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String junctions in flag manifold sigma model

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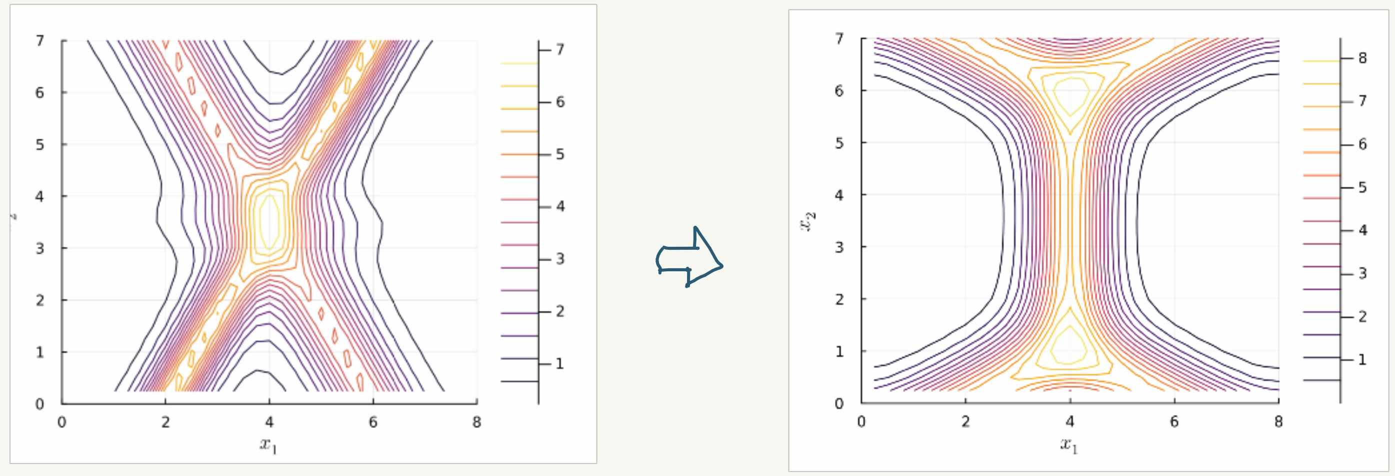
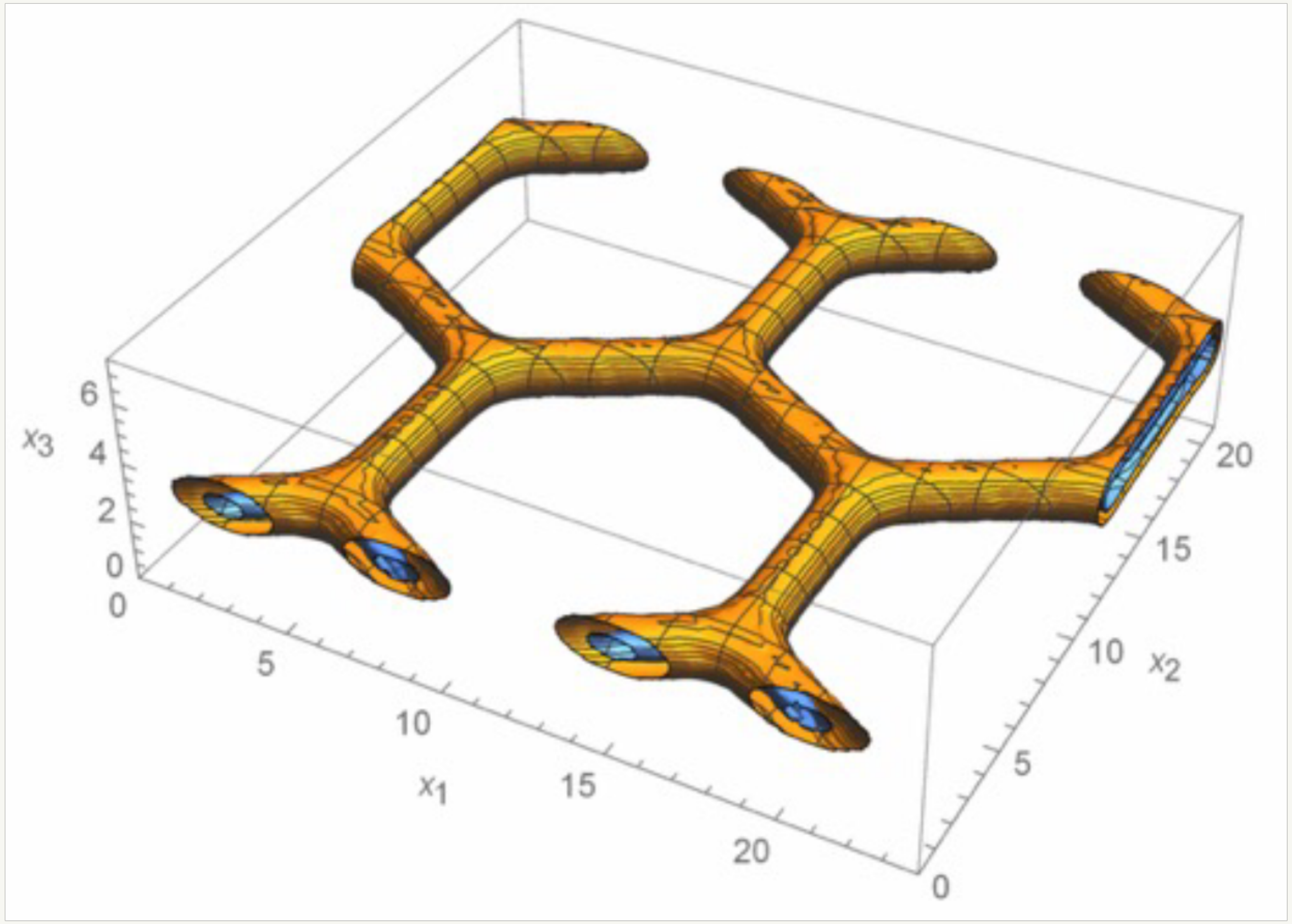




§1. Introduction & Motivation

- Faddeev and Niemi proposed that

  by using Cho-Duan-Ge-Faddeev-Niemi-Shabanov decomposition

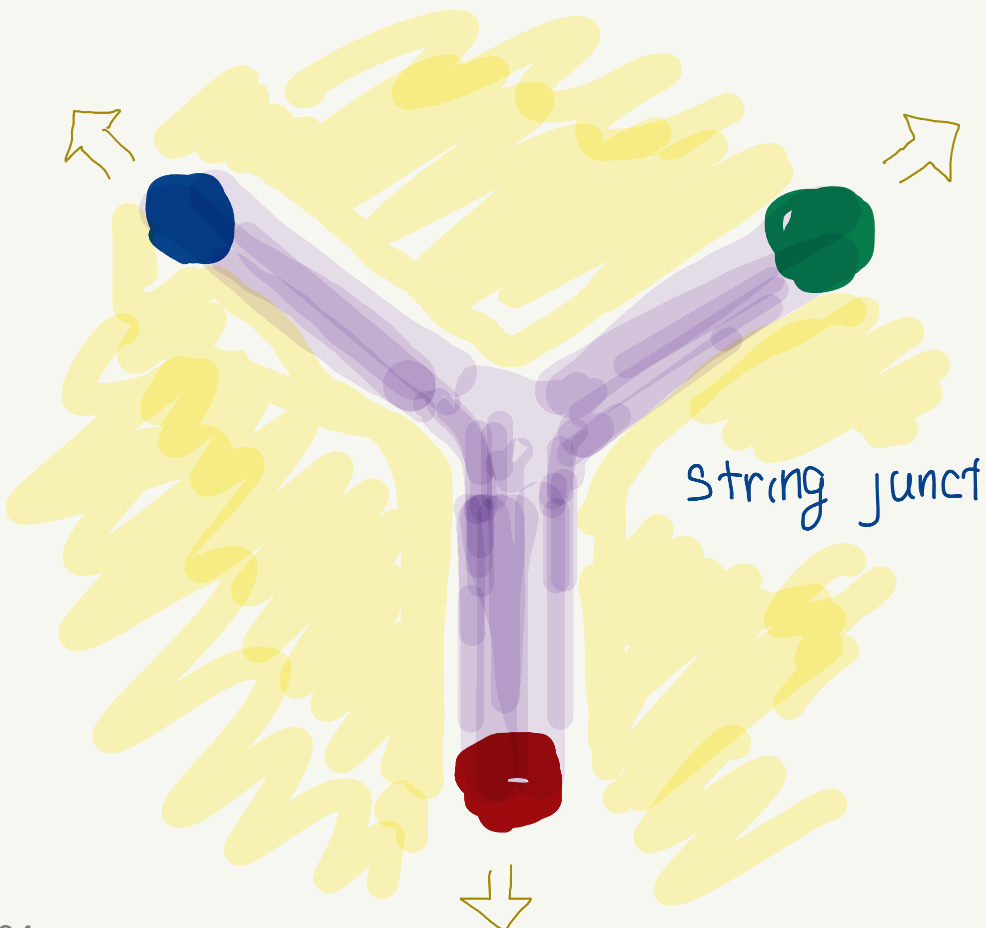
$$\begin{cases} SU(2) \text{ Yang-Mills theory} \\ \text{confined string} \\ \text{glueball} \end{cases} \Rightarrow \begin{cases} O(3) \simeq \dfrac{SU(2)}{U(1)} \text{ sigma model with four deriv term} \\ \text{topological string} \\ \text{Hopfion (knotted string)} \end{cases}$$

objection [Evslin-Giacomelli]

- For a more realistic case of QCD.

SU(3) Yang-Mills theory

$$\Rightarrow F_2 \simeq \frac{SU(3)}{U(1)^2} \text{ sigma model}$$



string junction

Lattice QCD simulations for string junction

[Matsufuru-Nemoto-Suganuma-Takahashi, Bali]

$\Rightarrow$ $\boxed{\text{three-string junction of a Y-shape in } F_2 \text{ manifold sigma model}}$?

§2. F_{N-1} - Faddeev-Skyrme model

- Lagrangian

with $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$

F_{N-1} manifold sigma model

four derivative term

$$-\mathcal{L} = \sum_{i=1}^{N} \left\{ \underbrace{\overbrace{\frac{f^2}{4}\,\mathrm{tr}[\partial_\mu P_i \partial^\mu P_i]}^{F_{N-1}\text{ manifold sigma model}} + \frac{1}{8g^2} f^i_{\mu\nu} f^{i\mu\nu}}_{\text{by Faddeev and Niemi}} + \frac{\mu^2}{4} \underbrace{\mathrm{tr}[(p_i - P_i)^2]}_{\text{potential term}} \right\}$$

by Faddeev and Niemi

potential term

needed to stabilize lump solution

P_i $(i=1,2,\dots,N)$: projection operators

$$P_i = U^\dagger p_i U, \quad U \in U(N) \quad \text{with} \quad p_i \equiv \mathrm{diag}.(\underbrace{0,\cdots,0,1}_{i},0,\dots,0)$$

with gauge redundancy of $U(1)^N$

$$U \sim U' = e^{i\Theta} U, \quad \Theta = \mathrm{diag}(\theta_1,\theta_2,\dots,\theta_N)$$

$$\Leftrightarrow \quad P_i^\dagger = P_i, \quad P_i P_j = \delta_{ij} P_i, \quad \sum_{i=1}^{N} P_i = \mathbb{1}_N$$

A set of these ops describes

the full flag manifold $\boxed{F_{N-1} \simeq \frac{SU(N)}{U(1)^{N-1}}}$

$f^i_{\mu\nu}$: field strength for $U(1)^N$

$\sum_i P_i = \mathbb{1}_N$

pure gauge

$$f^i_{\mu\nu} \equiv \partial_\mu a^i_\nu - \partial_\nu a^i_\mu , \quad a^i_\mu \equiv i\,\mathrm{tr}[P_i\, \partial_\mu U U^\dagger] , \qquad \sum_{i=1}^{N} a^i_\mu = i\, \partial_\mu \log(\det U)$$

overall $U(1)$ must be trivial.

- $\pi_2(F_{N-1}) = \mathbb{Z}^N / \mathbb{Z}$

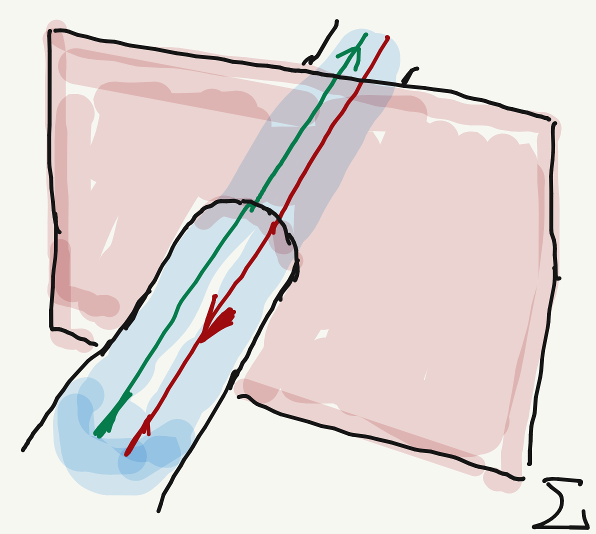
$$\vec{m} \equiv (m_1, m_2, \quad , m_N) , \qquad \sum_{i=1}^{N} m_i = 0$$

$$m_i \equiv \frac{1}{4\pi i} \oint_\Sigma dx^\mu \wedge dx^\nu f^i_{\mu\nu} \in \mathbb{Z}$$

$\Rightarrow$ topological soliton with codim $= 2$ $= \begin{cases} \text{lump in } d = 2+1 \\ \text{string in } d = 3+1 \end{cases}$

- $\langle i,j \rangle$-string

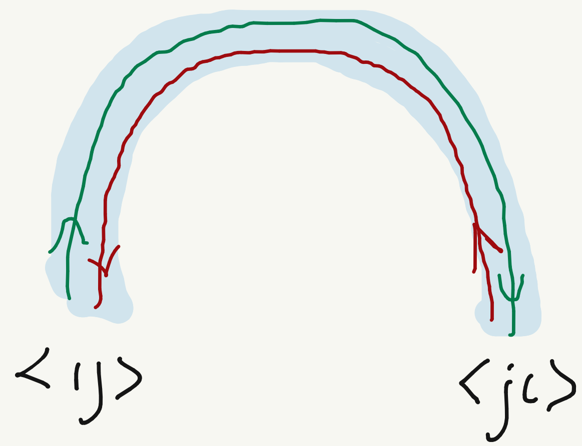
$$\vec{m} = \underset{i \neq j}{\vec{m}_{\langle i,j \rangle}} \equiv (0, \quad , 0, \underset{i}{1}, 0, \cdot \quad , 0, \underset{j}{-1}, 0, \cdot \cdot 0)$$



- annihilation

$$\vec{m}_{\langle ij \rangle} + \vec{m}_{\langle ji \rangle} = 0$$

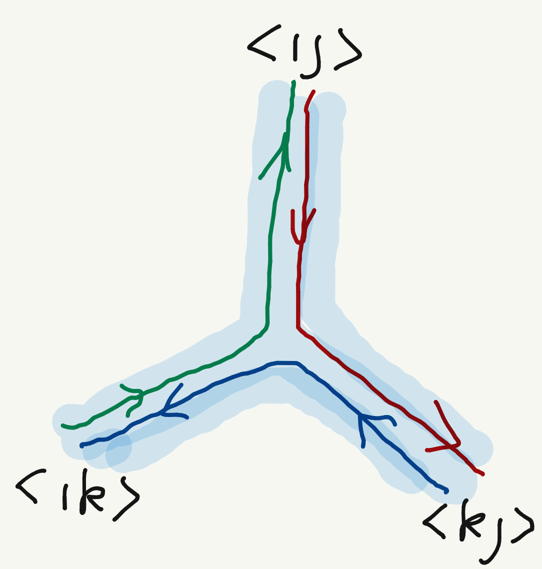
$(i \neq j)$



- Y-junction for $N \geq 3$

$$\vec{m}_{\langle ik \rangle} + \vec{m}_{\langle kj \rangle} = \vec{m}_{\langle ij \rangle}$$

$(k \neq i, j,\ i \neq j)$



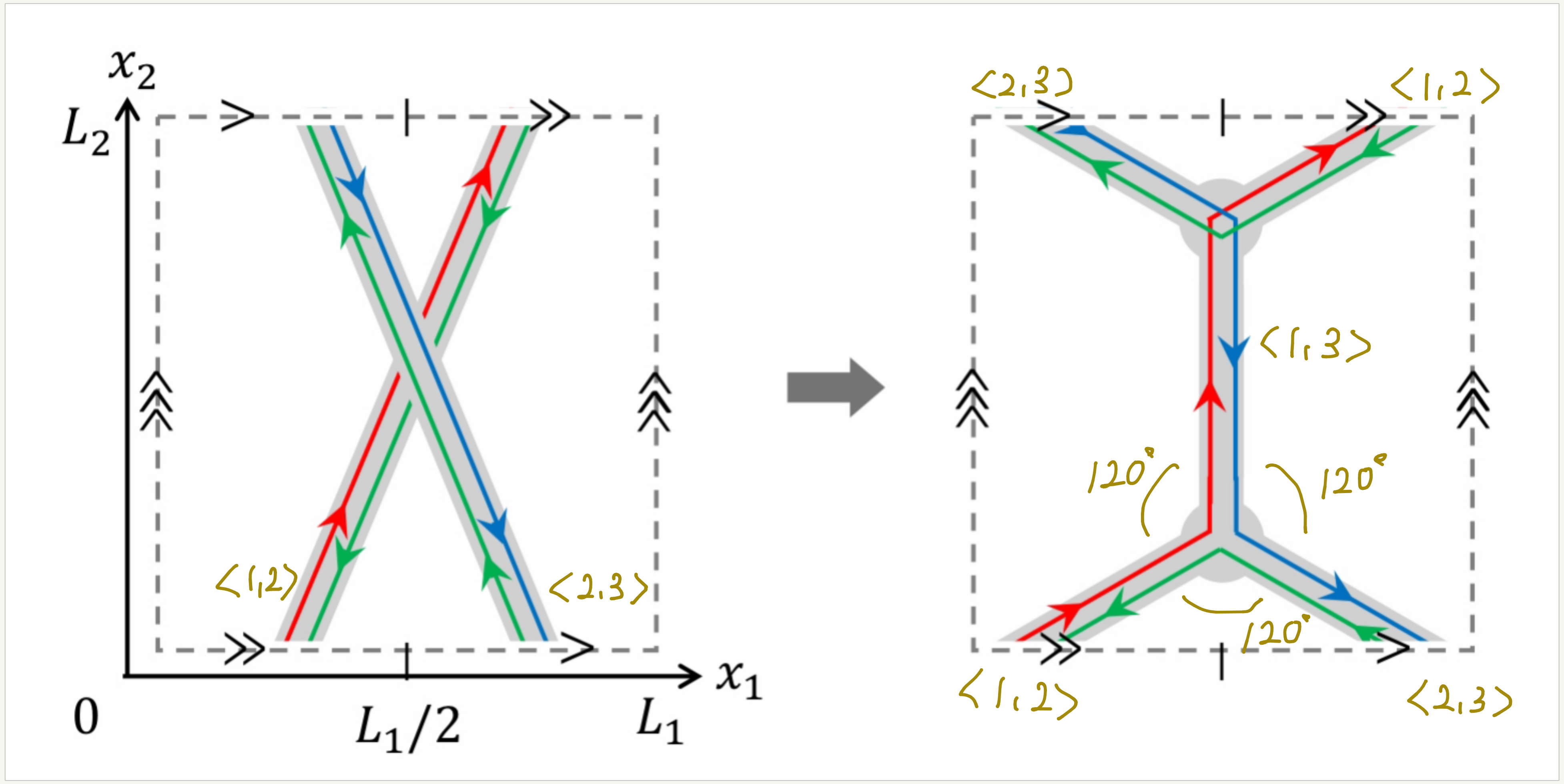
How to make this happen?

§3. BPS solution as an initial config.

Let us obtain a pair of string junctions in twisted T^3 base space

$$(x_1+L_1, x_2, x_3) \sim (x_1, x_2, x_3)$$
$$(x_1, x_2, x_3+L_3) \sim (x_1, x_2, x_3) \quad \text{for } (x_1,x_2,x_3) \in T^3$$
$$(x_1, x_2+L_2, x_3) \sim (x_1+\frac{L_1}{2}, x_2, x_3)$$

with twists of the flavor sym.



by numerical simulation using the steepest descent method.

How to take a topologically correct initial configuration? (N=3 case)

- Our F_2 model is S_3-symmetric and non-Kähler.

  $\Rightarrow$ common string tension $T_{\langle 12\rangle} = T_{\langle 13\rangle} = T_{\langle 23\rangle}$

  $\Rightarrow$ Inter-string forces are all atractive

continuas deformation

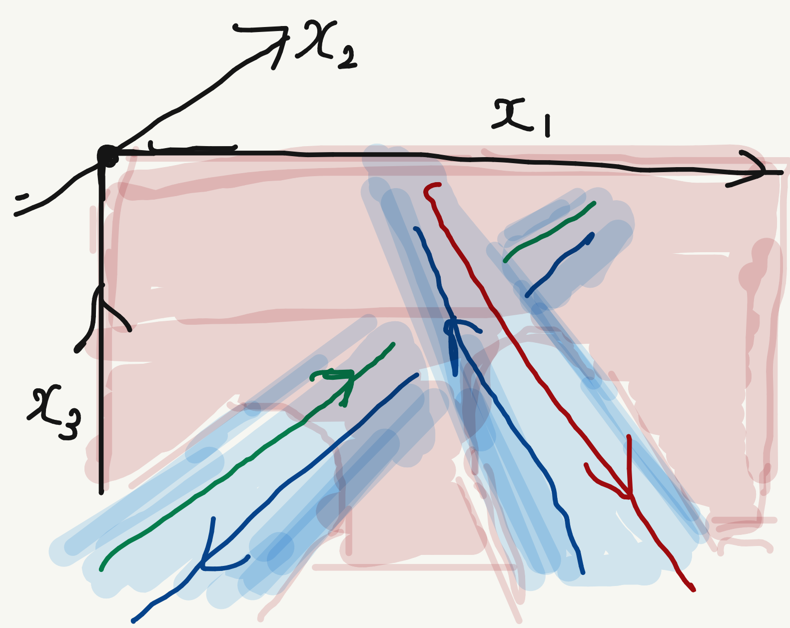
- Kähler F_2 manifold sigma model $= CP^2 \oplus CP^2$ under an orthogonality condition.
  which allows a BPS solution for a composite state of $\langle 12\rangle$-lump and $\langle 23\rangle$-lump.
  with $T_{\langle 13\rangle} = T_{\langle 12\rangle} + T_{\langle 23\rangle}$ in a x_1-x_3 plane

  $\Rightarrow$ There is no interaction between them

  $\Rightarrow$ Their positions are moduli parameters

Explicit solution using meromorphic functions of $z = x_1 + i x_3$

We can arrange them arbitrally for each x_2.

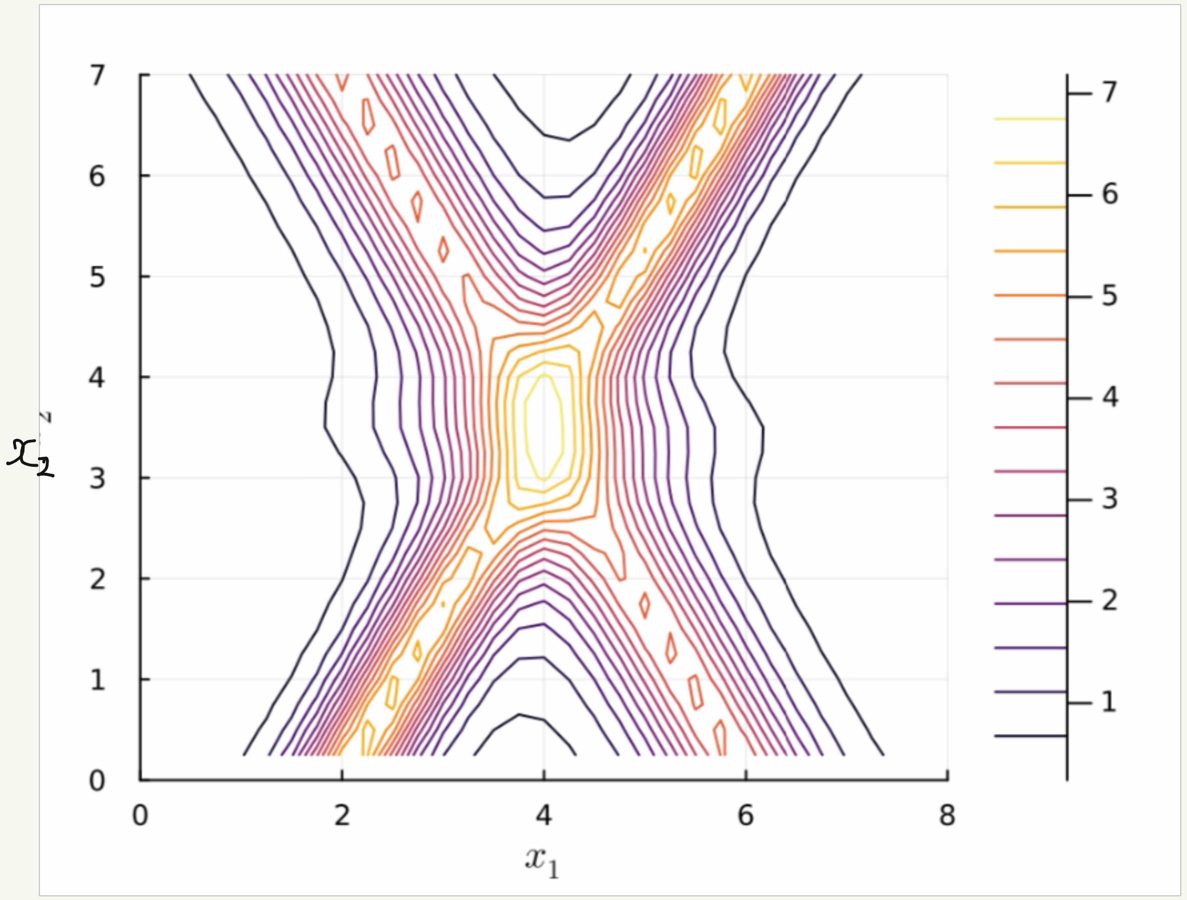


§4. Numerical results

We set $f = 1/g = \mu = 1$ and $L_1 : L_2 : L_3 = 8 : 7 : 5$

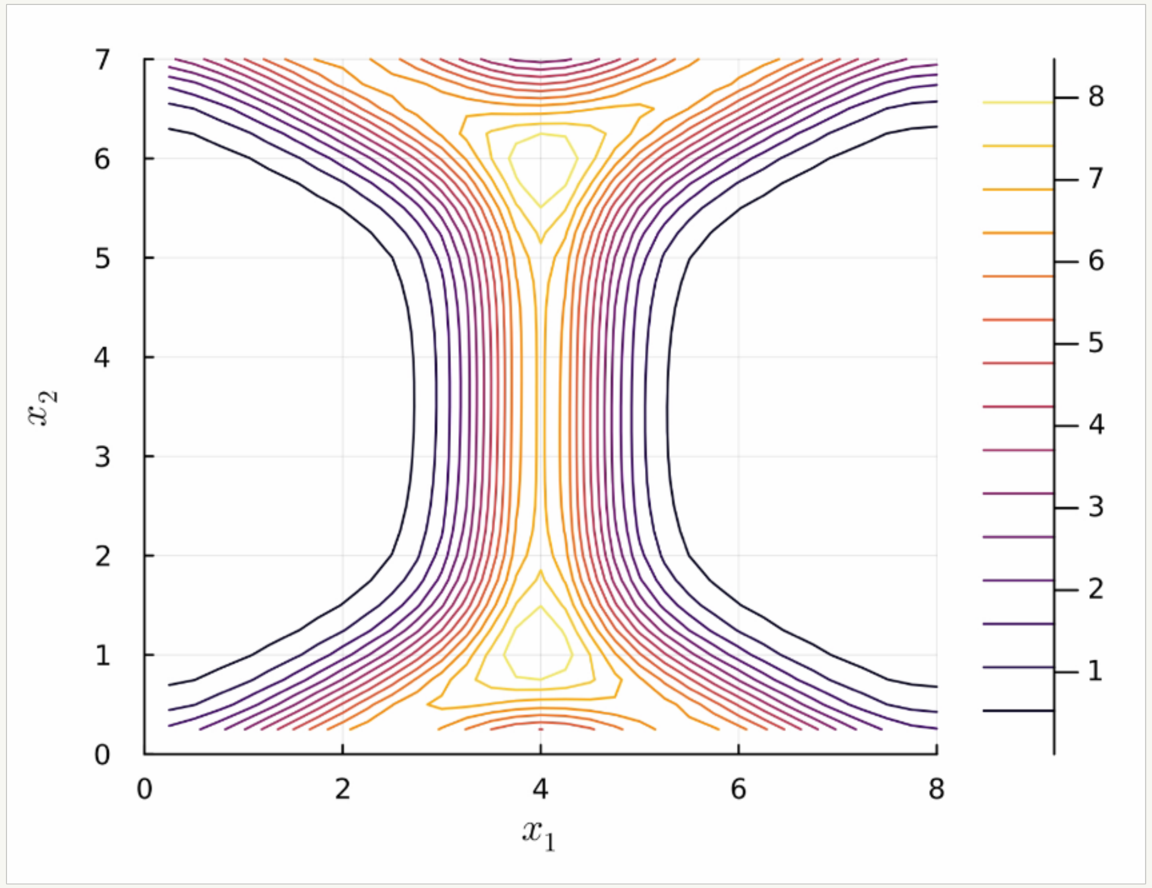
lattice spacing

Energy dencities at the $x_3 = \frac{L_3}{2}$ cutting plane with $L_1 = 8$, $a = \frac{1}{4}$



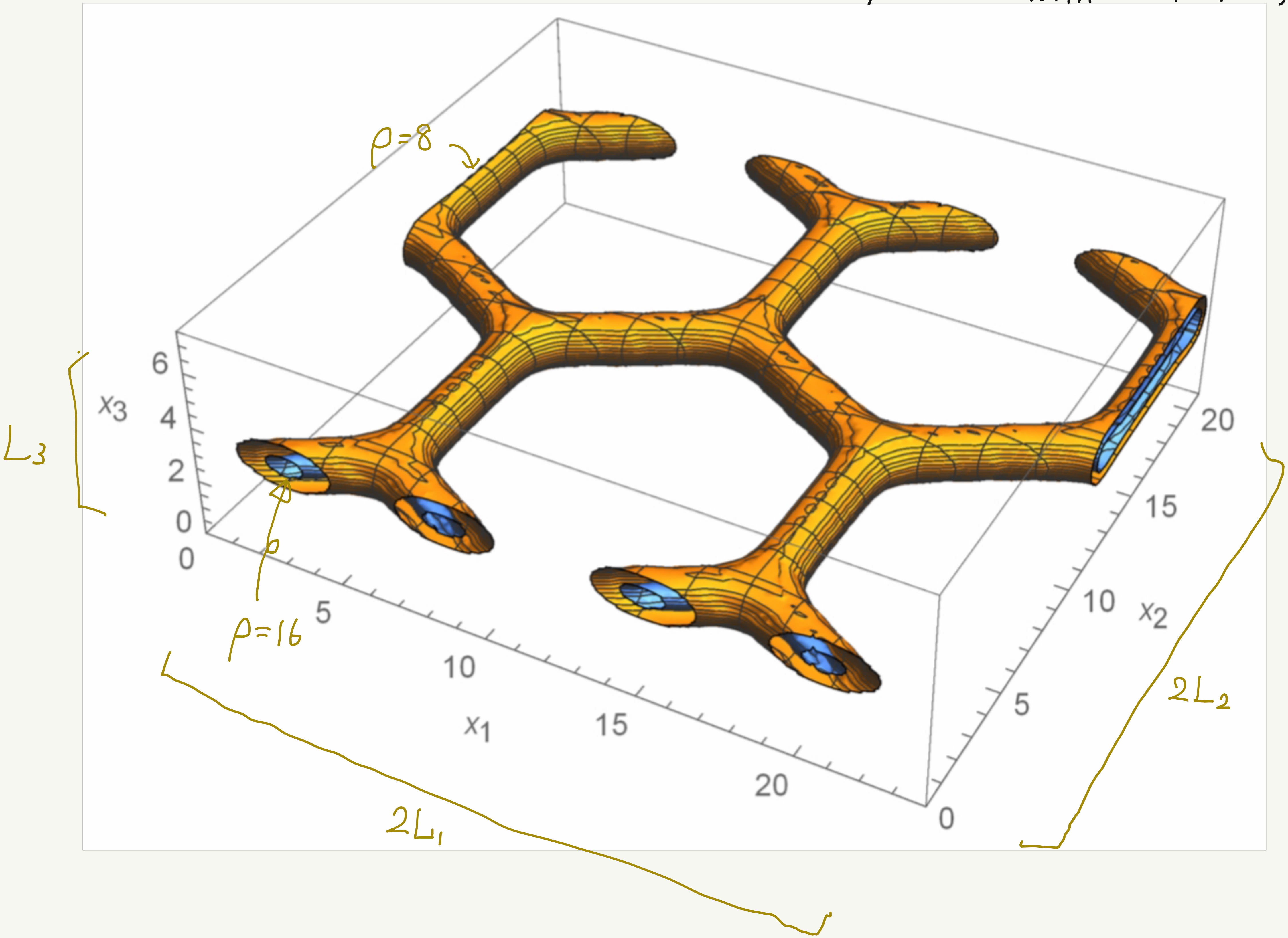
initial configuration
usig a BPS solution

the steepest descent method

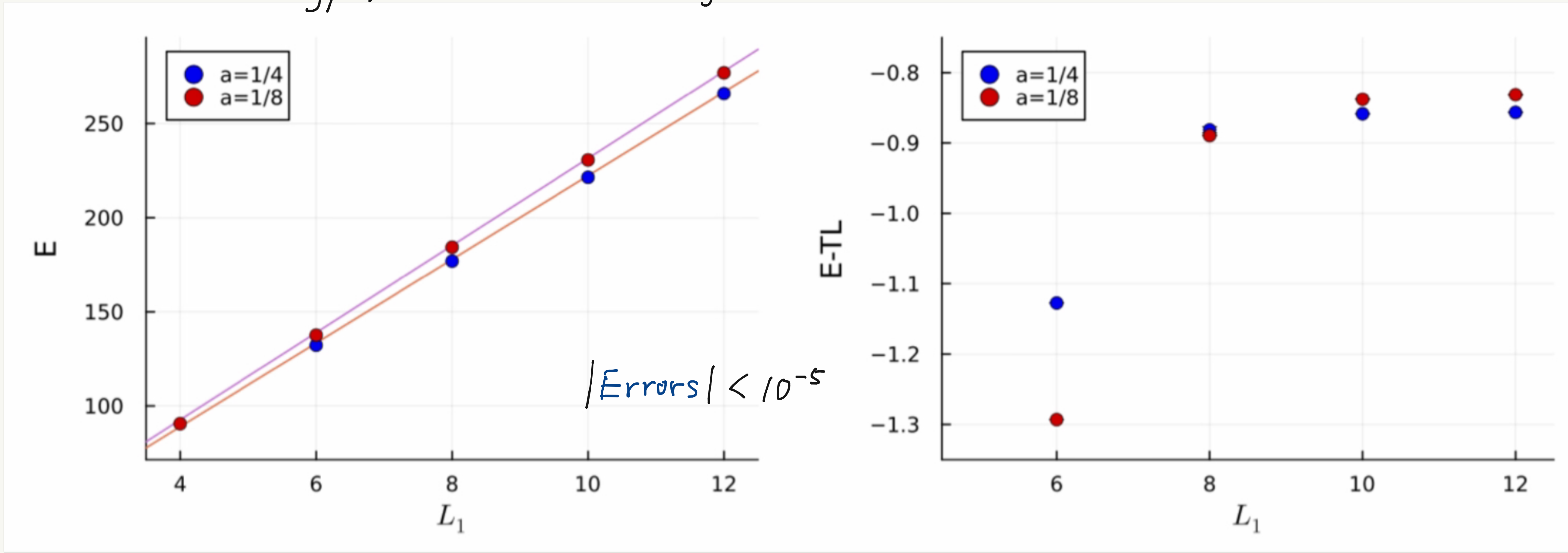


numerical result

The surface contour in the energy dencity ρ with $L_1 = 12$, $a = \frac{1}{8}$



total energy for the fundamental region of T^3



total length of strings with $L_2 = \frac{7}{8} L_1$

$$E \approx T \times \left(\frac{\sqrt{3}}{2} L_1 + L_2 \right) + M + \mathcal{O}(e^{-\# L_1})$$

string tension

"junction mass" [Komargodski and Zhong]

$$T = \begin{cases} 12.7706 & \text{for } a = \frac{1}{4} \\ 13.2993 & \text{for } a = \frac{1}{8} \end{cases}$$

§5. Summary

- We have numerically constructed a three-string junction of Y-shape in the F_2 sigma model with the four derivative Skyrme term and potential terms

- We have investigated the L_i dependence of the string junction.
  "Junction mass" is negative but small.

Future directions

- Hopfion with junctions? $\overset{?}{\Longleftrightarrow}$ glueballs in the SU(3)-Yang Mills theory

- Four-string junction of tetrapod-shape in the F_3 sigma model ?

- . .