Quantum focusing conjecture and the Page curve

松尾 善典

近畿大学

Based on arXiv:2308.05009

(For 2D gravity, arXiv:2403.19136 with A. Ishibashi, A. Tanaka)

場の理論と弦理論 2024 @ 京都大学 基礎物理学研究所

Quantum focusing conjecture is not violated even after Page time

Does quantum focusing conjecture (QFC) hold after Page time?

QFC $\Rightarrow$ S_{gen} is non-decreasing $\quad S_{\text{gen}} = \dfrac{\text{Area}}{4G_N} + S_{\text{rad}}$

After Page time $\begin{cases} \text{Area: decreasing} \\ S_{\text{rad}}\text{: decreasing} \end{cases}$ $\Rightarrow$ Violation of QFC?

Results from the island rule (formula)

Page time is an approximately null surface

Entanglement entropy is increasing after Page time

Quantum focusing conjecture is not violated

Plan of Talk

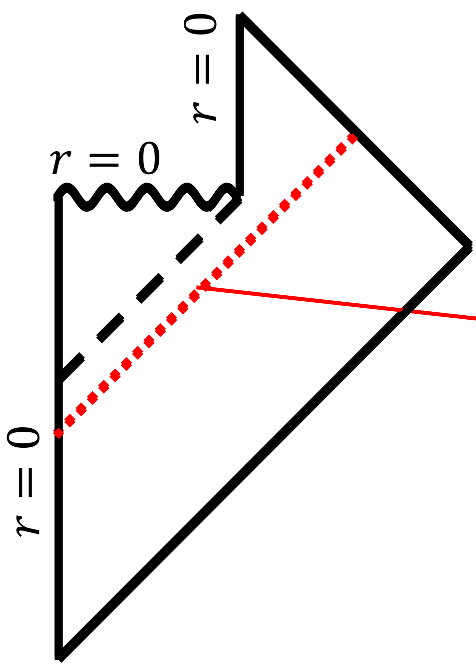
1. Quantum focusing conjecture (QFC)
    - Classical focusing is violated by quantum effects
    - QFC is inspired by generalized second law
    - Question: does QFC hold even after Page time?
2. Islands in evaporating black holes
3. Quantum focusing conjecture with/without islands

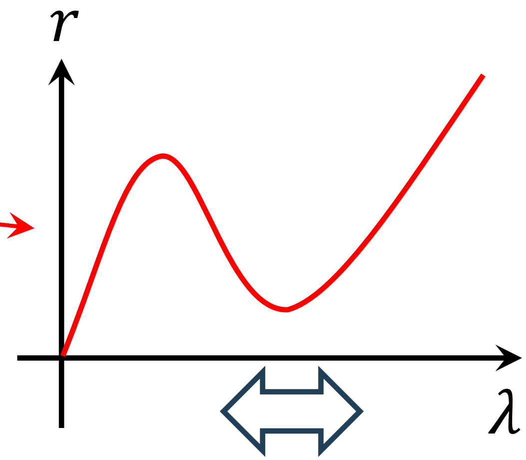
Classical focusing is violated by quantum effects

Classical focusing theorem holds if energy is positive

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma^2 - R_{\mu\nu}k^\mu k^\nu \leq 0 \qquad\qquad \theta = \frac{1}{r^2}\frac{dr^2}{d\lambda}$$

Negative energy appears in
evaporating black hole geometry





$\log r$ is convex $\Rightarrow$ Violation of classical focusing

Quantum focusing conjecture is proposed inspired by generalized second law

[Bousso-Fisher-Leuchenauer-Wall,'15]

Classical focusing $\Rightarrow$ θ is positive or ~~caustics occur~~

Event horizon has no caustics

Area of event horizon is non-decreasing

$$S_{\mathrm{BH}} = \frac{\mathrm{Area}}{4G_N}$$

BH entropy satisfies second law for classical black hole

Black hole emits Hawking radiation

Total entropy satisfies second law

$$S_{\mathrm{gen}} = \frac{\mathrm{Area}}{4G_N} + S_{\mathrm{rad}}$$

Classical focusing $\frac{d\theta}{\mathrm{d}\lambda} \leq 0$ $\Rightarrow$ Quantum focusing $\frac{d\Theta}{\mathrm{d}\lambda} \leq 0$

$$\theta = \frac{1}{r^2}\frac{dr^2}{d\lambda}$$

$$\Theta = \frac{1}{r^2}\frac{dS_{\mathrm{gen}}}{d\lambda}$$

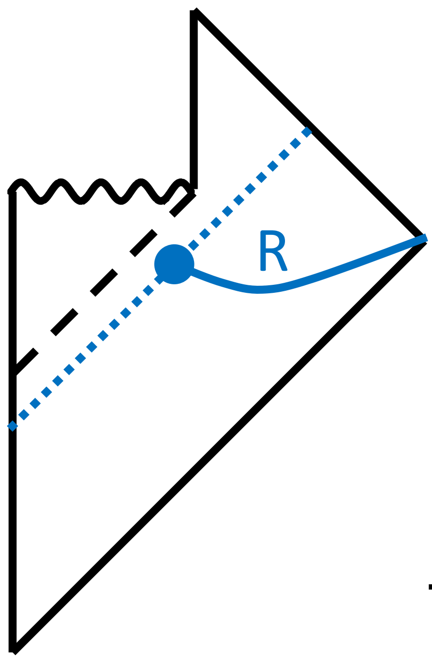
Question: does quantum focusing conjecture hold even after Page time?

Quantum focusing conjecture: $\frac{d\Theta}{d\lambda} \leq 0$ (along null surface)

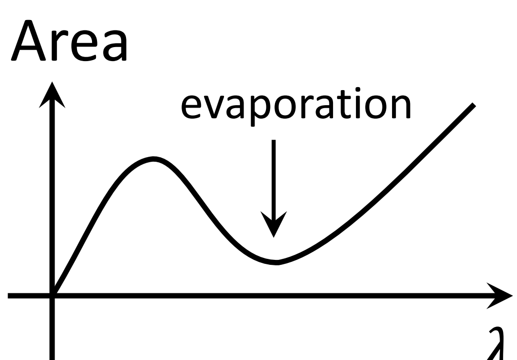
$$\Theta = \frac{1}{r^2}\frac{dS_{\text{gen}}}{d\lambda} \qquad S_{\text{gen}} = \frac{\text{Area}}{4G_N} + S_{\text{rad}}$$

Entanglement entropy of matters on R

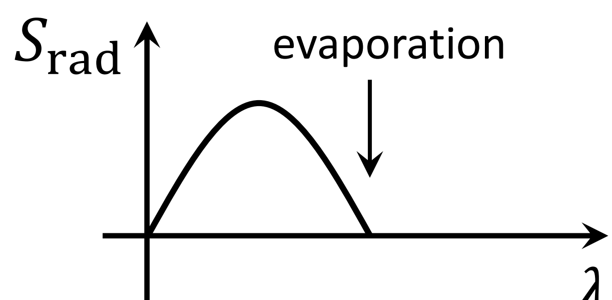
Gravity part of entanglement entropy on R



S_{gen} does not satisfy generalized second law if S_{gen} is defined by von Neumann entropy



Page curve



Plan of Talk

1. Quantum focusing conjecture

2. Islands in evaporating black holes

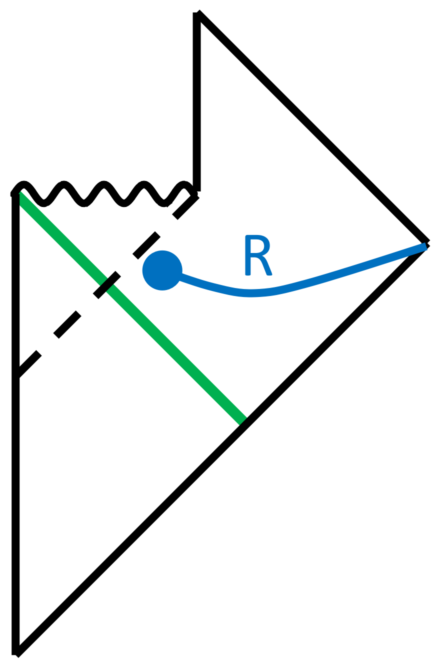
   - Hawking radiation is identified with a region outside BH
   - Island appears in replica trick with gravity
   - At early times, Entanglement entropy $\simeq$ Thermal entropy
   - Island gives entropy bound at Bekenstein-Hawking entropy

3. Quantum focusing conjecture with/without islands

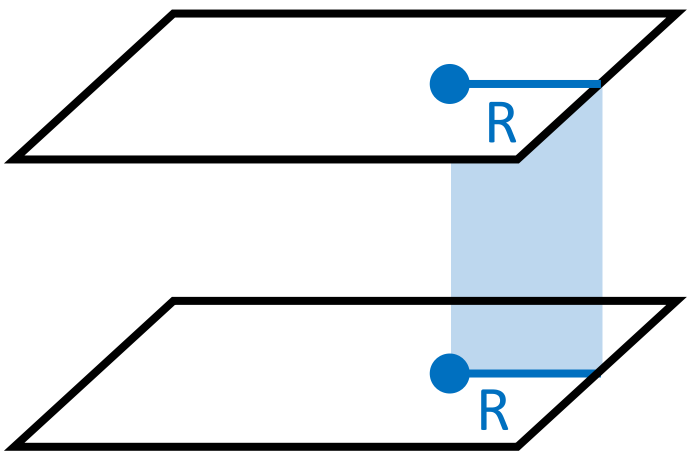
Hawking radiation is identified with
a region outside the black hole

We consider eternal (2-sided) Schwarzschild geometry

Entanglement entropy of Hawking radiation	=	Entanglement entropy of region R



We use replica trick for region R

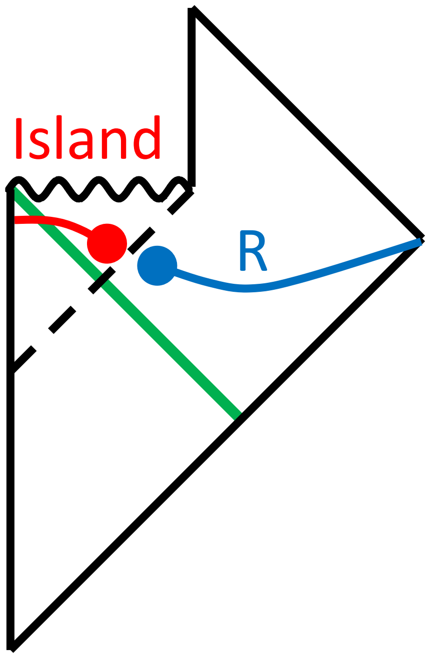


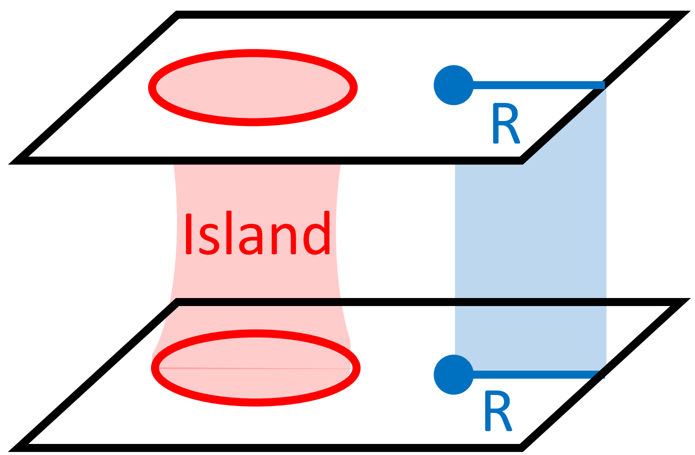
Island appears in replica trick with gravity

Replica trick with gravity

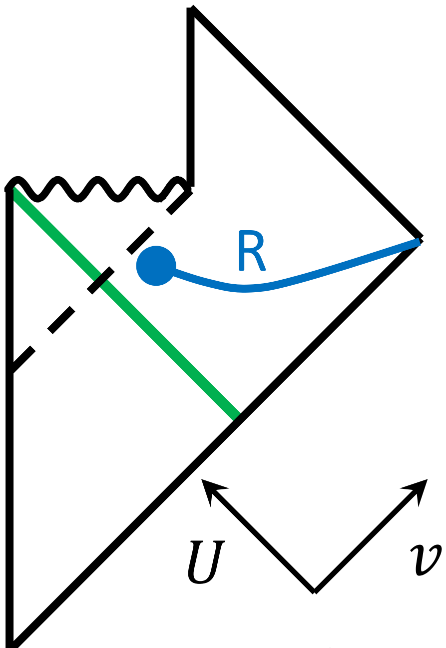
⇒ Wormhole geometry between replicas is allowed = Island

Entanglement entropy of R with gravity = Entanglement entropy of R∪I without gravity





No island at early times: Entanglement entropy $\simeq$ Thermal entropy



Entanglement entropy increases with time

$$S_{\mathrm{rad}} = \frac{c}{12}\log L^2$$

Unruh vacuum ➡ Eddington-Finkelstein coord.

$$ds^2 \simeq -e^{2\rho} dU dv + r^2 d\Omega^2 \qquad 2\rho \simeq \frac{v}{2r_h}$$

$$v = t + r_* \qquad U = -e^{(t-r_*)/2r_h}$$

Entanglement entropy increases with time

$$S_{\mathrm{rad}} = \frac{c}{12}\log e^{2\rho}(\Delta x)^2 \simeq \frac{c}{6}\rho = \frac{c}{24}\frac{v}{r_h} \sim \frac{c}{24}\frac{t}{r_h}$$

= thermal entropy of Hawking radiation

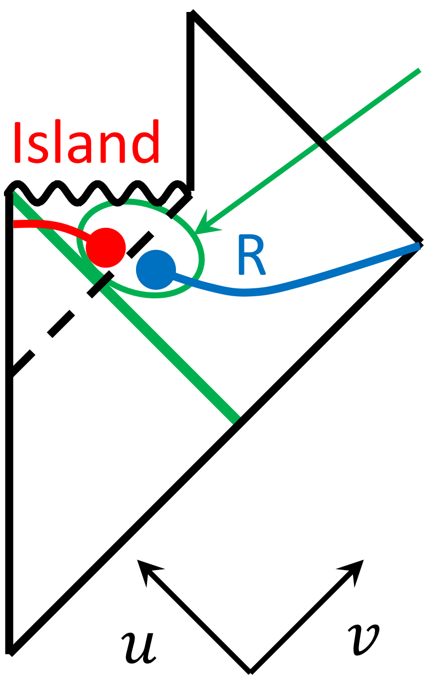
Island appears at late times: entropy bound at Bekenstein-Hawking entropy

$$v = t + r_*$$
$$u = t - r_*$$
$$U = -e^{-u/2r_h}$$

Area term + non-local term

$$S_{\rm rad} = \frac{\rm Area}{4G_N} + \frac{c}{3}\log L \simeq \frac{\pi r_h^2}{G_N}$$

(Bekenstein-Hawking entropy)



L: distance between R (U, v) and Island (U_a, v_a)

$$U_a = -\frac{1}{3}U \qquad v_a = -2r_h \log U = u$$

$$\frac{\rm Area}{4G_N} \simeq \frac{\pi r_h^2(v_0)}{G_N} - \frac{c}{48}\frac{u}{r_h} \simeq \frac{\pi r_h^2(v)}{G_N}$$

$$\frac{c}{6}\log L^2 \simeq -\frac{c}{24}\frac{u}{r_h} + \frac{c}{24}\frac{v}{r_h}$$

t-independent

Plan of Talk

1. Quantum focusing conjecture
2. Islands in evaporating black holes
3. Quantum focusing conjecture with/without islands
    - Results of entanglement entropy from island rule
    - Page time is an approximately null surface
    - Entanglement entropy decreases for timelike surface
    - Entanglement entropy increases for null surface

Results of entanglement entropy from island rule

Area term $\quad v = t + r_*\qquad u = t - r_*$

$$\frac{\text{Area}}{4G_N} = \frac{\pi r^2}{G_N} \simeq \frac{\pi r_0^2}{G_N} - \frac{c}{48}\frac{v}{r_h}$$

Entanglement entropy without islands

$$S_{\text{matter}} \simeq \frac{c}{24}\frac{v}{r_h} \simeq \frac{c}{24}\frac{t}{r_h}$$

Increases with time t

Increases with time v

Entanglement entropy with island

$$S_{\text{matter}} \simeq \frac{\pi r_0^2}{G_N} + \frac{c}{24}\frac{v}{r_h} - \frac{c}{16}\frac{u}{r_h} \simeq \frac{\pi r_0^2}{G_N} - \frac{c}{48}\frac{t}{r_h}$$

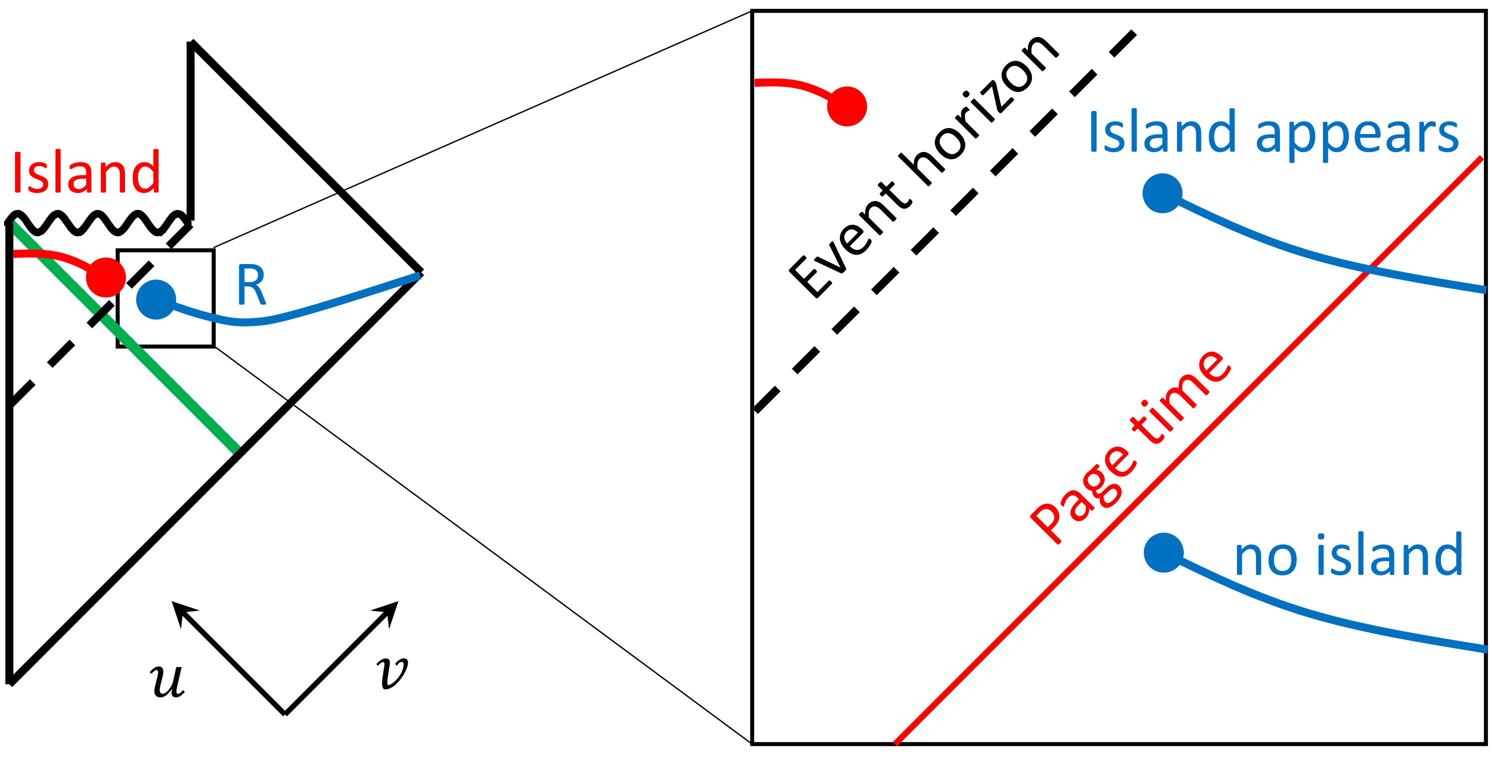
Increases with time v $\qquad$ decreases with time t

Page time is approximately null surface

Configuration with smaller S realizes

$$S^{(\text{no-island})} \simeq \frac{c}{24}\frac{v}{r_h} \qquad S^{(\text{island})} \simeq \frac{\pi r_0^2}{G_N} + \frac{c}{24}\frac{v}{r_h} - \frac{c}{16}\frac{u}{r_h}$$

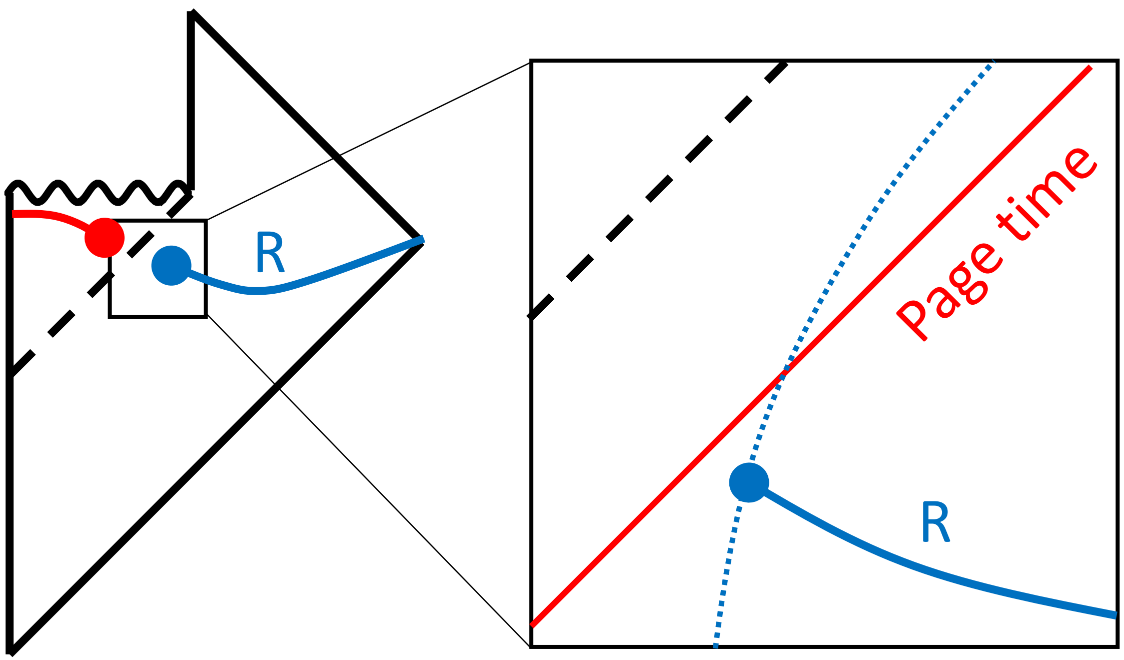
Def. of Page time: Island appears after the Page time



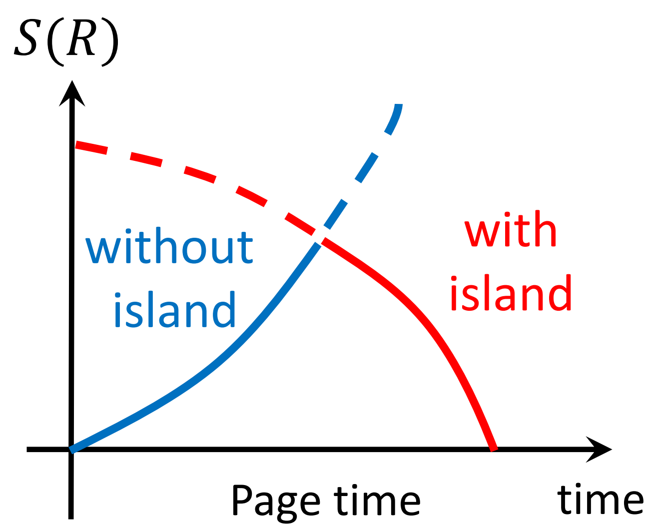
Entanglement entropy decreases along timelike surface after Page time

If endpoint of R moves along timelike surface:

$$S^{(\text{no-island})} \simeq \frac{c}{24}\frac{t}{r_h} \qquad S^{(\text{island})} \simeq \frac{\pi r_0^2}{G_N} - \frac{c}{48}\frac{t}{r_h}$$



Page curve reproduced

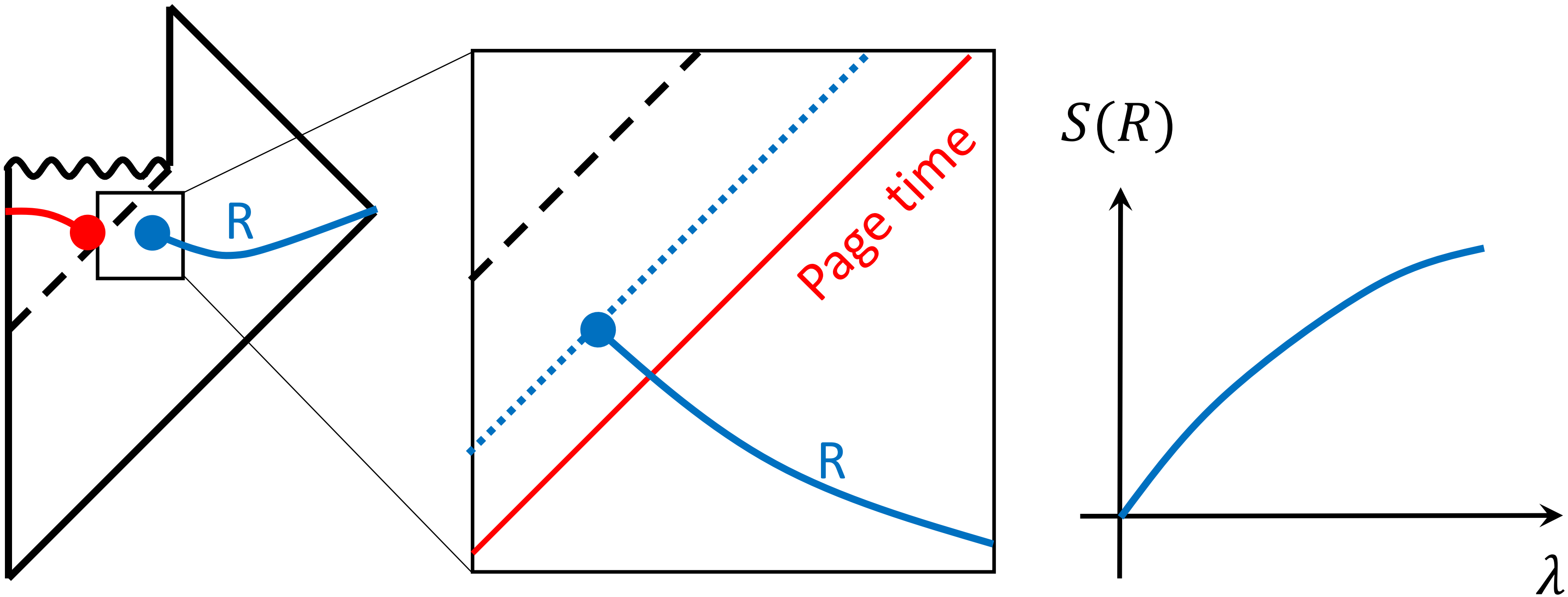


Entanglement entropy increases along outgoing null surface

If endpoint of R moves along null surface, $u = \text{const.}$,

$$S_{\text{gen}}^{(\text{no-island})} \simeq \frac{c}{48}\frac{v}{r_h} \qquad S_{\text{gen}}^{(\text{island})} \simeq \frac{\pi r_0^2}{G_N} + \frac{c}{48}\frac{v}{r_h} - \frac{c}{16}\frac{u}{r_h}$$

⇨ Quantum focusing conjecture is not violated



Quantum focusing conjecture is not violated even after Page time

Does quantum focusing conjecture (QFC) hold after Page time?

QFC $\Rightarrow$ S_{gen} is non-decreasing $\quad S_{\text{gen}} = \dfrac{\text{Area}}{4G_N} + S_{\text{rad}}$

After Page time $\begin{cases} \text{Area: decreasing} \\ S_{\text{rad}}\text{: decreasing} \end{cases}$ $\Rightarrow$ Violation of QFC?

Results from the island rule (formula)

Page time is an approximately null surface

Entanglement entropy is increasing after Page time

Quantum focusing conjecture is not violated

Thank you