YITP Workshop 場の理論と弦理論 2024 @ 8/5 2024

Long-range correlation induced by the inhomogeneous quenches in 2d CFT

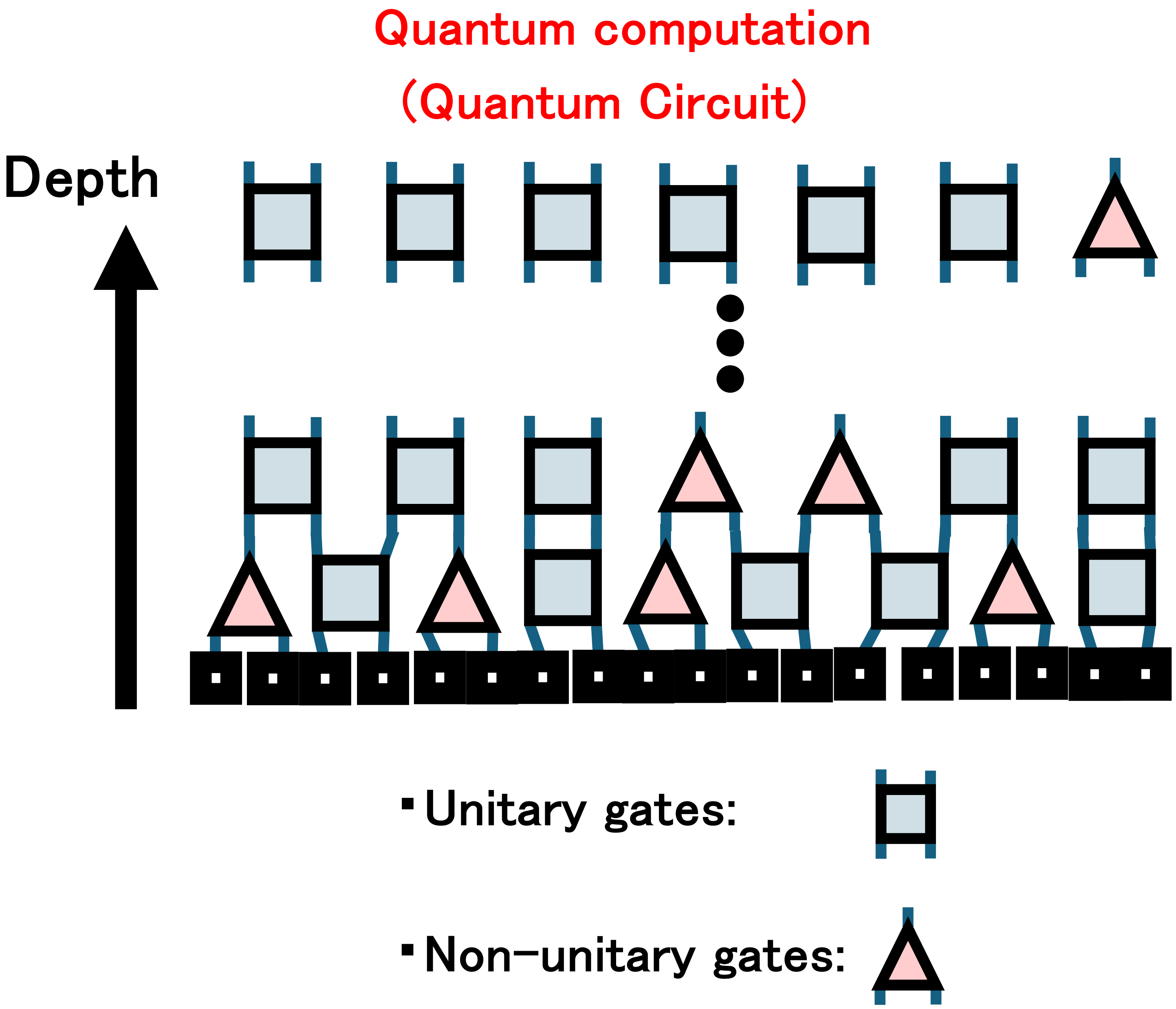
Masahiro Nozaki (KITS, UCAS & iTHEMS, Riken)

Based on the collaboration with

Chen Bai, Weibo Mao, Akihiro Miyata, and Mao Tian Tan

Work in progress

Background 1



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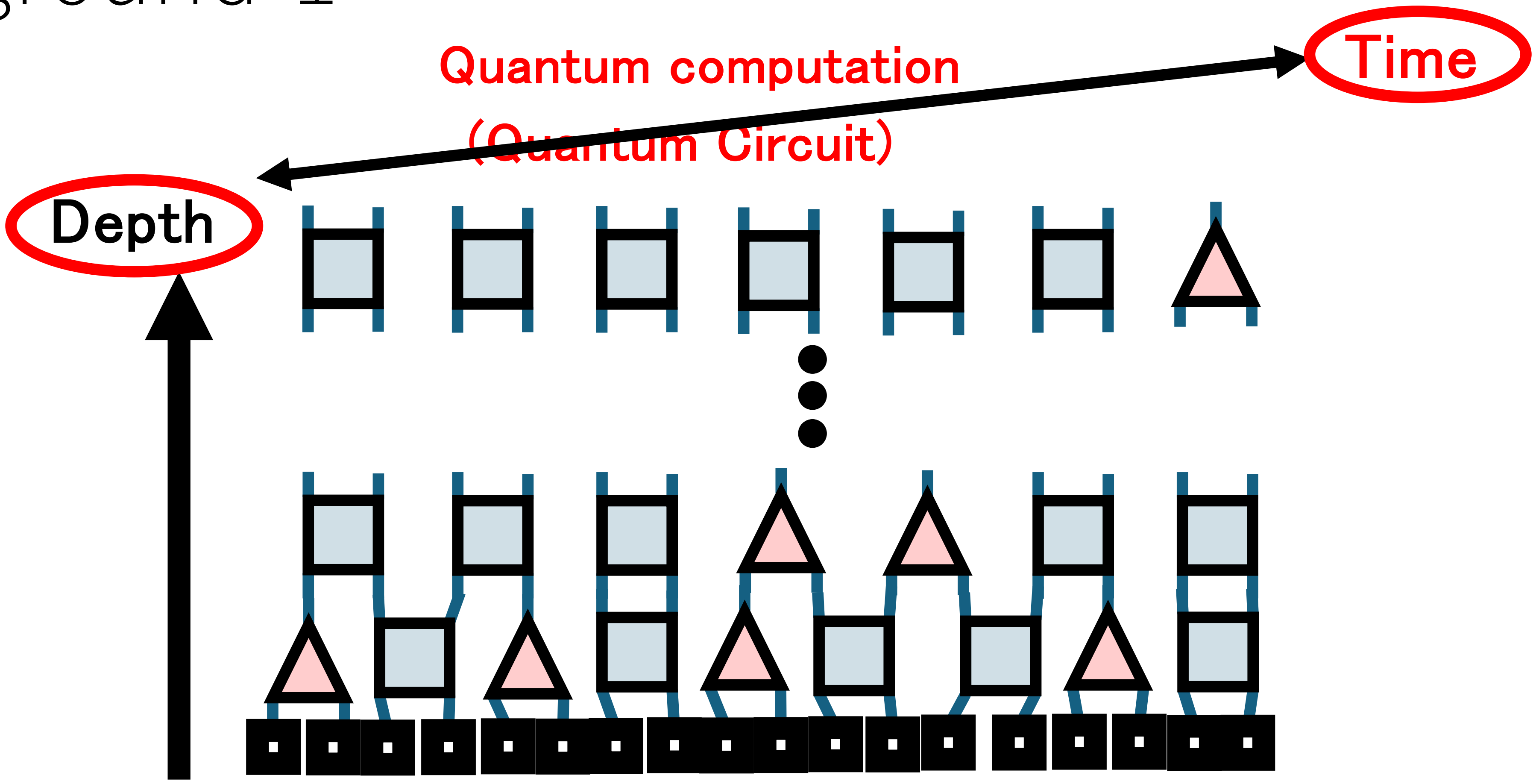

- Unitary gates:
- Non-unitary gates:

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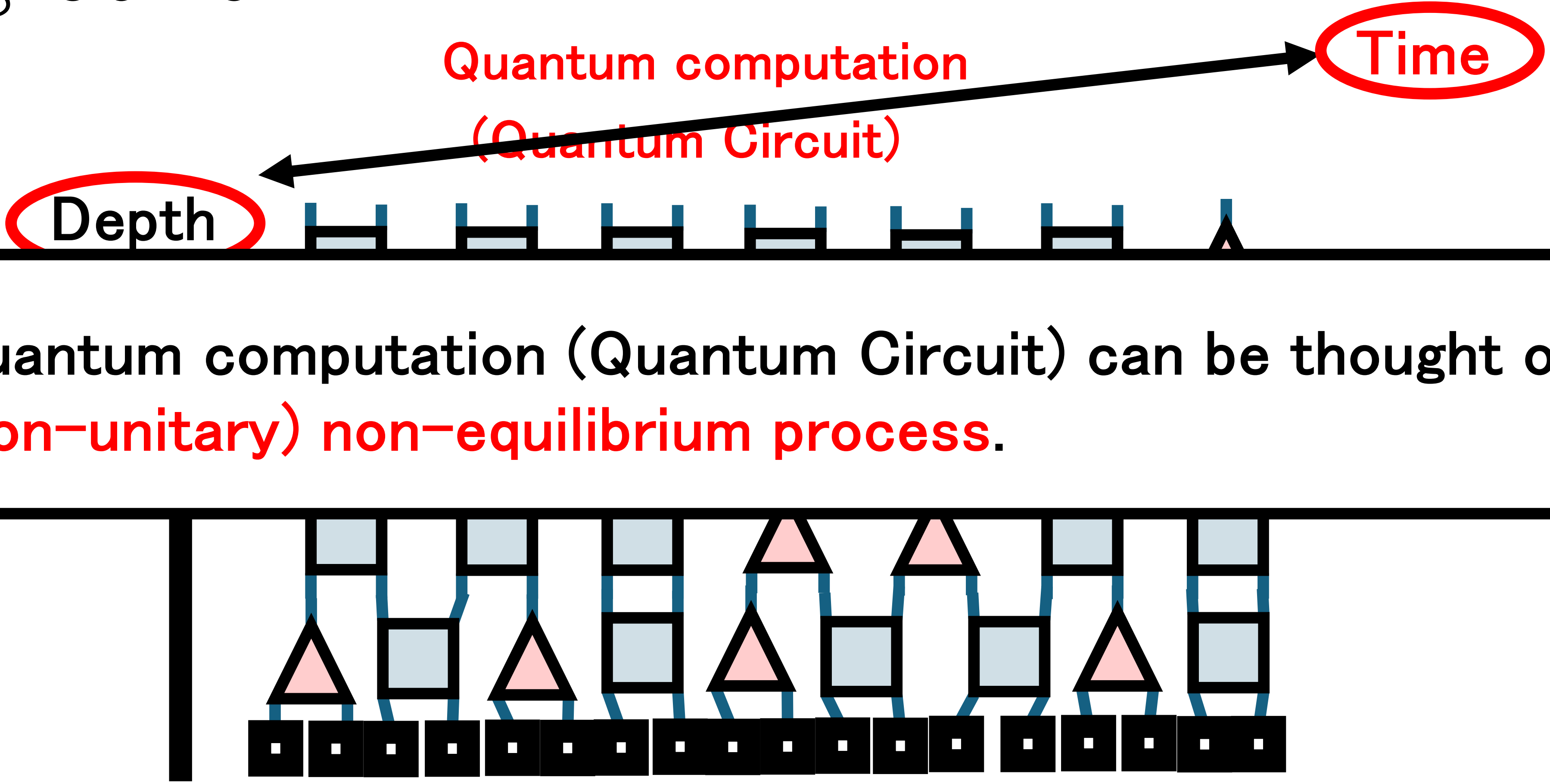

Quantum computation (Quantum Circuit) can be thought of as a (non-unitary) non-equilibrium process.

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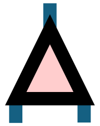
- Non-unitary gates: 

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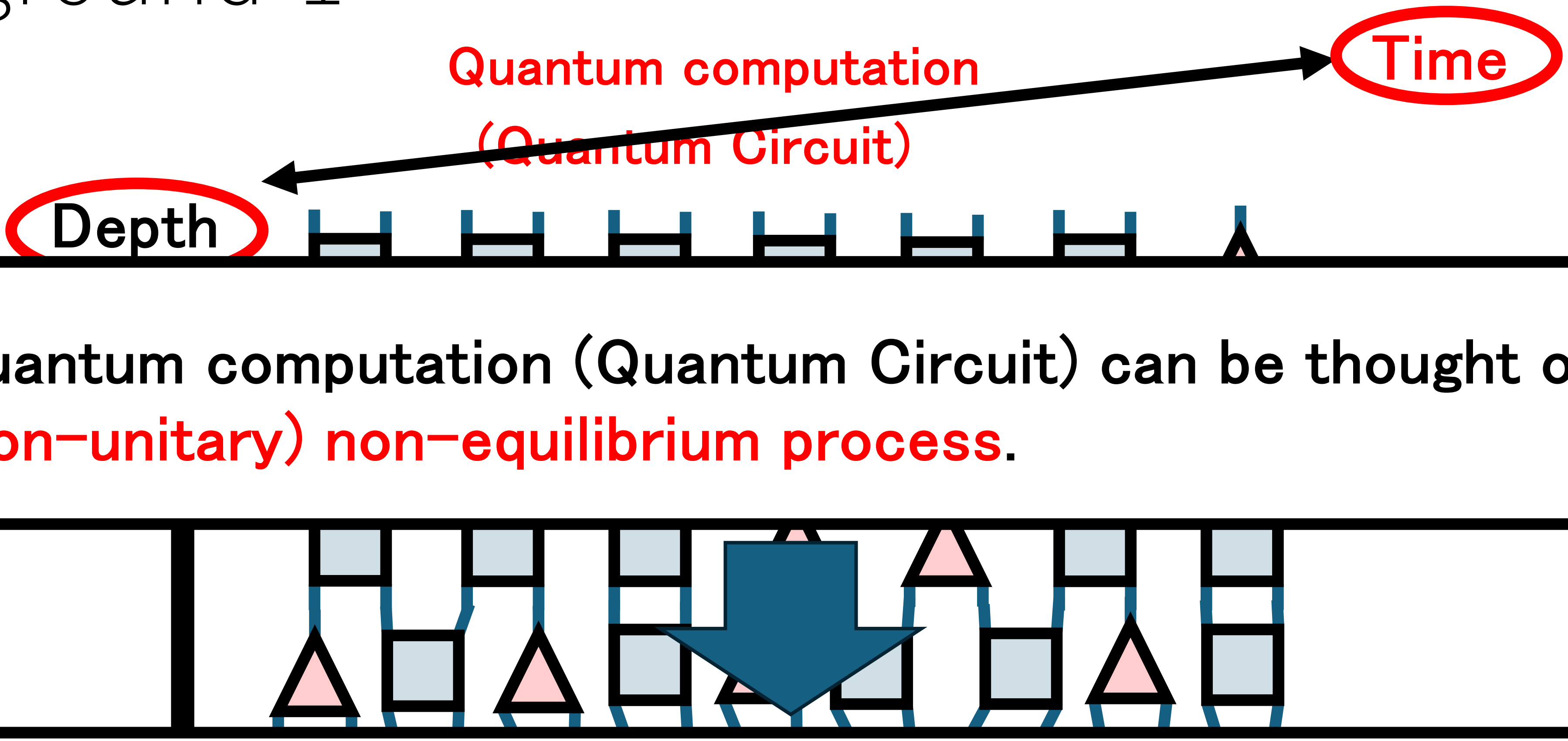

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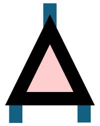
▪Non-unitary gates: 

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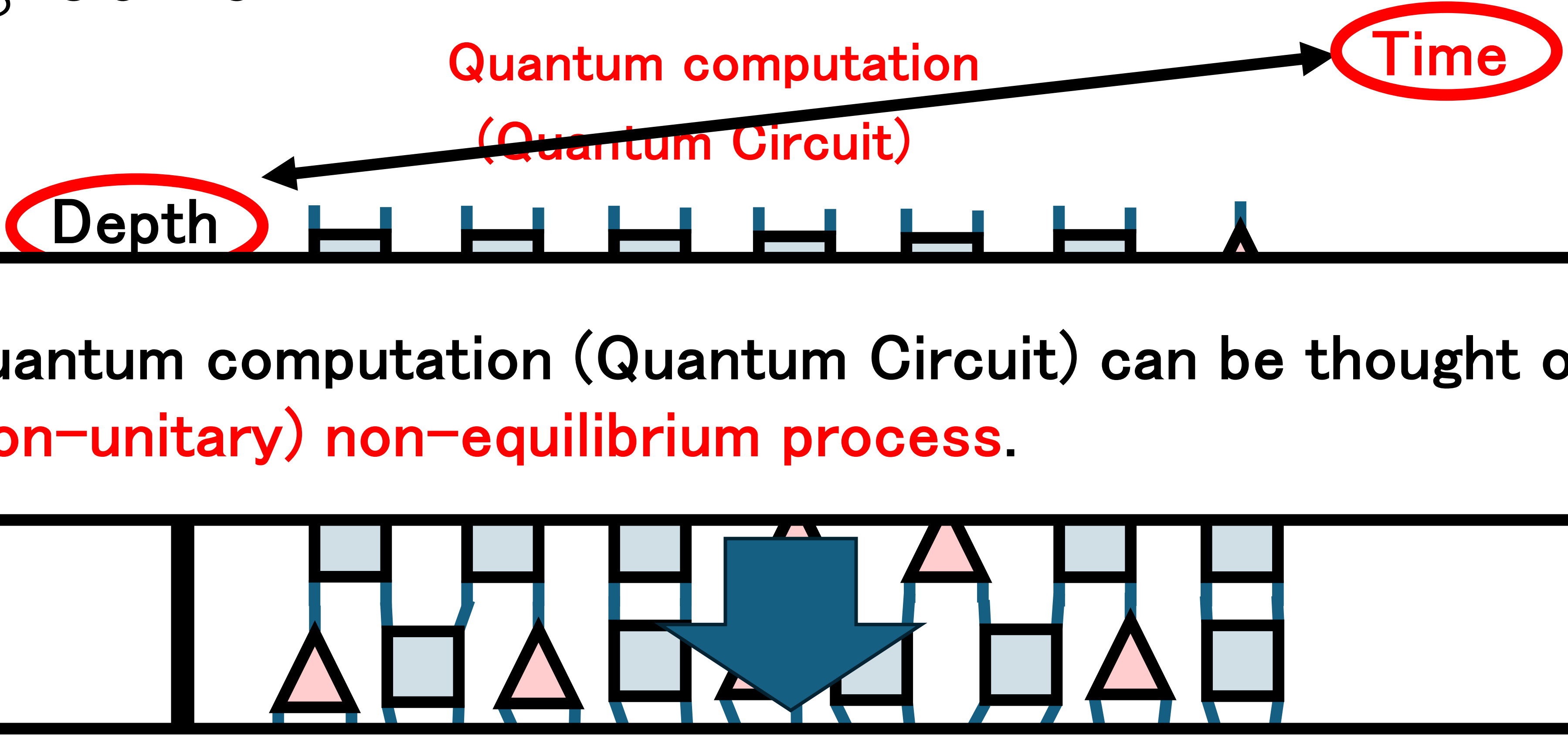


Quantum computation (Quantum Circuit) can be thought of as **a (non-unitary) non-equilibrium process**.

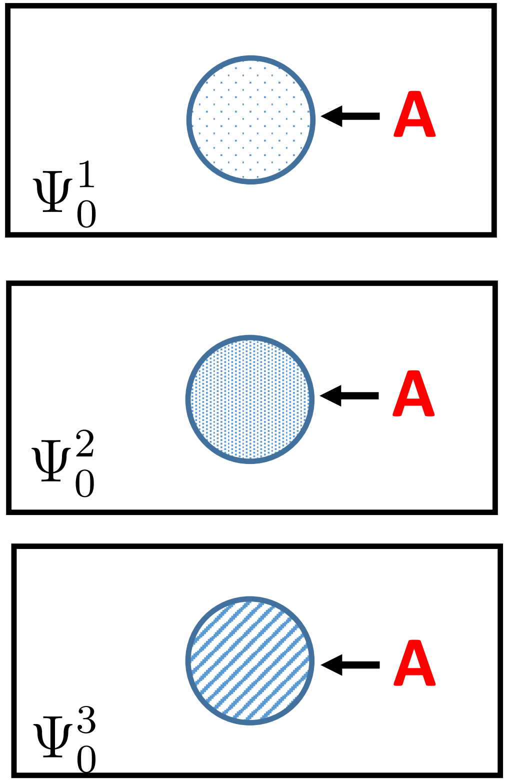
It would be important to find **a non-equilibrium phenomenon where the non-local correlation (a quantum property)** emerges even during the time evolution induced by the Hamiltonian with **the strong scrambling ability**.

- Non-unitary gates:

Background 2

Scrambling is one of the cutting-edge research topics.

This is closely relevant to **quantum thermalization**.



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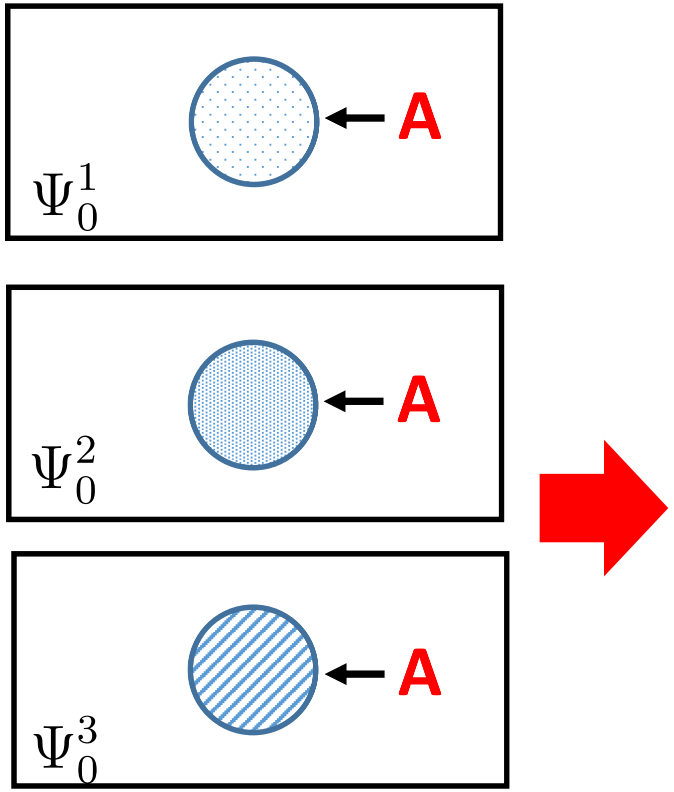

$\left|\Psi_0^i\right\rangle$ are initial states.

Local observables in A depend on initial condition.

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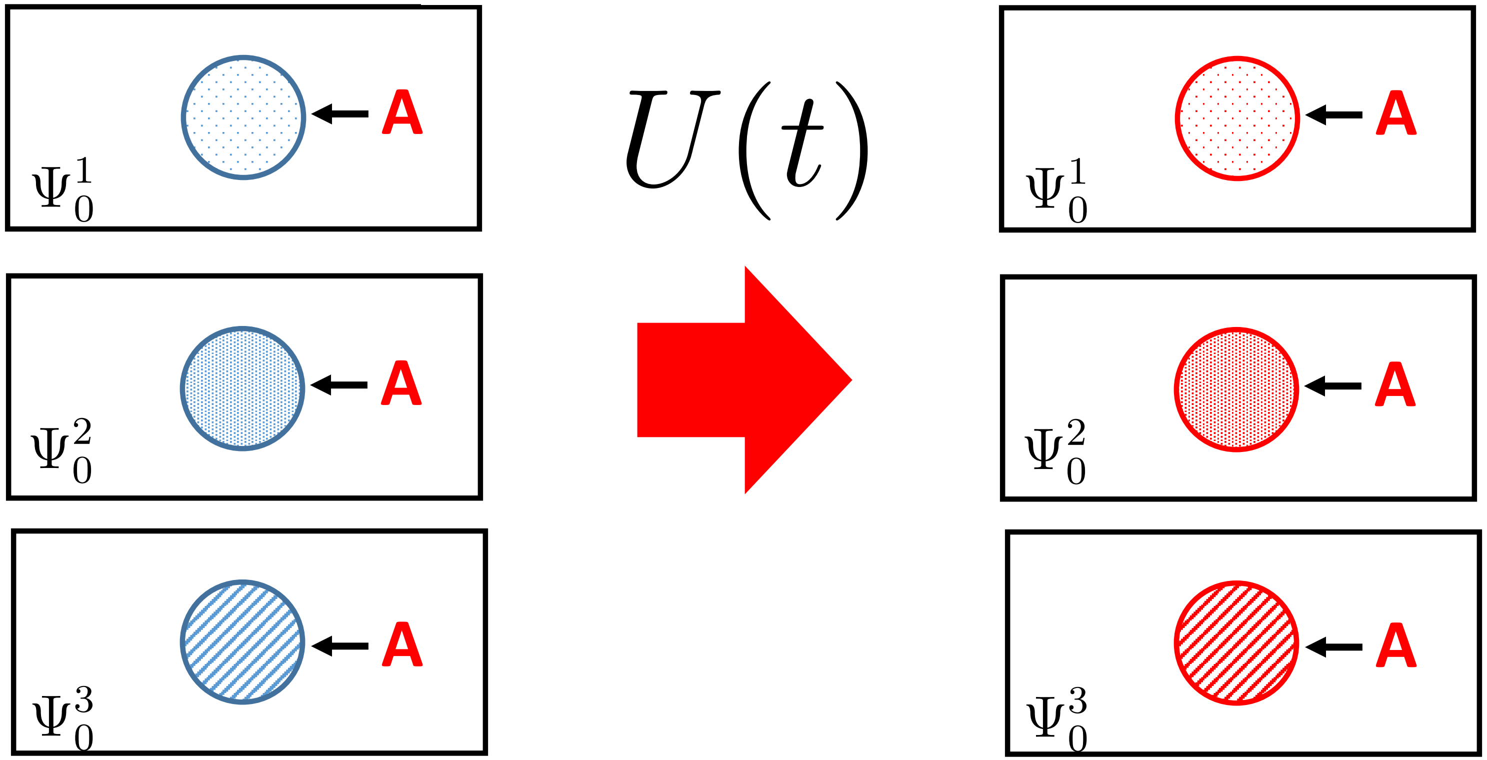
$$\left|\Psi^i(t)\right\rangle = U(t)\left|\Psi_0^i\right\rangle$$



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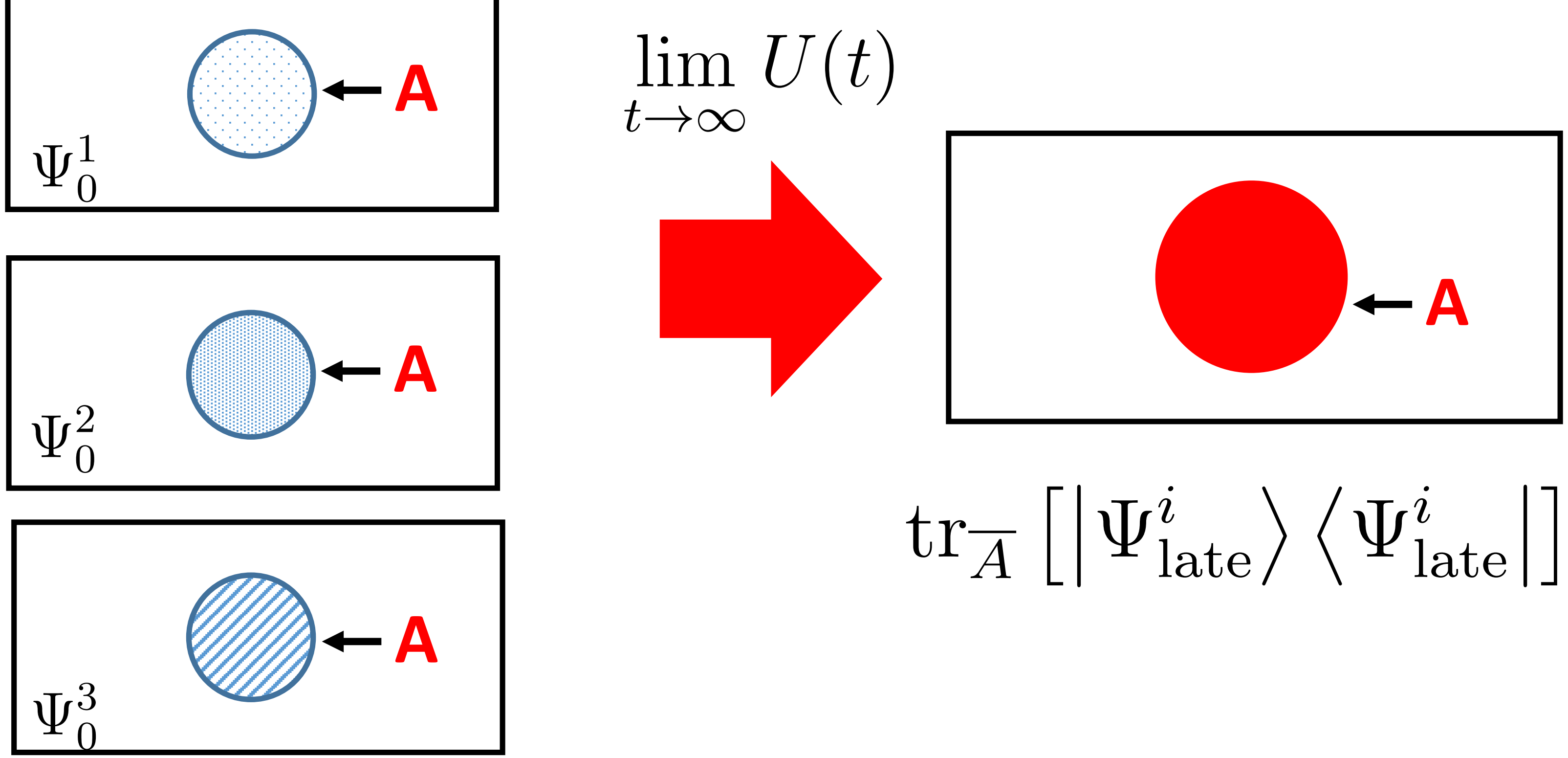
$$\left|\Psi^i_{\mathrm{late}}\right\rangle = \lim_{t\to\infty} U(t)\left|\Psi^i_0\right\rangle$$



$$\mathrm{tr}_{\overline{A}}\left[\left|\Psi^i_{\mathrm{late}}\right\rangle\left\langle\Psi^i_{\mathrm{late}}\right|\right] \approx \mathrm{tr}_{\overline{A}} e^{-\beta H}$$

Introduction

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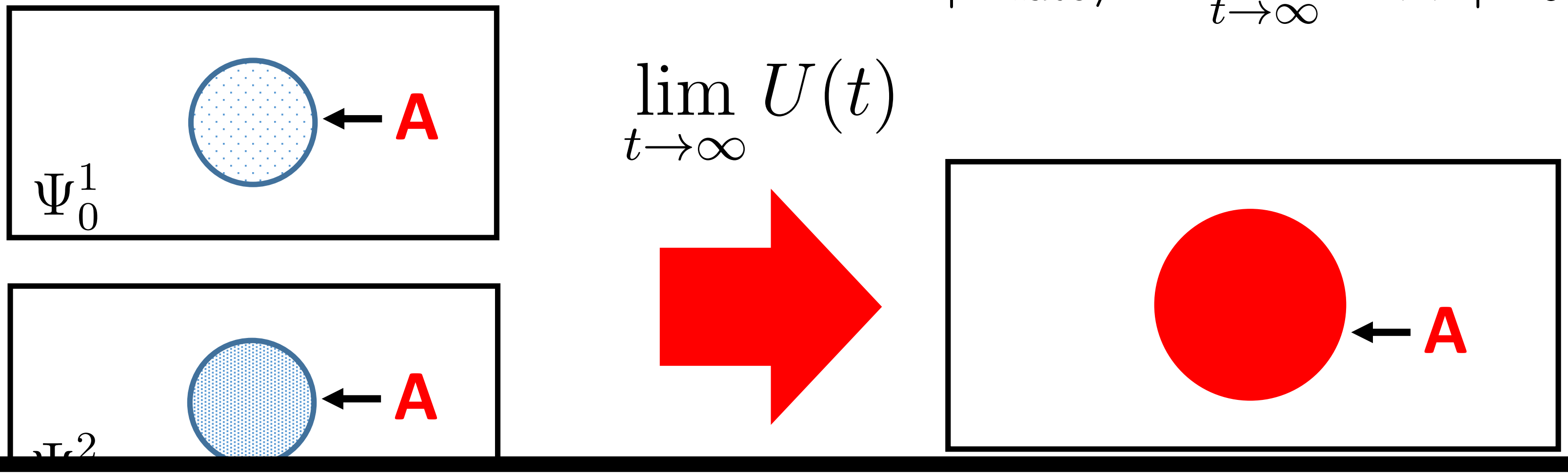

This is the process of (information) scrambling.

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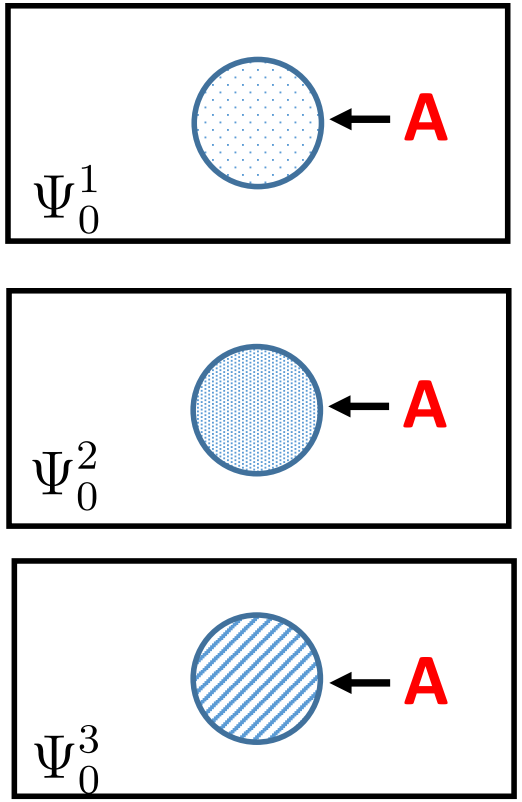

This late-time reduced density matrix associated to A is **independent of initial state.**

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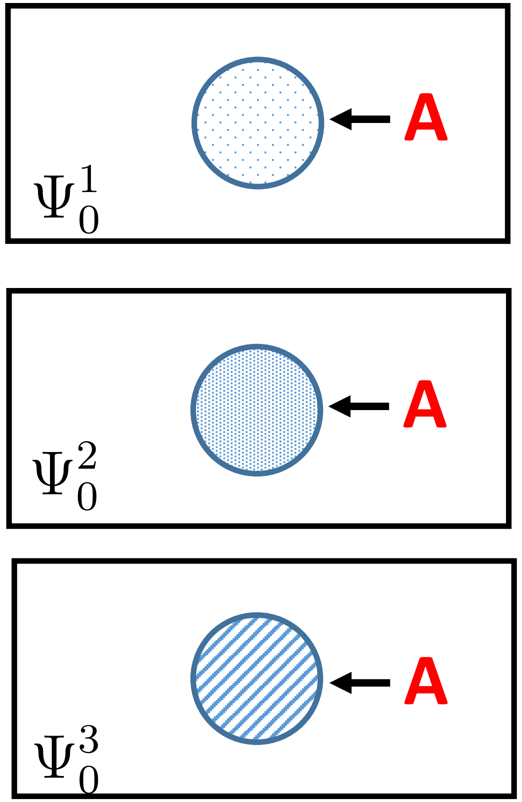

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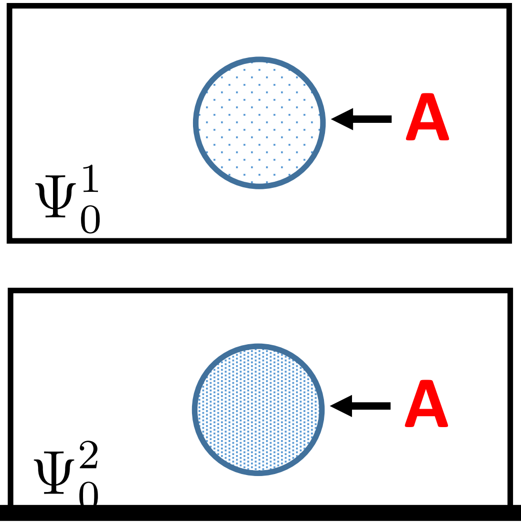


This late-time reduced density matrix associated to A is **independent of initial state.**

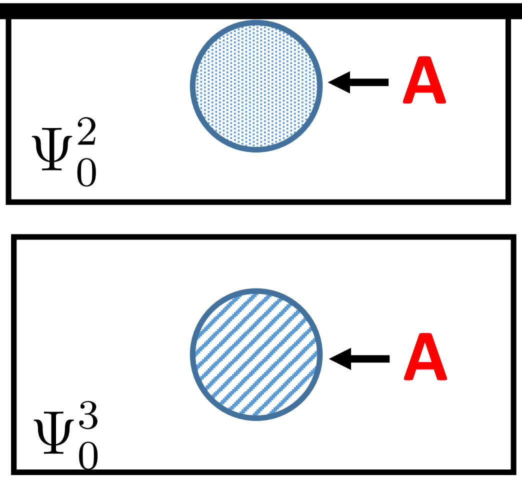
Information on the initial states is locally hidden (effectively lost).

Quantum thermalization (the thermalization of the subsystems) occurs because the reduced density matrices are approximated by the thermal state with the effective temperature.

Background 2

Scrambling is one of the cutting-edge research topics.

It is believed that if the Hamiltonian has a strong scrambling ability, information scrambling (quantum thermalization) occurs.



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Background 2

Scrambling is one of the cutting-edge research topics.

It is believed that 2d holographic CFT, CFT having gravity dual, has such strong scrambling ability.

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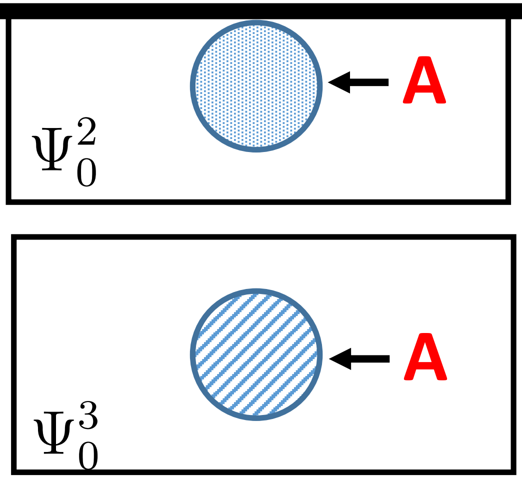

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Non-local correlation

Let us consider the behavior of non-local correlation during the quantum thermalization.

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Divide the Hilbert space into A, B, and the complement to A and B,

$$\frac{L}{2} > l_A + l_B > 0$$

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After quantum thermalization occurs.

$$S_A \approx l_A, S_B \approx l_B, S_{A\cup B} \approx (l_A + l_B)$$

,where $1 \gg \epsilon$: an effective inverse temperature

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$$I_{A,B} = S_A + S_B - S_{A\cup B}$$

$$S_A \approx l_A, S_B \approx l_B, S_{A\cup B} \approx (l_A + l_B) \Rightarrow \boxed{I_{A,B} \underset{1 \gg \epsilon}{\approx} 0}$$

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Non-local correlation

Let us consider the behavior of non-local correlation during the quantum thermalization,

The behavior of the reduced density: $\mathrm{tr}_{\overline{A}}\left(e^{-iHt}\left|\Psi\right\rangle\left\langle\Psi\right|e^{iHt}\right) \approx \mathrm{tr}_{\overline{A}}e^{-\epsilon H}$

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Non-local correlation

Let us consider the behavior of non-local correlation during the quantum thermalization,

The behavi

Non-local correlation is destroyed by the dynamics.

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Non-local correlation

Meaning of $\rho_{A\cup B} \underset{1\gg\epsilon}{\approx} \rho_A \otimes \rho_B$

➡ **No non-local correlation between A and B**

For example, $\langle \mathcal{O}(X_1 \in A)\mathcal{O}(X_2 \in B)\rangle \underset{1\gg\epsilon}{\approx} \langle \mathcal{O}(X_1 \in A)\rangle \times \langle \mathcal{O}(X_2 \in B)\rangle$

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Thus, quantum properties (here, non-local correlation) could be destroyed during quantum thermalization.

Motivation

Find the non-equilibrium phenomenon where the non-local correlation emerges during the time evolution induced by Holographic CFT (strongest scrambling QFT).

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Find the non-equilibrium phenomenon where the non-local correlation emerges during the time evolution induced by Holographic CFT (strongest scrambling QFT).

Idea

Put the holographic CFT on the curved background where at fixed points, the velocity of massless particles is zero.

Idea

The system considered:
Spatially-periodic one with the circumstance of L

Hamiltonian on flat space

Boundary state

We start from short-range entangled state: $|\Psi\rangle = \frac{1}{\sqrt{\langle \mathrm{B}| e^{-2\epsilon H} |\mathrm{B}\rangle}} e^{-\epsilon H} |\mathrm{B}\rangle$

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Evolve the system with $H_{\text{q-SSD}} = \int_0^L dx\, f(x) h(x) \qquad f(x) = 2\sin^2\left(\frac{q\pi x}{L}\right)$

Hamiltonian on the curved geometry

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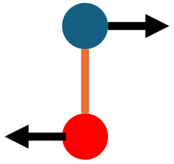
Hamiltonian on the curved geometry

Assumptions:

1. Entangled pairs, constructed of left- and right-moving particles, emerge at each spatial point:



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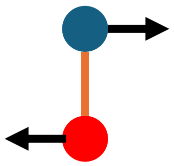

Entangled pair

2. Entanglement entropy associated with A

$\propto$ The number of entangled pair contributing to A

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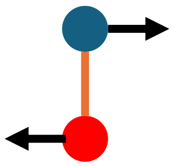

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3. The velocity of particles determined by the background:

Background: $ds^2 = -f^2(x)dt^2 + dx^2$ **Velocity:** $\frac{dx}{dt} = |f(x)|$

Idea

The system considered:
Spatially-periodic one with the circumstance of L

We start from short-range entangled state: $|\Psi\rangle = \frac{1}{\sqrt{\langle \mathrm{B}|\, e^{-2\epsilon H}\, |\mathrm{B}\rangle}} e^{-\epsilon H}\, |\mathrm{B}\rangle$

Evolve the system with $\underline{H_{\text{q-SSD}} = \int_0^L dx f(x) h(x) \qquad f(x) = 2\sin^2\left(\frac{q\pi x}{L}\right)}$

Hamiltonian on the curved geometry

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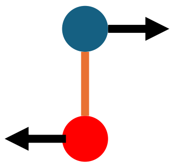

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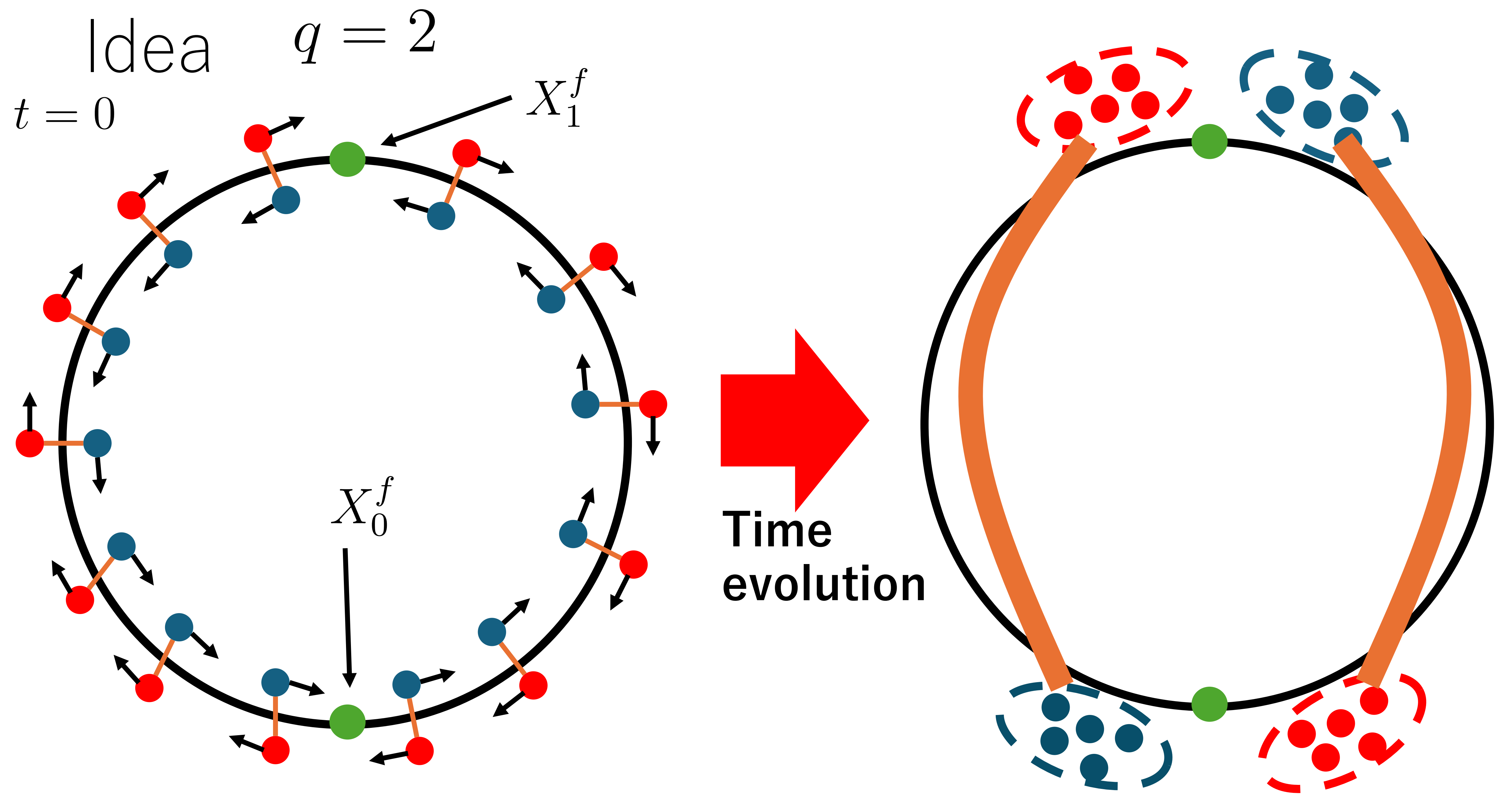
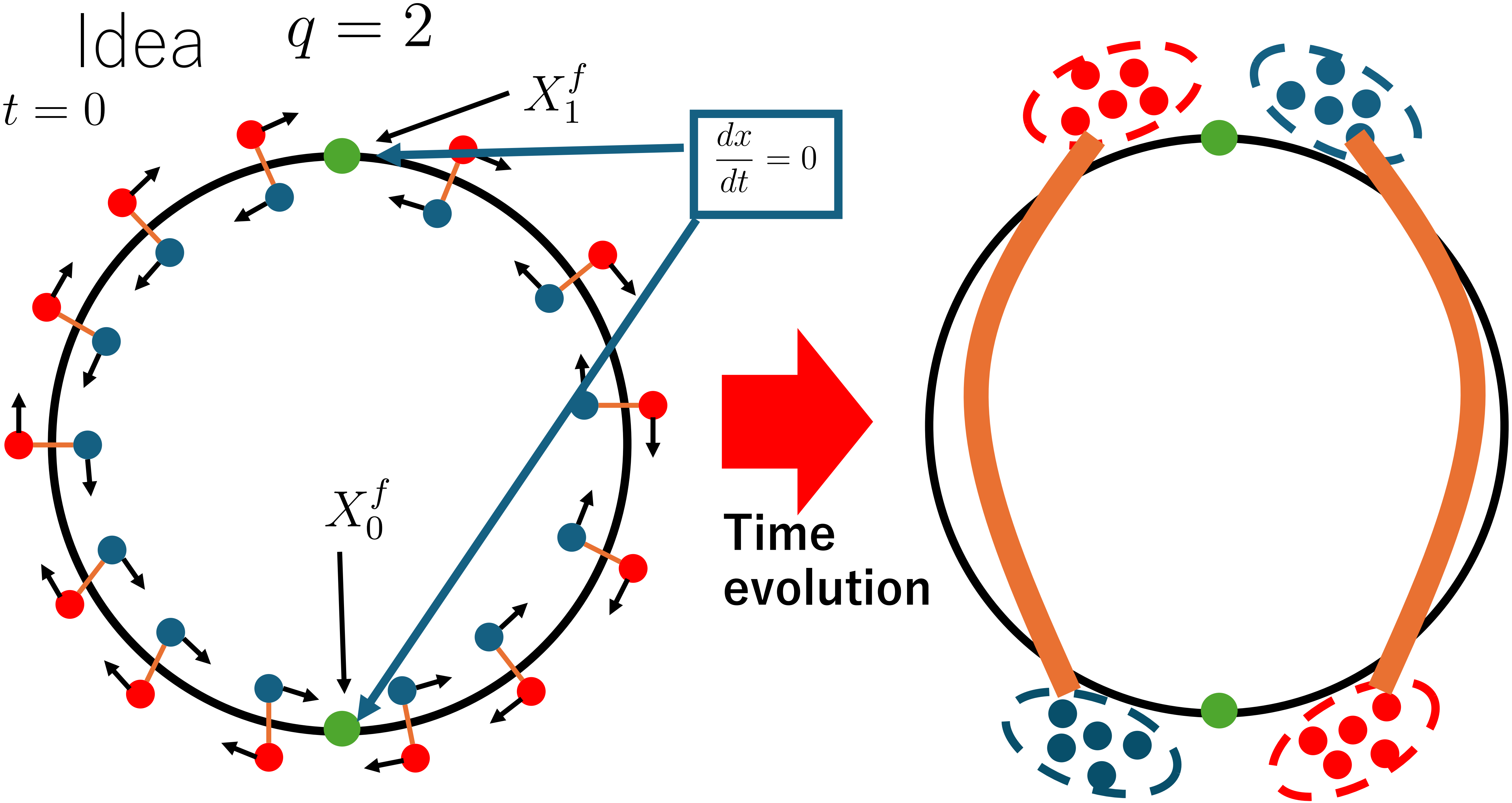
Background: $ds^2 = -f^2(x)dt^2 + dx^2$ **Velocity:** $\frac{dx}{dt} = |f(x)|$ $\boxed{\frac{dx}{dt} = 0 \text{ @ } x = X_m^f = \frac{mL}{q}}$



Idea
$q = 2$
$t = 0$
X_1^f
X_0^f
Time evolution



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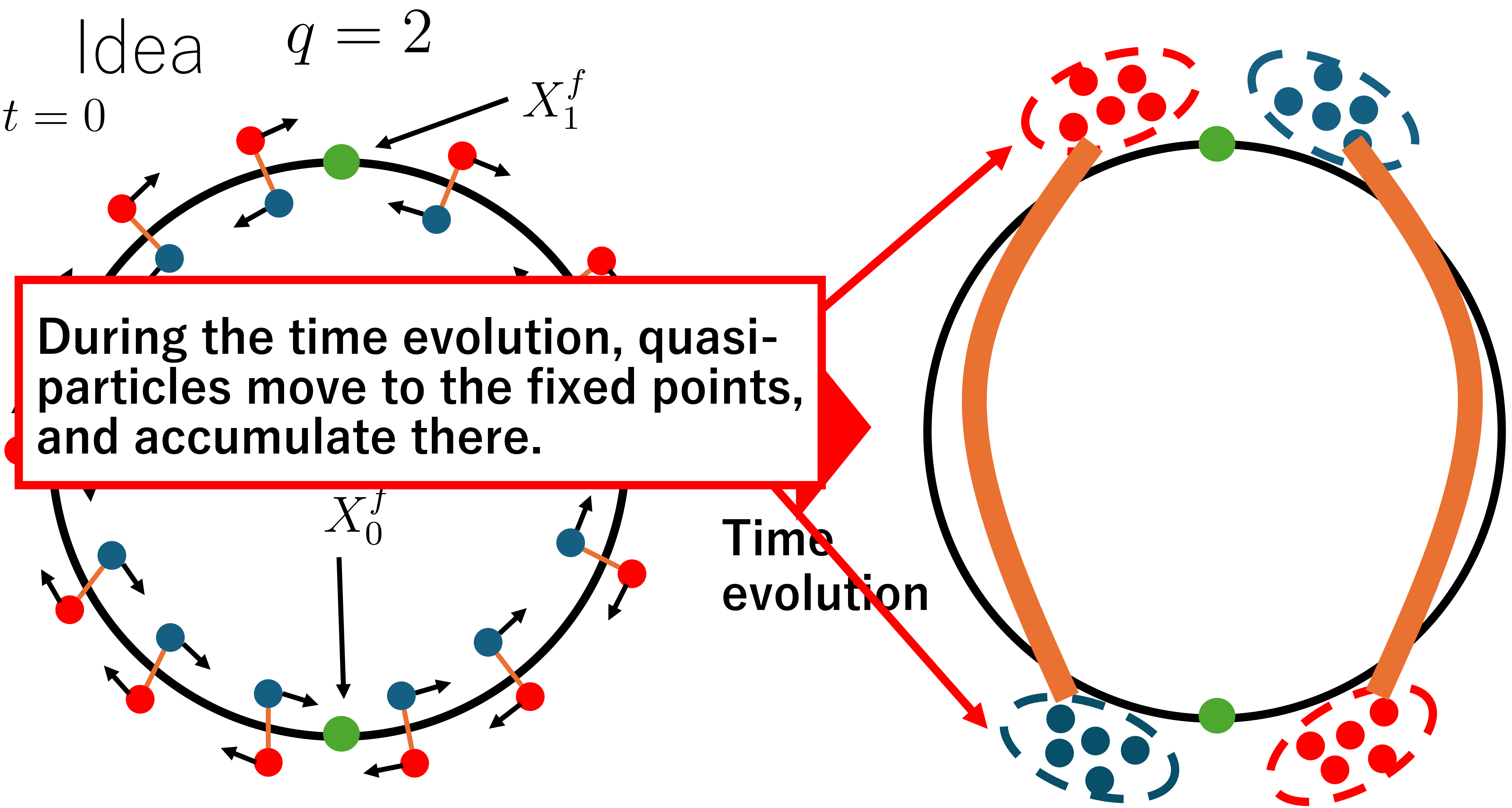
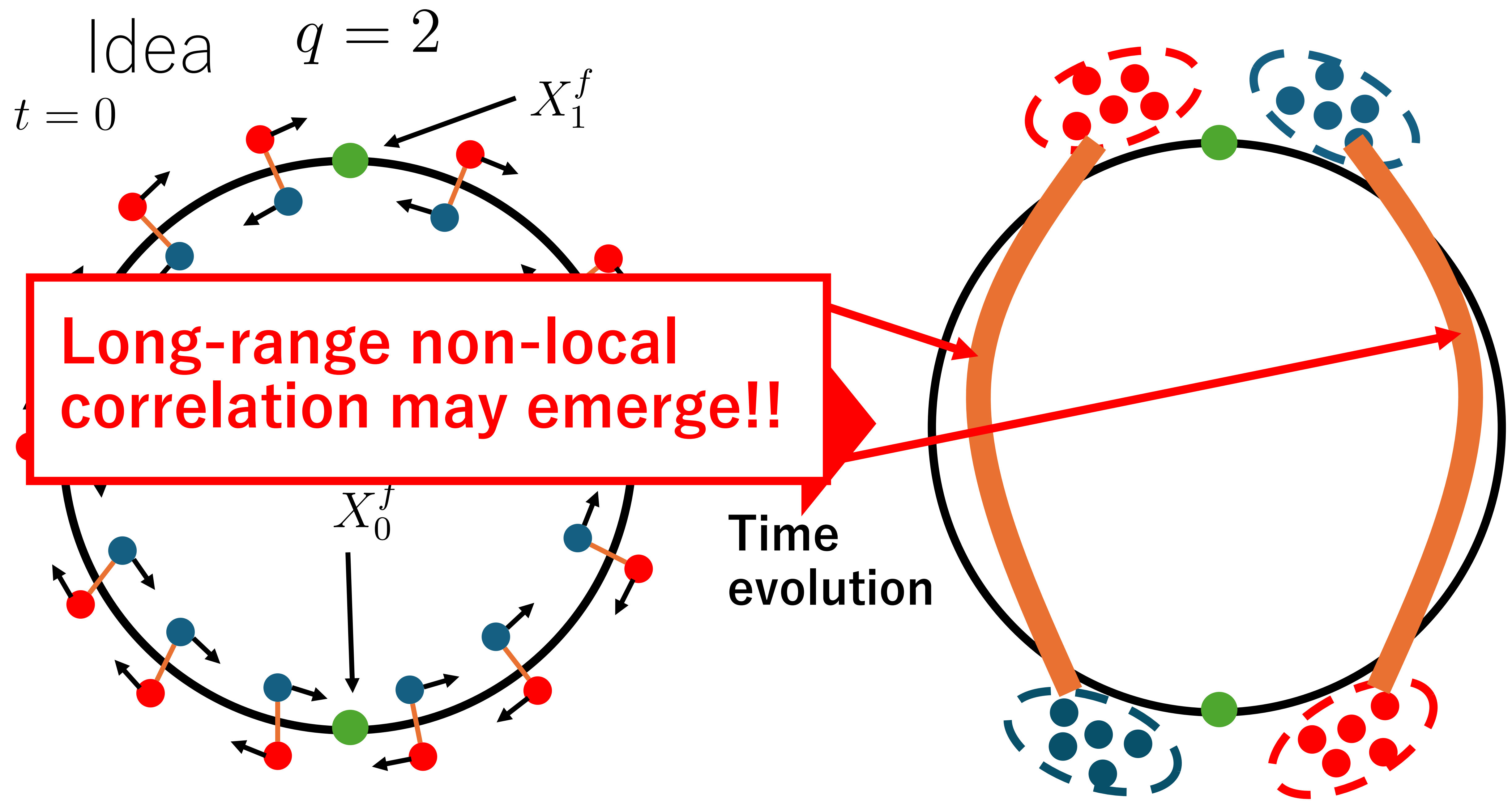
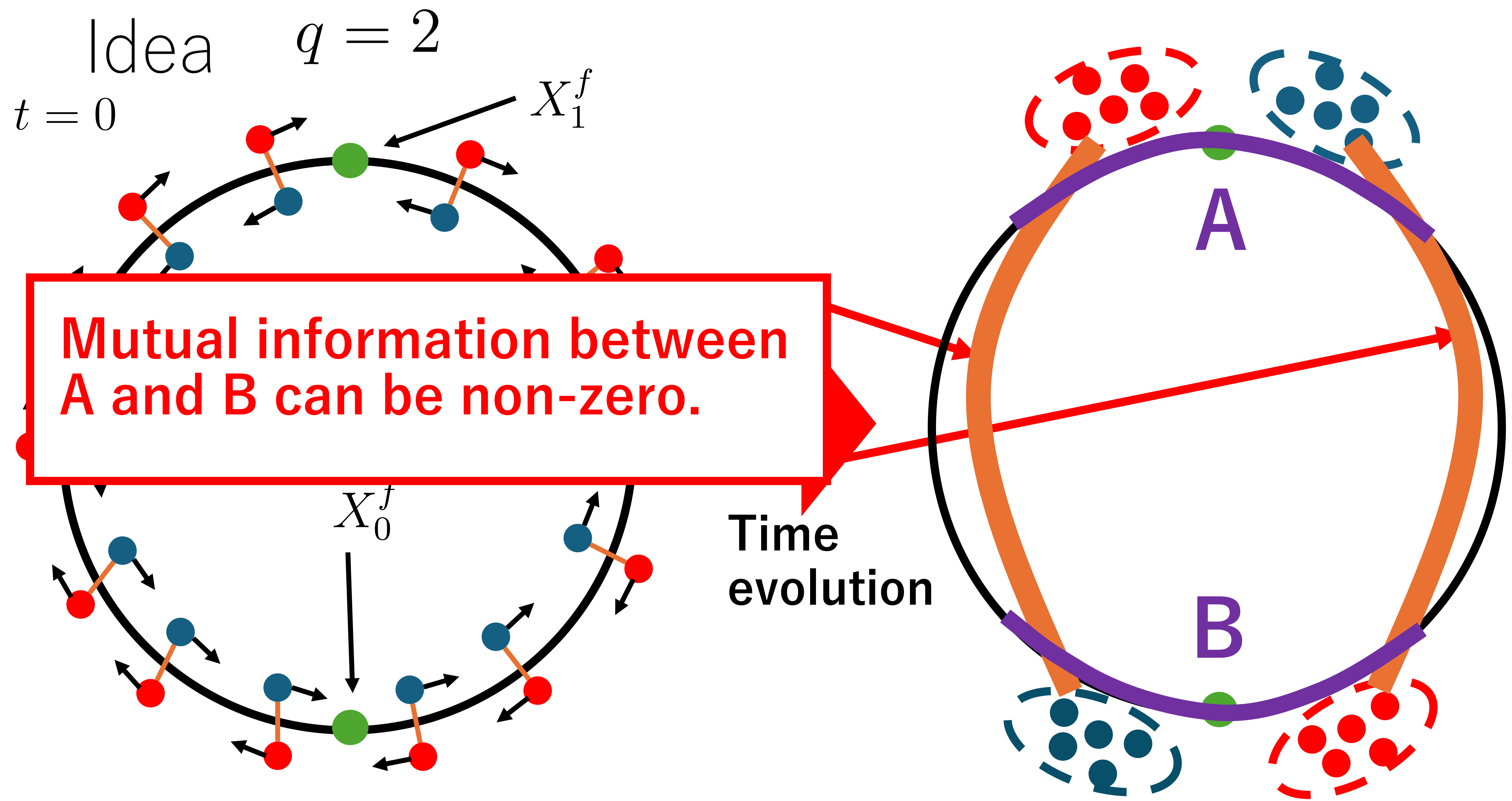

$q = 2$
$t = 0$
X_1^f
X_0^f
During the time evolution, quasi-particles move to the fixed points, and accumulate there.
Time evolution



Idea
$q = 2$
$t = 0$
X_1^f
X_0^f
Long-range non-local correlation may emerge!!
Time evolution


Idea
$q = 2$
$t = 0$
X_1^f
X_0^f
Mutual information between A and B can be non-zero.
Time evolution
A
B

Our findings

1. During the time evolution induced by $H_{\text{q-SSD}}$, the holographic Hamiltonian on the curved background,

The system with no correlation

The system with long-range correlation

2. During the time evolution induced by $H_{\text{q-SSD}}$, wormhole grows in time and **saturates to some size**.

Details of our project

Note

The parameter region considered in this talk is

$$L \gg l_{\mathcal{V}}, t \gg \epsilon \gg 1,$$

where these parameters are dimensionless and their unit is the lattice spacing.

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$$L \gg l_{\mathcal{V}}, t \gg \epsilon \gg 1.$$

System size

Time

Subsystem size

Inverse temperature

Note

All the theories considered in this talk are two-dimensional conformal field theories.

The setup considered

- System: Spatially-periodic one with circumstance of L

- State: $|\Psi(t)\rangle = \frac{1}{\sqrt{\langle \mathrm{B}| e^{-2\epsilon H} |\mathrm{B}\rangle}} e^{-itH_{\text{q-SSD}}} e^{-\epsilon H} |\mathrm{B}\rangle$ $\longrightarrow$ $S_A \sim \log \epsilon$

- Hamiltonian: $H = \int_0^L dx h(x)$ $\longleftarrow$ 2d CFT Hamiltonian on the flat space

$$H_{\text{q-SSD}} = 2\int_0^L dx \sin^2\left(\frac{q\pi x}{L}\right) h(x)$$ $\longleftarrow$ 2d CFT Hamiltonian on the curved background

$$ds^2 = -f^2(x)dt^2 + dx^2 \qquad \frac{dx}{dt} = |f(x)|$$

The quantities computed

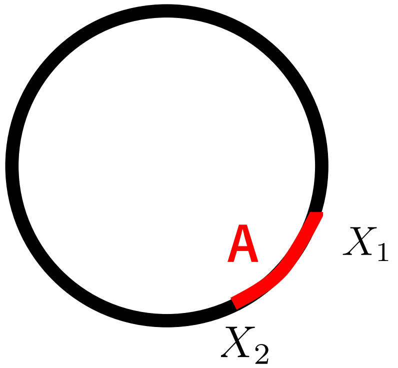
- Subsystems taken: The subsystem A contains 2 fixed points.
  The subsystem B contains 2 fixed points.

- Entanglement entropy (EE): $S_\alpha = -\mathrm{tr}_\alpha \rho_\alpha \log \rho_\alpha$

- Mutual information (MI) between A and B:

$$I_{A,B} + S_A + S_B - S_{A\cup B}$$

EE for the single interval



In the twist operator formalism,
EE for the single interval results in

$$S_A = \lim_{n\to 1} \frac{1}{1-n} \log \left[\frac{\langle \mathrm{B}| \, e^{-\epsilon H} \sigma_{n,H}(X_1) \overline{\sigma}_{n,H}(X_2) e^{-\epsilon H} \, |\mathrm{B}\rangle}{\langle \mathrm{B}| \, e^{-2\epsilon} \, |\mathrm{B}\rangle} \right].$$

Operator in Heisenberg picture: $\mathcal{O}_H = e^{itH_{\text{q-SSD}}} \mathcal{O} e^{-itH_{\text{q-SSD}}}$

$\sigma_n, \overline{\sigma}_n$: Primary operators with $(h_n, h_n) = \left(\frac{c(n^2-1)}{24n}, \frac{c(n^2-1)}{24n} \right)$

EE for the single interval

Transformation of primary operator

$$e^{\epsilon H} e^{\tau H_{\text{q-SSD}}} \mathcal{O}(w,\overline{w}) e^{-\tau H_{\text{q-SSD}}} e^{-\epsilon H} = \left|\frac{dw^{\text{New,q}}}{dw}\right|^{2h_{\mathcal{O}}} \mathcal{O}(w^{\text{New,q}},\overline{w}^{\text{New,q}}).$$

$$w^{\text{New},q} = \epsilon + \frac{L}{2q\pi}\log\left[\frac{Lz^q+\pi q\tau\left(z^q-1\right)}{L+\pi q\tau\left(z^q-1\right)}\right] \quad \overline{w}^{\text{New},q} = \epsilon + \frac{L}{2q\pi}\log\left[\frac{L\overline{z}^q+\pi q\tau\left(\overline{z}^q-1\right)}{L+\pi q\tau\left(\overline{z}^q-1\right)}\right]$$

$$(w,\overline{w}) = (T+iX, T-iX),\ (z,\overline{z}) = (e^{\frac{2\pi w}{L}}, e^{\frac{2\pi \overline{w}}{L}})$$

$\sigma_n, \overline{\sigma}_n$: Primary operators with $(h_n,h_n)=\left(\frac{c(n^2-1)}{24n},\frac{c(n^2-1)}{24n}\right)$

EE for the single interval

After the transformation of the twist operators,
EE for the single interval results in

$$S_A \approx \frac{c}{12}\log\left[\prod_{i=1,2}\left(\frac{\left(L^2-(2q\pi t)^2\sin^2\left(\frac{q\pi X_i}{L}\right)\right)^2-(4q\pi Lt)^2\sin^4\left(\frac{q\pi X_i}{L}\right)}{L^4}\right)\right]$$

$$+\lim_{n\to 1}\frac{1}{1-n}\log\left[\frac{\langle \mathrm{B}|\, e^{-2\epsilon H_0}\sigma_n\left(w^{\mathrm{New,q}}_{X_1},\overline{w}^{\mathrm{New,q}}_{X_1}\right)\overline{\sigma}_n\left(w^{\mathrm{New,q}}_{X_2},\overline{w}^{\mathrm{New,q}}_{X_2}\right)|\mathrm{B}\rangle}{\langle \mathrm{B}|\, e^{-2\epsilon H_0}\,|\mathrm{B}\rangle}\right].$$

In 2d holographic CFT, this is determined by the geodesic length on some background.

EE for the single interval

$$+\lim_{n\to 1}\frac{1}{1-n}\log\left[\frac{\langle \mathrm{B}|\, e^{-2\epsilon H_0}\sigma_n\left(w_{X_1}^{\mathrm{New,q}},\overline{w}_{X_1}^{\mathrm{New,q}}\right)\overline{\sigma}_n\left(w_{X_2}^{\mathrm{New,q}},\overline{w}_{X_2}^{\mathrm{New,q}}\right)|\mathrm{B}\rangle}{\langle \mathrm{B}|\, e^{-2\epsilon H_0}\,|\mathrm{B}\rangle}\right]$$

In 2d holographic CFT, this is determined by the geodesic length on some background.

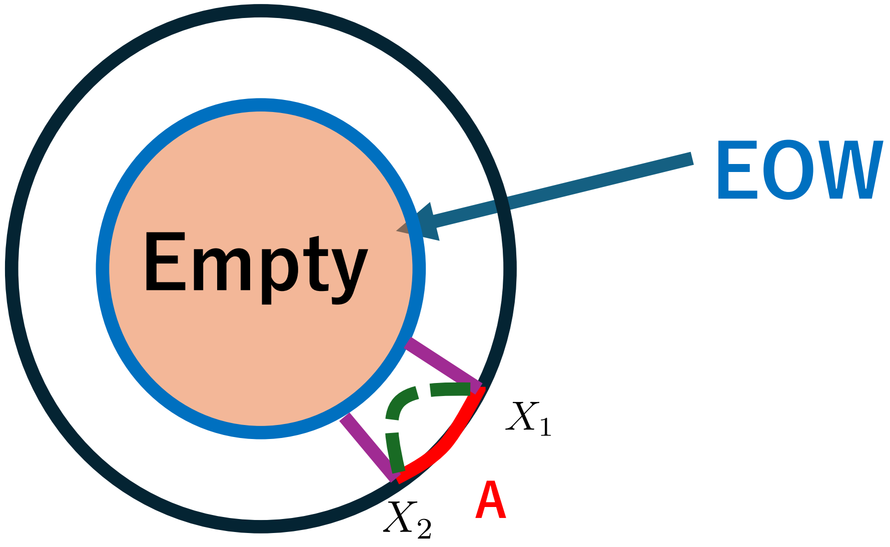
After employing the method of image, this is determined by the four-point function on the torus with L and 4ϵ.

In high temperature region, $\frac{L}{4\epsilon}>1$, this is determined by the geodesic length on geometry with the end of the world brane (EOW).

EE for the single interval

$$+\lim_{n\to 1}\frac{1}{1-n}\log\left[\frac{\langle \mathrm{B}|\, e^{-2\epsilon H_0}\sigma_n\left(w^{\mathrm{New,q}}_{X_1},\overline{w}^{\mathrm{New,q}}_{X_1}\right)\overline{\sigma}_n\left(w^{\mathrm{New,q}}_{X_2},\overline{w}^{\mathrm{New,q}}_{X_2}\right)|\mathrm{B}\rangle}{\langle \mathrm{B}|\, e^{-2\epsilon H_0}\,|\mathrm{B}\rangle}\right]$$

The subsystem A contains 2 fixed points.

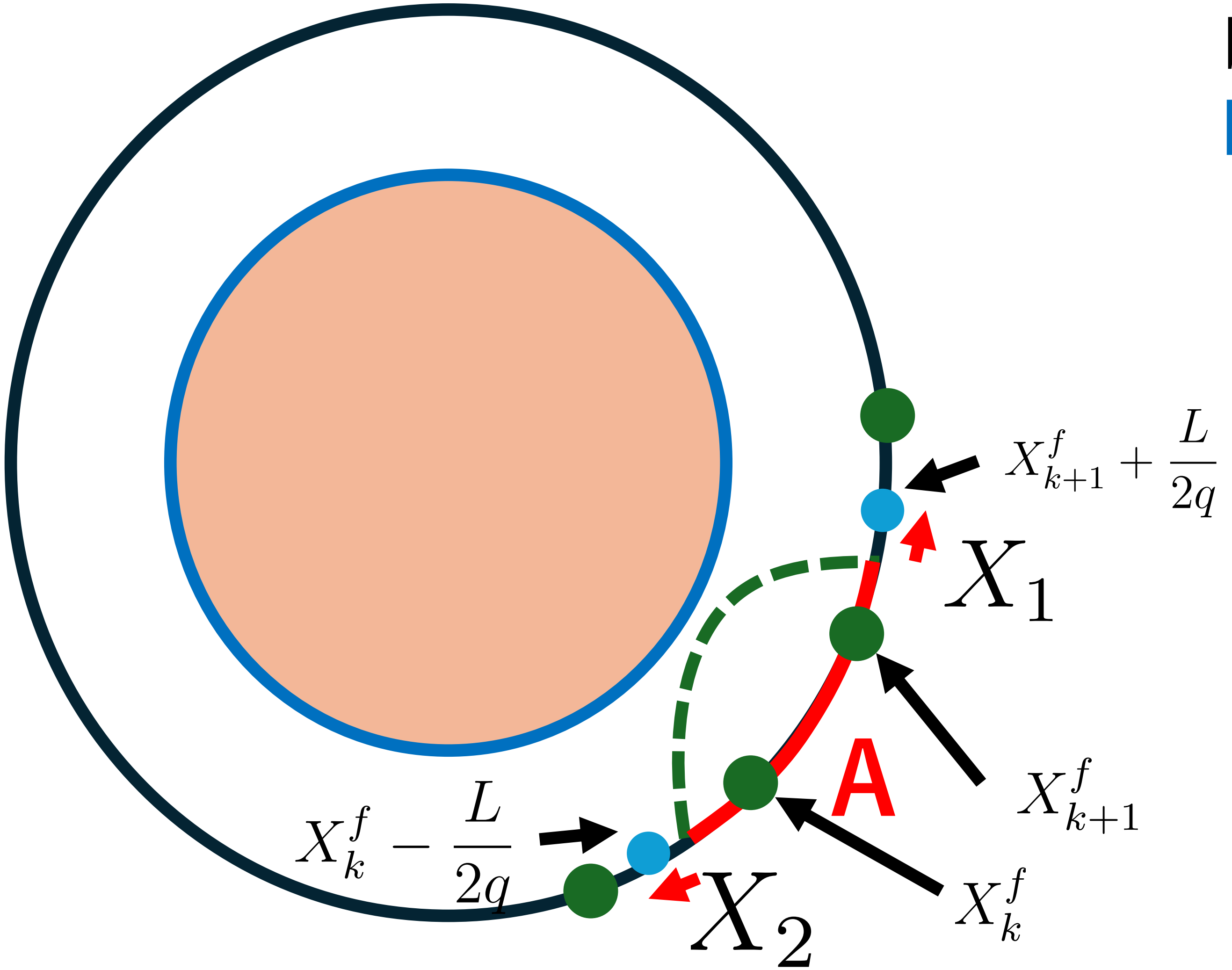


Minimal one of || and

$$+\lim_{n\to 1}\frac{1}{1-n}\log\left[\frac{\langle \mathrm{B}|\, e^{-2\epsilon H_0}\sigma_n\left(\underline{w^{\mathrm{New,q}}_{X_1},\overline{w}^{\mathrm{New,q}}_{X_1}}\right)\overline{\sigma}_n\left(\underline{w^{\mathrm{New,q}}_{X_2},\overline{w}^{\mathrm{New,q}}_{X_2}}\right)|\mathrm{B}\rangle}{\langle \mathrm{B}|\, e^{-2\epsilon H_0}\,|\mathrm{B}\rangle}\right]$$

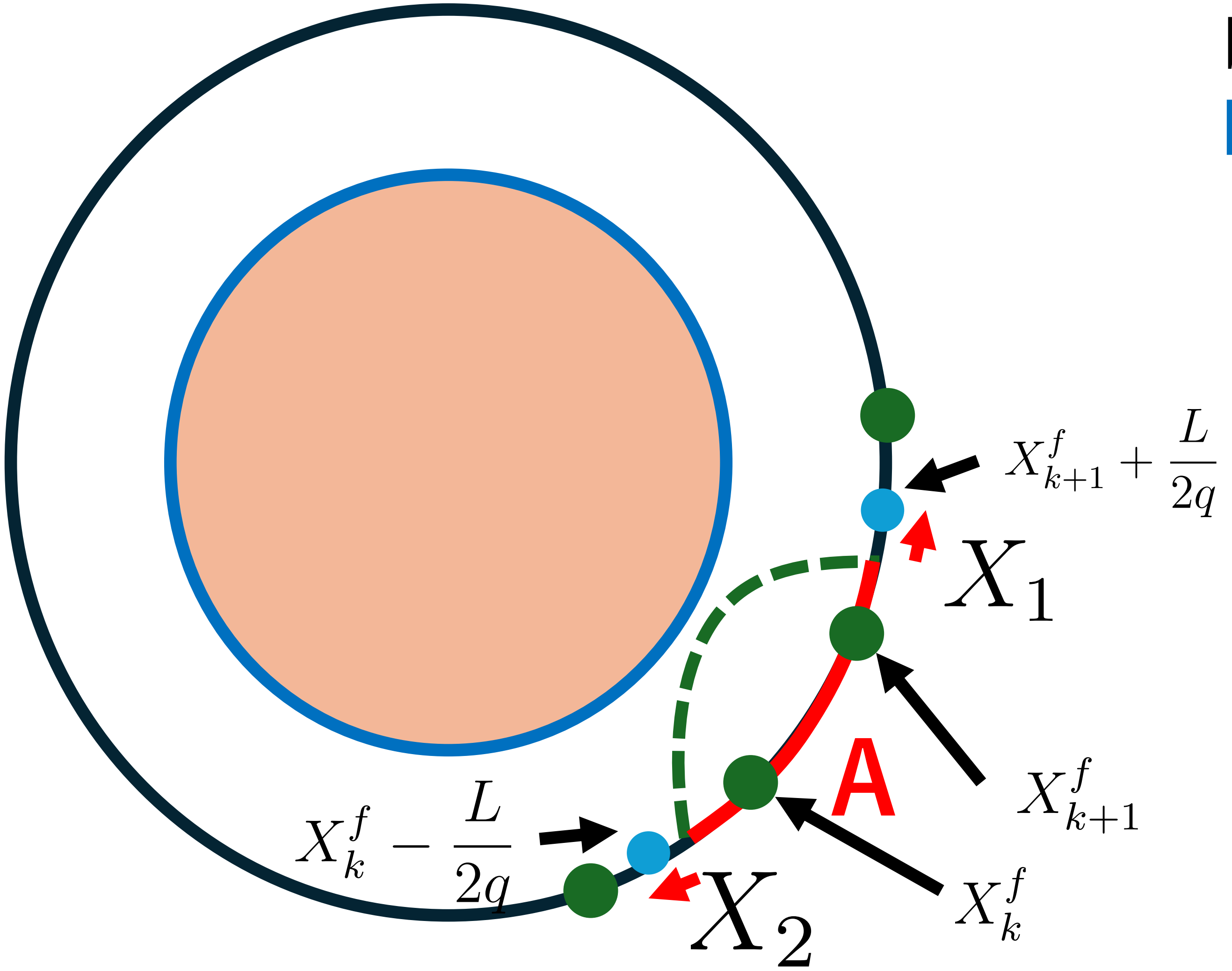
$$X^{\text{New, q}}=\frac{w^{\mathrm{New,q}}-\overline{w}^{\mathrm{New,q}}}{2i}$$

During the time evolution, boundaries of A move to **blue points**.



$$+\lim_{n\to 1}\frac{1}{1-n}\log\left[\frac{\langle \mathrm{B}|\, e^{-2\epsilon H_0}\sigma_n\left(w_{X_1}^{\mathrm{New,q}},\overline{w}_{X_1}^{\mathrm{New,q}}\right)\overline{\sigma}_n\left(w_{X_2}^{\mathrm{New,q}},\overline{w}_{X_2}^{\mathrm{New,q}}\right)|\mathrm{B}\rangle}{\langle \mathrm{B}|\, e^{-2\epsilon H_0}\,|\mathrm{B}\rangle}\right]$$

$$X^{\mathrm{New,\ q}}=\frac{w^{\mathrm{New,q}}-\overline{w}^{\mathrm{New,q}}}{2i}$$

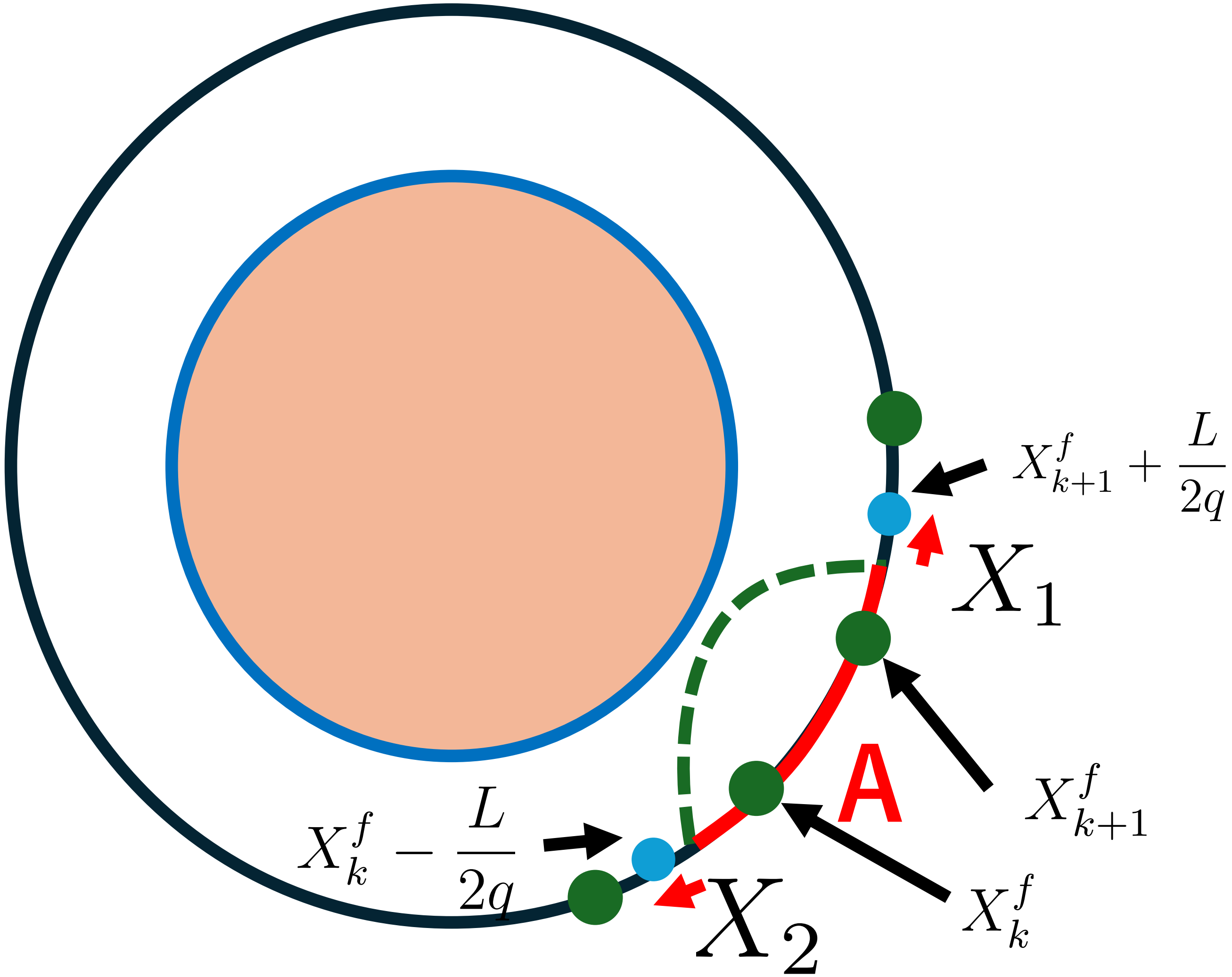


During the time evolution, boundaries of A move to **blue points**.

Effective size of A $\propto \frac{2\mathrm{L}}{\mathrm{q}}$

$$+\lim_{n\to 1}\frac{1}{1-n}\log\left[\frac{\langle \mathrm{B}|\, e^{-2\epsilon H_0}\sigma_n\left(w^{\mathrm{New,q}}_{X_1},\overline{w}^{\mathrm{New,q}}_{X_1}\right)\overline{\sigma}_n\left(w^{\mathrm{New,q}}_{X_2},\overline{w}^{\mathrm{New,q}}_{X_2}\right)|\mathrm{B}\rangle}{\langle \mathrm{B}|\, e^{-2\epsilon H_0}\,|\mathrm{B}\rangle}\right]$$

$$X^{\text{New, q}}=\frac{w^{\mathrm{New,q}}-\overline{w}^{\mathrm{New,q}}}{2i}$$



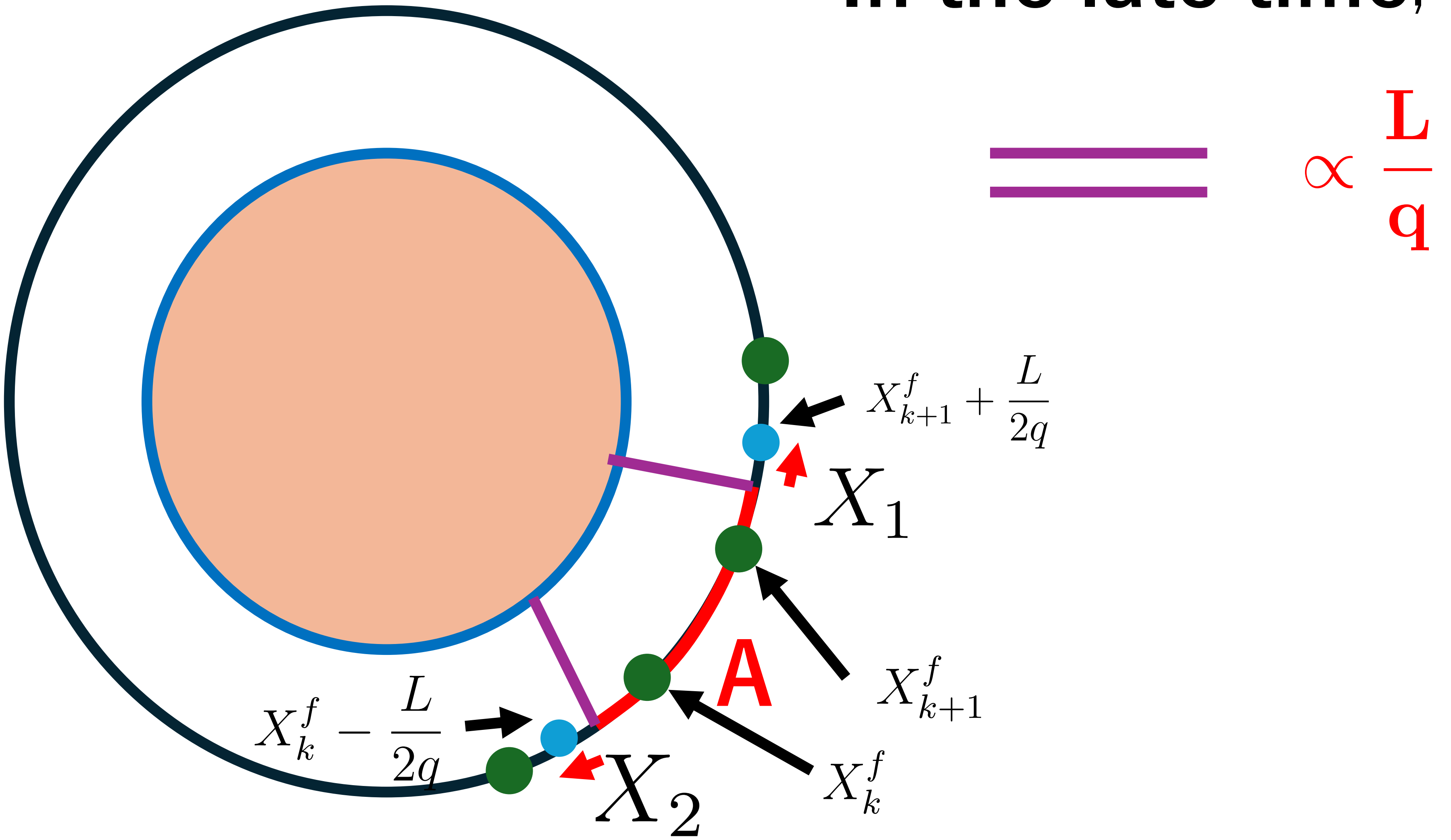
During the time evolution, boundaries of A move to **blue points**.

Effective size of A $\propto \frac{2\mathrm{L}}{\mathrm{q}}$

$\propto \frac{2\mathrm{L}}{\mathrm{q}}$

$$X^{\text{New, q}} = \frac{w^{\text{New,q}} - \overline{w}^{\text{New,q}}}{2i}$$

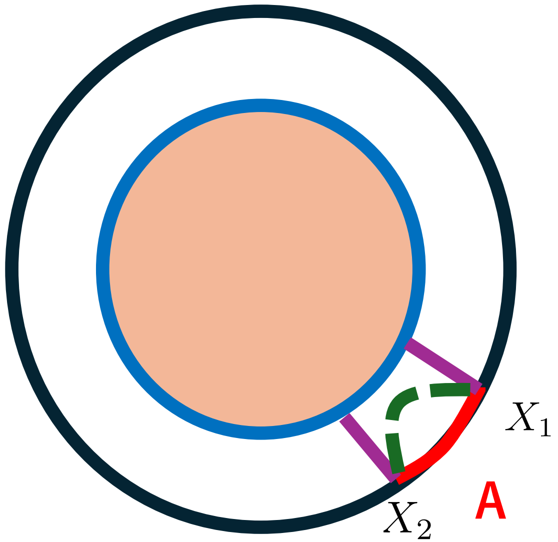
In the late time,



In the late time regime

$\propto \frac{\mathbf{L}}{\mathbf{q}}$

$\propto \frac{\mathbf{2L}}{\mathbf{q}}$



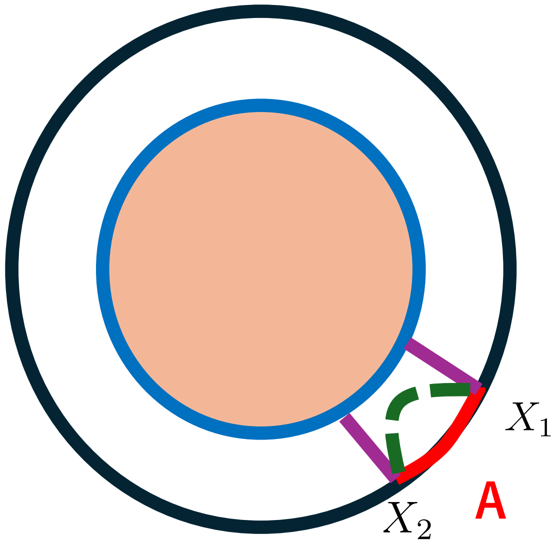
$$+\lim_{n\to 1}\frac{1}{1-n}\log\left[\frac{\langle \mathrm{B}|\, e^{-2\epsilon H_0}\sigma_n\left(w^{\mathrm{New,q}}_{X_1},\overline{w}^{\mathrm{New,q}}_{X_1}\right)\overline{\sigma}_n\left(w^{\mathrm{New,q}}_{X_2},\overline{w}^{\mathrm{New,q}}_{X_2}\right)|\mathrm{B}\rangle}{\langle \mathrm{B}|\, e^{-2\epsilon H_0}\,|\mathrm{B}\rangle}\right]$$

is determined by

In the late time regime

$\propto \frac{\mathbf{L}}{\mathbf{q}}$

$\propto \frac{\mathbf{2L}}{\mathbf{q}}$



$$S_A \approx \frac{c}{3}\cdot\frac{\pi L}{4q\epsilon}$$

$$+\lim_{n\to 1}\frac{1}{1-n}\log\left[\frac{\langle \mathrm{B}|\, e^{-\cdots} \cdots {}^{\mathrm{w,q}}, \overline{w}_{X_2}^{\mathrm{New,q}}\big)\,|\mathrm{B}\rangle}{\langle \mathrm{B}|\, e^{-2\epsilon H_0}\,|\mathrm{B}\rangle}\right]$$

is determined by

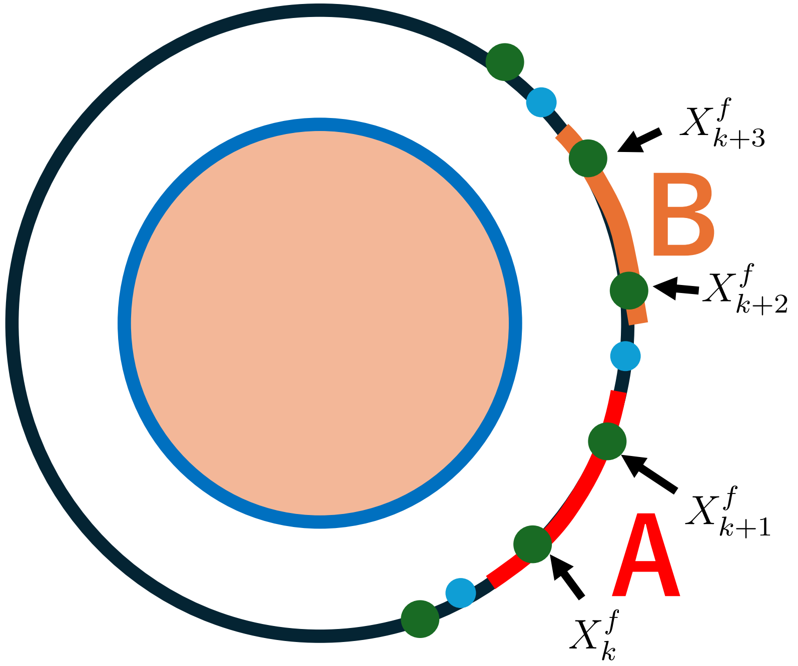
Mutual information

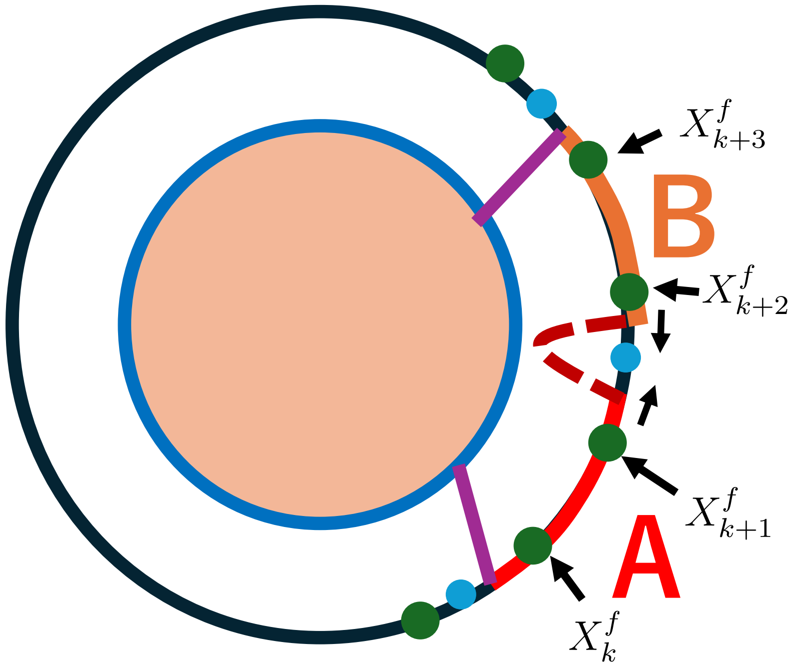
The subsystem A contains 2 fixed points.
The subsystem B contains 2 fixed points.

$$S_{\alpha=A,B} \approx \quad \approx \frac{c}{3}\cdot\frac{\pi L}{4q\epsilon}$$

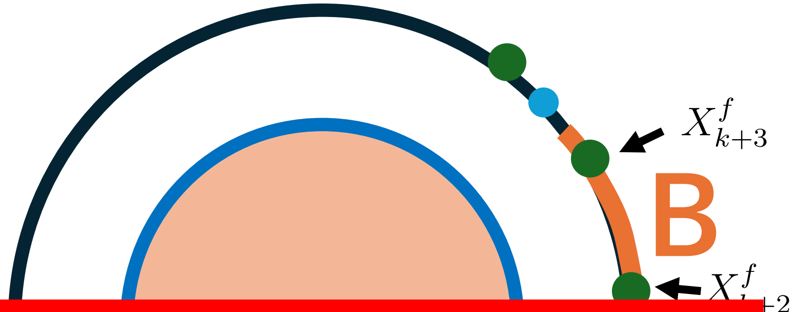
In the late time regime

$$S_{A\cup B} \approx \quad + \quad \mathcal{O}(1)$$





Mutual information

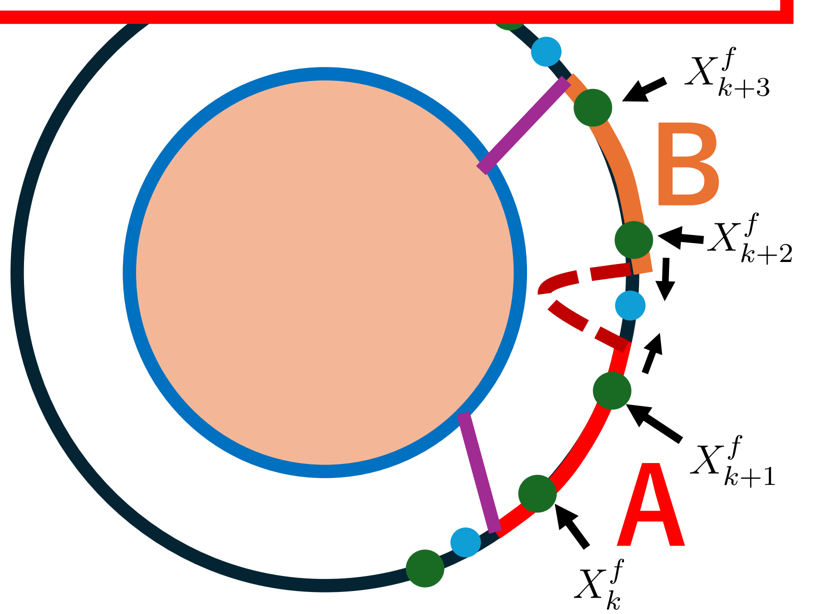


$$I_{A,B} = S_A + S_B - S_{A\cup B}$$

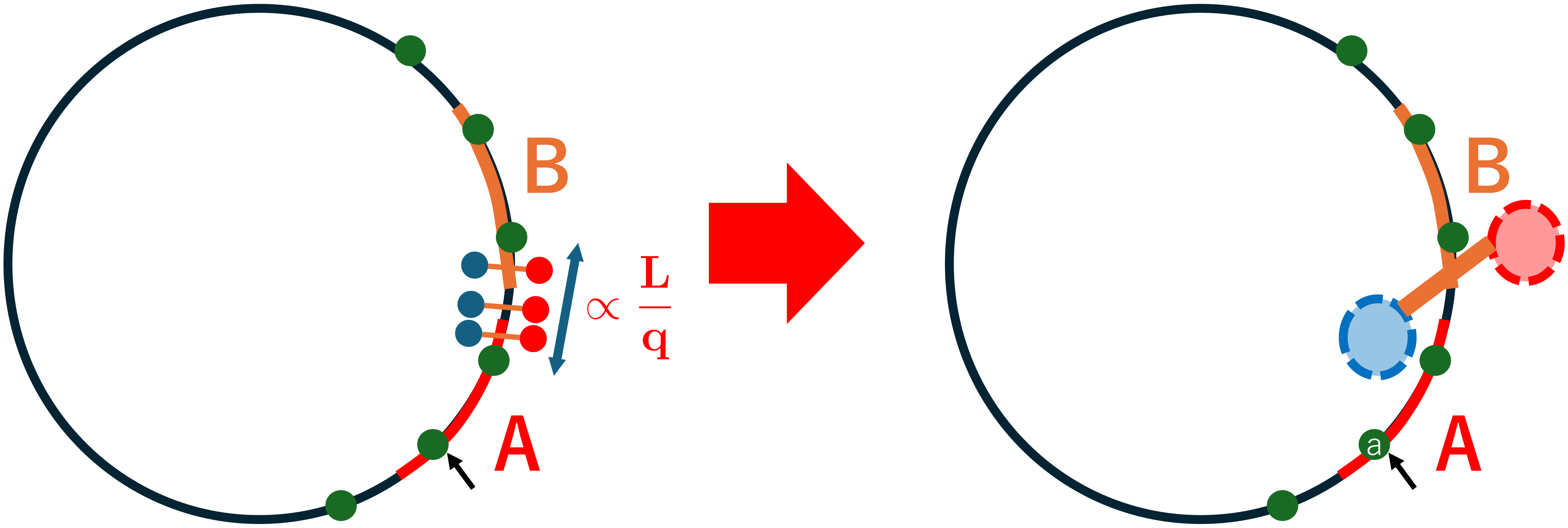
$$\approx \quad + \quad - \quad = \quad \approx \frac{c}{3}\cdot\frac{\pi L}{4q\epsilon} \propto \frac{\mathbf{L}}{\mathbf{q}}$$

In the late time regime

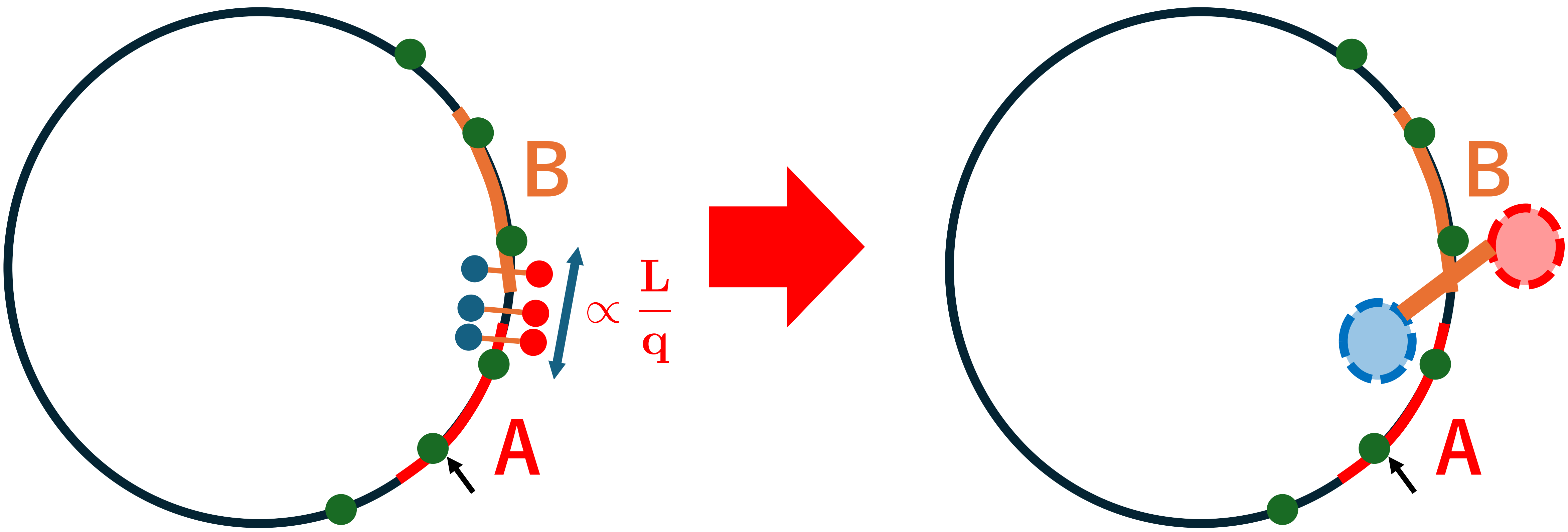
$$S_{A\cup B} \approx \quad + \quad \mathcal{O}(1)$$



In quasi-particle picture

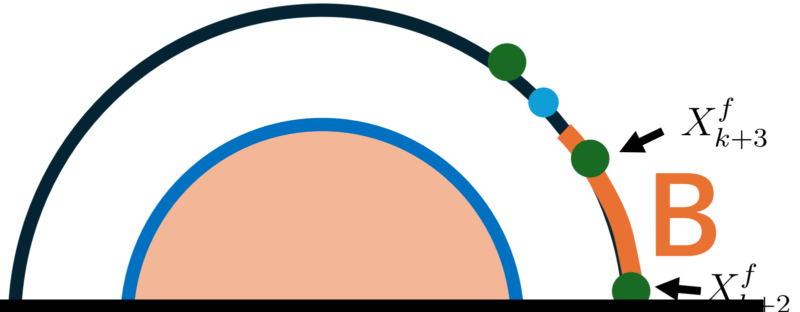


In quasi-particle picture



In the large time region, $\frac{L}{q}$ Bell pairs contributes to non-local correlation between A and B.

Mutual information



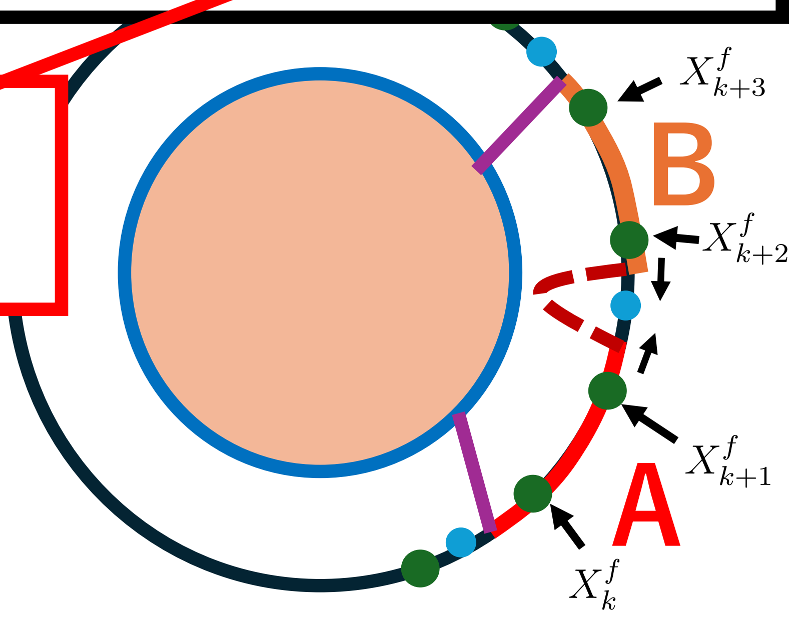
$$I_{A,B} = S_A + S_B - S_{A\cup B}$$

$$\approx \frac{c}{3}\cdot\frac{\pi L}{4q\epsilon} \propto \frac{\mathbf{L}}{\mathbf{q}}$$

In the late time regime

This MI is due to $\frac{\mathbf{L}}{\mathbf{q}}$ Bell pairs.

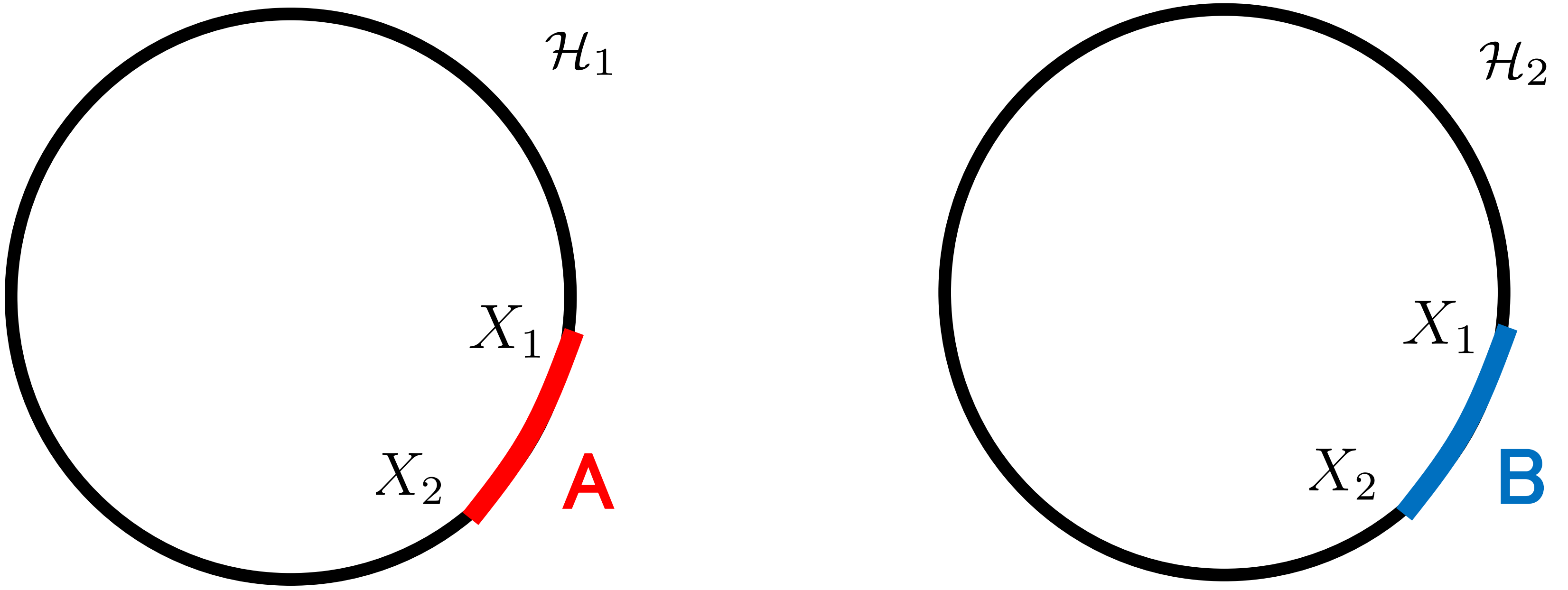
$\mathcal{O}(1)$



Wormhole growth

If the spatially-periodic system in $|\Psi(t)\rangle = \frac{1}{\sqrt{\mathrm{tr} e^{-2\epsilon H}}} e^{-\frac{(it+\epsilon)}{2}(H^1+H^2)} \sum_a |a\rangle_{\mathcal{H}_1} \otimes |a\rangle_{\mathcal{H}_2}$

Subsystem: AUB



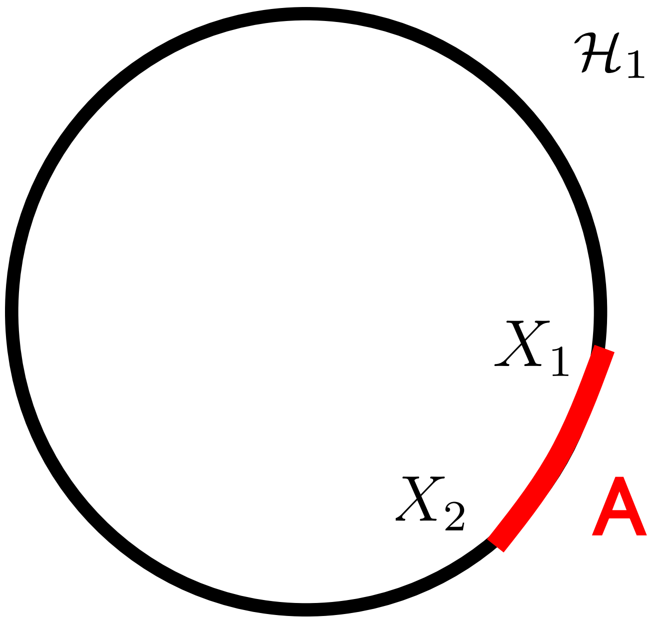
Wormhole growth

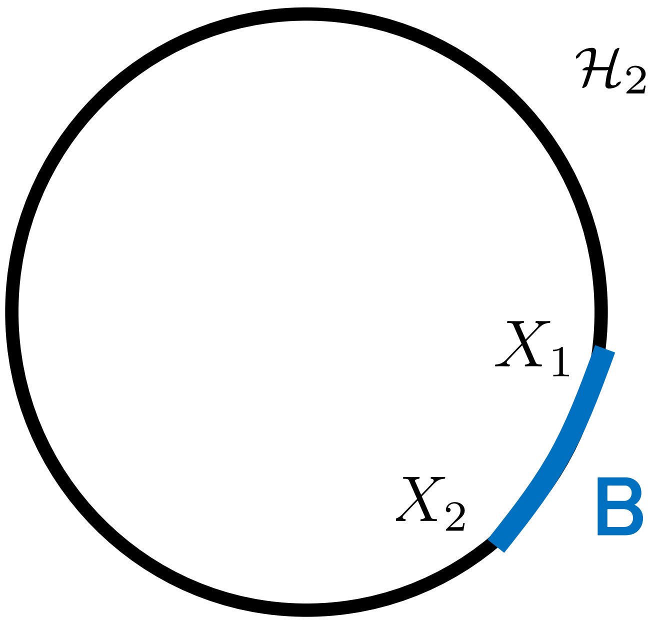
Uniform Hamiltonian acting on $\mathcal{H}_i$

If the spatially-periodic system in $|\Psi(t)\rangle = \frac{1}{\sqrt{\mathrm{tr} e^{-2\epsilon H}}} e^{-\frac{(it+\epsilon)}{2}(H^1+H^2)} \sum_a |a\rangle_{\mathcal{H}_1} \otimes |a\rangle_{\mathcal{H}_2}$

Eigenstates of H^i

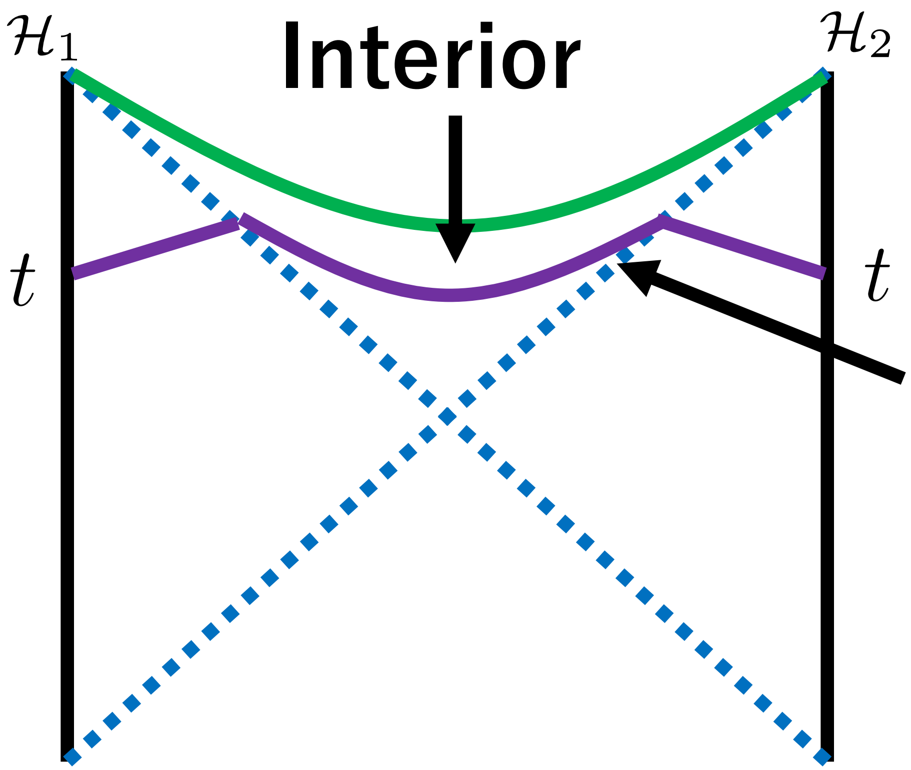
Subsystem: AUB





Wormhole growth

If the spatially-periodic system in $|\Psi(t)\rangle = \frac{1}{\sqrt{\mathrm{tr} e^{-2\epsilon H}}} e^{-\frac{(it+\epsilon)}{2}(H^1+H^2)} \sum_a |a\rangle_{\mathcal{H}_1} \otimes |a\rangle_{\mathcal{H}_2}$



In the early-time region,
$S_{A\cup B}$ is determined by the
geodesic penetrating the interior of black hole.
This geodesic linearly grows in time.

Wormhole growth

If the spatially-periodic system in $|\Psi(t)\rangle = \frac{1}{\sqrt{\mathrm{tr} e^{-2\epsilon H}}} e^{-\frac{(it+\epsilon)}{2}(H^1+H^2)} \sum_a |a\rangle_{\mathcal{H}_1} \otimes |a\rangle_{\mathcal{H}_2}$

In the late-time region,

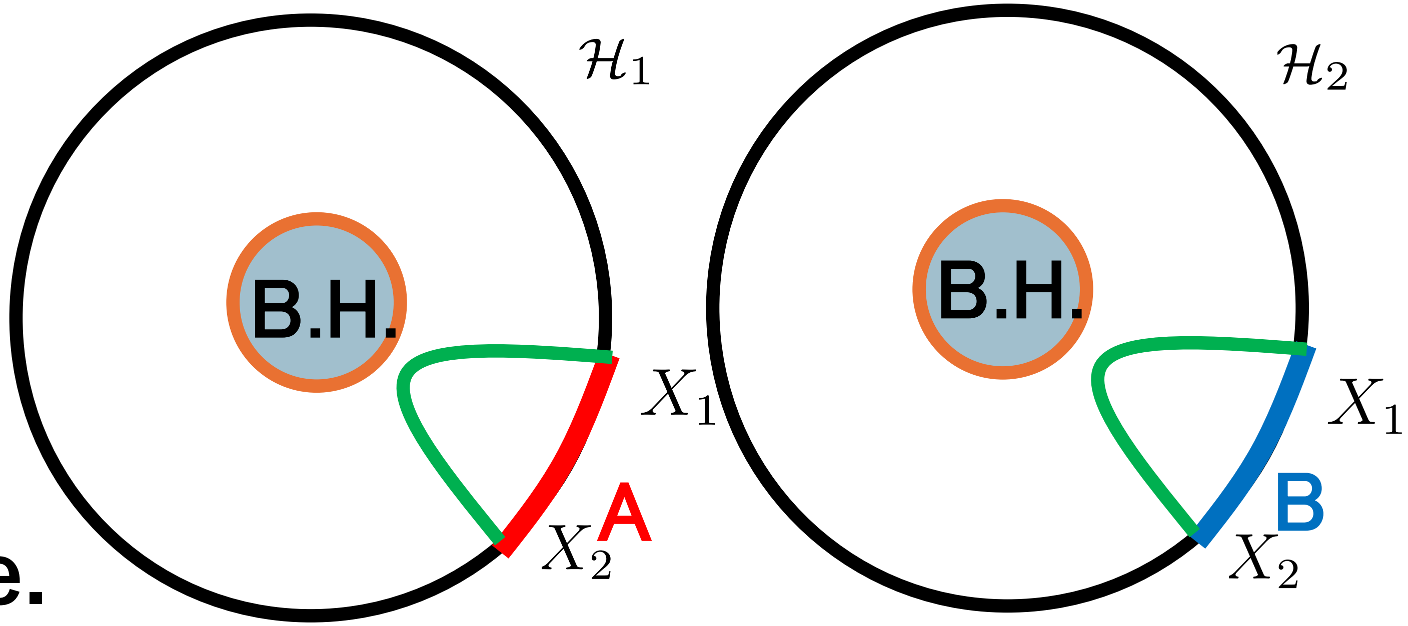
$S_{A\cup B}$ **is determined by the**

geodesics connecting

the spatial points

on the same Hilbert space.

This is independent of time.



Wormhole growth

If the spatially-periodic system in $|\Psi(t)\rangle = \frac{1}{\sqrt{\mathrm{tr}e^{-2\epsilon H}}} e^{-\frac{(it+\epsilon)}{2}(H^1+H^2)} \sum_a |a\rangle_{\mathcal{H}_1} \otimes |a\rangle_{\mathcal{H}_2}$

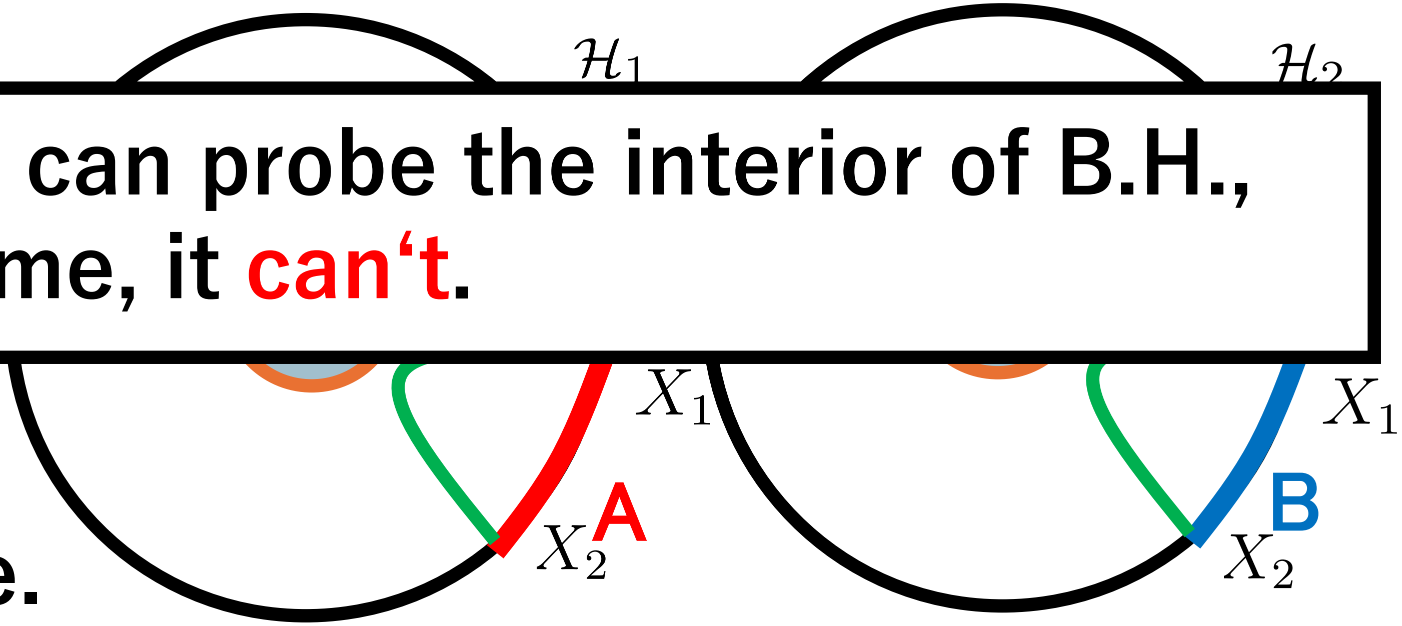
In the late-time region,

In early time regime, $S_{A\cup B}$ can probe the interior of B.H., while in the late time regime, it can‘t.

the spatial points

on the same Hilbert space.

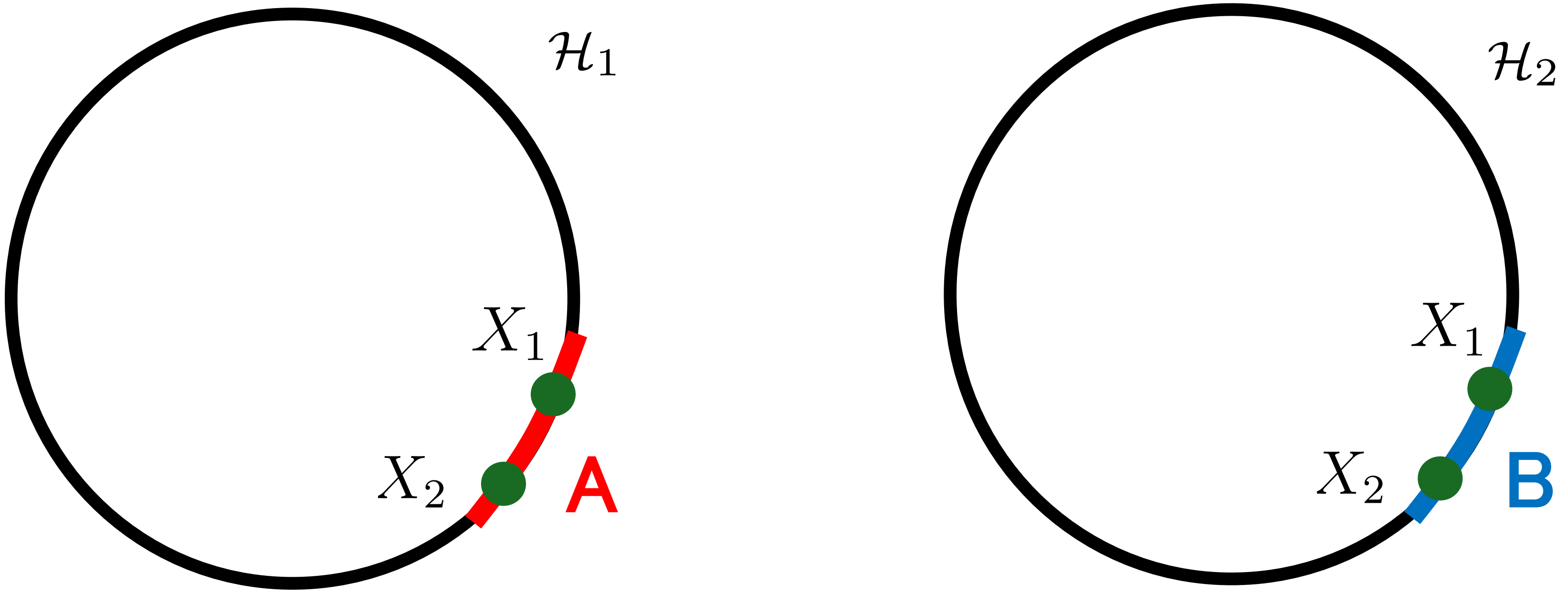
This is independent of time.



Wormhole growth

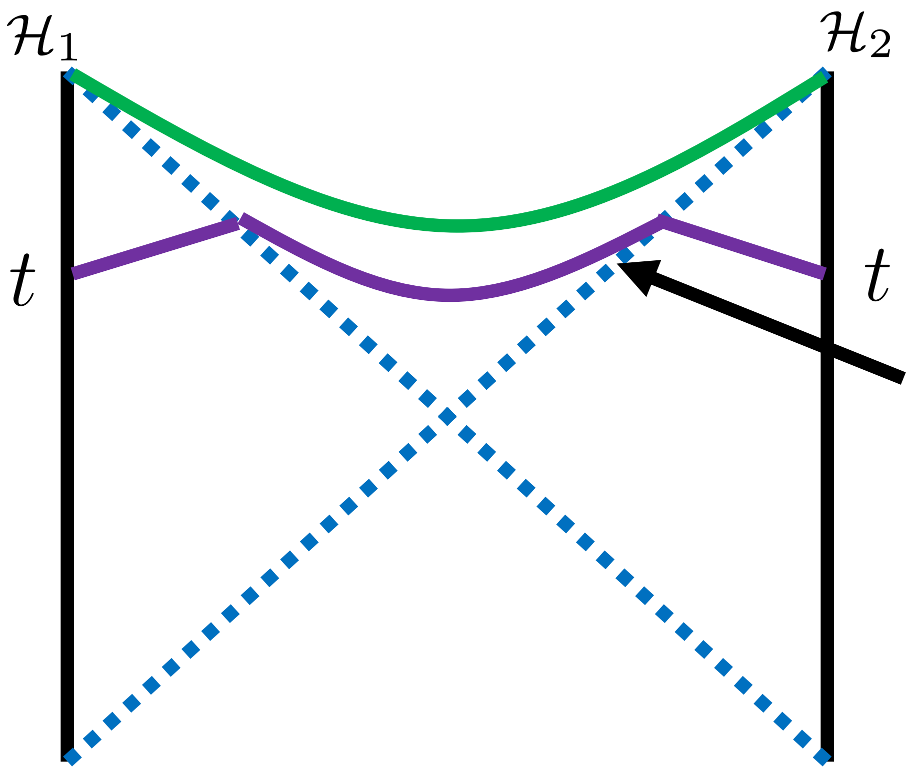
$$|\Psi(t)\rangle = \frac{1}{\sqrt{\mathrm{tr} e^{-2\epsilon H}}} e^{-\frac{(it+\epsilon)}{2}(H^1+H^2)} \sum_a |a\rangle_{\mathcal{H}_1} \otimes |a\rangle_{\mathcal{H}_2} \Rightarrow |\Psi(t)\rangle = \frac{1}{\sqrt{\mathrm{tr} e^{-2\epsilon H}}} e^{-\frac{(it)}{2}\left(\underline{H^1_{\text{q-SSD}}+H^2_{\text{q-SSD}}}\right)} e^{-\frac{(\epsilon)}{2}(H^1+H^2)} \sum_a |a\rangle_{\mathcal{H}_1} \otimes |a\rangle_{\mathcal{H}_2}$$

Subsystem: AUB (including two fixed points)



Wormhole growth

$$|\Psi(t)\rangle = \frac{1}{\sqrt{\mathrm{tr} e^{-2\epsilon H}}} e^{-\frac{(it+\epsilon)}{2}\left(H^1+H^2\right)} \sum_a |a\rangle_{\mathcal{H}_1} \otimes |a\rangle_{\mathcal{H}_2} \Rightarrow |\Psi(t)\rangle = \frac{1}{\sqrt{\mathrm{tr} e^{-2\epsilon H}}} e^{-\frac{(it)}{2}\left(H^1_{\text{q-SSD}}+H^2_{\text{q-SSD}}\right)} e^{-\frac{(\epsilon)}{2}\left(H^1+H^2\right)} \sum_a |a\rangle_{\mathcal{H}_1} \otimes |a\rangle_{\mathcal{H}_2}$$



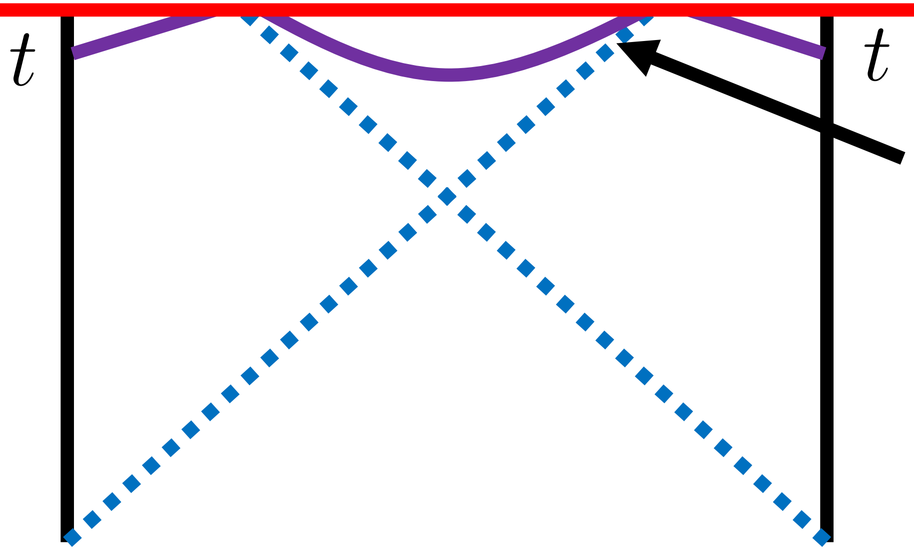
Even in the late-time regime, $S_{A\cup B}$ is determined by the geodesic penetrating the interior of black hole.

This geodesic saturates to some value.

Wormhole growth

$$|\Psi(t)\rangle = \frac{1}{\sqrt{\mathrm{tr}\, e^{-2\epsilon H}}} e^{-\frac{(it+\epsilon)}{2}(H^1+H^2)} \sum |a\rangle_{\mathcal{H}_1} \otimes |a\rangle_{\mathcal{H}_2} \Rightarrow |\Psi(t)\rangle = \frac{1}{\sqrt{\mathrm{tr}\, e^{-2\epsilon H}}} e^{-\frac{(it)}{2}(H^1_{\text{q-SSD}}+H^2_{\text{q-SSD}})} e^{-\frac{(\epsilon)}{2}(H^1+H^2)} \sum |a\rangle_{\mathcal{H}_1} \otimes |a\rangle_{\mathcal{H}_2}$$

Even in the late time regime, $S_{A\cup B}$ can probe the interior of B.H..



$S_{A\cup B}$ **is determined by the geodesic penetrating the interior of black hole.**

This geodesic saturates to some value.

Summary and future direction

1. During the time evolution induced by $H_{\text{q-SSD}}$

system evolves:

No non-local correlation $\Rightarrow$ **Long-range non-local correlation**

2. In the case of the thermofield double state, during the time evolution induced by $H_{\text{q-SSD}}$, **EE can probe the interior of B.H..**

Future direction

- Holographic complexity
- Beyond holographic CFT
- Magic
- Simulation in Lab..