Recent Developments in 2D BCFT & ICFT

Yuya Kusuki (Kyushu University)

Andreas Karch

Hirosi Ooguri

Haoyu Sun

Zixia Wei

Mianqi Wang

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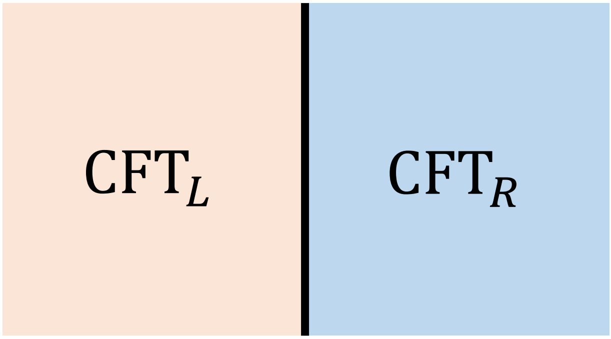
Contents

- Review of ICFT
  - Why should we study interface?
  - CFT data in the presence of interface
- Universal asymptotic formula
- Universal bounds
- Discussion

Interface in 2D



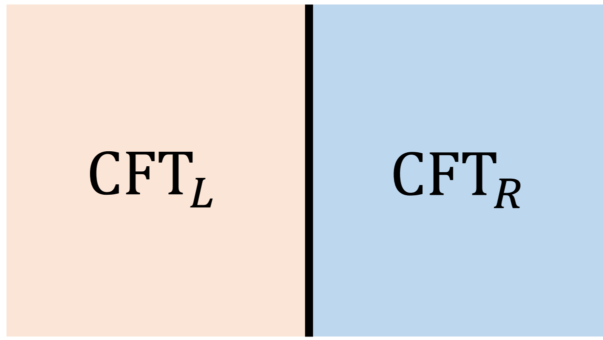
interface $\mathcal{J}$

Definition

Interface = Non-local object

such as boundary, defect, symmetry line, etc.
Interface is possible between two different CFTs

Interface in 2D



interface $\mathcal{J}$

Definition

Interface = Non-local object

$\Rightarrow$Why consider Interface?

Effects from Defect/Boundary

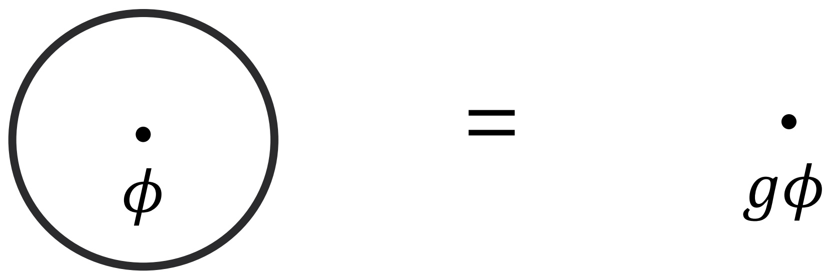
defect

boundary

Condensed matter always involves defect & boundary

⇒Understanding of effects from interface is important.

Modern Approach to Symmetry



topological line

Action on Hilbert space

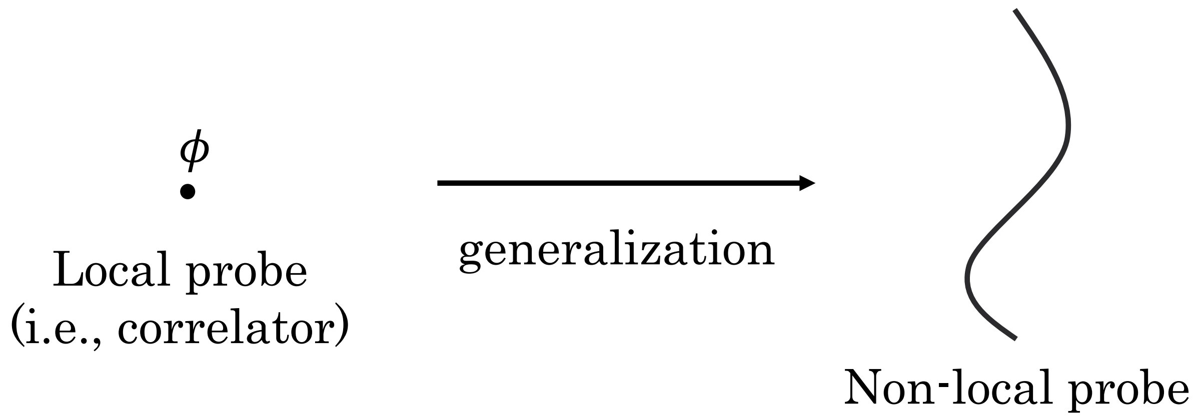
Modern understanding of Symmetry

Global Symmetry = **<u>Topological Defect</u>**

$\Rightarrow$ new concept: non-invertible symmetry

e.g., Kramers-Wannier duality defect

Extracting Non-local Info.



Non-local property in QFT can be detected by non-local probe.

e.g., Renyi entropy = $\mathbb{Z}_n$ topological defect
$\Rightarrow$ probe whether a given state is pure or mixed

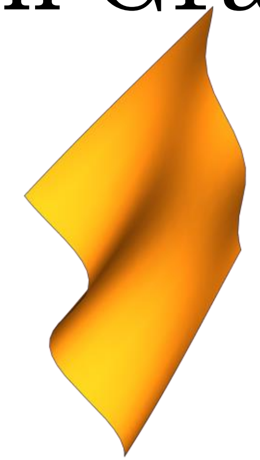
'‘t Hooft anomaly can be detected by boundary & interface

Approach to Quantum Gravity

interface

=

AdS/CFT



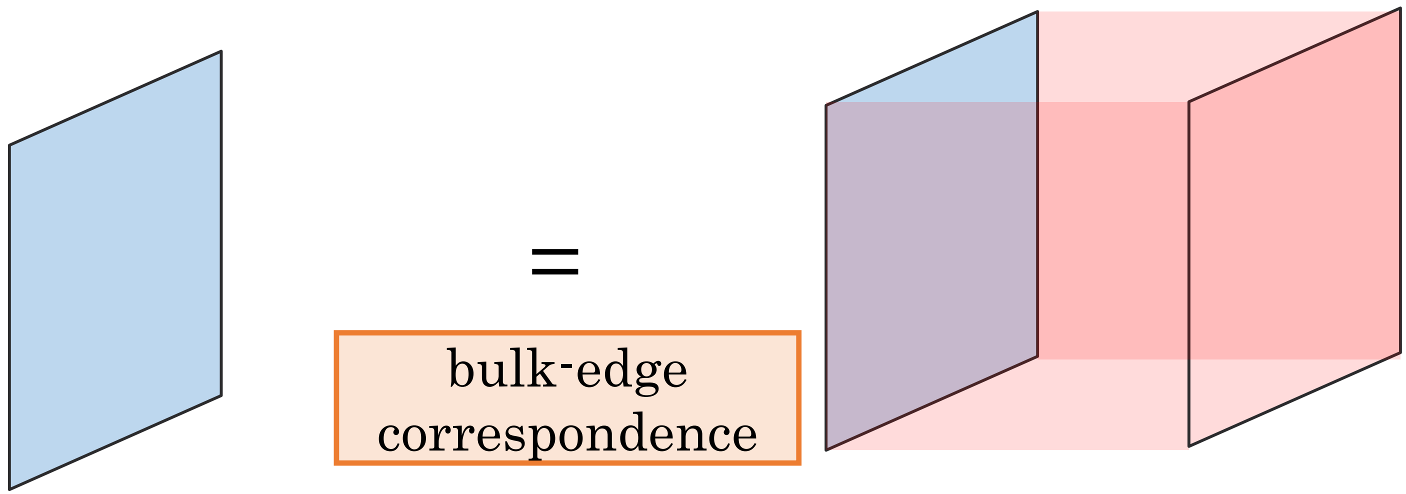
brane

Holographic dual of ICFT involves **brane**

⇒ holographic approach to quantum gravity

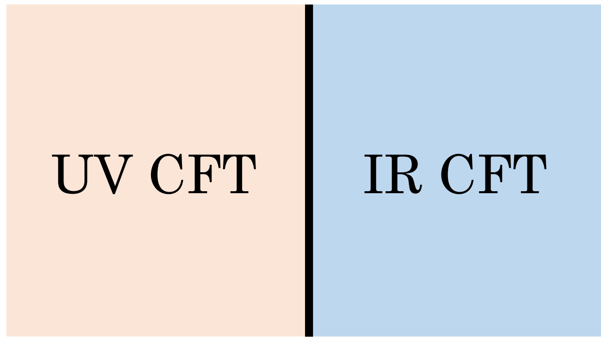
- understanding braneworld holography
- providing tractable model of QG
  (i.e., island model)

Classification of Topological Phase



- BCFT = TO_3
  (Ishibashi state = reduced density matric)
- BCFT = SPT_3
  (no symmetric bdy ⇔ ‘t Hooft anomaly ⇔ SPT)
- BCFT = SPT_2
- ICFT appears in more general setup

Simple way to address RG



RG brane = interface between UV & IR CFT

allows us to extract information about RG flow, such as UV/IR map.

(e.g., Gaiotto RG brane between minimal models)

This talk

Question

What is **universal** and what is **theory-dependent**?

This talk

Question

What is **universal** and what is **theory-dependent**?

Universality in 2D CFT

- Universal asymptotic spectrum (Cardy formula)

$$log\ \rho(\Delta) \simeq 2\pi\sqrt{\frac{c}{3}\left(\Delta - \frac{c}{12}\right)}, \qquad \Delta \gg c$$

- Universal bound on gap (Hellerman bound)

$$\Delta_1 - \Delta_0 \leq \frac{c}{6} + O(c^0)$$

This talk

Question

What is **universal** and what is **theory-dependent**?

Universality 2D Interface CFT?

I will introduce two approaches,

- Conformal bootstrap
  $\Rightarrow$universal ICFT data
- AdS/CFT
  $\Rightarrow$universal bound on ???

This talk

Question

What is **universal** and what is **theory-dependent**?

Universality 2D Interface CFT?

I will introduce two approaches,

- Conformal bootstrap
  $\Rightarrow$universal ICFT data
- AdS/CFT
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What is ICFT data?

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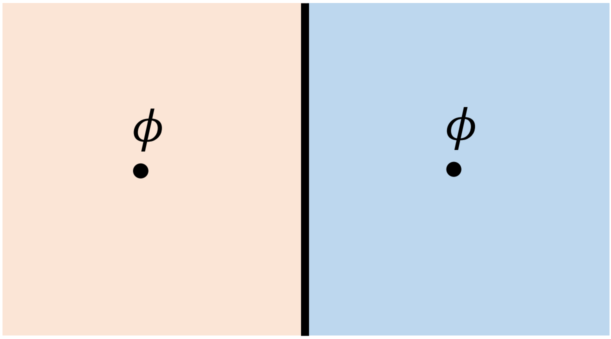
CFT data

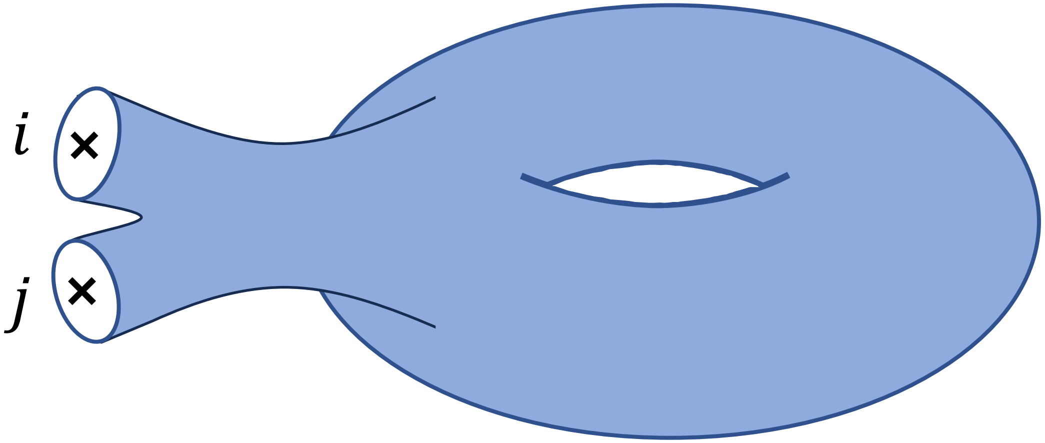


Definition

Basic object in QFT = Correlation function

CFT data is data that determines all correlators.

Correlator in CFT

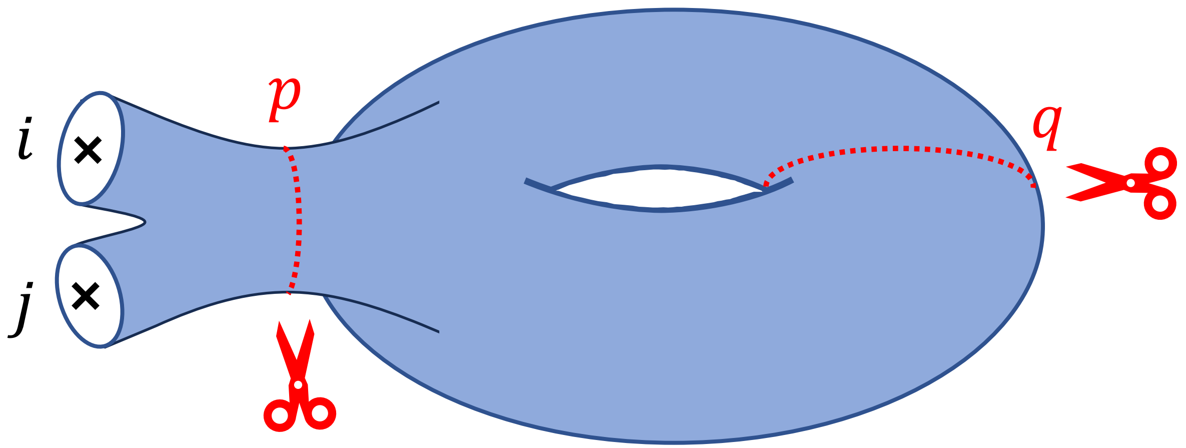


Review

How to evaluate correlation function in CFT?

Let us take an example, $\langle \phi_i \, \phi_j \rangle_{\mathbb{T}^2}$

Correlator in CFT



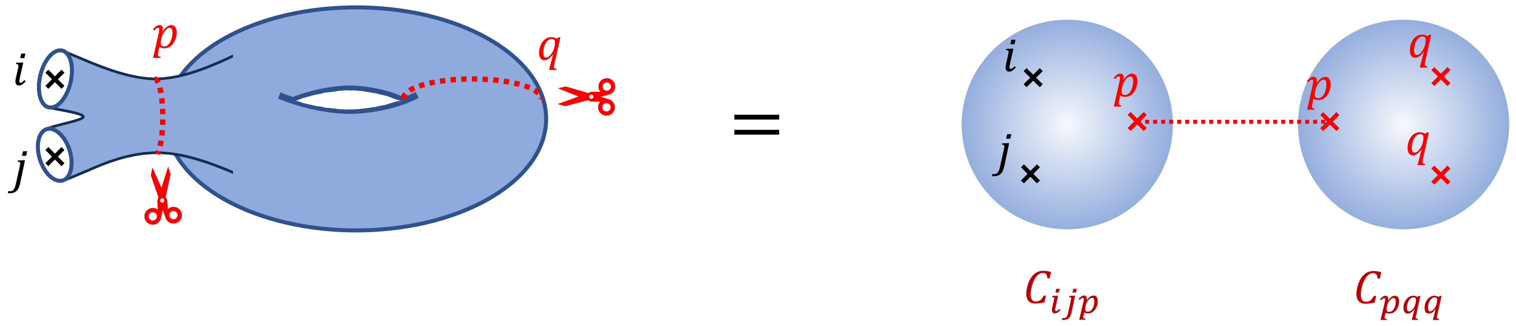
Pants decomposition = Cutting by $\mathbb{I}$

$$\mathbb{I} \equiv \sum_{p} \sum_{\boldsymbol{n}} |p, \boldsymbol{n}\rangle\langle p, \boldsymbol{n}|$$

p: primary label

$\boldsymbol{n}$: descendant label

Correlator in CFT

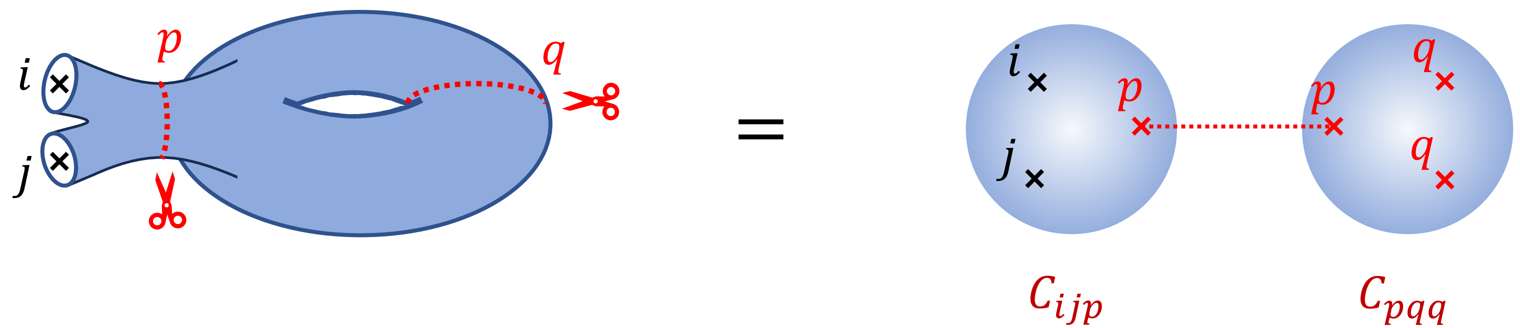


$$\langle \phi_i \; \phi_j \rangle_{\mathbb{T}^2} = \sum_{p,q} \sum_{\boldsymbol{n},\boldsymbol{m}} \langle \phi_i \phi_j \phi_p^{\boldsymbol{n}} \rangle \langle \phi_p^{\boldsymbol{n}} \phi_q^{\boldsymbol{m}} \phi_q^{\boldsymbol{m}} \rangle$$

Virasoro symmetry completely fixes 3pt. function, except for primary 3pt. function $C_{ijk} \equiv \langle \phi_i | \phi_j(1) | \phi_k \rangle$

$$\langle \phi_i^{\boldsymbol{n}} \phi_j^{\boldsymbol{m}} \phi_k^{\boldsymbol{l}} \rangle = C_{ijk} \times \text{known special function}$$

Correlator in CFT



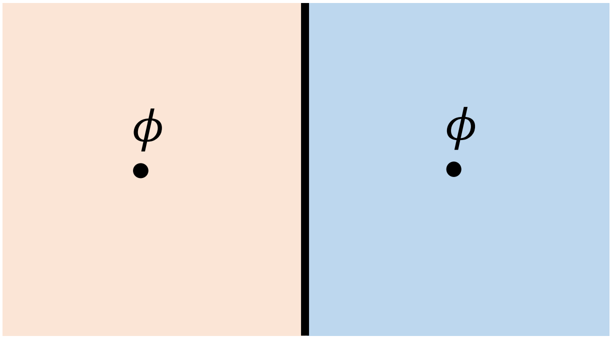
$$\langle \phi_i \, \phi_j \rangle_{\mathbb{T}^2} = \sum_{p,q} C_{ijp} C_{pqq} \, \mathcal{F}(p,q|\mathbb{z}) \overline{\mathcal{F}(p,q|\mathbb{z})}$$

theory dependent

special function (i.e., Virasoro block)

Any correlation function can be evaluated from $\{C_{ijk}\}$

Correlator in CFT



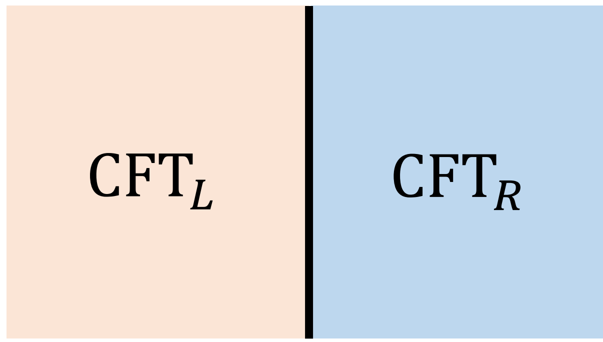
Review

information fixing all correlation functions

CFT data = $\{C_{ijk}\}$ (i, j, k= primaries)

$\Rightarrow$ Let us generalize it into ICFT
(before that, we first define interface explicitly)

Conformal Interface



interface $\mathcal{J}$

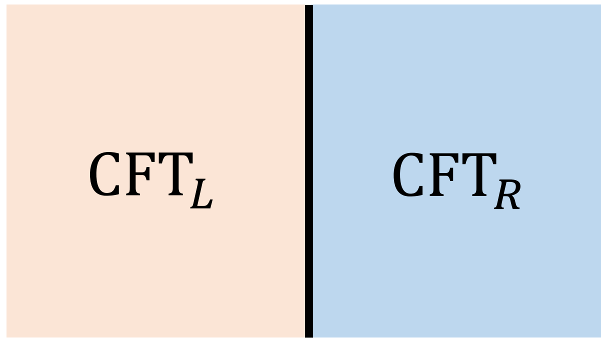
<u>Definition</u>

We define “conformal interface”, as a solution to

$$\left(L_n^{(L)} - \bar{L}_{-n}{}^{(L)}\right)\mathcal{J} = \mathcal{J}\left(L_n^{(R)} - \bar{L}_{-n}{}^{(R)}\right)$$

- making it tractable
- including interesting interfaces shown in Intro.

Conformal Interface



interface $\mathcal{J}$

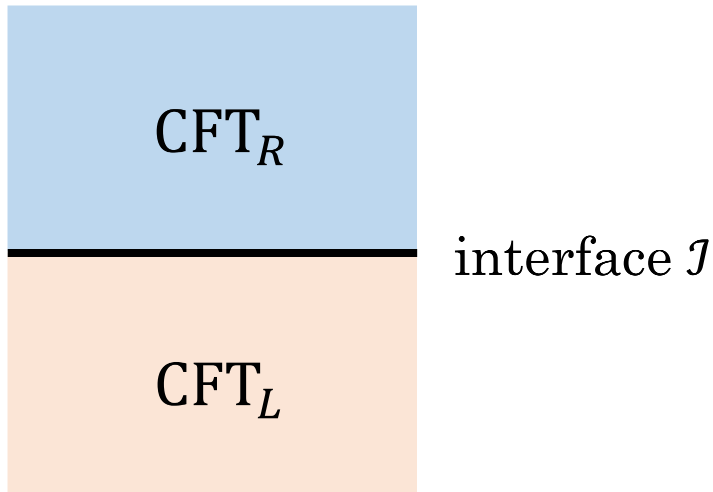
Assumption

No absorption & emission of energy on $\mathcal{J}$:

$$T^{(L)} - \bar{T}^{(L)} = T^{(R)} - \bar{T}^{(R)} \quad \text{on } \mathcal{J}$$

(which is satisfied in all examples in Intro.)

Conformal Interface

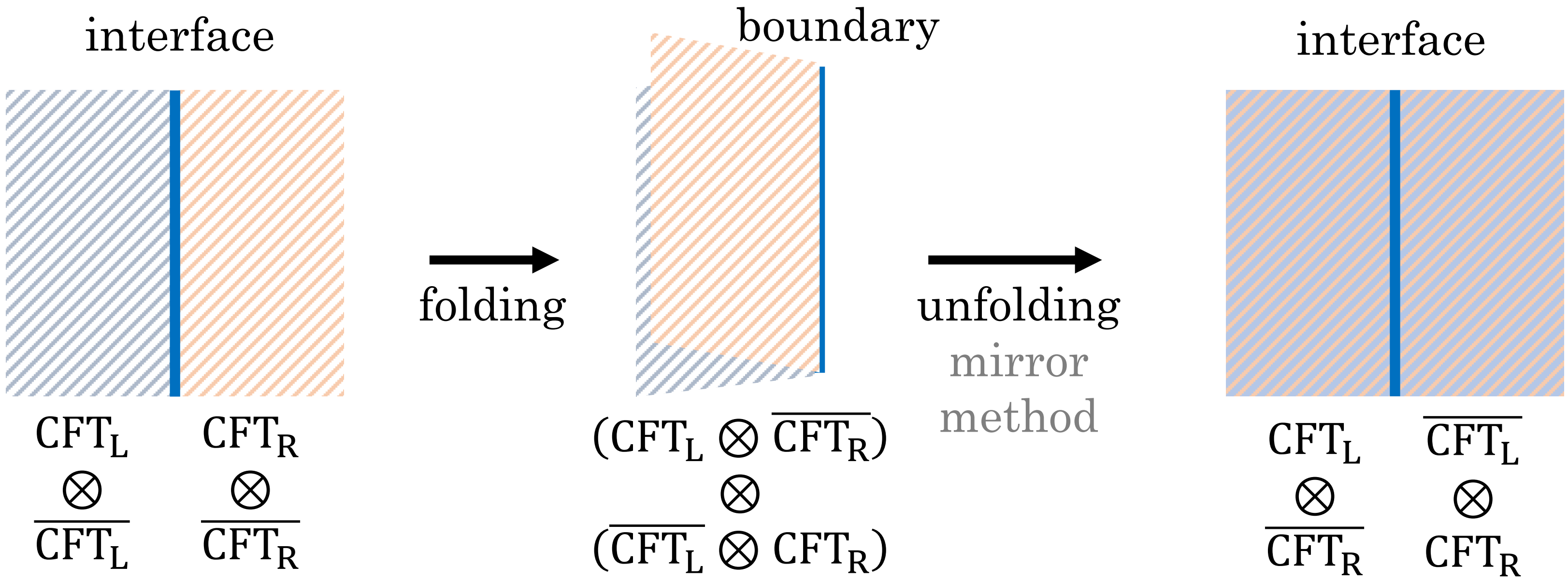


Assumption(Operator formalism)

Map from states in CFT_L to states in CFT_R, satisfying

$$\left(L_n^{(L)} - \bar{L}_{-n}{}^{(L)}\right)\mathcal{J} = \mathcal{J}\left(L_n^{(R)} - \bar{L}_{-n}{}^{(R)}\right)$$

Conformal Interface



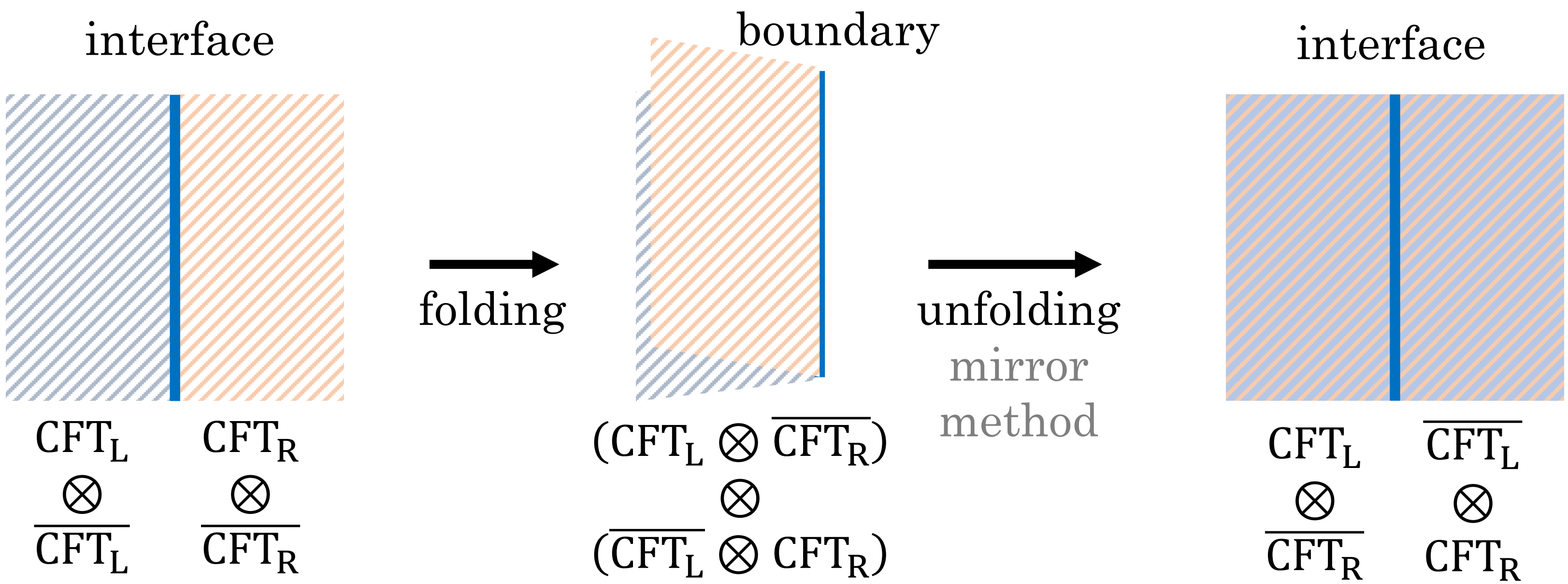
<u>Alternative Interpretation of Assumption</u>

$$T^{(L)} - \bar{T}^{(L)} = T^{(R)} - \bar{T}^{(R)}$$

$$\Updownarrow$$

$$T^{(L)} + \bar{T}^{(R)} = \bar{T}^{(L)} + T^{(R)}$$

Conformal Interface



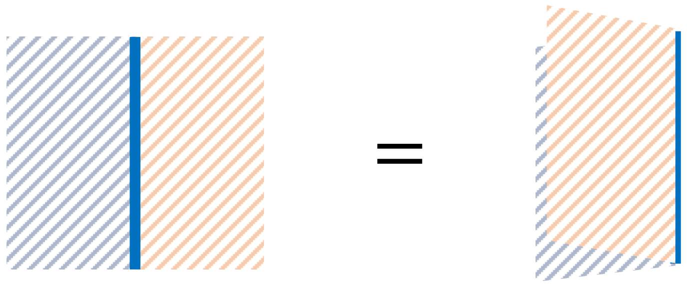
<u>Alternative Interpretation of Assumption</u>

Define T_{tot} as stress tensor of product CFT, then

$$T_{\mathrm{tot}} = \overline{T_{\mathrm{tot}}} \quad \text{on } \mathcal{I}$$

$\Rightarrow$ Conformal boundary condition

ICFT = BCFT



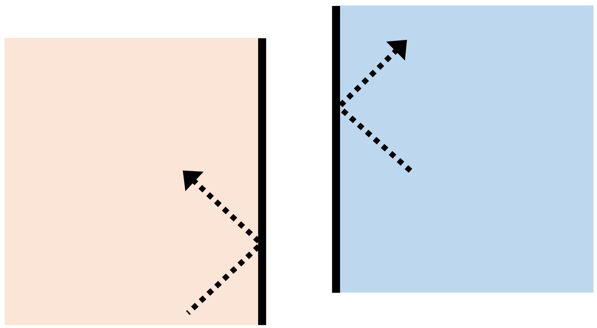
Implication from ICFT = BCFT

- Interface breaks symmetry into **single** Virasoro.
  ⇒ called conformal interface
- Interface is a solution to conformal boundary condition, which can be expanded in Ishibashi basis,

$$|\mathcal{I}\rangle = g \sum_i C_i |i\rangle\rangle$$

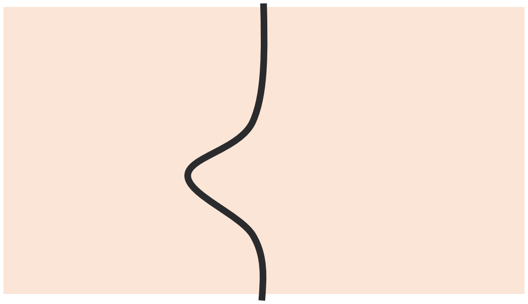
Trivial Solutions

- Totally reflective interface (=boundary)



$$\left(L_n^{(L)} - \bar{L}_{-n}{}^{(L)}\right)\mathcal{J} = 0$$
$$\mathcal{J}\left(L_n^{(R)} - \bar{L}_{-n}{}^{(R)}\right) = 0$$

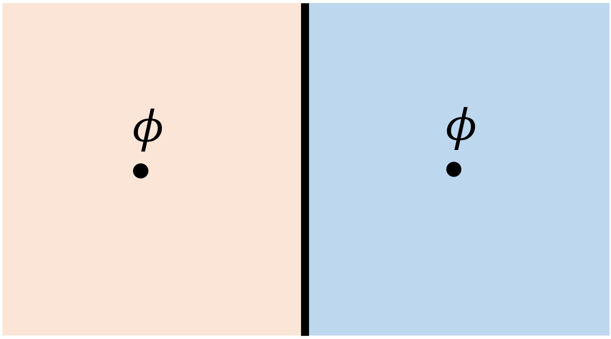
- Totally transmissive interface (=topological interface)



Possible to deform with no cost

$$L_n^{(L)}\mathcal{J} = \mathcal{J}L_n^{(R)}$$
$$\bar{L}_{-n}^{(L)}\mathcal{J} = \mathcal{J}\bar{L}_{-n}^{(R)}$$

Correlator in ICFT

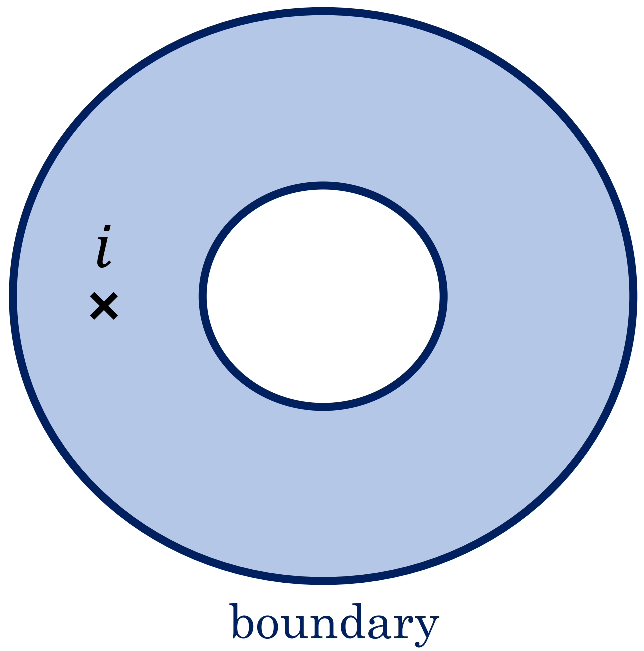


How to evaluate correlation functions in the presence of **interfaces**?

$\downarrow$ translated

How to evaluate correlation functions in the presence of **boundaries**? $\Rightarrow$ Let us focus on BCFT

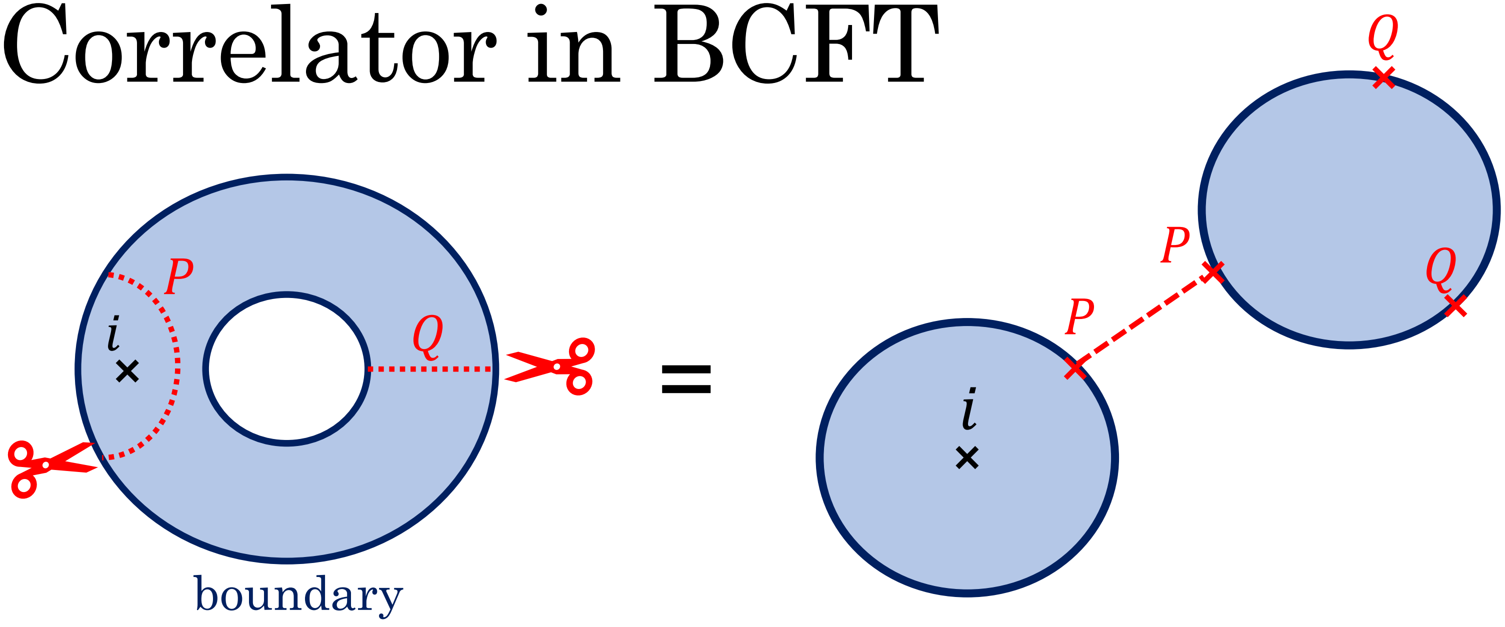
Correlator in BCFT



How to evaluate correlation function in BCFT?

Let us take an example, $\langle \phi_i \rangle_{I\times S^1}$

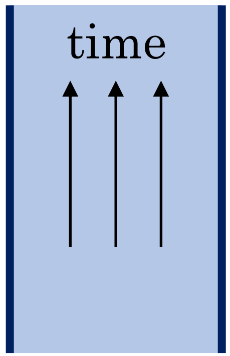
Correlator in BCFT



$$\langle \phi_i \rangle_{I\times S^1} = \sum_{P,Q} C_{iP} C_{PQQ}\, \mathcal{F}(P,Q|\mathbb{z})\overline{\mathcal{F}(P,Q|\mathbb{z})}$$

$\Rightarrow$ New ingredients: $\{C_{iP}\}$ & $\{C_{PQR}\}$?

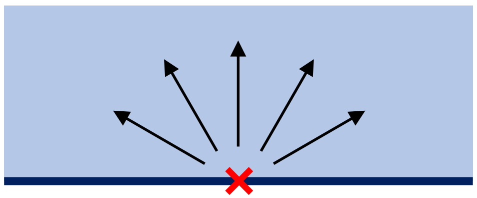
Correlator in BCFT



$|P\rangle \in \mathcal{H}_{\text{open}}\ (\neq \mathcal{H}_{\text{closed}})$

=

state=operator



P

boundary primary

labeled by a capital letter

Definition

Boundary primary = h.w.s. in open string Hilbert space

- h.w.r. of single Virasoro in BCFT
  $\Rightarrow$ transformation law is the same as standard primary
- Introducing new OPE coef. $\{C_{iP}\}$ & $\{C_{PQR}\}$

ICFT data

	CFT	ICFT = BCFT
Symmetry	$Vir \otimes \overline{Vir}$	Vir
CFT data	$\{C_{ijk}\}$	$\{C_{ijk}\}$ & $\{C_{iJ}\}$ & $\{C_{IJK}\}$

Message in this Part

BCFT (ICFT) is much **richer** than CFT,
but is **less tractable**.

$\Rightarrow$ BCFT (ICFT) remains largely unexplored.

This talk

Question

What is **universal** and what is **theory-dependent**?

Universality 2D Interface CFT?

I will introduce two approaches,

- Conformal bootstrap
  ⇒universal ICFT data
- AdS/CFT
  ⇒universal bound on special bulk-bdy OPC coef.

What is ICFT data?

Universality in ICFT

[YK]

[Karch, YK, Ooguri, Sun, Wang]× 2

- **Universal asymptotic formula** for $\{C_{iJ}\}$ & $\{C_{IJK}\}$

  Generalization of Cardy formula.

- **Universal bound** on 1pt function

$$C_{d\mathbb{I}} \equiv \left\langle B \middle| L_{-2}^{(L)} L_{-2}^{(R)} \right\rangle = \langle 0 | L_{2}^{(L)} \mathcal{I} L_{-2}^{(R)} | 0 \rangle$$

folded picture　　unfolded picture

  This looks universal because $\langle T(z_1)T(z_2)T(z_3)\cdots\rangle$ is completely fixed by Virasoro symmetry.

  But interface **mixes** $\{T^{(L)}, T^{(R)}, \bar{T}^{(L)}, \bar{T}^{(R)}\}$, therefore, $C_{d\mathbb{I}}$ highly depends on the details of interface.

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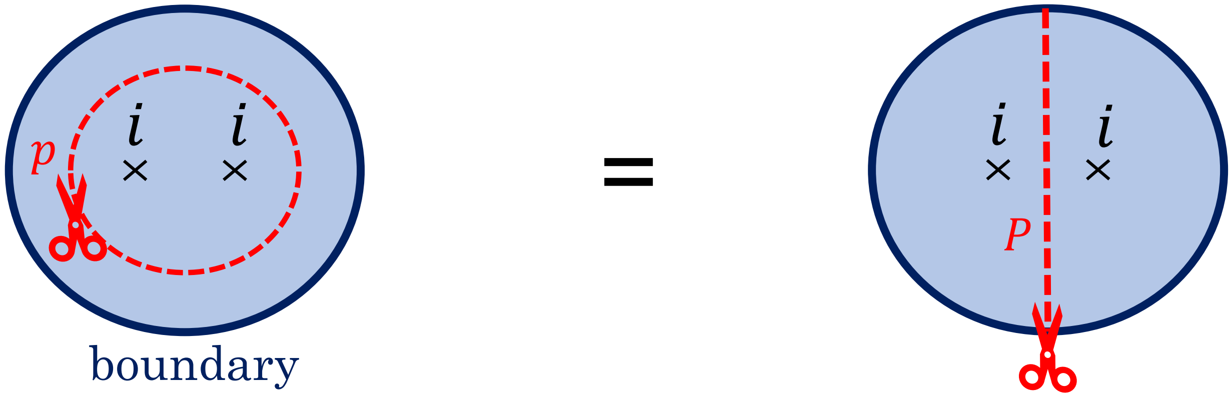
Universal Asymptotic Formula

[YK], [Numasawa-Tsiares]



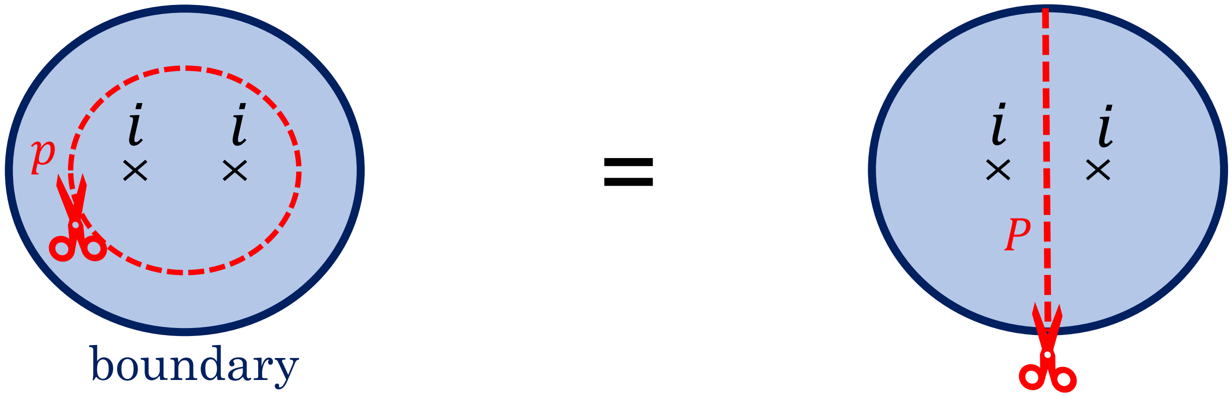
$$\int \mathrm{d}h_p \ \rho^{\mathrm{bulk}}(h_p, h_p)\overline{C_{iip}C_{p0}}\mathcal{F}^{ii}_{ii}(p|1-z) = \int \mathrm{d}h_P \ \rho^{\mathrm{bdy}}(h_P)\overline{(C_{iP})^2}\mathcal{F}^{ii}_{ii}(P|z)$$

Conformal Bootstrap

Two different quantizations lead to constraints on CFT data.

Universal Asymptotic Formula

[YK], [Numasawa-Tsiares]



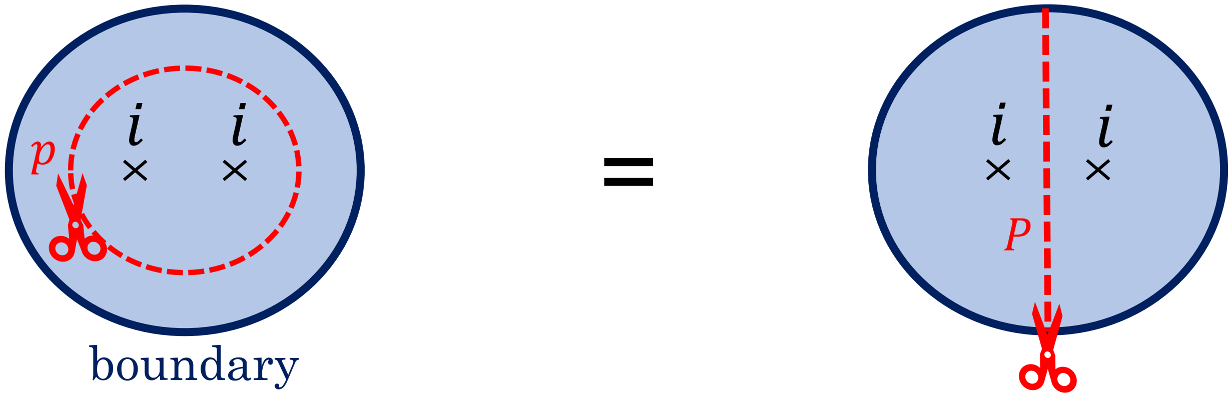
$$\int \mathrm{d}h_p \; \rho^{\mathrm{bulk}}(h_p, h_p)\overline{C_{iip}C_{p0}}\mathcal{F}^{ii}_{ii}(p|1-z) = \int \mathrm{d}h_P \; \rho^{\mathrm{bdy}}(h_P)\overline{(C_{iP})^2}\mathcal{F}^{ii}_{ii}(P|z)$$

$1 - z \ll 1$

$$\mathcal{F}^{ii}_{ii}(0|1-z) \simeq \int \mathrm{d}h_P \; \rho^{bdy}(h_P)\overline{(C_{iP})^2}\mathcal{F}^{ii}_{ii}(P|z)$$

Universal Asymptotic Formula

[YK], [Numasawa-Tsiares]



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$$\mathcal{F}^{ii}_{ii}(0|1-z) \simeq \int \mathrm{d}h_P\ \rho^{bdy}(h_P)\overline{(C_{iP})^2}\mathcal{F}^{ii}_{ii}(P|z)$$

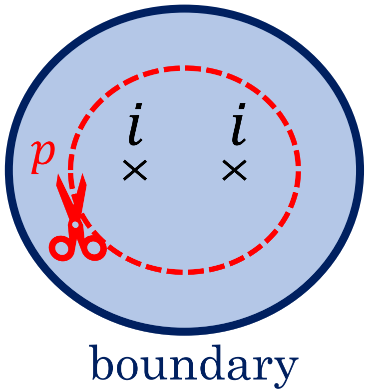
Fusion transformation

$$\mathcal{F}^{ii}_{ii}(0|1-z) = \int \mathrm{d}h_P\, F_{0P}\begin{bmatrix} i & i \\ i & i \end{bmatrix}\mathcal{F}^{ii}_{ii}(P|z)$$

theory-independent

Universal Asymptotic Formula

[YK], [Numasawa-Tsiares]



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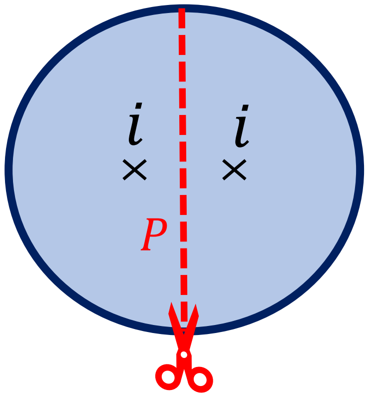

$$\int \mathrm{d}h_p \ \rho^{\mathrm{bulk}}(h_p, h_p)\overline{C_{iip}C_{p0}}\mathcal{F}_{ii}^{ii}(p|1-z) = \int \mathrm{d}h_P \ \rho^{\mathrm{bdy}}(h_P)\overline{(C_{iP})^2}\mathcal{F}_{ii}^{ii}(P|z)$$

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$\wr|$

Fusion transformation

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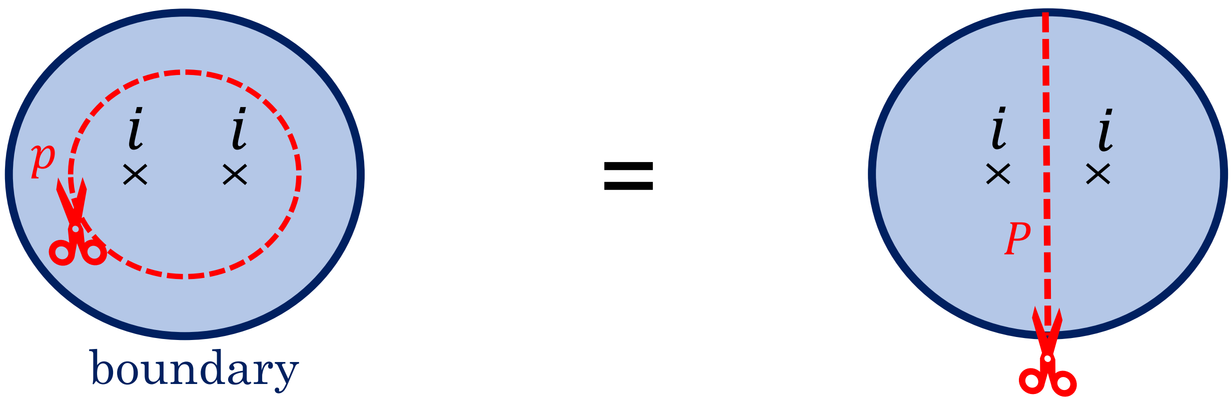
theory-independent

Universal Asymptotic Formula

[YK], [Numasawa-Tsiares]



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$1 - z \ll 1$

$$\mathcal{F}^{ii}_{ii}(0|1-z) \simeq \int \mathrm{d}h_P\ \rho^{bdy}(h_P)\overline{(C_{iP})^2}\mathcal{F}^{ii}_{ii}(P|z)$$

$$\wr\wr \quad h_P \gg c$$

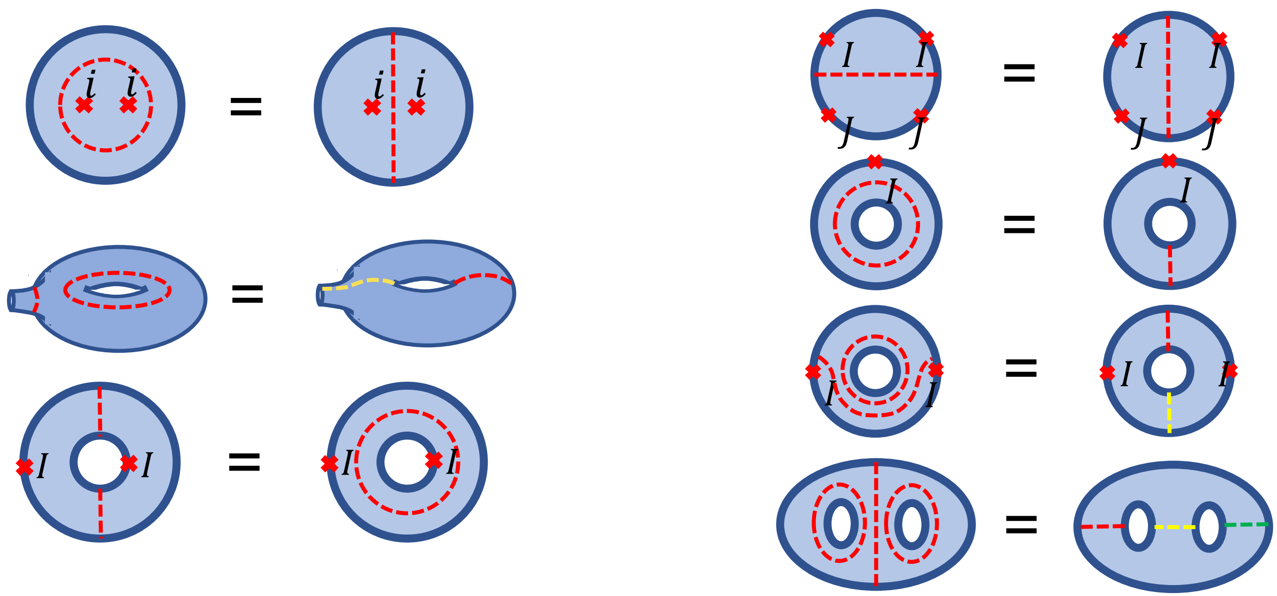
Fusion transformation

$$\mathcal{F}^{ii}_{ii}(0|1-z) = \int \mathrm{d}h_P\, F_{0P}\begin{bmatrix} i & i \\ i & i \end{bmatrix}\mathcal{F}^{ii}_{ii}(P|z)$$

theory-independent

Universal Asymptotic Formula

[YK], [Numasawa-Tsiares]



Result

C_{ijk}, C_{iJ}, C_{IJK} = fusion matrix, if $\max\{h_i\}, \{h_I\} \gg c$

Note: fusion matrix does not depend on theory

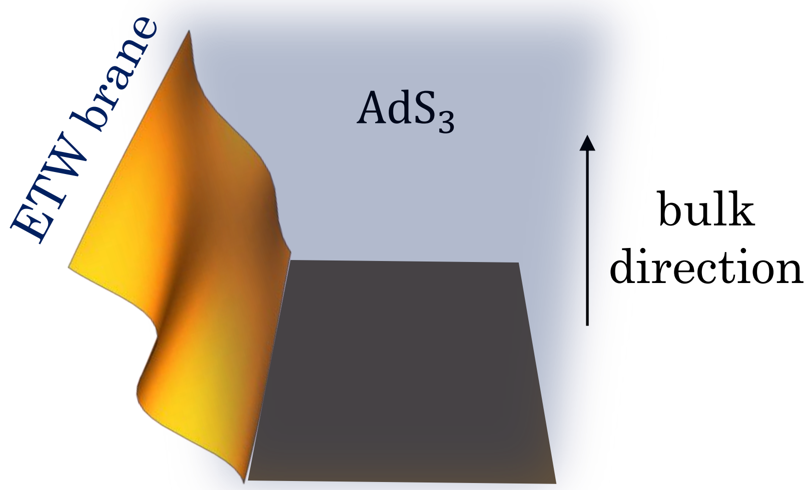
Application

[YK], [YK, Wei]

AdS/BCFT

$$I_{grav} = -\frac{1}{16\pi G_N}\int_M d^3x\,\sqrt{g}(R-2\Lambda) + \sum_i m_i \int dl_i - \frac{1}{8\pi G_N}\int_Q d^2x\,\sqrt{h}(K-T)$$

E-H action massive particle ETW brane



Asymptotic bdy = BCFT$_2$

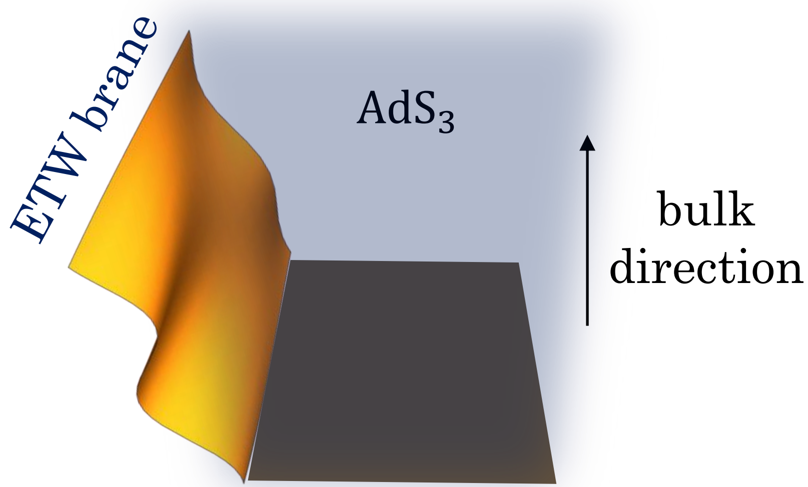
Application

[YK], [YK, Wei]

AdS/BCFT

$$I_{grav} = -\frac{1}{16\pi G_N}\int_M d^3x\,\sqrt{g}(R-2\Lambda) + \sum_i m_i \int dl_i - \frac{1}{8\pi G_N}\int_Q d^2x\,\sqrt{h}(K-T)$$

E-H action | massive particle | ETW brane



Asymptotic bdy = BCFT$_2$

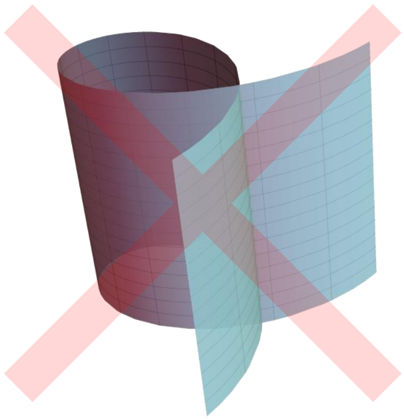
Extra assumption

$$C_{p\mathbb{I}} = \delta_{p\mathbb{I}}$$

$\Rightarrow$ **extension** of validity regime of asymptotic formula

Application

[YK], [YK, Wei]



Implication from **extended asymptotic formula**

Self-interacted brane is impossible

When attempting to create self-interaction,
black hole is always formed

Contents

- Review of ICFT
  - Why should we study interface?
  - CFT data in the presence of interface
- Universal asymptotic formula
- Universal bounds
- Discussion

This talk

Question

What is **universal** and what is **theory-dependent**?

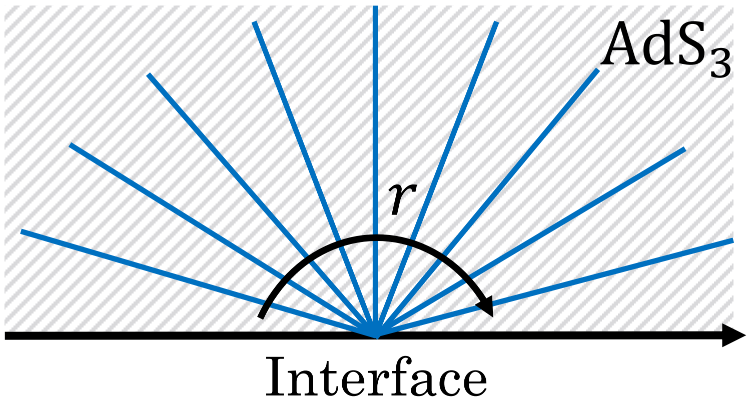
Universality 2D Interface CFT?

I will introduce two approaches,

- Conformal bootstrap
  ⇒universal
- **AdS/CFT**
  ⇒universal bound on special bulk-bdy OPC coef.

Why?

AdS/ICFT



Partitioning AdS_3 into slices of AdS_2

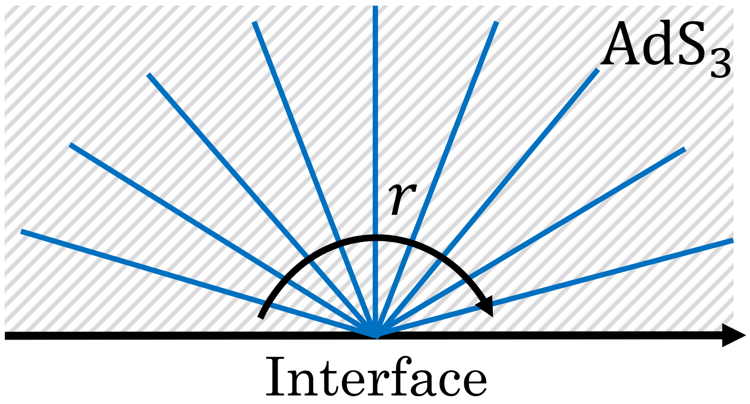
Holographic dual of generic interface

$$ds^2 = e^{2A(r)} \frac{dx^2 - dt^2}{x^2} + dr^2, \qquad r \in (-\infty, \infty)$$

Details of interface is encoded in $A(r)$;

- AdS: $e^{2A(r)} = \cosh r$
- Janus: $e^{2A(r)} = \frac{1}{2}\left(1 + \sqrt{1 - 2\gamma^2} \cosh 2r\right)$

AdS/ICFT



Partitioning AdS$_3$ into slices of AdS$_2$

Holographic dual of generic interface

$$ds^2 = e^{2A(r)} \frac{dx^2 - dt^2}{x^2} + dr^2, \qquad r \in (-\infty, \infty)$$

Generic interface is obtained from generic $A(r)$

$\Rightarrow$ universal information of interface is encoded as universal property of $A(r)$

AdS/ICFT

Generic interface is obtained from generic $A(r)$

⇒ universal information of interface is encoded as universal property of $A(r)$

Comment

It is difficult to construct a generic interface on the CFT side.

Only a few examples of non-topological interfaces e.g., free CFT, Gaiotto brane, …

⇒It is impossible to extract universal information from examining various examples

Universality in ICFT

[YK]

[Karch, YK, Ooguri, Sun, Wang]× 2

- **Universal asymptotic formula** for $\{C_{iJ}\}$ & $\{C_{IJK}\}$

  Generalization of Cardy formula.

- **Universal bound** on 1pt function

What?

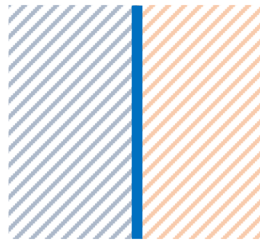
$$C_{d\mathbb{I}} \equiv \left\langle B \middle| L_{-2}^{(L)} L_{-2}^{(R)} \right\rangle = \langle 0| L_{2}^{(L)} \mathcal{I} L_{-2}^{(R)} |0\rangle$$

folded picture　　　unfolded picture

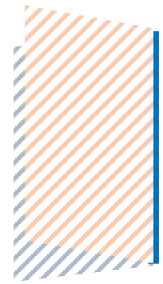
This looks universal because $\langle T(z_1)T(z_2)T(z_3)\cdots\rangle$ is completely fixed by Virasoro symmetry.

But interface **mixes** $\{T^{(L)}, T^{(R)}, \bar{T}^{(L)}, \bar{T}^{(R)}\}$, therefore, $C_{d\mathbb{I}}$ highly depends on the details of interface.

ICFT $\overset{?}{=}$ BCFT



$\overset{?}{=}$

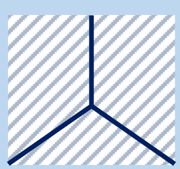


BCFT picture obscures unique properties of ICFT

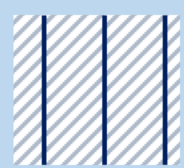
- **Energy transmission** from L to R
- **Entanglement** between L & R

which are encoded in BCFT but make sense only in ICFT

BCFT picture is not applicable in ICFT with

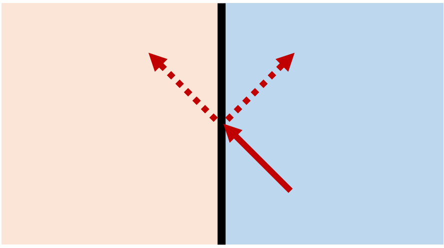


junction



multiple interface

Energy Transmission



Important quantity in ICFT

$C_{d\mathbb{I}} \equiv \left\langle B \middle| L_{-2}^{(L)} L_{-2}^{(R)} \right\rangle = \langle 0 | L_2^{(L)} \mathcal{I} L_{-2}^{(R)} | 0 \rangle$

$$\langle T_L(z) T_R(w) \rangle = \frac{c_{LR}}{2(z-w)^4}$$

[Quella, Runkel, Watts]

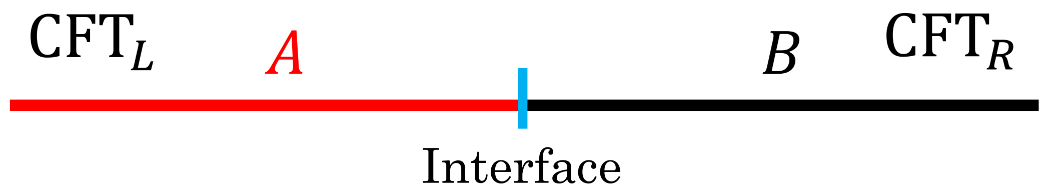
(This is ICFT data and NOT fixed by Virasoro symmetry)

Physical interpretation

$$\mathcal{T}_{L/R} = \frac{\text{transmitted energy}}{\text{incident energy}} = \frac{c_{LR}}{c_{L/R}}$$

[Meineri, Penedones, Rousset]

Entanglement



Important quantity in ICFT

Amount of entanglement (# of EPR pairs) across interface

$$S_A \equiv -\mathrm{tr}\rho_A \log \rho_A = \frac{c_{\mathrm{eff}}}{3} \log \frac{|A|}{\epsilon}$$

[Sakai, Satoh]

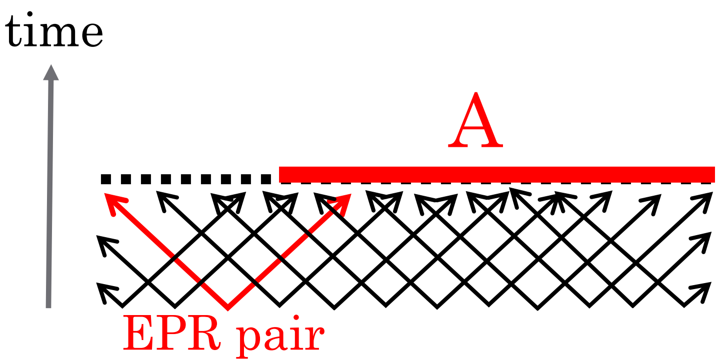
Alternative interpretation

$$\mathcal{T}_{L/R}^{\mathrm{info}} = \frac{\text{transmitted information}}{\text{incident information}} = \frac{c_{\mathrm{eff}}}{c_{L/R}}$$

[Wen, Wang, Ryu]

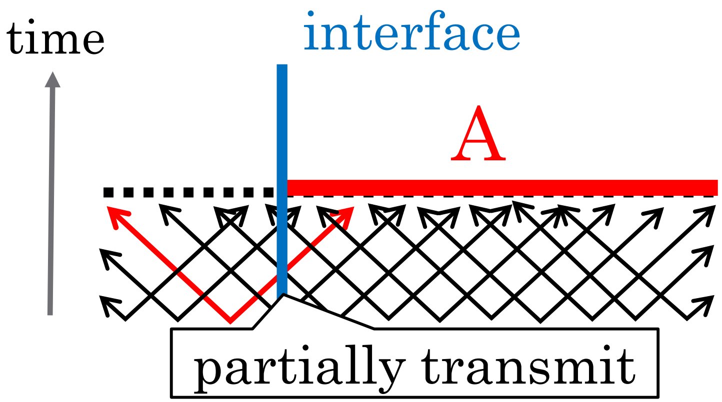
c_{eff} as Information Transmission

[Wen, Wang, Ryu], [Karch, YK, Ooguri, Sun, Wang]



EE under global quench

$$S_A = \#\, c\, t$$



EE under global quench

$$S_A = \#\, c_{\text{eff}}\, t$$

$$\Rightarrow \frac{c_{\text{eff}}}{c} = \text{ information transmission}$$

Universal Bounds

- Holographic c_{eff}

  [Ryu, Takayanagi]

- Holographic c_{LR}

  [Bachas, Baiguera, Chapman, Policastro, Schwartzman]

Point: Both are determined by $A(r)$

- Generalized holographic c theorem (NEC)

$$A''(r) \leq e^{-2A(r)}$$

  $\Rightarrow$ upper bound on c_{eff} [Karch, YK, Ooguri, Sun, Wang '23]

- Comparison between holographic c_{eff} & c_{LR}

  $\Rightarrow$ upper bound on c_{LR} [Karch, YK, Ooguri, Sun, Wang '24]

Universal Bounds

[Karch, YK, Ooguri, Sun, Wang]× 2

- Holographic c_{eff}
  [Ryu, Takayanagi]
- Holographic c_{LR}
  [Bachas, Baiguera, Chapman, Policastro, Schwartzman]

Point: Both are determined by $A(r)$

Result

$$0 \leq c_{LR} \underset{①}{\leq} c_{\mathrm{eff}} \underset{②}{\leq} \min(c_L, c_R)$$

① The amount of energy transmission can never exceed the amount of information transmission

② Any interface decreases entanglement

Universal Bounds

[Karch, YK, Ooguri, Sun, Wang]× 2

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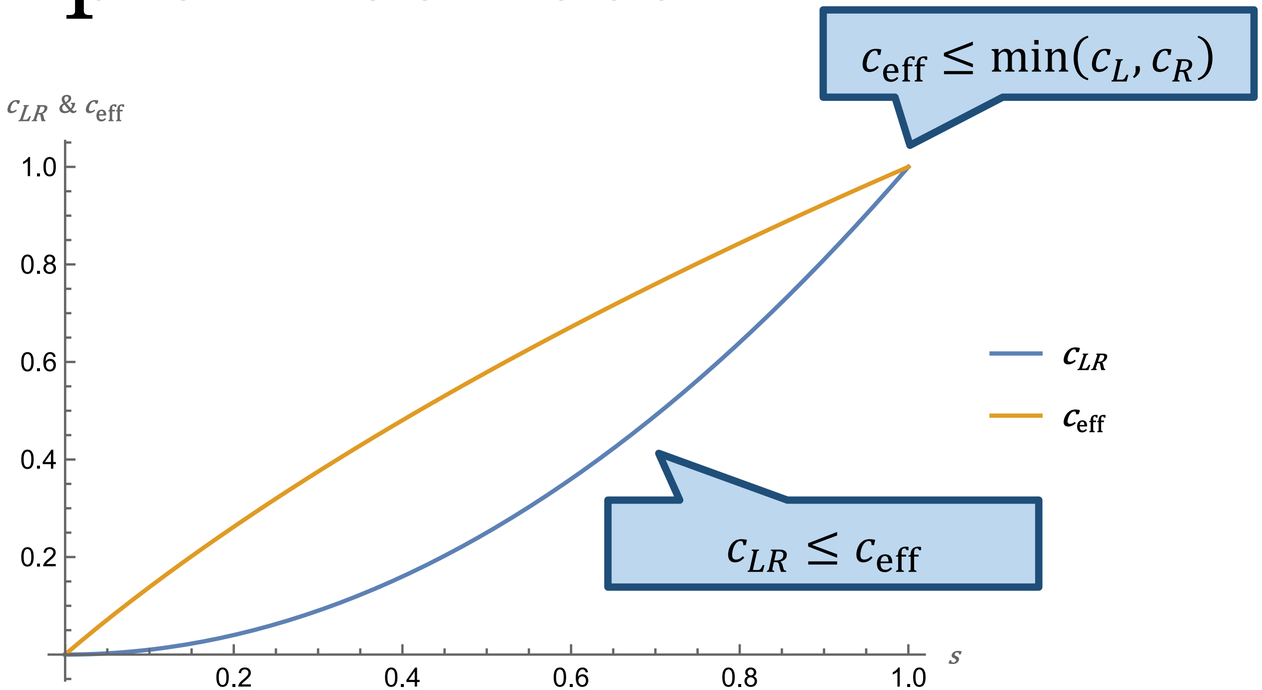
Result

$$0 \leq c_{LR} \underset{①}{\leq} c_{\text{eff}} \underset{②}{\leq} \min(c_L, c_R)$$

① The amount of energy transmission can never exceed the amount of information transmission

② Any interface decreases entanglement (proven in CFT)

Example: free boson

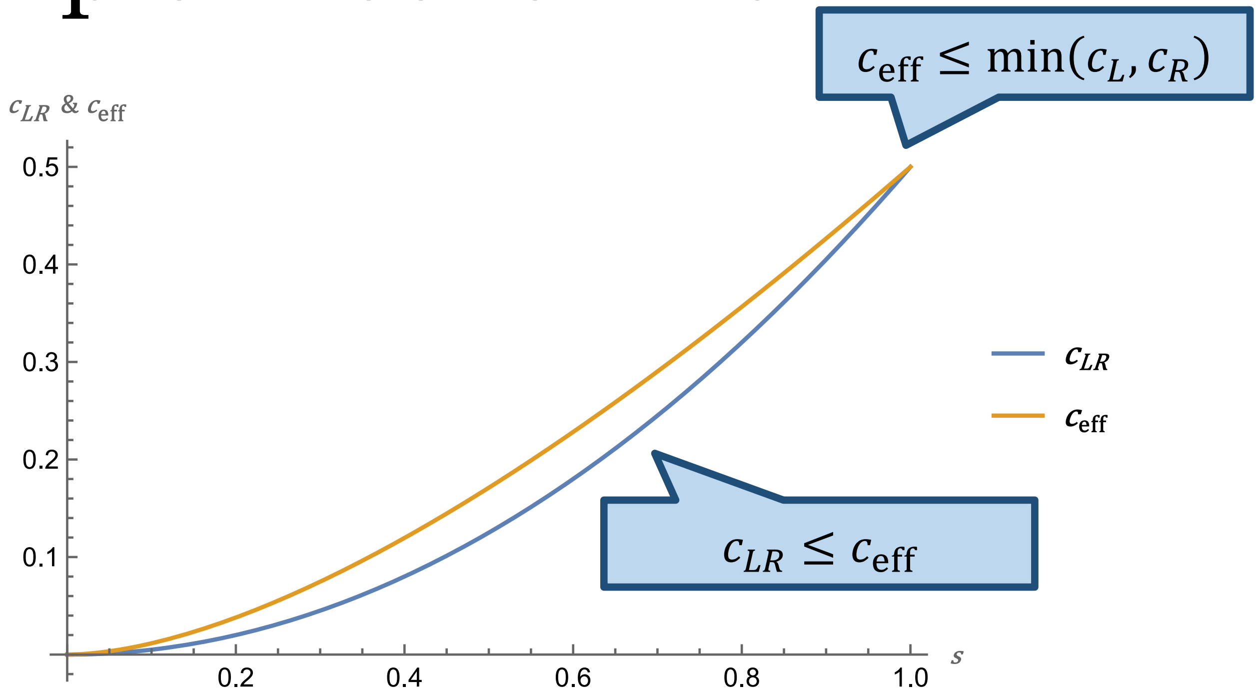


c_{LR} & c_{eff} are parametrized by one parameter $s \in (0,1)$;

$c_{LR} = s^2$

$c_{\text{eff}} = \frac{1}{2} + s + \frac{3}{\pi^2}\left((s+1)\log(s+1)\log s\ + (s-1)\mathrm{Li}_2(1-s) + (s+1)\mathrm{Li}_2(-s)\right)$

Example: free fermion

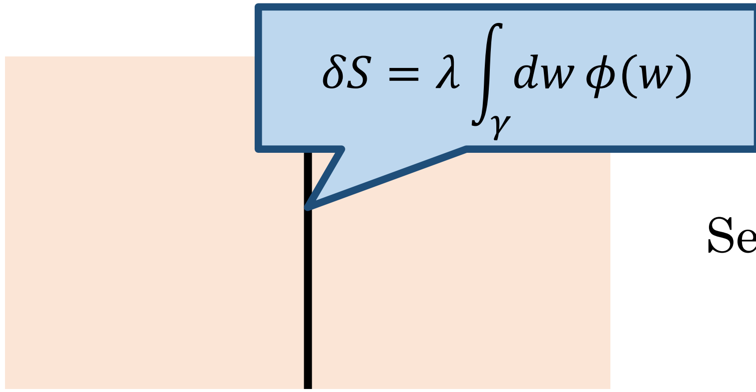


c_{LR} & c_{eff} are parametrized by one parameter $s \in (0,1)$;

$$c_{LR} = \frac{s^2}{2}$$

$$c_{\mathrm{eff}} = \frac{s-1}{2} - \frac{3}{\pi^2}\Big((s+1)\log(s+1)\log s \;+ (s-1)\mathrm{Li}_2(1-s) + (s+1)\mathrm{Li}_2(-s)\Big)$$

Example: defect perturbation



Setup: general CFT

$$\mathrm{CFT}_L = \mathrm{CFT}_R$$

Perturbation by defect field ϕ

$$\delta S = \lambda \int_\gamma dw\, \phi(w)$$

It has been shown in [Brehm],

$$c_{\text{eff}} = c\left(\mathcal{T} + \frac{1}{4}(1-\mathcal{T})\right) + O(\lambda^2)$$

This is consistent with λ expansion of c_{eff} in free fermion.

Since $c_{LR} = c\mathcal{T}$ and $0 \le \mathcal{T} \le 1$,

$$c_{LR} \le c_{\text{eff}}$$

Contents

Future direction

Application of universal asymptotic formula to
- Averaged BCFT / ICFT

Application of universal bounds to
- Weak measurement
- Pseudo entropy
- Numerical bootstrap [WIP]

Higher-dimensional generalization
- Asymptotic formula using TFT approach [WIP]
- Universal bounds (partly done in our works) [WIP]

Many other applications of ICFT

Example from my (ongoing) works:

Symmetry resolution & Asymmetry

[YK, Mirciano, Ooguri, Pal] & [WIP with same members]

Characterizing TO & SPT

[Liu, YK, Kudler-Flam, Sohal, Ryu] & [WIP with Liu, Ohyama, Ryu]

Exactly marginal deformation

[WIP with Komatsu, Ooguri]

If you are curious whether interface can be applied to XXX, please feel free to ask me ;)

Thank you

Appendix

- Calculation of EE in ICFT
- Universal Bound on c_{eff}
  - Holographic proof
  - Entropic proof
- Universal Bound on c_{LR}
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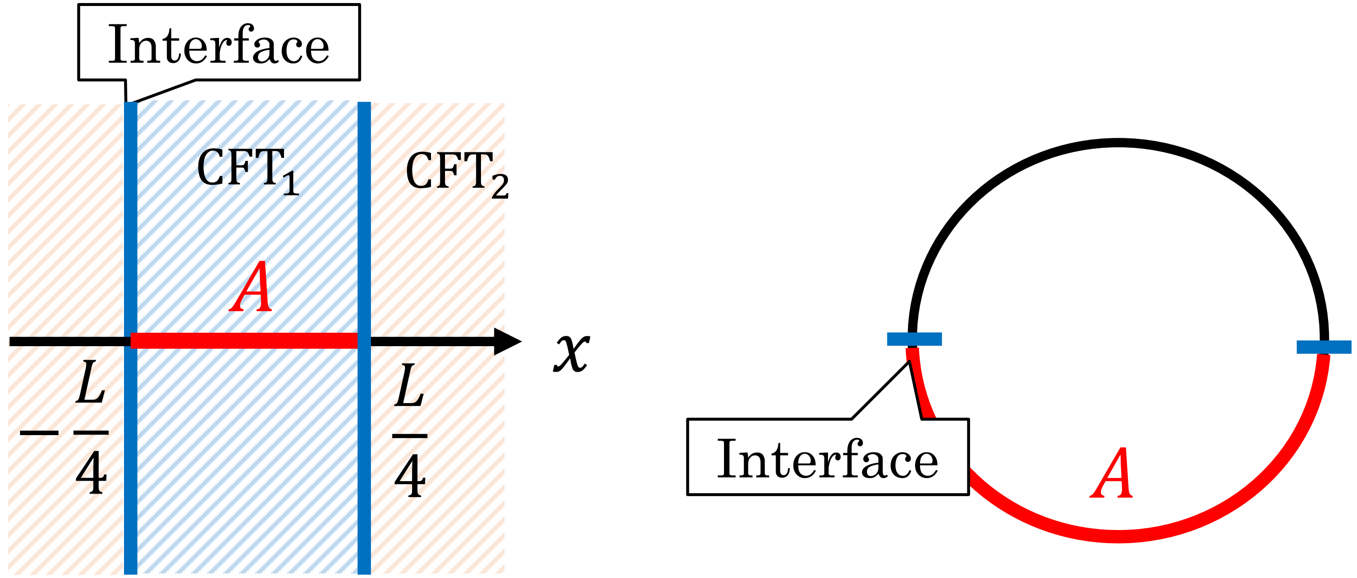
  - Check in non-holographic CFTs
- Higher dimensional generalization

Calculation of EE

Our setup



Calculation of EE

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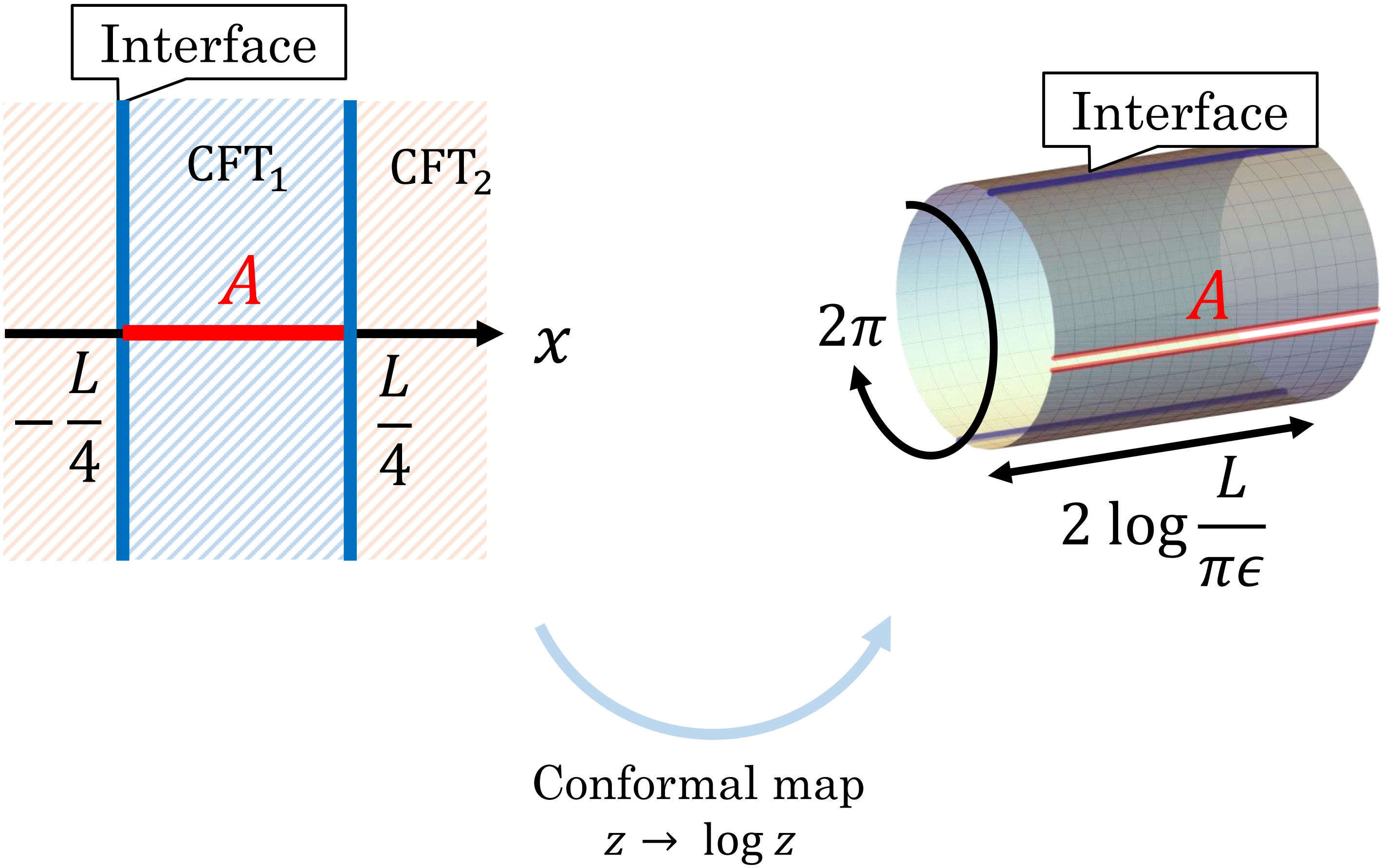
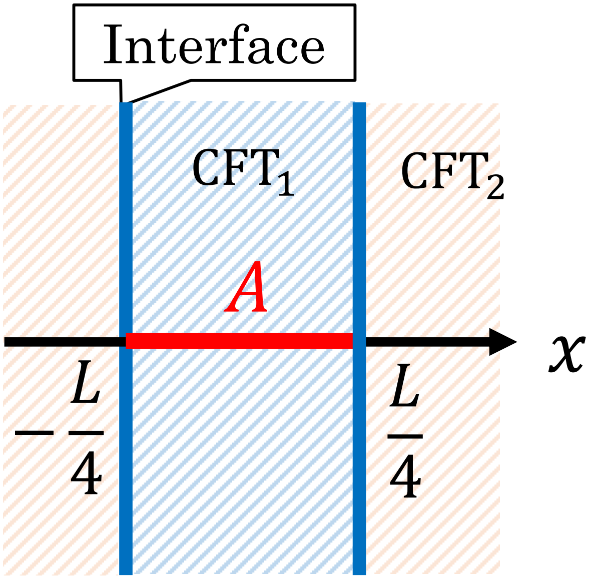
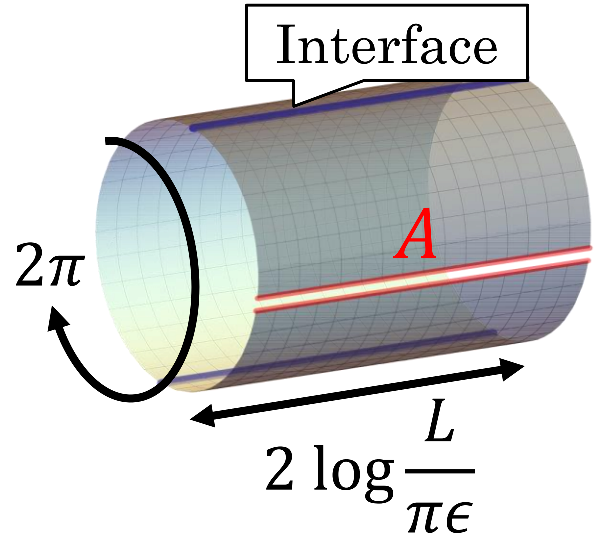


Calculation of EE





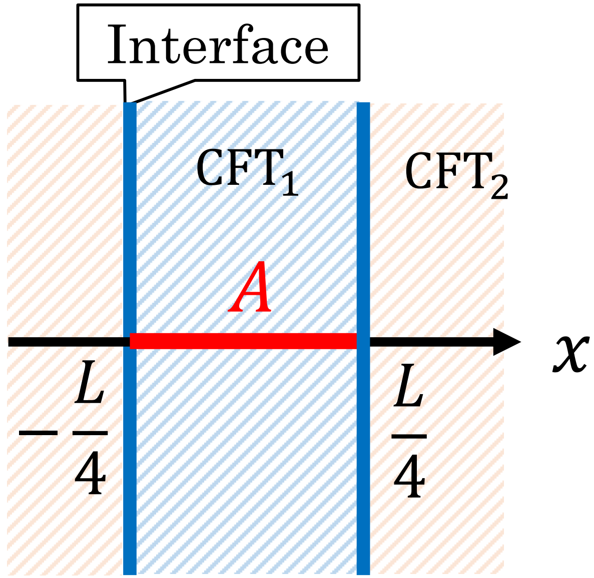
Replica trick

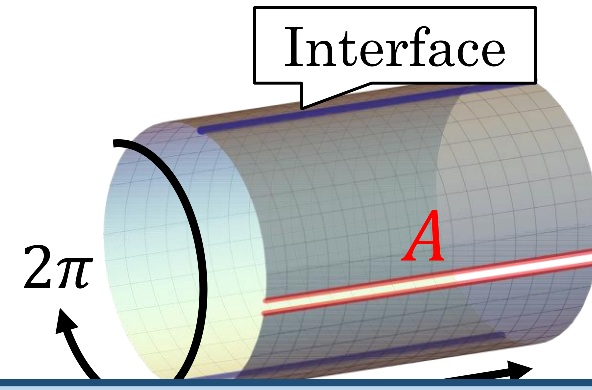
$$S_A = \lim_{n\to 1} S_A^{(n)}, \qquad S_A^{(n)} \equiv \frac{1}{1-n}\log\frac{Z_n}{(Z_1)^n}$$

where the replica partition function is

$$Z_n = \mathrm{tr}\left(e^{-\frac{\beta}{2}H^{(1)}}\,\mathcal{J}\,e^{-\frac{\beta}{2}H^{(2)}}\mathcal{J}^\dagger\right)^n, \quad \beta \equiv \frac{\pi^2}{\log\frac{L}{\pi\epsilon}}, \quad H^{(i)} \equiv L^{(i)} - \frac{c_i}{24}$$

Calculation of EE





Hamiltonian in BCFT is conventionally defined on $[0, \pi]$,

$$H_{\mathrm{conv}}^{(i)} = \pi\left(L^{(i)} - \frac{c_i}{24}\right)$$

But for convenance, we use

$$\beta H^{(i)} = \frac{2\pi}{2\log\frac{L}{\pi\epsilon}} H_{\mathrm{conv}}^{(i)}$$

Replica trick

$$S_A = \lim_{n\to 1} S_A^{(n)},$$

where the replica partition function is

$$Z_n = \mathrm{tr}\left(e^{-\frac{\beta}{2}H^{(1)}}\,\mathcal{J}\,e^{-\frac{\beta}{2}H^{(2)}}\mathcal{J}^{\dagger}\right)^n, \quad \beta \equiv \frac{\pi^2}{\log\frac{L}{\pi\epsilon}}, \quad H^{(i)} \equiv L^{(i)} - \frac{c_i}{24}$$

Calculation of EE

Open-closed duality

$$Z_n = \mathrm{tr}\left(e^{-\frac{\beta}{2}H^{(1)}}\,\mathcal{I}\,e^{-\frac{\beta}{2}H^{(2)}}\mathcal{I}^\dagger\right)^n = \langle B|e^{-\frac{(2\pi)^2}{2n\beta}H_n}|B\rangle$$

where H_n is the Hamiltonian of the theory with $2n$ interfaces.
(B is some boundary condition imposed on entangling surface.)

In the high-temperature limit $\beta = \frac{\pi^2}{\log\frac{L}{\pi\epsilon}} \xrightarrow[\epsilon\to 0]{} 0$,
only the vacuum ($\equiv \Delta_n^0$)propagates through the cylinder,

$$Z_n \sim e^{-\frac{(2\pi)^2}{2n\beta}\Delta_n^0}$$

Thus, we have

$$S_A^{(n)} = \frac{1}{1-n}\log\frac{Z_n}{(Z_1)^n} = \frac{2}{1-n}\left(\frac{\Delta_n^0}{n} - n\Delta_1^0\right)\log\frac{l}{\epsilon}$$

Calculation of EE

Just a formal expression
No simple formula is known

Open-closed duality

$$Z_n = \mathrm{tr}\left(e^{-\frac{\beta}{2}H^{(1)}}\,\mathcal{J}\,e^{-\frac{\beta}{2}H^{(2)}}\mathcal{J}^\dagger\right)^n = \langle B|e^{-\frac{(2\pi)^2}{2n\beta}H_n}|B\rangle$$

where H_n is the Hamiltonian of the theory with $2n$ interfaces.

(Def) Effective central charge

$$c_{\mathrm{eff}} = \lim_{n\to 1}\frac{12n}{1-n^2}\left(n\Delta_1^0 - \frac{\Delta_n^0}{n}\right),$$

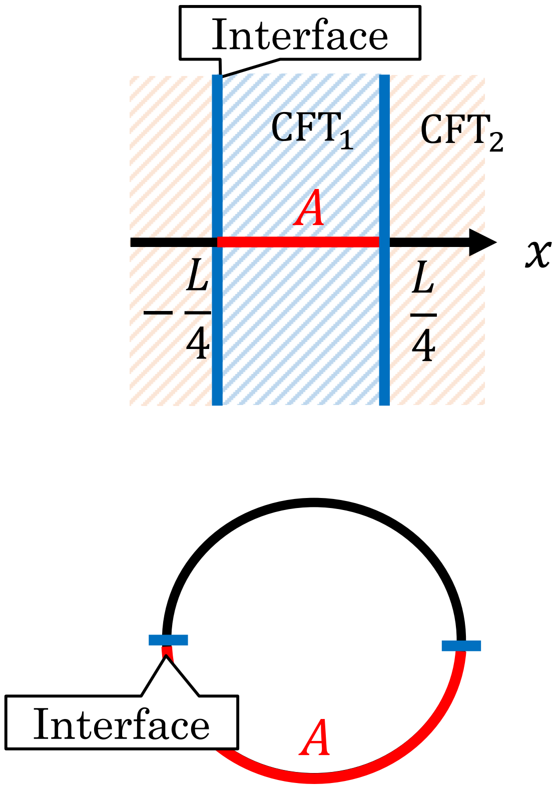
where Δ_n^0 is the vacuum energy in the $2n$-interface Hilbert space.

⇒Is this useful to give insights into general interfaces?

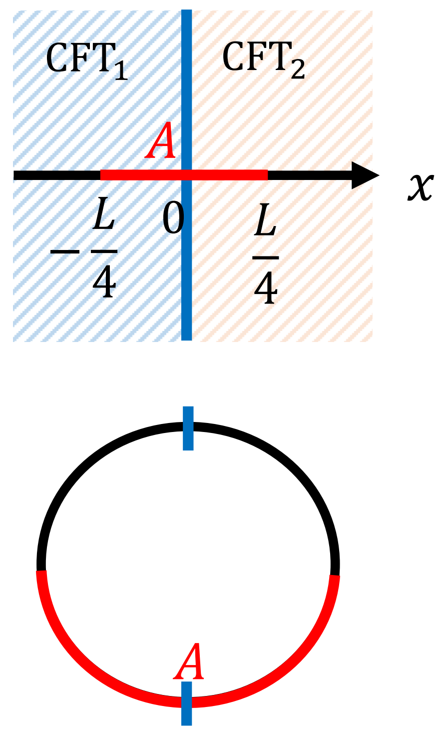
😵 No nice property is known for Δ_n^0.

EE in various setups

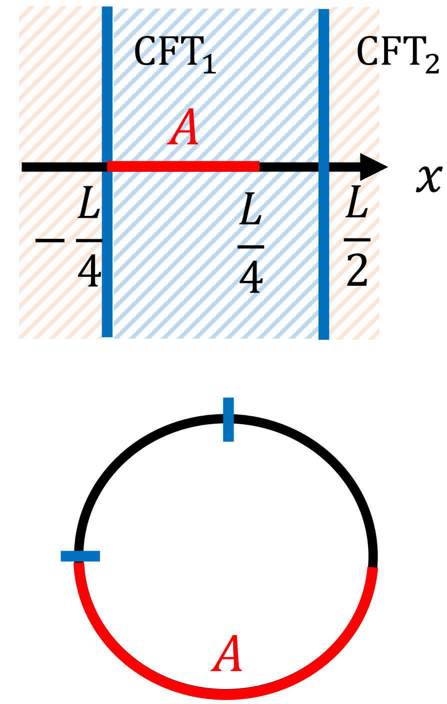
(i) one-side



(ii) symmetric



(iii) asymmetric



EE in various setups

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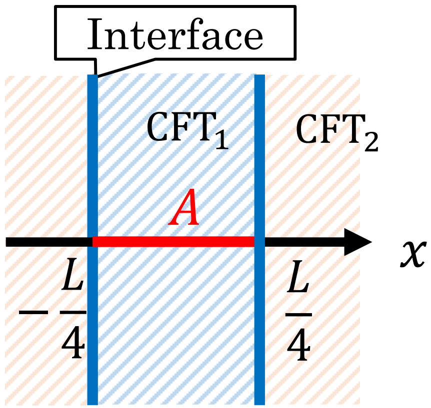
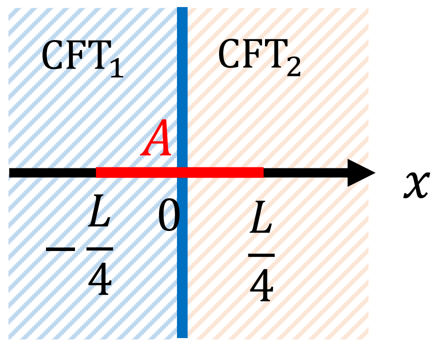


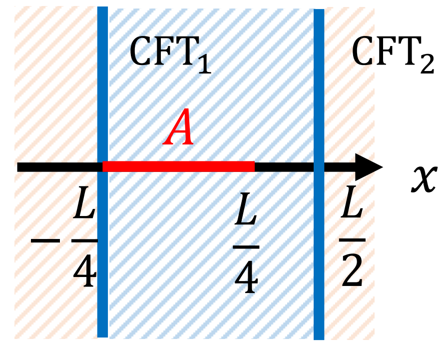
$$S_A = \frac{c_{\mathrm{eff}}}{3} \log \frac{L}{\pi \epsilon}$$

(ii) symmetric



$$S_A = \frac{c_1 + c_2}{6} \log \frac{L}{\pi \epsilon}$$

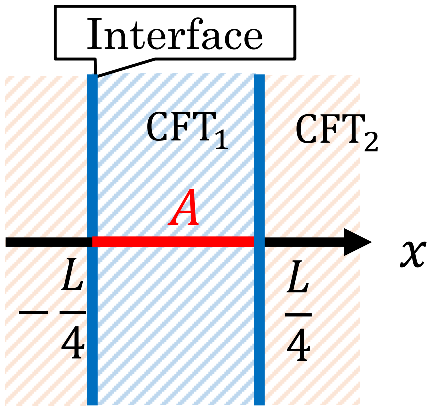
(iii) asymmetric



$$S_A = ?$$

EE in various setups

(i) one-side



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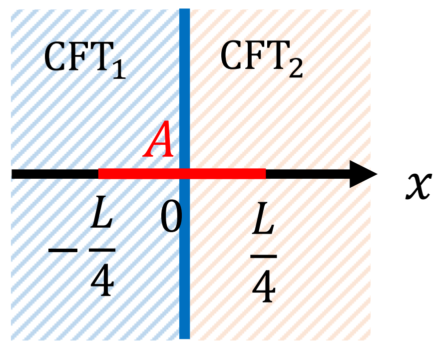

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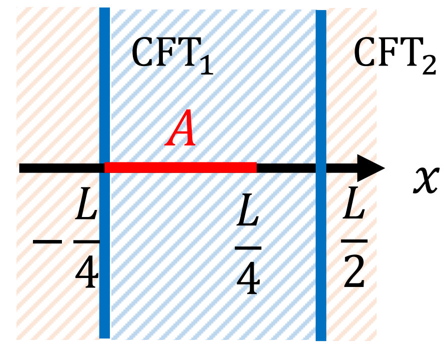
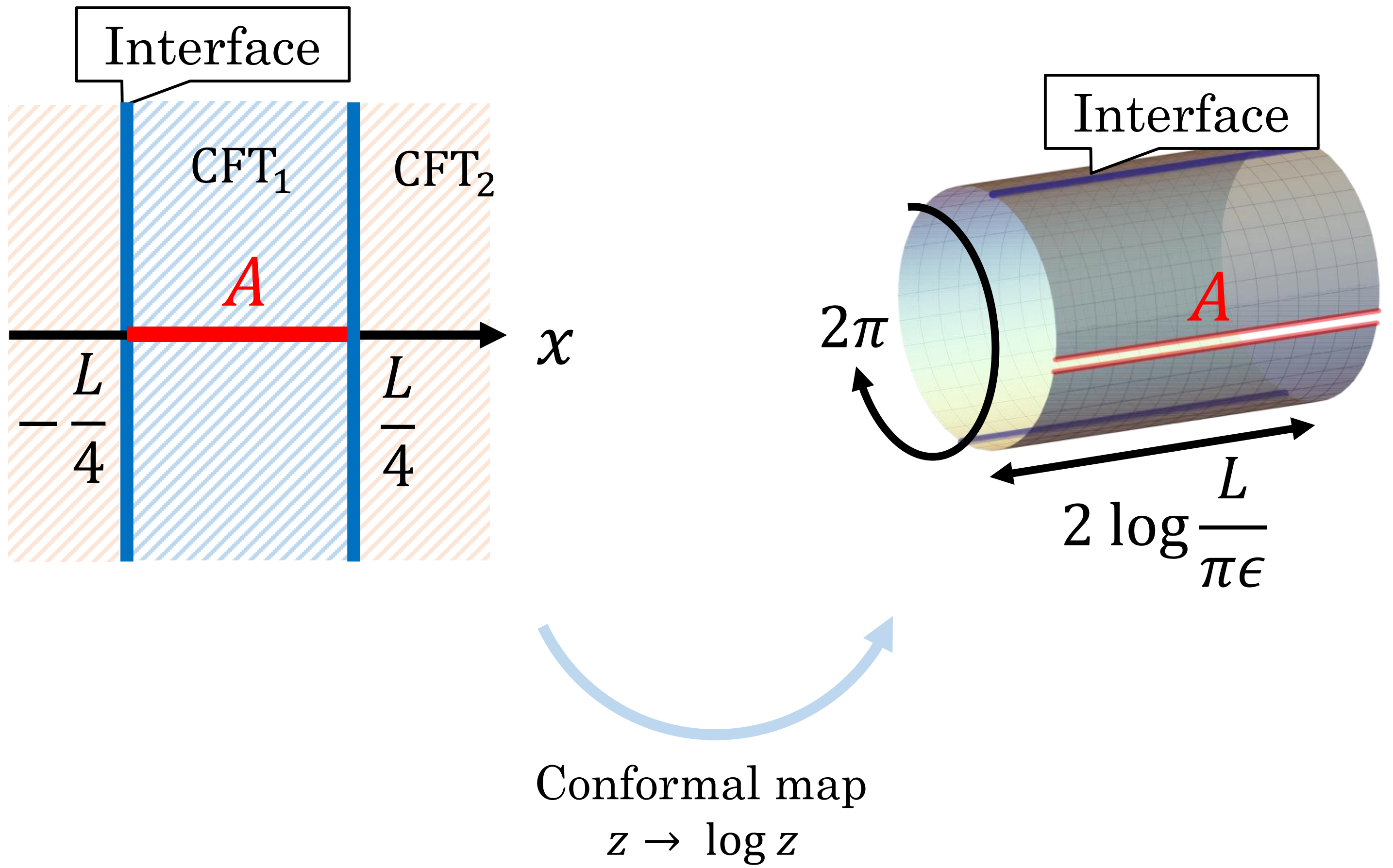


$$S_A = \frac{c_1 + c_{\mathrm{eff}}}{6} \log \frac{L}{\pi\epsilon}$$

Conjectured from the gravity side [Karch, Luo, Sun] & [Karch, Wang]

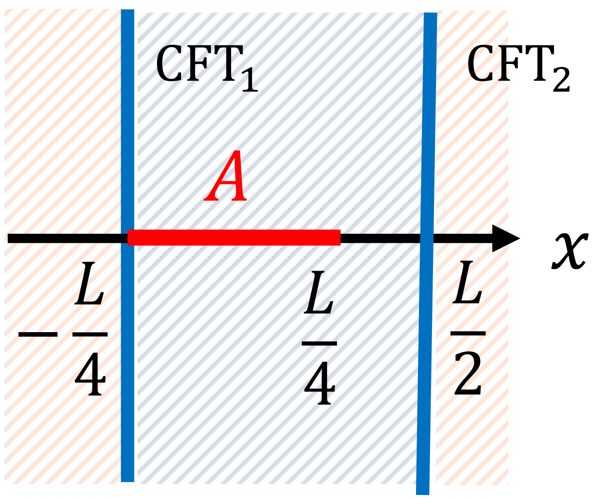
EE in one-side setup

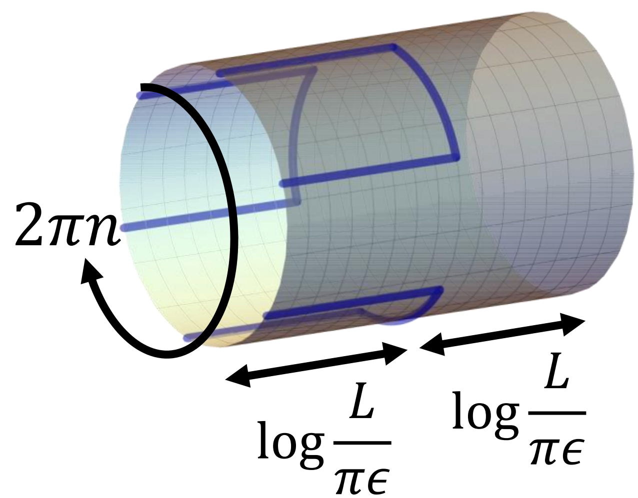
Our setup



EE in asymmetric setup

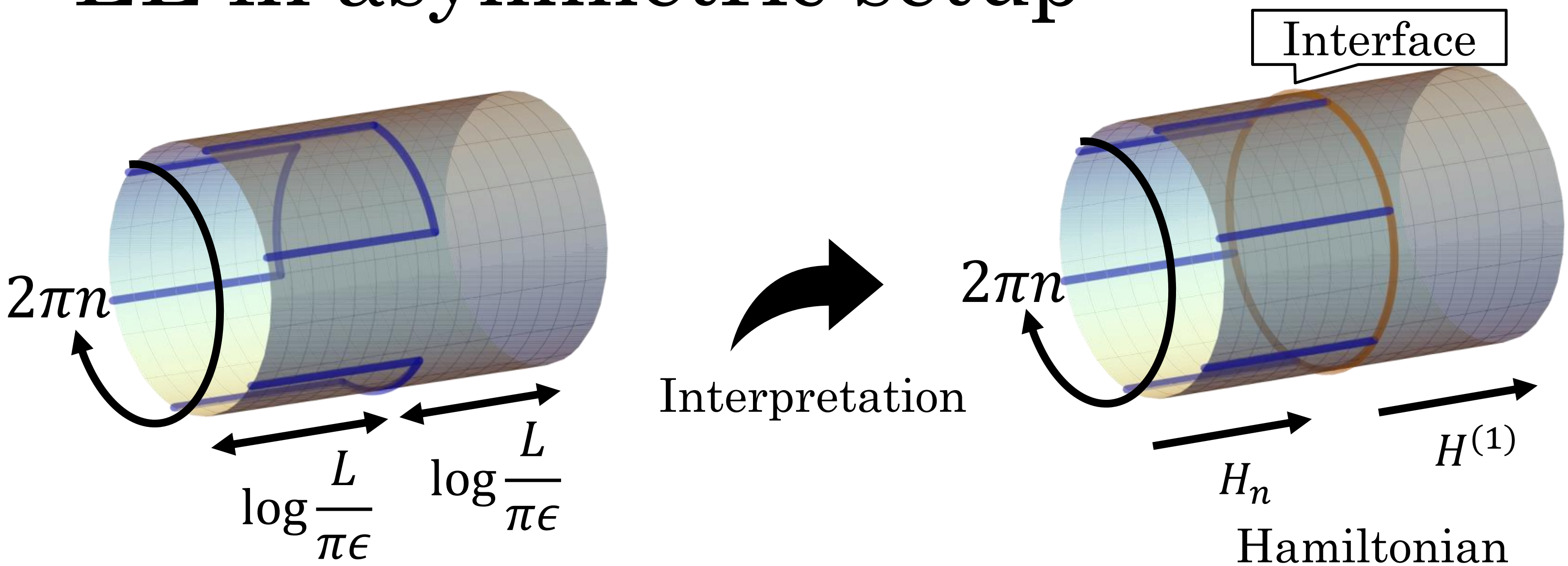
Our setup





Conformal map
$z \to \log z$

EE in asymmetric setup



(Result)

$$Z_n = \langle B| e^{-\frac{1}{n}\log\frac{L}{\pi\epsilon}H_n}\, \mathcal{J}\, e^{-\frac{1}{n}\log\frac{L}{\pi\epsilon}H^{(1)}} |B\rangle$$
$$\sim\ e^{-\frac{1}{n}\log\frac{L}{\pi\epsilon}\Delta_n^0} \times e^{-\frac{1}{n}\log\frac{L}{\pi\epsilon}\Delta_{CFT_1}^0}$$

$$\Rightarrow \quad S_A = \frac{c_1+c_{\mathrm{eff}}}{6}\log\frac{L}{\pi\epsilon}$$

Conjecture from gravity is now proven!

Appendix

- Calculation of EE in ICFT
- Universal Bound on c_{eff}
  - Holographic proof
  - Entropic proof
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  - Check in non-holographic CFTs
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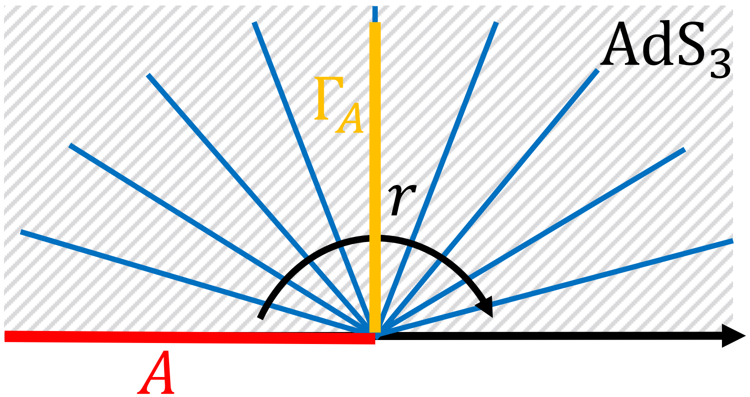
Holographic EE in ICFT



$$ds^2 = e^{2A(r)} \frac{dx^2 - dt^2}{x^2} + dr^2$$

Relation between c and $A(r)$:

$$c_L = \frac{3}{2G_N} \frac{1}{A'(\infty)}$$

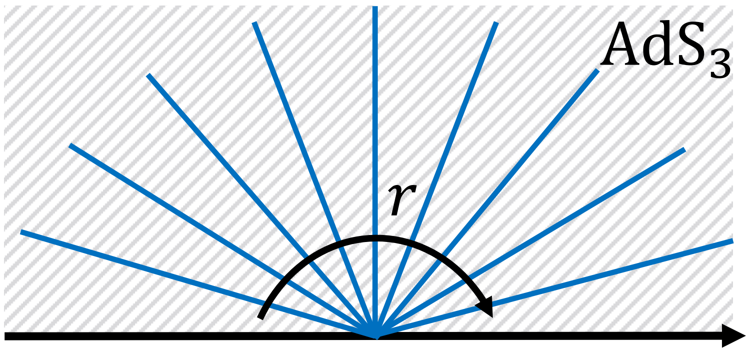
$$c_R = -\frac{3}{2G_N} \frac{1}{A'(-\infty)}$$

$$c_{\text{eff}} = \frac{3}{2G_N} e^{A(r_{\min})}$$

R-T formula

where $r_{\min}$ is r that leads to the minimum value of $A(\mathrm{r})$.

Holographic EE in ICFT



$$ds^2 = e^{2A(r)}\frac{dx^2 - dt^2}{x^2} + dr^2$$

Generalization of holographic c-theorem (=NEC):

$$A''(r) \le -ke^{-2A(r)}$$

where k is the sign of the constant curvature of the 2d slice.

Remark:

- $k = 0$ leads to the standard c-theorem.
- $k = -1$ makes the inequality weaker, so nobody is interested in this case. But now we make use of this!

Calculation of EE in ICFT

Define

$$F(r) \equiv A'(r)^2 + e^{-2A(r)}$$

Then,

$$F'(r) = 2\big(A''(r) - e^{-2A(r)}\big)A'(r)$$

Calculation of EE in ICFT

Define

$$F(r) \equiv A'(r)^2 + e^{-2A(r)}$$

Then,

$$F'(r) = 2\left(A''(r) - e^{-2A(r)}\right)A'(r)$$

Proposition 1

When approaching infinity, we reach AdS_3, i.e. $A(r) \approx \cosh(r)$.

$\Rightarrow \quad F(\pm\infty) = A'(\pm\infty)^2$

Calculation of EE in ICFT

Define

$$F(r) \equiv A'(r)^2 + e^{-2A(r)}$$

Then,

$$F'(r) = 2\underbrace{\left(A''(r) - e^{-2A(r)}\right)}_{<0\ (\because \mathrm{NEC})}A'(r)$$

Proposition 1

When approaching infinity, we reach AdS_3, i.e. $A(r) \approx \cosh(r)$.

$\Rightarrow \quad F(\pm\infty) = A'(\pm\infty)^2$

Proposition 2

$\Rightarrow \quad F'(r_{\min}) = A'(r_{\min}) = 0$

$\Rightarrow \quad F(r_{\min}) = e^{-2A(r_{\min})}$

$F(r_{\min})$ is the maximum of $F(r)$

Calculation of EE in ICFT

Proposition 1

$$F(\pm\infty) = A'(\pm\infty)^2$$

Proposition 2

$$F(r_{\min}) = e^{-2A(r_{\min})}$$

$F(r_{\min})$ is the maximum of $F(r)$

Proposition 3

$$c_L = \frac{3}{2G_N}\frac{1}{A'(\infty)}$$

$$c_R = -\frac{3}{2G_N}\frac{1}{A'(-\infty)}$$

$$c_{\text{eff}} = \frac{3}{2G_N}e^{A(r_{\min})}$$

(Result) Universal upper bound

$$F(r_{\min}) \geq F(\pm\infty)$$

$$\Rightarrow \quad e^{-2A(r_{\min})} \geq A'(\pm\infty)^2$$

$$\Rightarrow \quad c_{\text{eff}} \leq \min(c_L, c_R)$$

AdS version of holographic c-theorem

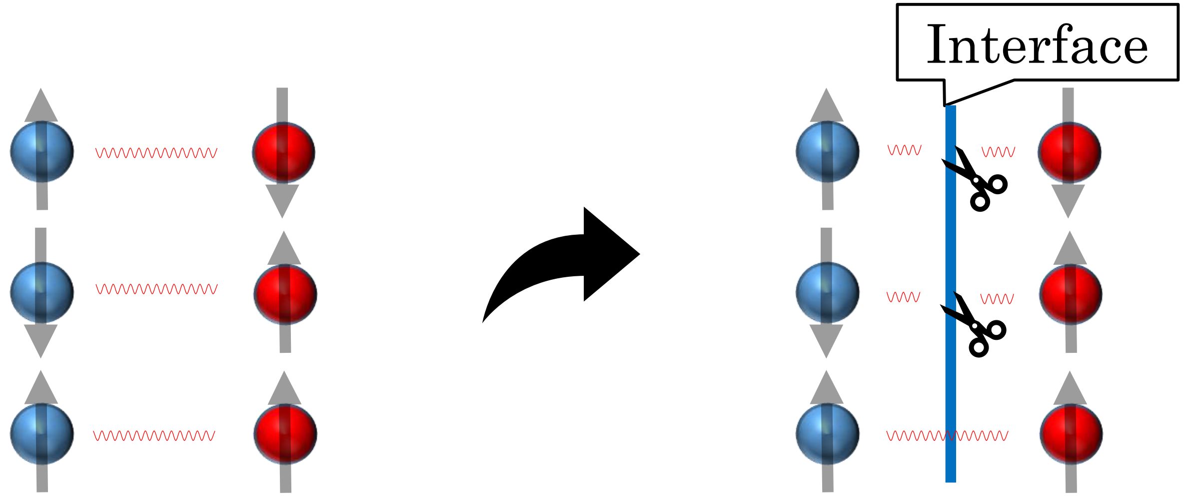
Result

(Result) Universal upper bound

$$c_{\mathrm{eff}} \leq \min(c_L, c_R)$$

(Interpretation)

<u>**Any interface decreases entanglement.**</u>



Result

(Result) Universal upper bound

$$c_{\text{eff}} \leq \min(c_L, c_R)$$

(Interpretation)

Any interface decreases entanglement.

Proof without AdS/CFT?

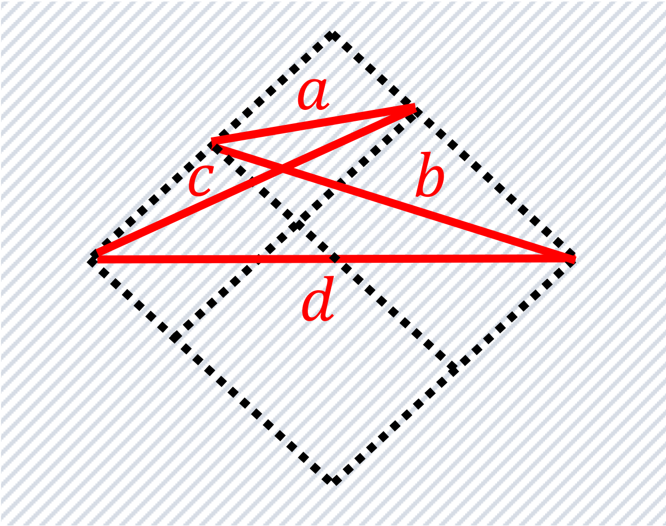
$\Rightarrow$ Entropic c-theorem (see details in our paper or App.)

Upper bound from CFT

How can we show the upper bound on c_{eff} on the CFT side?

On the gravity side, the **holographic c-theorem** is the key!

$\Rightarrow$The c-theorem may be useful on the CFT side.

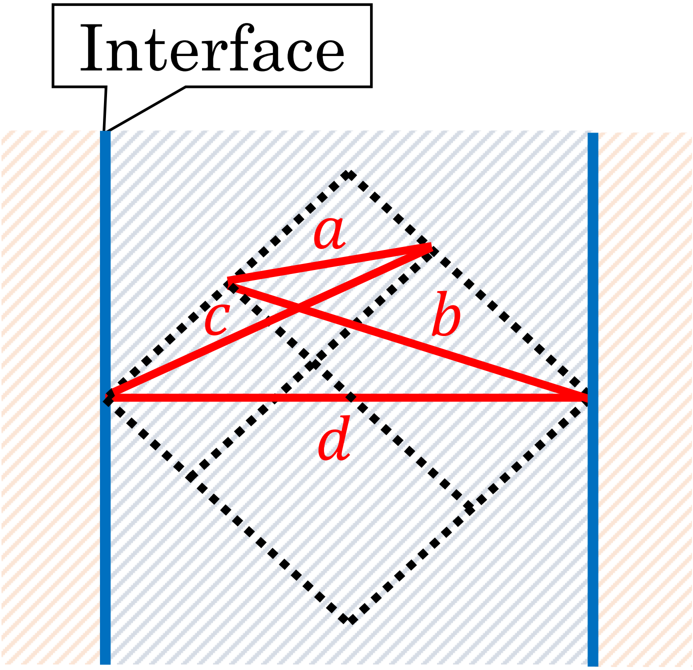


Key tools to derive entropic c-theorem:

(1) $S(b) + S(c) \geq S(a) + S(d)$

(2) $|a||d| = |b||c|$

Entropic c-theorem



Key tools to derive entropic c-theorem:

(1) $S(b) + S(c) \geq S(a) + S(d)$

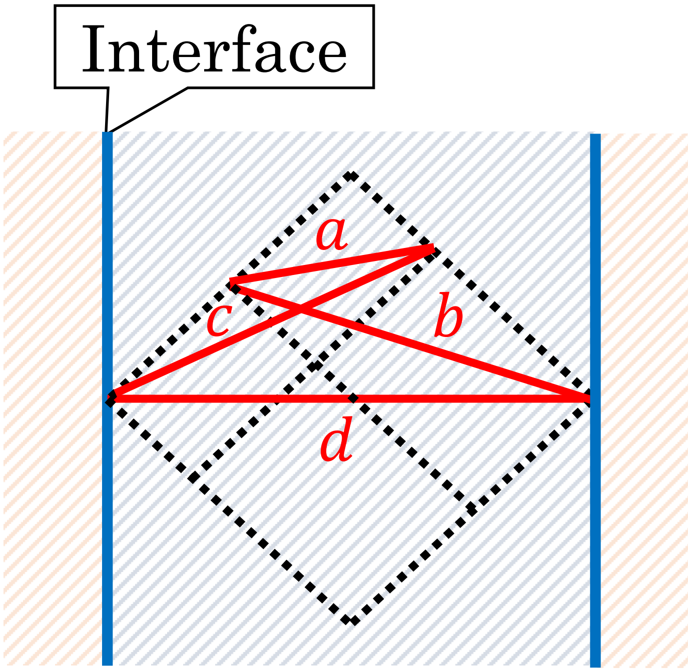
(2) $|a||d| = |b||c|$

Remark:

These two still work in ICFT.

$S(b)$ and $S(c)$ are evaluated using the formula (iii).

Entropic c-theorem



Key tools to derive entropic c-theorem:

(1) $S(b) + S(c) \geq S(a) + S(d)$

(2) $|a||d| = |b||c|$

(Result)

$$\frac{c_1 - c_{\text{eff}}}{6} \log \frac{|d|}{|a|} \geq 0, \qquad |d| \gg |a|$$

$$\Rightarrow \quad c_{\text{eff}} \leq \min(c_1, c_2)$$

CFT version of the proof

Appendix

- Calculation of EE in ICFT
- Universal Bound on c_{eff}
  - Holographic proof
  - Entropic proof
- Universal Bound on c_{LR}
  - Holographic proof
  - Check in non-holographic CFTs
- Higher dimensional generalization

Holographic c_{LR}

- metric

$$ds^2 = \Omega(\theta)^2 \left(\frac{dx^2 - dt^2}{x^2} + d\theta^2 \right), \qquad \theta \in (-\pi/2, \pi/2)$$

- effective AdS radius

$$l(\theta) \equiv \frac{\Omega(\theta)^2}{\sqrt{\Omega(\theta)^2 + \Omega'(\theta)^2}}$$

- effective brane tension ($\equiv \sigma(\theta)$)

$$8\pi G_N \frac{d\sigma}{d\theta} = \frac{\Omega(\theta)|l'(\theta)|}{l(\theta)^2 \sqrt{\Omega(\theta)^2 - l(\theta)^2}}$$

Gravity calculation of c_{LR}

$$c_{LR} = \frac{3}{G_N} \left(\frac{1}{l_L} + \frac{1}{l_R} + 8\pi G_N \sigma \right)^{-1}$$

Simple bound [Karch, YK, Ooguri, Sun, Wang]

Gravity calculation of c_{LR}

$$c_{LR} = \frac{3}{G_N}\left(\frac{1}{l_L} + \frac{1}{l_R} + 8\pi G_N \sigma\right)^{-1}$$

σ is complicated but has simple bound;

$$8\pi G_N \int d\sigma = \int_{-\pi/2}^{\pi/2} d\theta \frac{\Omega(\theta)|l'(\theta)|}{l(\theta)^2\sqrt{\Omega(\theta)^2 - l(\theta)^2}}$$

$$\geq \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \frac{|l'(\theta)|}{l(\theta)^2} = \int \frac{|dl|}{l^2} \geq \left|\frac{1}{l_L} - \frac{1}{l_{\min}}\right| + \left|\frac{1}{l_{\min}} - \frac{1}{l_R}\right|$$

Thus, we obtain

$$c_{LR} \leq \frac{3 l_{\min}}{2 G_N} = c_{\mathrm{eff}}$$

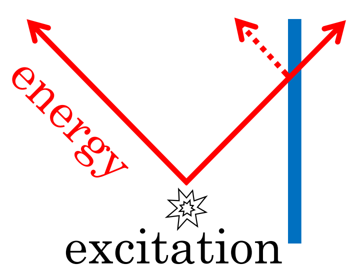
Result

(Result) Universal lower bound

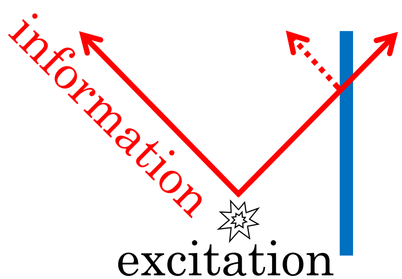
$$c_{LR} \leq c_{\text{eff}}$$

(Interpretation)

The amount of energy transmission can never exceed the amount of information transmission.



$\leq$



Amount of transmitted energy

Amount of transmitted information
(= # of transmitted EPR pairs)

Result

(Result) Universal lower bound

$$c_{LR} \leq c_{\mathrm{eff}}$$

(Interpretation)

The amount of energy transmission can never exceed the amount of information transmission.

Is the inequality sharp?

Only two possibilities:

- $c_{LR} = c_{\mathrm{eff}} = 0$ (⇔ totally reflective)
- $c_{LR} = c_{\mathrm{eff}} = c_L = c_R$ (⇔ totally transmissive)

Result

(Result) Universal lower bound

$$c_{LR} \leq c_{\text{eff}}$$

(Interpretation)

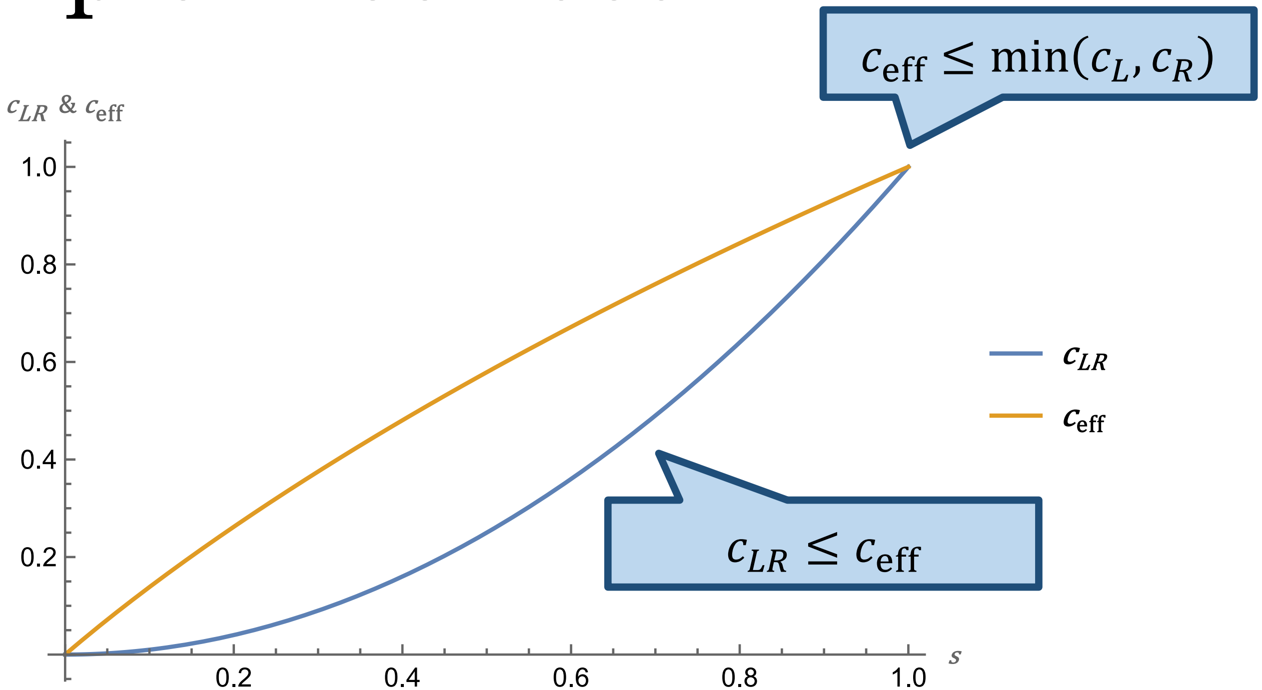
The amount of energy transmission can never exceed the amount of information transmission.

Beyond holography?

$\Rightarrow$ One can check in some examples:

- free boson
- free fermion
- defect perturbation

Example: free boson

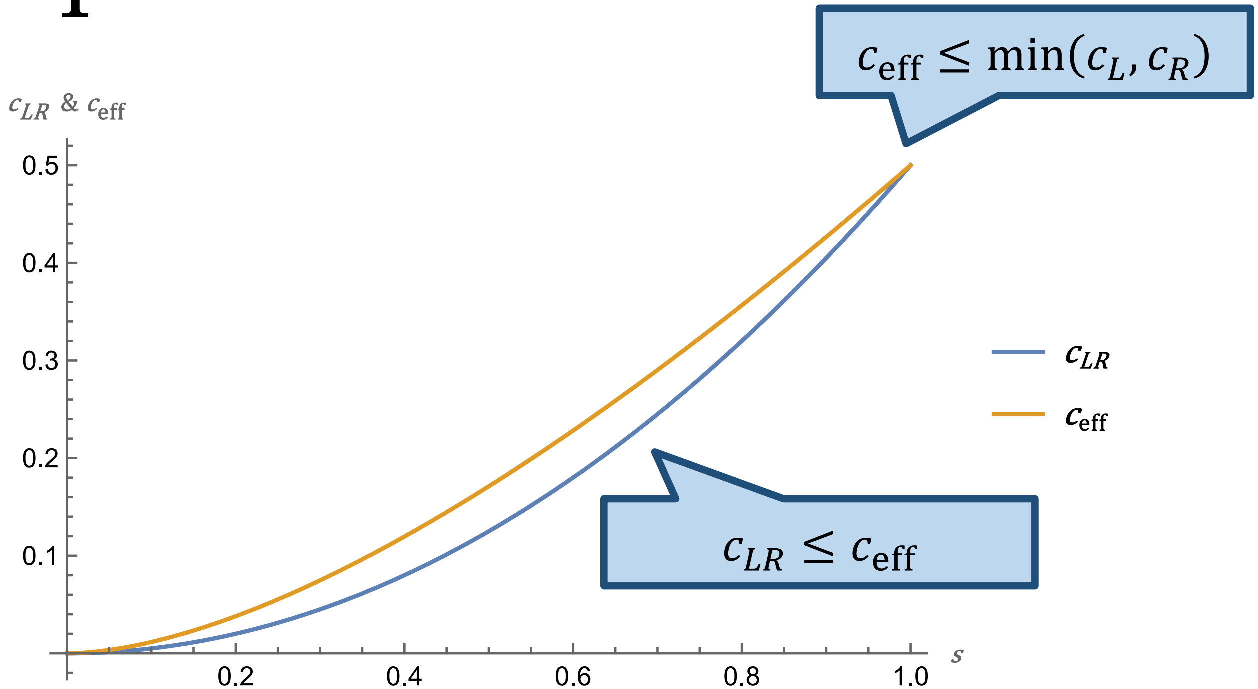


c_{LR} & c_{eff} are parametrized by one parameter $s \in (0,1)$;

$c_{LR} = s^2$

$$c_{\text{eff}} = \frac{1}{2} + s + \frac{3}{\pi^2}\left((s+1)\log(s+1)\log s \; + (s-1)\text{Li}_2(1-s) + (s+1)\text{Li}_2(-s)\right)$$

Example: free fermion

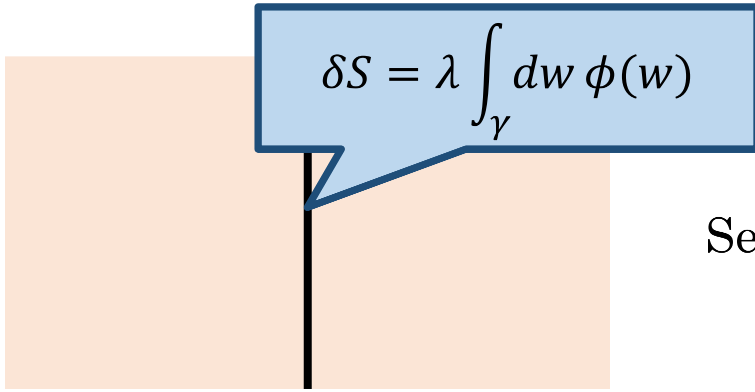


c_{LR} & c_{eff} are parametrized by one parameter $s \in (0,1)$;

$$c_{LR} = \frac{s^2}{2}$$

$$c_{\text{eff}} = \frac{s-1}{2} - \frac{3}{\pi^2}\Big((s+1)\log(s+1)\log s\; + (s-1)\mathrm{Li}_2(1-s) + (s+1)\mathrm{Li}_2(-s)\Big)$$

Example: defect perturbation



Setup: general CFT

$$\mathrm{CFT}_L = \mathrm{CFT}_R$$

Perturbation by defect field ϕ

$$\delta S = \lambda \int_\gamma dw\, \phi(w)$$

It has been shown in [Brehm],

$$c_{\text{eff}} = c\left(\mathcal{T} + \frac{1}{4}(1 - \mathcal{T})\right) + O(\lambda^2)$$

This is consistent with λ expansion of c_{eff} in free fermion.

Since $c_{LR} = c\mathcal{T}$ and $0 \leq \mathcal{T} \leq 1$,

$$c_{LR} \leq c_{\text{eff}}$$

Comment

In three examples, c_{eff} is a function of c_{LR},
which means that c_{eff} and c_{LR} have the same information.

Is this true?

😟 Answer is NO.

While c_{LR} is given by an integration over the **entire region**,
c_{eff} depends only on the **minimal value** of $A(r)$.

$\Rightarrow$ Relation between c_{LR} & c_{eff} is highly nontrivial.

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c_{eff} in higher d

Question:

What is a higher-dimensional analog of c_{eff}?

Assume d is even (few modifications in odd d)

General form of EE

$$S_A = s_{d-2}\left(\frac{l}{\epsilon}\right)^{d-2} + s_{d-4}\left(\frac{l}{\epsilon}\right)^{d-4} + \cdots + C\log\frac{l}{\epsilon} + s_0$$

Candidate

c_{eff} in higher d

Setup:

CFT on $\mathbb{R} \times S^{d-1}$ with interface along equator $\mathbb{R} \times S^{d-2}$

Holographic ICFT:

$$ds^2 = e^{2A(r)} ds^2_{\text{AdS}_d} + dr^2$$

Holographic EE:

$$S_A = \cdots + (-1)^{\frac{d}{2}-1} 4\, c_{\text{eff}} \log \frac{2}{\epsilon} + s_0$$

where

$$c_{\text{eff}} = \frac{\pi^{\frac{d}{2}}}{\Gamma\left(\frac{d}{2}\right)} \frac{e^{(d-1)A(r_{\min})}}{4\pi G_N^{(d-1)}}$$

c_{eff} in higher d

Proposition 1

$$c_L = \frac{\pi^{\frac{d}{2}}}{\Gamma\left(\frac{d}{2}\right)} \frac{\left(A'(\infty)\right)^{1-d}}{4\pi G_N^{(d-1)}}$$

$$c_R = \frac{\pi^{\frac{d}{2}}}{\Gamma\left(\frac{d}{2}\right)} \frac{\left(A'(-\infty)\right)^{1-d}}{4\pi G_N^{(d-1)}}$$

$$c_{\text{eff}} = \frac{\pi^{\frac{d}{2}}}{\Gamma\left(\frac{d}{2}\right)} \frac{e^{(d-1)A(r_{\min})}}{4\pi G_N^{(d-1)}}$$

Proposition 2

NEC holds in **any dimension**

$$\Rightarrow \quad e^{-2A(r_{\min})} \geq A'(\pm\infty)^2$$

(Result) Universal upper bound in **any dimension**

$$c_{\text{eff}} \leq \min(c_L, c_R)$$

Higher-dimension

- Higher dimensional analog of universal formula

$$S_A = \frac{c_L + c_{\mathrm{eff}}}{6} \log \frac{L}{\pi\epsilon}$$

✓ True

- Higher dimensional analog of

$$c_{\mathrm{eff}} \leq \min(c_L, c_R)$$

✓ True

Our candidate for c_{eff} satisfies all higher dimensional analogs of universal formulae.

$\Rightarrow$higher dimensional analog of c_{eff} is defined by

$$S_A = \cdots + (-1)^{\frac{d}{2}-1} 4\, c_{\mathrm{eff}} \log \frac{2}{\epsilon} + s_0$$

c_{eff} in odd d

Setup:

CFT on $\mathbb{R} \times S^{d-1}$ with interface along equator $\mathbb{R} \times S^{d-2}$

Holographic ICFT:

$$ds^2 = e^{2A(r)} ds^2_{\text{AdS}_d} + dr^2$$

Holographic EE:

Not log term

$$S_A = \cdots + (-1)^{\frac{d-1}{2}} 2\pi\, c_{\text{eff}}$$

where

Same as even d

$$c_{\text{eff}} = \frac{\pi^{\frac{d}{2}}}{\Gamma\left(\frac{d}{2}\right)} \frac{e^{(d-1)A(r_{\min})}}{4\pi G_N^{(d-1)}}$$