New Phases in QCD from Topological Solitons Stabilized by Background Fields

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References (QCD) Collaborators

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QCD in Strong Magnetic Field:

[1] M.Eto, K.Nishimura & MN, 2304.02940
[2] M.Eto, K.Nishimura & MN, *JHEP*, 2311.01112
[3] Y.Amari, M.Eto & MN, 2403.06778
[4] Y.Amari, MN & R.Yokokura, 2406.14419
Domain-wall Skyrmion Phase
[5] Z.Qiu & MN, *JHEP*, 2403.07433 Vortex Skyrmion Phase
[6] M.Eto & MN, *JHEP*, 2207.00211 quantum creation
[7] Z.Qiu & MN, *JHEP*, 2304.05089 quasi-crystal
[8] MN & S.Sasaki 2404.12066 supersymmetry

QCD in Rapid Rotation:

[9] M.Eto, K.Nishimura & MN, *JHEP*, 2112.01381
[10] M.Eto, K.Nishimura & MN, *JHEP*, 2310.17511 ←Domain-wall Skyrmion Phase
[11] M.Eto, K.Nishimura & MN, *JHEP*, 2312.10927

References (Cond-mat) Collaborators

Yuki Amari (Keio U.), R.Calum (Edge Hill U.), S.B.Gudnason (Henan U.), Seungho Lee, Se Kwon Kim (KAIST), Yutaka Akagi (U. Tokyo), Yasha Shnir (Dubna), Toshiaki Fujimori (Dokkyo U.)

Chiral Magnets:

[12] C.Ross & MN, *PRB* 2205.11417

[13] Y.Amari & MN, *JHEP* 2307.11113

[14] Y.Amari, C.Ross & MN, *PRB* 2311.05174

[15] S.B.Gudnason, Y.Amari & MN, 2406.19056

[16] S.Lee, T.Fujimori, MN & S.K.Kim, 2407.04007

$\mathbb{C}P^2$ Magnets:

[17] Y.Amari, Y.Akagi, S.B.Gudnason, MN & Y.Shnir, *PRB* 2204.01476

Main messages

(1) Unstable topological solitons can be stabilized by a background (gauge) field.

(2) When the background is strong enough, the soliton’s energy becomes negative.
➔ Inhomogeneous (Solitonic) Phases

(3) Examples in QCD & Cond-mat.

(4) Maybe in Early Universe as well?

Main messages

(1) Unstable topological solitons can be stabilized by a background (gauge) field.

(2) When the background is strong enough, the soliton's energy becomes negative.
➔ Inhomogeneous (Solitonic) Phases

Not today

(3) Examples in QCD & Cond-mat.

(4) Maybe in Early Universe as well?

In future

The stability of topological solitons

$\vec{n} = (n_1, n_2, n_3), \vec{n}^2 = 1$

Criteria: The conventional Derrick's scaling argument.

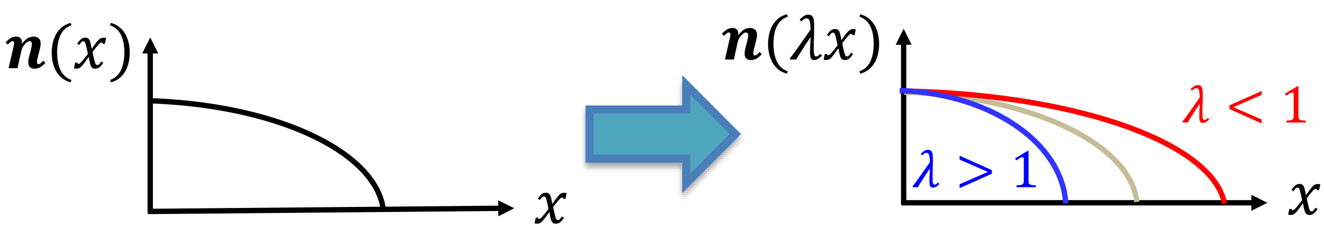
e.g. **O(3) model:** $\mathcal{L} = \overset{E_2}{\partial_\mu \vec{n} \partial^\mu \vec{n}} - \overset{E_0}{V(n)} + \kappa \overset{E_4}{\left(\partial_\mu \vec{n} \times \partial_\nu \vec{n}\right)^2}$

Skyrme term

$x \to x' = \lambda x \quad n(x) \to n(\lambda x)$

$$E(\lambda) = E_0(\lambda) + E_2(\lambda) + E_4(\lambda)$$
$$= \lambda^{-d} E_0(\lambda = 1) + \lambda^{2-d} E_2(\lambda = 1) + \lambda^{4-d} E_4(\lambda = 1)$$

$$E'(\lambda)|_{\lambda=1} = -dE_0 + (2-d)E_2 + (4-d)E_4 = 0$$



The stability of topological solitons

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Skyrme term

Spatial dim

$d = 2$: $\boxed{E'(\lambda)|_{\lambda=1} = -2E_0 + 2E_4 = 0}$

$E_0 = 0, E_4 = 0$: lump (scale invariant)

$E_0 \neq 0, E_4 \neq 0$: baby Skyrmion

$E_0 \neq 0, E_4 = 0$: unstable to shrink

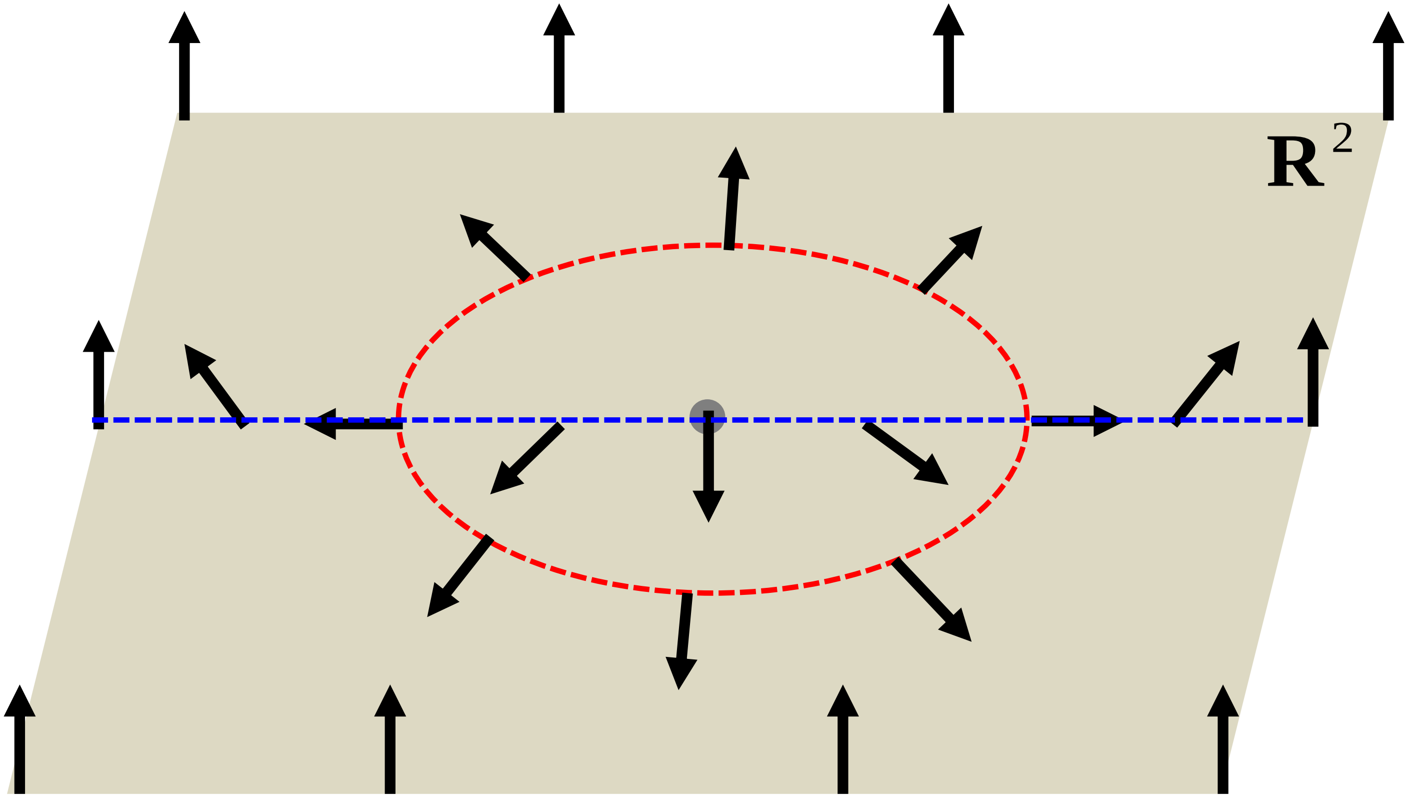
$d = 3$: $\boxed{E'(\lambda)|_{\lambda=1} = -3E_0 - E_2 + E_4 = 0}$

$E_2 \neq 0, E_4 \neq 0$: Skyrmion & Hopfion

$E_2 \neq 0, E_4 = 0$: unstable to shrink

2D (baby) Skyrmion (or lump)

$\pi_2(S^2) \cong \mathbb{Z}$



$$\pi_2(S^2) \cong \mathbb{Z}$$

Amazon.co.jp: STK スイカ 吊り下げスイカネット 園芸ネット 果物袋 カボチャ メロン 丈夫 通気 栽培用 (30cm（普通、ロープ付き）, 10PCS) : DIY・工具・ガーデン

The stability of topological solitons

Criteria: The conventional Derrick's scaling argument.

We should think it more carefully in the presence of a background (gauge) field.

Examples:

1. **Condensed matter systems**
   (e.g. chiral magnets,
   ultracold atomic gases with synthetic gauge field)

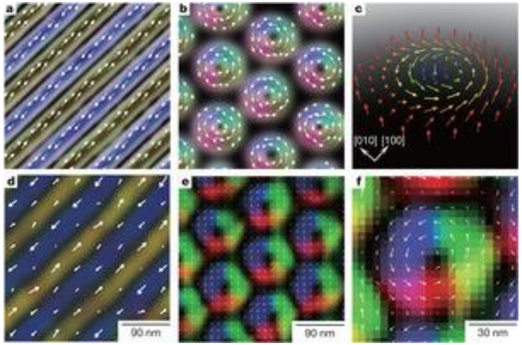
2. **QCD (or field theory) under external fields**
   e.g. magnetic field / rotation (relevant to QGP, neutron stars)

1. Examples in Cond-mat

Magnetic Skyrmions in chiral magnets

Important for nanotechnology such as
low-energy consumption magnetic memories.

a thin film of $Fe_{0.5}Co_{0.5}Si$
X. Z. Yu *et.al*
Nature 465, 901 (2010)



O(3) model +
Dzyaloshinsky-Moriya (DM) interaction

$$\mathcal{H}_{\mathrm{DM}} = \kappa \mathbf{n} \cdot (\nabla \times \mathbf{n})$$

Derrick's theorem

$$\underbrace{E_{1(\mathrm{DM})}}_{<0} + \underbrace{2E_{0(\mathrm{pot})}}_{>0} = 0$$

< 0 $\qquad$ > 0: prefers shrinking

Negative energy **prevents shrinking**
: possible only for either of (anti-)Skyrmion

O(3) model + a background $SU(2)$ gauge field

B. J. Schroers, SciPost Phys. 7 (2019) 030 [1905.06285].

$$\mathcal{H} = \frac{1}{2}(D_k \mathbf{n})^2 \qquad A_\mu^a = \left(A_0^a = 0, A_k^a\right)$$

$\mu = 0, 1, 2$: spacetime index

$a = 1, 2, 3$: $SU(2)$ adj index

$$= \frac{1}{2}\partial_k \mathbf{n} \cdot \partial_k \mathbf{n} + \boxed{\boldsymbol{A}_k \cdot (\mathbf{n} \times \partial_k \mathbf{n})} + \frac{1}{2}(\boldsymbol{A}_k \times \mathbf{n})^2$$

$\mathcal{H}_{\rm DM}$ DM interaction potential $V_{\rm tot}$

$SU(2)$ field strength $F_{12} = \kappa^2 \sigma_3$

Cf. Magnetized D-brane

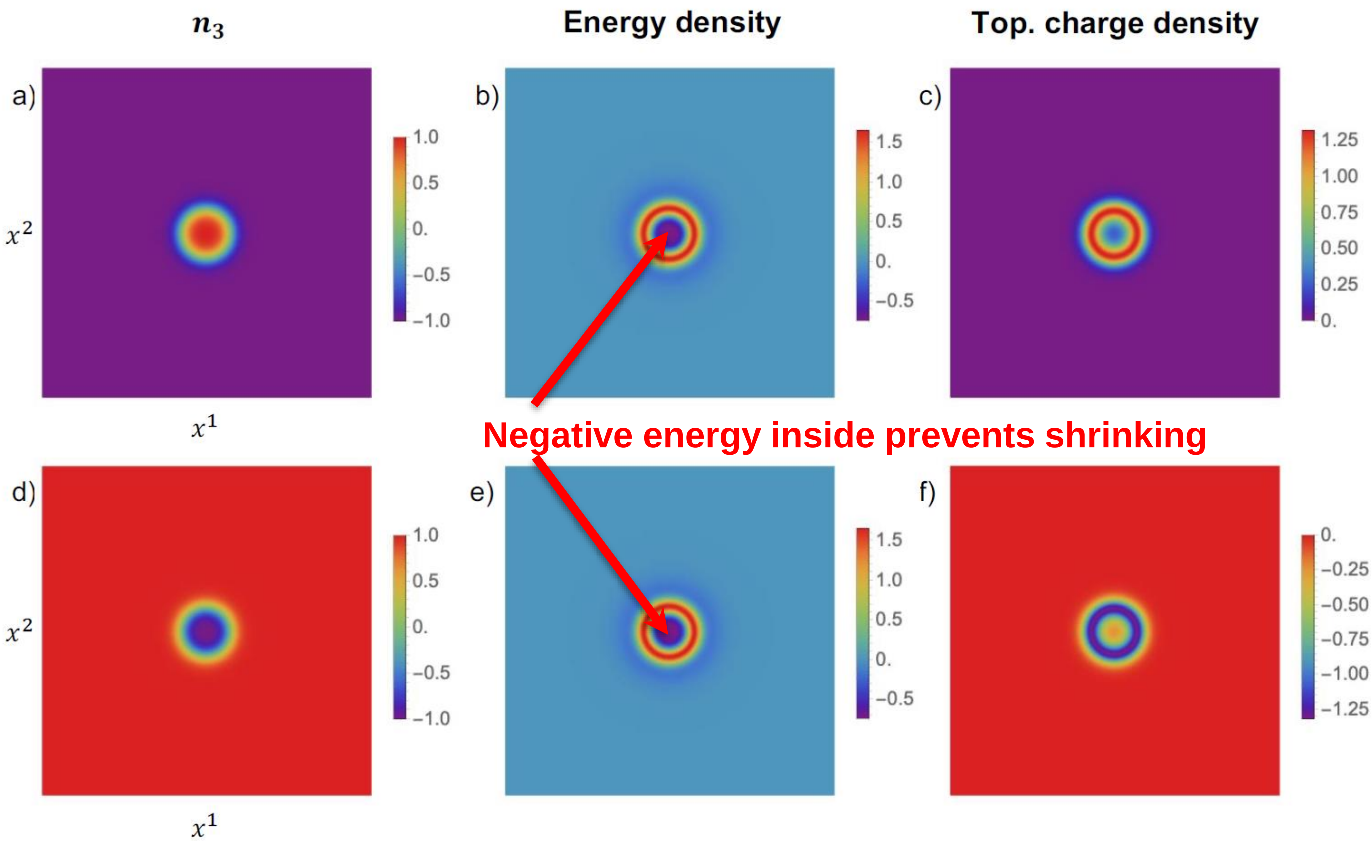
$$V(\overrightarrow{n}) = m^2(1 - n_3^2)$$

Skyrmion & anti-Skyrmion are stable (without Zeeman).

Skyrmion
@ the ferro g.s.

$$n_3 = -1$$

(Anti-Skyrmion is unstable)



Anti-Skyrmion
@ the ferro g.s.

$$n_3 = +1$$

(Skyrmion is unstable)

Ultracold atomic gas with a synthetic gauge field

Lin, Spiermal et.al, Nature 462 ('09)

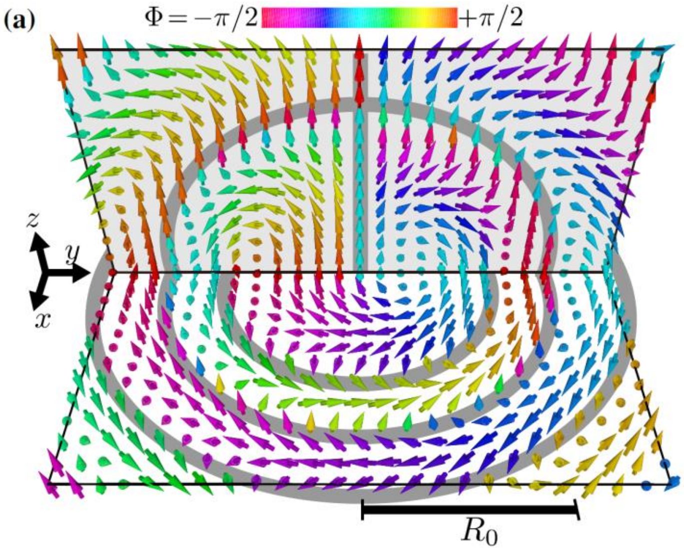
3D Skyrmions
(without Skyrme term)

T. Kawakami, T. Mizushima, MN, and K. Machida
PRL 109, 015301 (2012)

$$\mathcal{H}_0 = \int d^3 \boldsymbol{r} [\boldsymbol{D}_{\mu\nu} \Psi_\nu(\boldsymbol{r})]^\dagger \cdot [\boldsymbol{D}_{\nu\eta} \Psi_\eta(\boldsymbol{r})]$$

$$\boldsymbol{D}_{\mu\nu} = (-i \boldsymbol{\nabla} \sigma_0 + \boldsymbol{A})_{\mu\nu}$$

$$\boldsymbol{A} = \kappa_\perp (\sigma_x \hat{\boldsymbol{x}} + \sigma_y \hat{\boldsymbol{y}}) + \kappa_z \sigma_z \hat{\boldsymbol{z}}$$



2. Examples in Sigma Model & QCD

$U(1)$ gauged $O(3)$ [or $\mathbb{C}P^1$] model

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + D_\mu\vec{n}\cdot D^\mu\vec{n} - V(n) \qquad \vec{n} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}, \quad \vec{n}^2 = v^2$$

gauge tr. $n_1 + in_2 \to e^{-i\theta(x)}(n_1 + in_2)\,, \quad n_3 \to n_3$

$$D_\mu n_1 = \partial_\mu n_1 + eA_\mu n_2\,, \quad D_\mu n_2 = \partial_\mu n_2 - eA_\mu n_1\,, \quad D_\mu n_3 = \partial_\mu n_3$$

Well-known model?

1.With a potential $V(n) = m^2 n_3^2$, half gauged-lumps (merons) are BPS and stable. B.J.Schroers(‘95)

1’. $V(n) = m^2(n_3 - c)^2$, fractional gauged-lumps

2. Without potential, lumps are unstable to expand.

2. **Without potential, lumps are unstable to expand.**

Background gauge field (constant magnetic field)

$$\mathcal{A}_0 = 0\,, \quad \mathcal{A}_1 = -\frac{By}{2}\,, \quad \mathcal{A}_2 = \frac{Bx}{2}$$

$$F_{12} = B(= -B_3)$$

Y. Amari, M.Eto & MN, 2403.06778

We find

1. **For non-dynamical gauge field,**
   $k = 1$ **stable &** $k = -1$ **unstable** *(similar to magnetic Skyrmion)*
2. **For dynamical gauge field, both are stable.**
   $k = 1$ **lighter &** $k = -1$ **heavier**

3. For higher k, droplet behavior: Area $\sim k$

Non-dynamical gauge field

$$E[\vec{n}] = \int d^2x \, |D_i \vec{n}|^2$$

$$= \int d^2x \left[\underbrace{|\partial_i \vec{n}|^2}_{E_2} + \underbrace{2e\mathcal{A}_i \varepsilon_{ab} \partial_i n_a n_b}_{E_1} + \underbrace{e^2 \mathcal{A}_i^2 (n_1^2 + n_2^2)}_{E_0} \right]$$

Derick's Scaling

$$\boldsymbol{\delta \equiv E_1 + 2E_0 = 0}$$

< 0 > 0: **prefers shrinking**

$$\vec{n}^{(\lambda)}(x) = \vec{n}(\lambda x)\,, \quad \partial_i \vec{n}^{(\lambda)}(x) = \lambda \partial_i' \vec{n}(\lambda x)$$

$$\mathcal{A}_i^{(\lambda)}(x) = \lambda^{-1} \mathcal{A}_i(\lambda x)$$

$$e(\lambda) = E\left[\vec{n}^{(\lambda)}(x)\right] = E_2 + \lambda^{-2} E_1 + \lambda^{-4} E_0 \qquad \frac{de(\lambda)}{d\lambda}\Big|_{\lambda=0} = 0$$

Axisymmetric ansatz

$$n_1 = v\frac{x}{r}\sin\Theta(r)\,, \quad n_2 = v\frac{y}{r}\sin\Theta(r)\,, \quad n_3 = v\cos\Theta(r)$$

EOM $$\Theta'' + \frac{\Theta'}{r} - \frac{(2-eBr^2)^2}{4r^2}\sin\Theta\cos\Theta = 0$$

$$\boldsymbol{k = +1:\ n_3(\infty) = -v,\ \ n_3(0) = +v}$$
$$\boldsymbol{k = -1:\ n_3(\infty) = +v,\ \ n_3(0) = -v}$$

$$E_0 = \frac{\pi e^2 B^2 v^2}{2}\int_0^\infty dr\, r^3 \sin^2\Theta \ > \mathbf{0}$$

$$\boldsymbol{\delta \equiv E_1 + 2E_0 = 0}$$

$$E_1 = -2\pi e B v^2 \int_0^\infty dr\, r\sin^2\Theta \ \begin{cases} < \mathbf{0 \text{ for } \boldsymbol{B} > 0\text{:} \ \ OK} \\ > \mathbf{0 \text{ for } \boldsymbol{B} < 0\text{:}} \end{cases}$$

Axisymmetric ansatz **Charge conjugation**

$$n_1 = v\frac{x}{r}\sin\Theta(r)\,, \qquad n_2 = -v\frac{y}{r}\sin\Theta(r)\,, \qquad n_3 = v\cos\Theta(r)$$

EOM $$\Theta'' + \frac{\Theta'}{r} - \frac{(2 + eBr^2)^2}{4r^2}\sin\Theta\cos\Theta = 0$$

$$\boldsymbol{k = -1\text{: } n_3(\infty) = -v,\ \ n_3(0) = +v}$$
$$\boldsymbol{k = +1\text{: } n_3(\infty) = +v,\ \ n_3(0) = -v}$$

$$E_0 = \frac{\pi e^2 B^2 v^2}{2}\int_0^\infty dr\, r^3 \sin^2\Theta \ > \mathbf{0}$$

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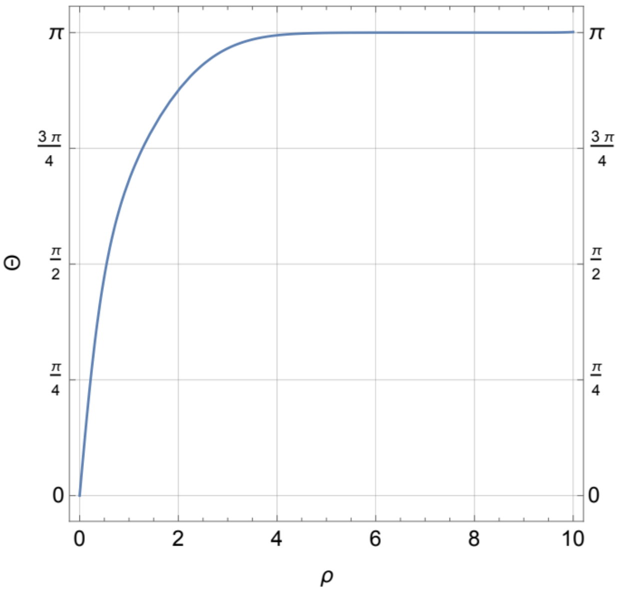
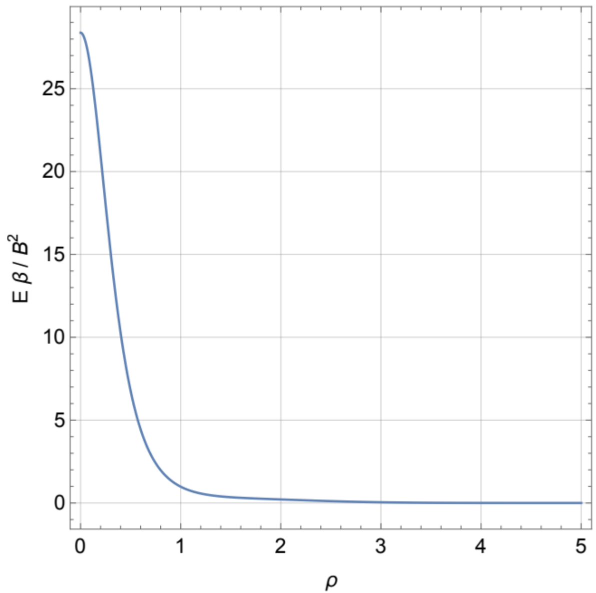
$$E_1 = +2\pi e B v^2 \int_0^\infty dr\, r \sin^2\Theta \ \begin{cases} < \mathbf{0} \textbf{ for } \boldsymbol{B} < \mathbf{0}\text{:} \ \ \textbf{OK} \\ > \mathbf{0} \textbf{ for } \boldsymbol{B} > \mathbf{0} \end{cases}$$

Stable solution

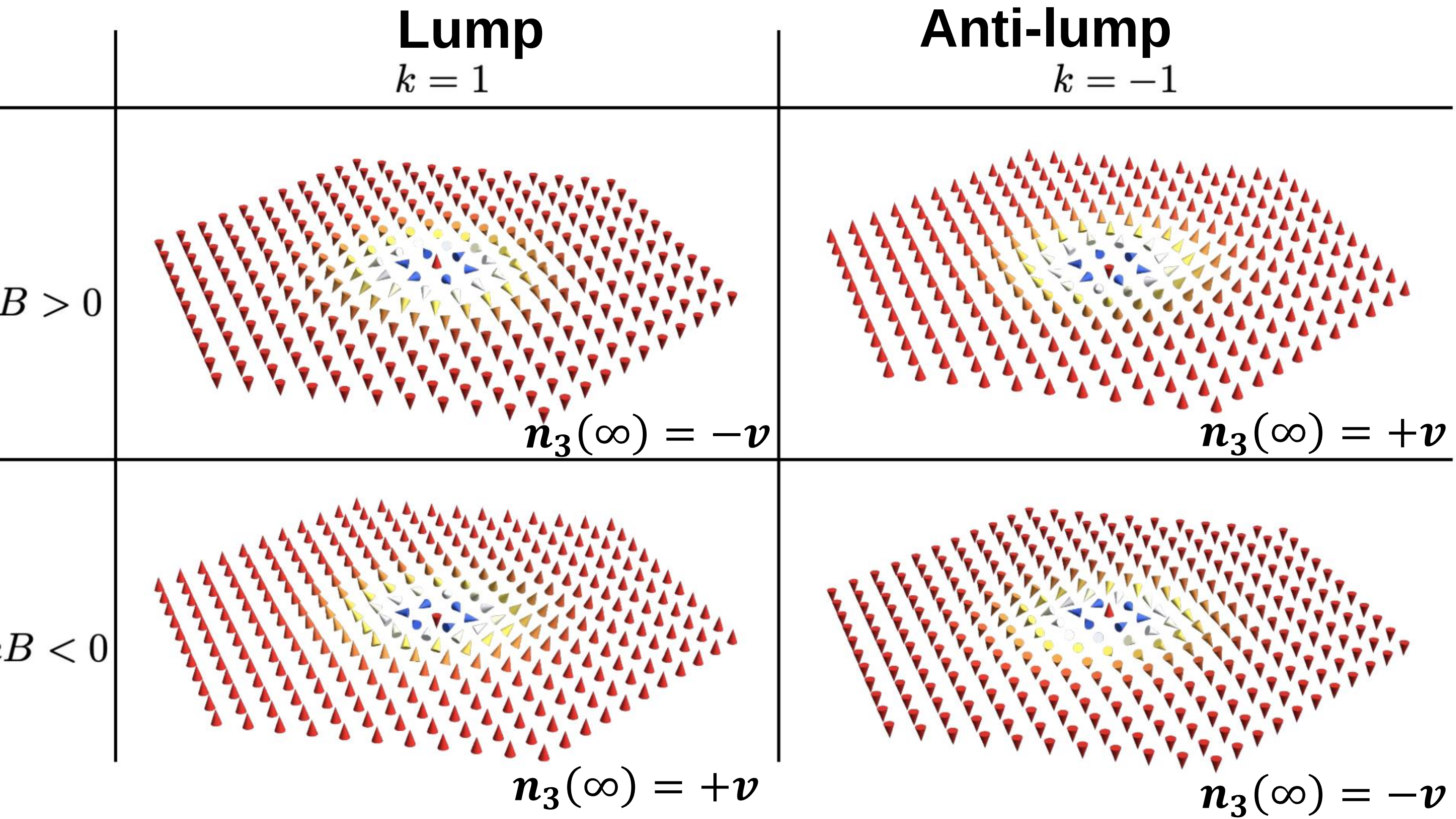
$$n_1 = v\frac{x}{r}\sin\Theta(r)\,, \quad n_2 = v\frac{y}{r}\sin\Theta(r)\,, \quad n_3 = v\cos\Theta(r)$$



Profile



Energy density



Lump
$k = 1$
Anti-lump
$k = -1$
$B > 0$
$n_3(\infty) = -v$
$n_3(\infty) = +v$
$B < 0$
$n_3(\infty) = +v$
$n_3(\infty) = -v$

Profiles

(1) Mass: $\frac{M}{2\pi v^2} = 3.62188$

$< 4 = \frac{M_{BPS}}{2\pi v^2}$: **90.5% of BPS lump mass**

(2) Size: $r_0 = \frac{0.544784}{\sqrt{eB}}$ **40% of** $\tilde{r}_0 = \sqrt{\frac{2}{eB}}$

(3) Magnetic flux inside a superconducting ring:

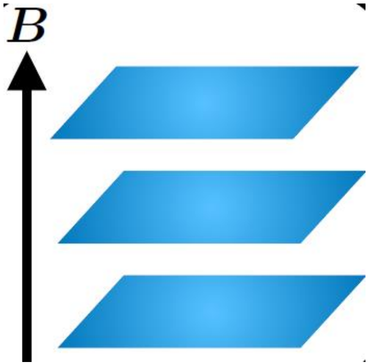
$$\Phi = \pi r_0^2 B \simeq 0.15 \times \frac{2\pi}{e}$$ **15% of** $\tilde{\Phi} = \frac{2\pi}{e}$

(4) As for stability, very similar to magnetic skyrmions

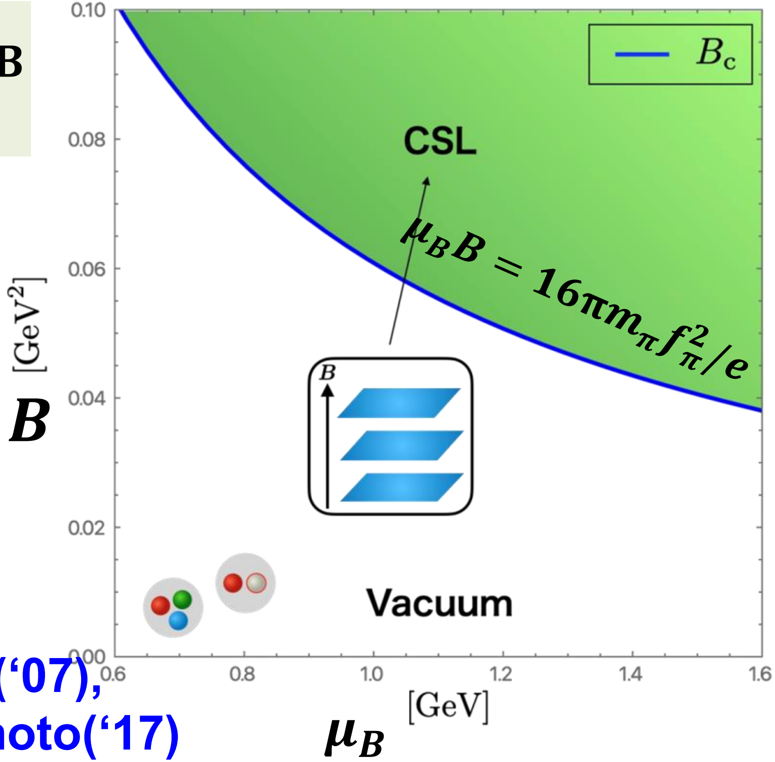
Why do we consider this model? ➔ It appears in QCD !

Chiral Soliton Lattice(CSL) phase

chemical pot. μ_B
magnetic field B



Tension<0



Son & Stephanov('07),
Brauner & Yamamoto('17)

Solitons carry baryon #

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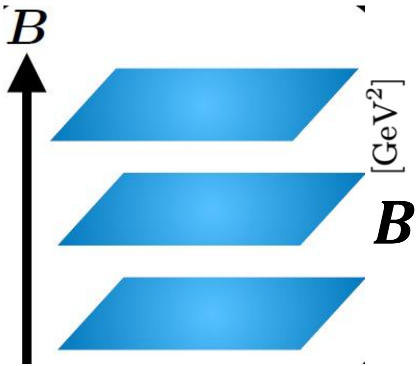

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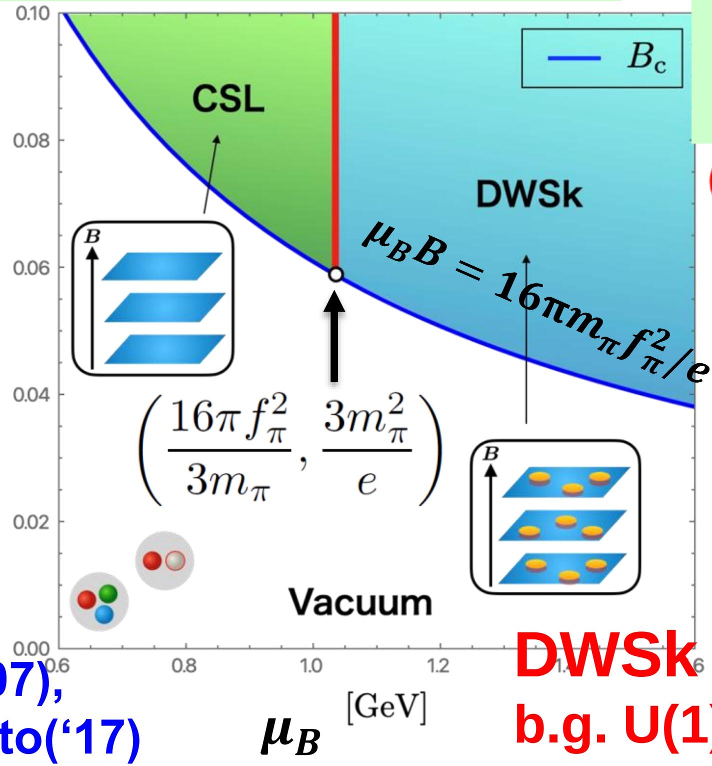
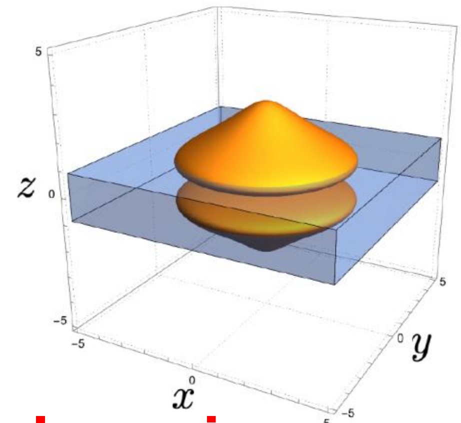


Domain-Wall Skyrmion Crystal (DW-SkX) phase

@ $\mu_B \geq 1.03$ GeV



DWSk = lump in
b.g. U(1)-gauged O(3) model

Solitons carry baryon #

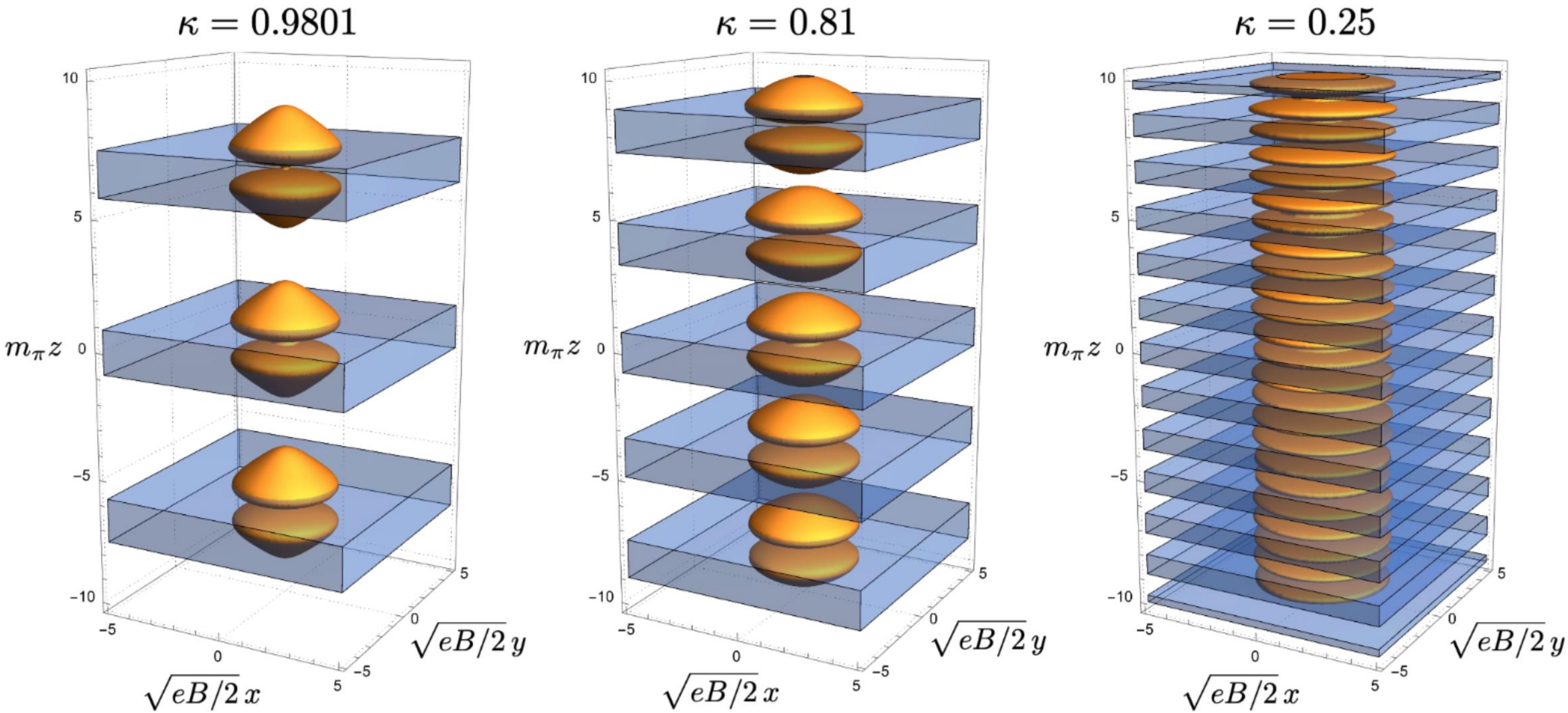
M.Eto, K.Nishimura & MN, 2304.02940

Macarons

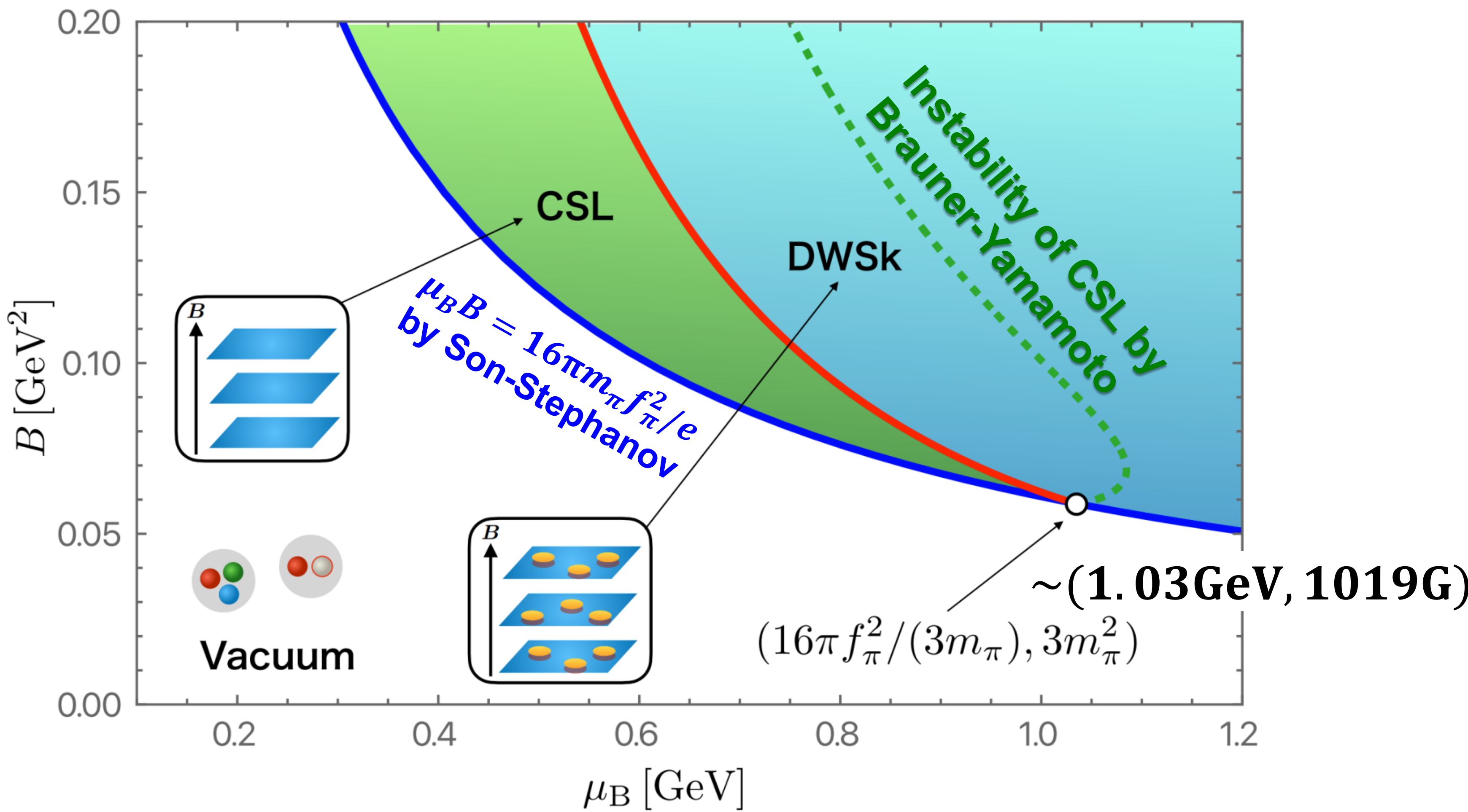
small $\mu_B B$

Pancake tower

large $\mu_B B$



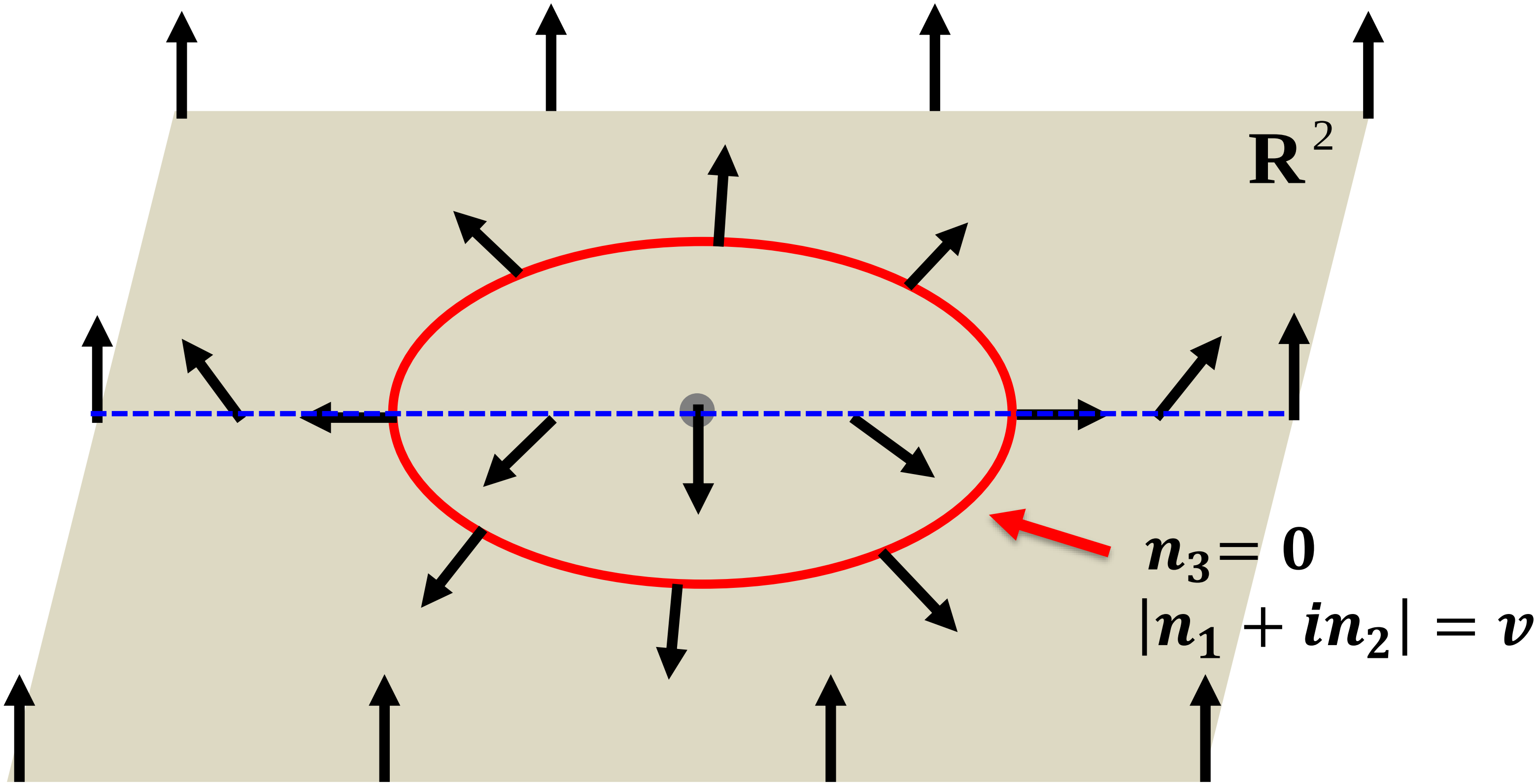
M.Eto, K.Nishimura & MN, *JHEP*, arXiv: [2311.01112](2311.01112)



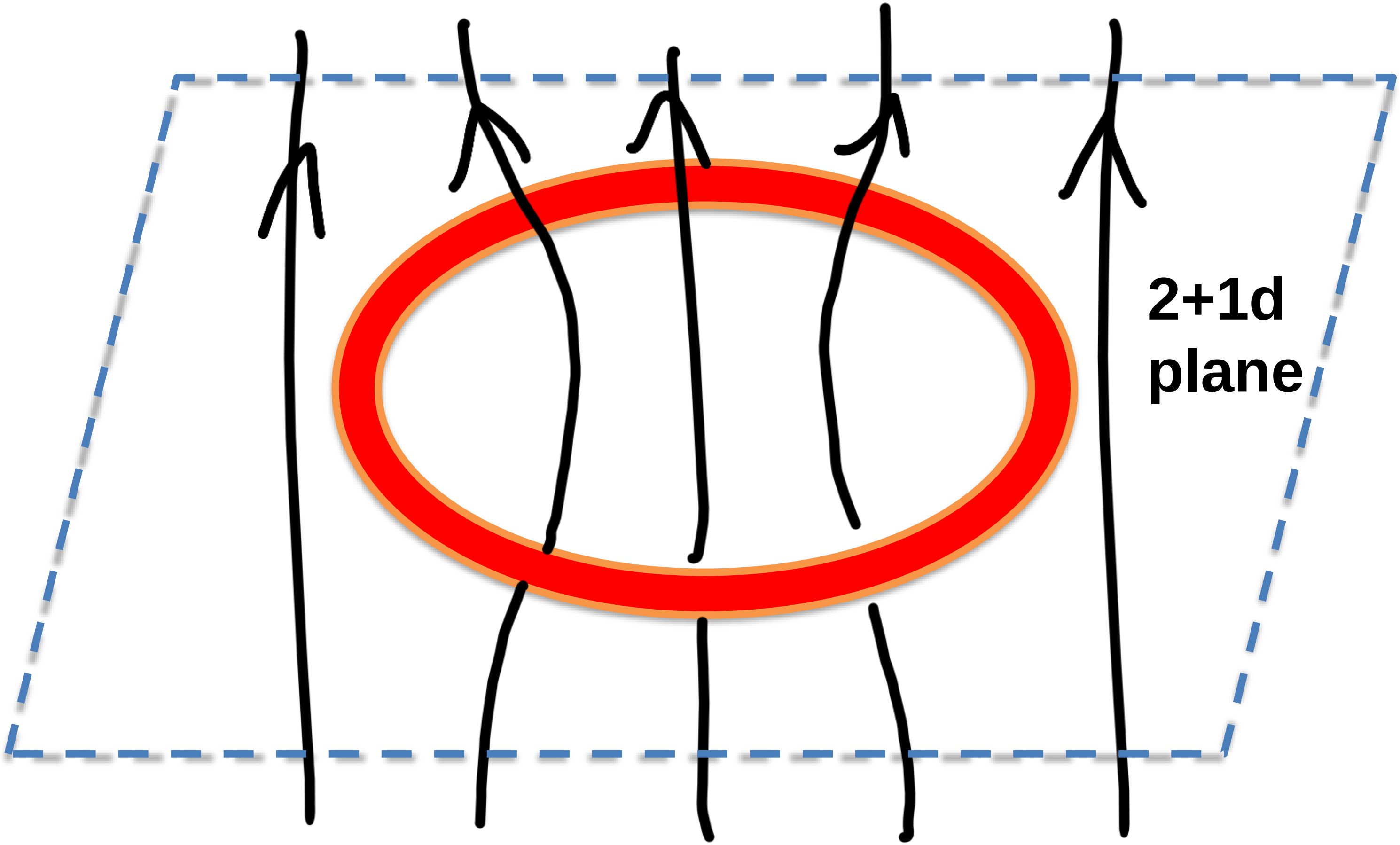
M.Eto, K.Nishimura & MN, *JHEP*, arXiv: 2311.01112

Lump = a superconducting ring

$\pi_2(S^2) \cong \mathbb{Z}$



Superconducting ring in magnetic field



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