-\frac{8\pi^2}{Ng^2}} e^{i\vec{\alpha}_i \cdot \vec{\sigma} + i\theta/N}$$



$L_3 \ll \Lambda^{-1}$: restricted to $\vec{\sigma} = S^{-1}\vec{\sigma}$

$\vec{\alpha}_i \cdot \vec{\sigma}_k = \frac{2\pi k}{N}$

2d center-vortex gas

Shift-invariant N vacua:

$$\vec{\sigma} = \vec{\sigma}_k = \frac{2\pi k}{N} (\vec{\mu}_1 + \dots + \vec{\mu}_{N-1})$$

$$k = 0, \dots, N-1$$

Center-vortex amplitude

$$[\mathcal{V}] \sim e^{-\frac{8\pi^2}{Ng^2}} e^{i\frac{2\pi k}{N} + i\theta/N}$$

$$Z_{\mathbb{R}^2 \times (S^1)_3} = \int_{\substack{\vec{\sigma}(x, x_3 + L_3) \\ = S^{-1}\vec{\sigma}(x, x_3)}} \mathcal{D}\vec{\sigma} e^{-S_{3d}[\vec{\sigma}]} \approx \sum_{\substack{\vec{\sigma} = \vec{\sigma}_k \\ k \in \mathbb{Z}_N}} e^{-S_{3d}[\vec{\sigma}]} = \sum_{k \in \mathbb{Z}_N} e^{\#V_{2d} e^{-\frac{8\pi^2}{Ng^2}} \cos\left(\frac{\theta + 2\pi k}{N}\right)} = Z_{2d}$$

Microscopically: monopole in $\mathbb{R}^2 \times (S^1)_3$

- BPS/KK monopole in 3d effective theory:

magnetic charge $\vec{\alpha}_i \Rightarrow \nabla^2 \vec{\sigma} \sim 2\pi \vec{\alpha}_i \delta^{(3)}(x - x_*)$

boundary condition: $\vec{\sigma}(x, x_3 + L_3) = S^{-1} \vec{\sigma}(x, x_3)$

$\Rightarrow$ “mirror image”: infinite chain of BPS/KK monopoles

$$\vec{\sigma} \sim \sum_{n \in \mathbb{Z}} \frac{\vec{\alpha}_{i-n \pmod{N}}}{|x - x_* - nL_3 \hat{x}_3|}$$

- A proper expression (with good convergence):

$$\vec{\sigma} \sim \sum_{k \in \mathbb{Z}} \left[\sum_{\ell \in \mathbb{Z}_N} \vec{v}_{i-\ell \pmod{N}} \left\{ \frac{1}{|x - x_* - (Nk + \ell)L_3 \hat{x}_3|} - \frac{1}{|x - x_* - (Nk + \ell + 1)L_3 \hat{x}_3|} \right\} \right]$$

$\vec{v}_i$: weight vector of defining representation

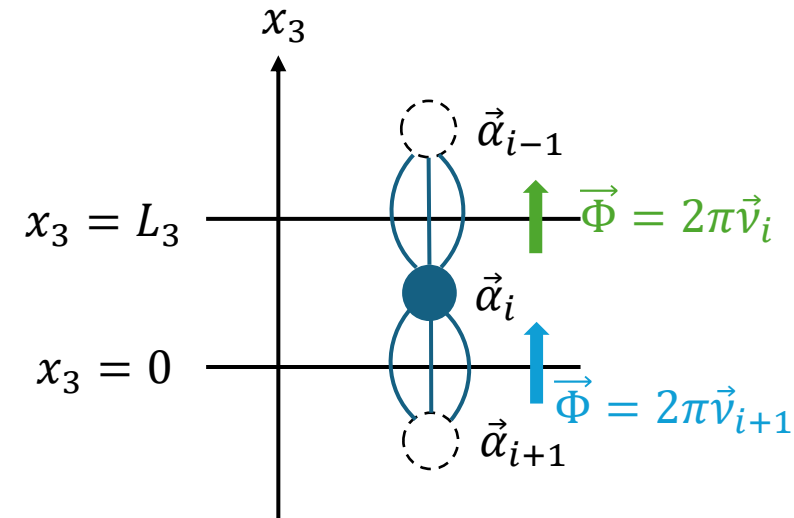
$$\vec{\alpha}_i = \vec{v}_i - \vec{v}_{i+1}$$

outgoing magnetic flux

$$\vec{\Phi} = 2\pi \vec{v}_i$$

incoming magnetic flux

$$\vec{\Phi} = 2\pi \vec{v}_{i+1}$$



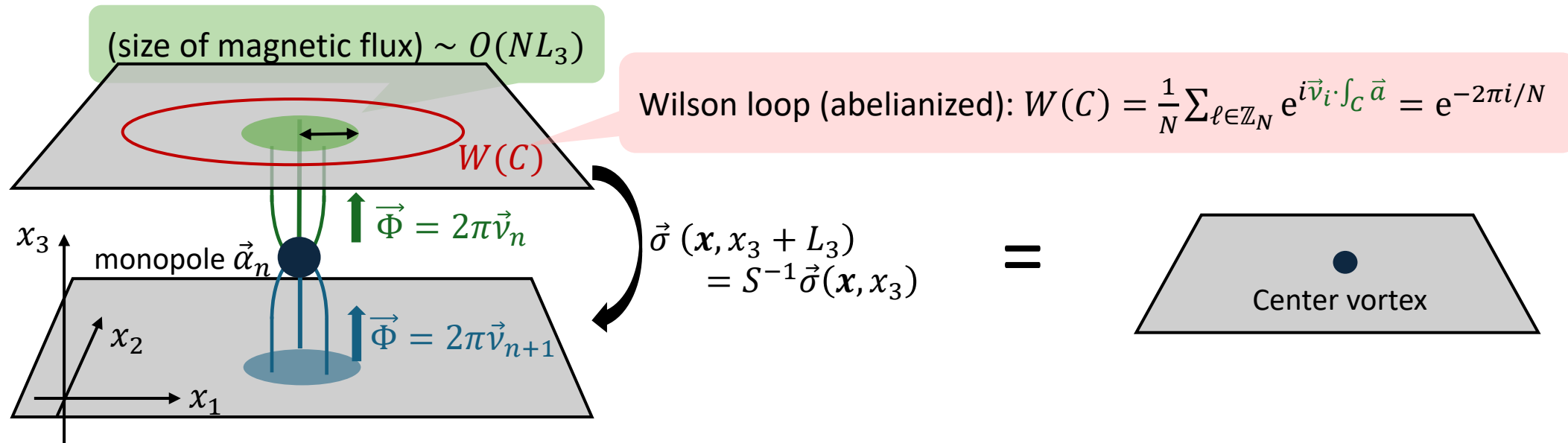
3d BPS/KK monopoles become 2d center vortex

- The magnetic flux (of size $O(NL_3)$) is indeed center vortex:
Wilson loop acquires $e^{-2\pi i/N}$ phase.

- **3d BPS/KK monopole-instanton = 2d center-vortex-instanton:**

The 3d/2d semiclassical confinement mechanisms are essentially same!

- **“monopole as junction of center vortex” (realizing the old expectation!)**
- Species of monopole (BPS/KK) is included in extended moduli $x_3 \in [0, NL_3)$



Summary

Quark confiners: monopole and center vortex

Weak-coupling semiclassical realizations:

3d monopole semiclassics

[Ünsal '07, Ünsal-Yaffe '08,...]

SU(N) Yang-Mills on $\mathbb{R}^3 \times S^1$ with
“center-stabilizing deformation”

⇒ **confinement by 3d monopole gas**



2d center vortex semiclassics

[Tanizaki-Ünsal '22, ...]

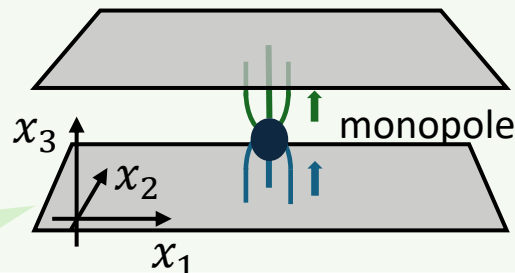
SU(N) Yang-Mills on $\mathbb{R}^2 \times T^2$ with ‘t
Hooft flux

⇒ **confinement by 2d center-vortex gas**

This work: Consider an interpolating setup on $(\mathbb{R}^2 \times S^1) \times S^1$

Monopole in $\mathbb{R}^2 \times S^1$

“monopole as junction of
center vortices”



=

Center vortex in 2d

