Non-invertible flavor symmetries in magnetized extra dimensions

Hajime Otsuka (Kyushu University)

Reference :

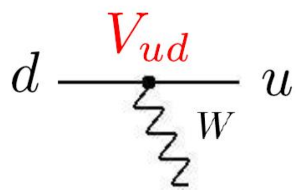
T. Kobayashi (Hokkaido U.) and H.O.,, 2408.13984 [hep-th]

場の理論と弦理論 2024 @ YITP, Aug. 8, 2024

Flavor puzzle

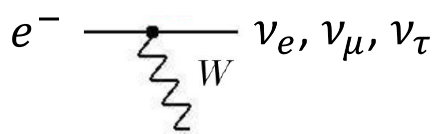
- Origin of flavor and CP : important issue in the SM

PDG ('20)



$$V_{\mathrm{CKM}} = \begin{pmatrix} 0.97401 \pm 0.00011 & 0.22650 \pm 0.00048 & 0.00361^{+0.00011}_{-0.00009} \\ 0.22636 \pm 0.00048 & 0.97320 \pm 0.00011 & 0.04053^{+0.00083}_{-0.00061} \\ 0.00854^{+0.00023}_{-0.00016} & 0.03978^{+0.00082}_{-0.00060} & 0.999172^{+0.000024}_{-0.000035} \end{pmatrix}$$

NuFIT 5.0 (2020)



$$|U|^{\text{w/o SK-atm}}_{3\sigma} = \begin{pmatrix} 0.801 \to 0.845 & 0.513 \to 0.579 & 0.143 \to 0.156 \\ 0.233 \to 0.507 & 0.461 \to 0.694 & 0.631 \to 0.778 \\ 0.261 \to 0.526 & 0.471 \to 0.701 & 0.611 \to 0.761 \end{pmatrix}$$

- Hierarchical structure of quarks/lepton masses

2	5	100	1270	4180	172000	Mass
u	*d*	*s*	*c*	*b*	*t*	[MeV]

→ Non-trivial flavor structure of quarks and leptons

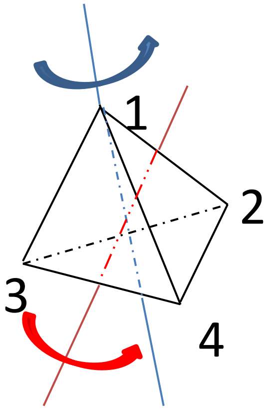
"Traditional" flavor symmetry

- Field transformations:

$$\phi_i \xrightarrow{g} \rho_i(g)\phi_i \qquad g \in G_{\rm flavor}$$

- Given $G_{\rm flavor}$, Abelian and non-Abelian symmetries are proposed.

Ex., $U(1)_{\rm FN}$
A_4 : Tetrahedral sym.



Question :

通常はInvertible symmetryを考えるが、non-invertible symmetry？

Main message

- Non-invertible symmetries
  in type IIB magnetized D-branes on $T^2/\mathbb{Z}_N$

  - Fusion algebras of $\mathbb{Z}_N$-invariant operators :

$$\text{ex.,}\ U_{\mathbb{Z}_2}^{(\lambda_1)} U_{\mathbb{Z}_2}^{(\lambda_2)} = U_{\mathbb{Z}_2}^{(\lambda_1+\lambda_2)} + U_{\mathbb{Z}_2}^{(\lambda_1-\lambda_2)}$$

  - Representations of chiral zero-modes

- Chiral zero-modes will correspond to
  3 generations of quarks/leptons
  Non-invertible symmetries → flavor symmetries

6D QED on T^2 with magnetic flux

$$\mathcal{L} = -\frac{1}{4} F_{mn} F^{mn} + i \bar{\Psi} \Gamma^m D_m \Psi \qquad m, n = x^\mu, y_1, y_2$$

$\boldsymbol{T^2} = \mathbb{C}/\Lambda$

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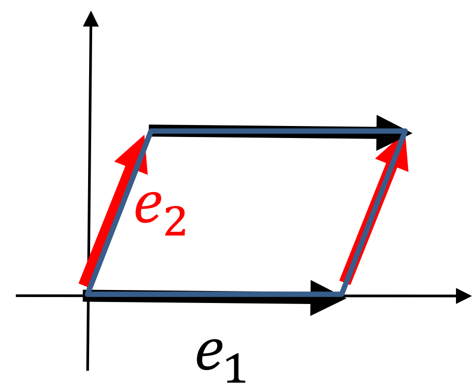

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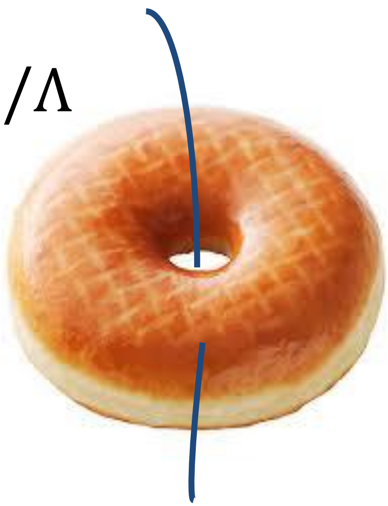

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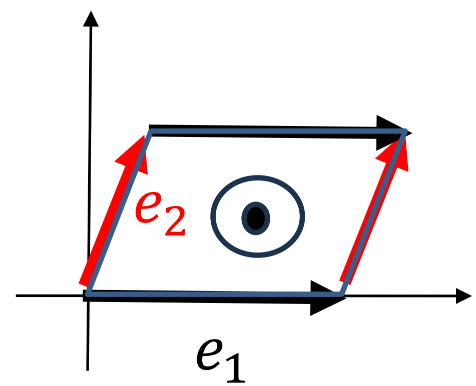

- *U(1)* magnetic flux

$$F_{y_1 y_2} = 2\pi i M \qquad (M \in \mathbb{Z})$$

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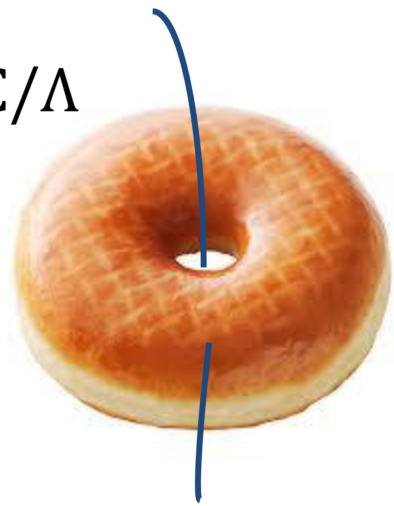

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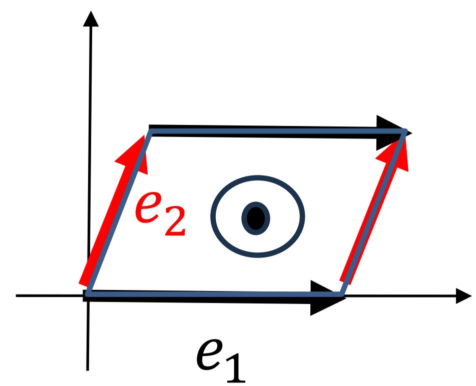


- *U(1)* magnetic flux

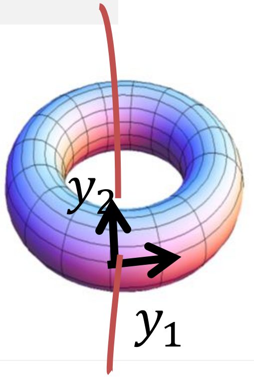
$$F_{y_1 y_2} = 2\pi i M \qquad (M \in \mathbb{Z})$$

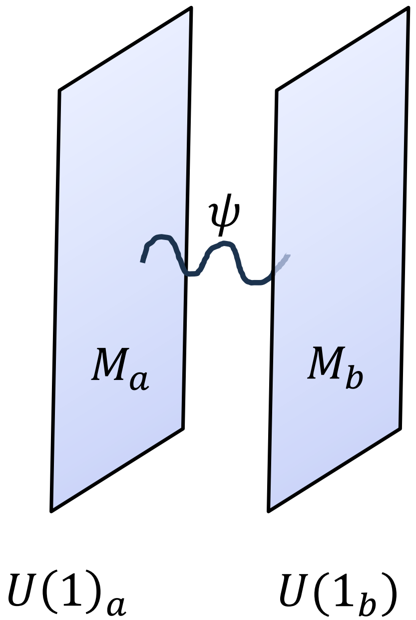
- *Hamiltonian*

$$\hat{H} = \left(\hat{p} - q\hat{A}\right)^2 \qquad E_n = \omega\left(n + \frac{1}{2}\right) \qquad \omega = |qF_{12}|$$

Magnetized D-branes in type IIB string

- *Two stacks of D5-branes wrapping T^2 with magnetic flux*





- *Effective action*

  $U(2)$ SUSY Yang-Mills theory

  $$U(2) \to U(1)_a \times U(1)_b$$

- *U(1) magnetic flux*

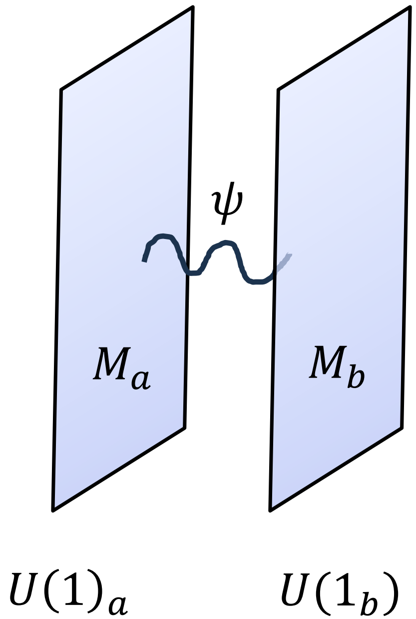
  $$F_{y_1 y_2} = 2\pi \begin{pmatrix} M_a & 0 \\ 0 & M_b \end{pmatrix}$$

- *Matter* :

  $\psi(x,y)$: bifundamental rep. of $U(1)_a \times U(1)_b$
  with charge (1, -1)

Chiral matters on T^2

D.Cremades, L.E.Ibanez, F.Marchesano (2004)



$\psi(x,y)$ bifundamental rep. of $U(1)_a \times U(1)_b$
with charge (1, -1)

- *Mの flux を感じる($M \equiv M_a - M_b > 0$)*
  T^2 上のIndex theorem :

$$\frac{1}{2\pi}\int \mathrm{tr}F = M \in \mathbb{Z}$$

M 個の Chiral matters(zero-modes) $\psi^{j,M}$(y)が存在

$$j = 0,1,2,\ldots,M-1$$

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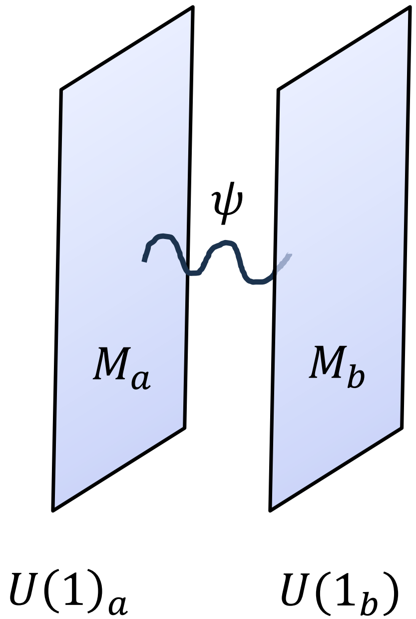

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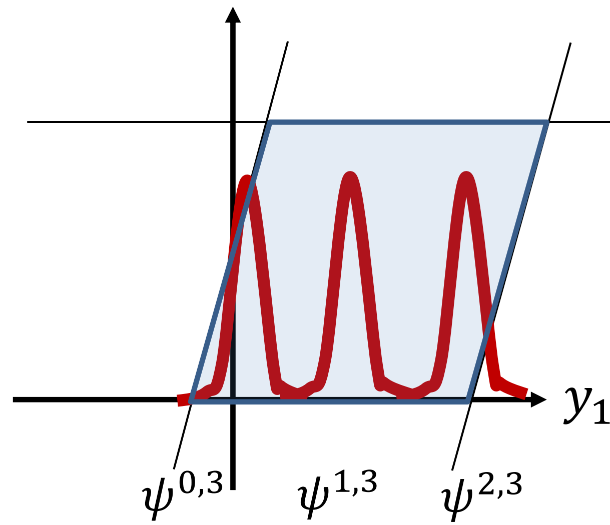
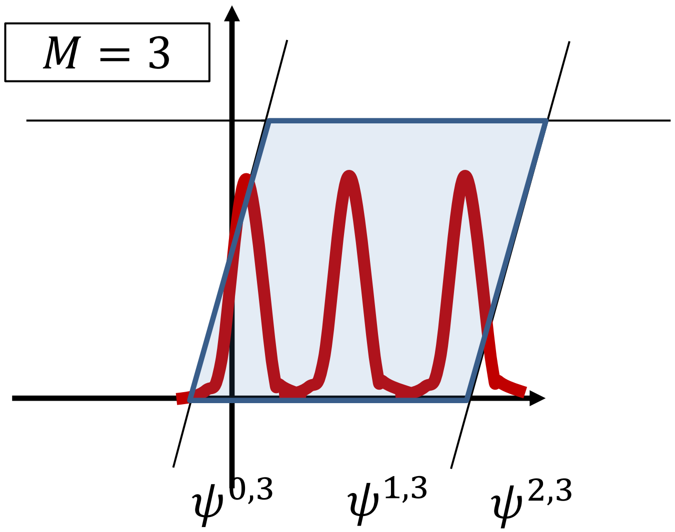
$\boxed{M = 3}$



Flavor symmetry of chiral matters on T^2



H. Abe, K.Choi, T.Kobayashi, H.Ohki (2009),
M.Berasaluce-Gonzalez, P.G.Camara, F.Marchesano, (2012)

Flavor symmetry :

M 個の Chiral matters $\psi^{j,M}$を入れ替える対称性

$$H_M \simeq \left(\mathbb{Z}_M \times \mathbb{Z}_M^{(Z)}\right) \rtimes \mathbb{Z}_M^{(C)}$$

$$\mathbb{Z}_M = e^{\frac{2\pi i}{M}} \mathrm{diag}(1,1,\ldots)$$

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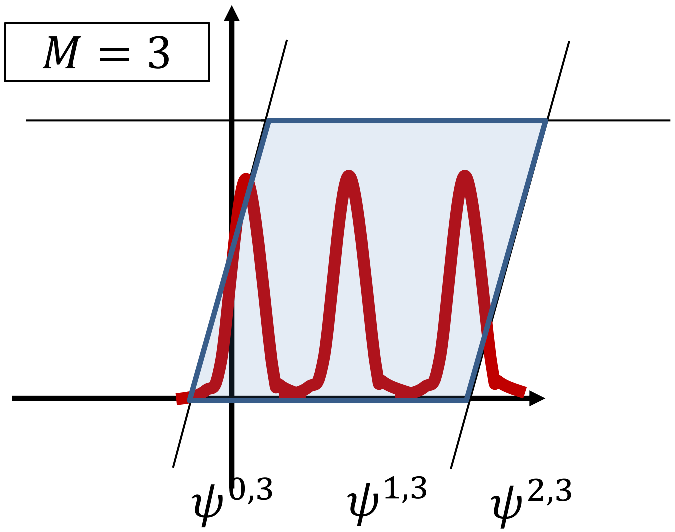

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$$\mathbb{Z}_M = e^{\frac{2\pi i}{M}} \mathrm{diag}(1,1,\dots)$$

Under the discrete translations :

$$y_1 \to y_1 + \frac{1}{M}, \qquad y_2 \to y_2 + \frac{1}{M}$$

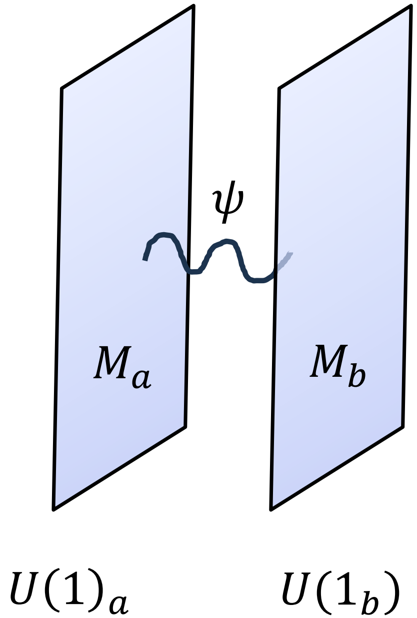
The chiral zero-modes $\psi^{j,M}$ transform as (Generators : $\underline{e^{\frac{1}{M}D_{y_1}}}$ and $\underline{e^{\frac{1}{M}D_{y_2}}}$) :

$$Z : e^{\frac{1}{M}D_{y_1}}\psi^{j,M} = e^{2\pi i \frac{j}{M}}\psi^{j,M} \qquad (Z)^M = (C)^M = \mathbb{I}$$

$$C : e^{\frac{1}{M}D_{y_2}}\psi^{j,M} = \psi^{j+1,M} \qquad CZ = e^{\frac{2\pi i}{M}}ZC$$

Quantum mechanical system for the magnetized D-branes

D.Cremades, L.E.Ibanez, F.Marchesano (2004)
T.h.Abe, Y.Fujimoto, T.Kobayashi, T.Miura, K.Nishiwaki, M. Sakamoto (2014)



- *Hamiltonian*

$$\hat{H} = (\hat{p} - A)^2 = \left(\hat{p} - \frac{1}{2} F \hat{y}\right)^2$$

$$[\hat{y}_i, \hat{p}_j] = i\delta_{ij} \qquad i, j = 1,2$$

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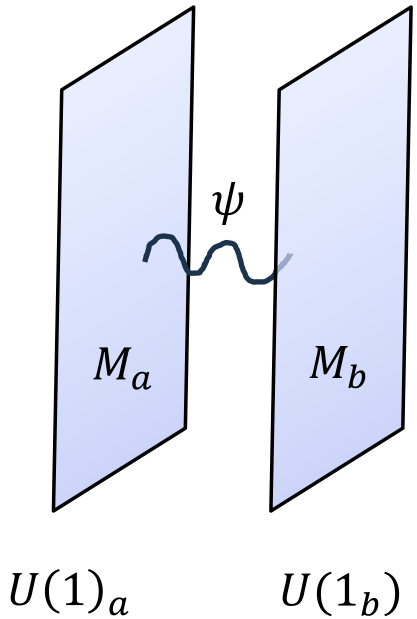

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$$\{\hat{y}_1, \hat{y}_2, \hat{p}_1, \hat{p}_2\} \rightarrow \{\hat{Y}, \hat{P}, \hat{T}_1, \hat{T}_2\}$$

Harmonic oscillator : $\hat{H} = \frac{1}{2}\hat{P}^2 + \frac{\omega}{2}\hat{Y}^2$

$[\hat{Y}, \hat{P}\,] = i$, $[\hat{T}_1, \hat{T}_2] = 2\pi i M$, (others=0)

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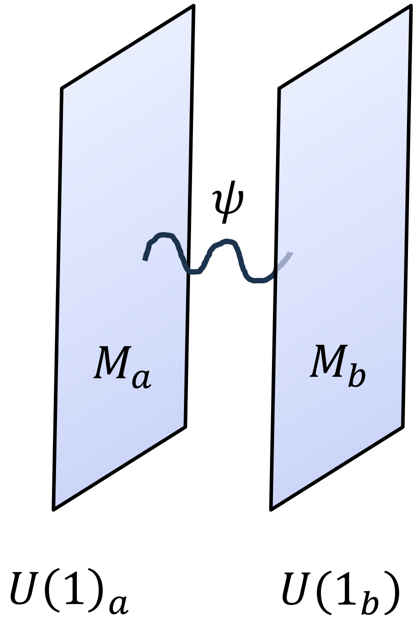

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$[\hat{Y}, \hat{P}\,] = i$, $[\hat{T}_1, \hat{T}_2] = 2\pi i M$, (others=0)

$\hat{T}_1, \hat{T}_2$は、Hamitonianに現れない保存量だが、statesに制限を与える

Quantum mechanical system for the magnetized D-branes

- *Constraints on states (～boundary conditions of wavefunctios)*

$$e^{i\hat{T}_a}|\psi\rangle = |\psi\rangle$$

two operators $\hat{T}_1, \hat{T}_2$ は、degenerate statesを区別

- *We choose the eigenstate of $\hat{T}_2$ for the state s.t.*

$$\left|\psi^{j,M}\right\rangle = \left|\frac{j}{M}\right\rangle = e^{-i\frac{j}{M}\hat{T}_1}|0\rangle$$

$$e^{i\hat{T}_2}\left|\frac{j}{M}\right\rangle = e^{2\pi i\, j}\left|\frac{j}{M}\right\rangle$$

- *Schrödinger eq.*

$$\hat{H}\left|\psi^{j,M}\right\rangle = E_n|\psi^{j,M}\rangle$$

For two-dimensional spinor,

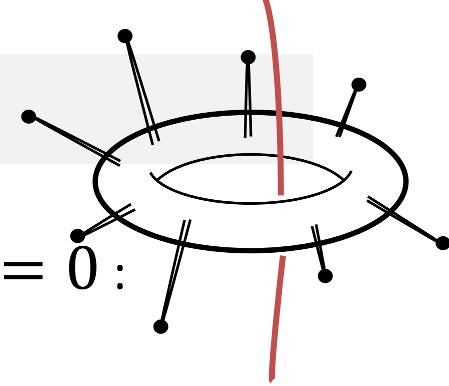
$$E_n = \omega\, n$$

$n = 0$: zero modes
$n \neq 0$: KK modes

$T^2/\mathbb{Z}_N$ orbifoldsへ拡張

→ Non-invertible symmetries

Orbifolding



- *Two-dimensional rotation around the origin* $y_1 = y_2 = 0$:

$$\boldsymbol{y} \to R_\theta \boldsymbol{y} \qquad R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

- *In the operator formalism:*

T.h.Abe, Y.Fujimoto, T.Kobayashi, T.Miura, K.Nishiwaki, M. Sakamoto (2014)

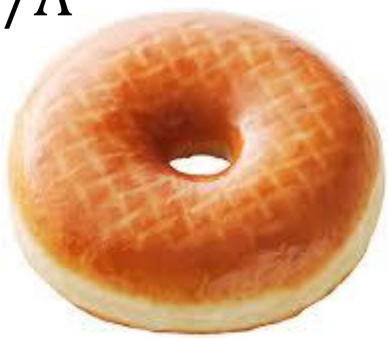
$$\hat{\boldsymbol{y}} \to \hat{U}_\theta \hat{\boldsymbol{y}} \hat{U}_\theta^\dagger = R_\theta \hat{\boldsymbol{y}}$$
$$\hat{\boldsymbol{p}} \to \hat{U}_\theta \hat{\boldsymbol{p}} \hat{U}_\theta^\dagger = R_\theta \hat{\boldsymbol{p}}$$
$$\hat{H} \to \hat{H} \qquad \text{due to } [R_\theta, F] = 0$$

- *However, boundary conditions generated by* $\hat{T}_1$ and $\hat{T}_2$ are modified

$$\hat{T}_a \to \hat{U}_\theta \hat{T}_a \hat{U}_\theta^\dagger = \left(R_\theta^T e_a\right)^T \left(\hat{\boldsymbol{p}} - \frac{1}{2} F^T \hat{\boldsymbol{y}}\right)$$

Ex., $\mathbb{Z}_2$ - twist

$T^2 = \mathbb{C}/\Lambda$



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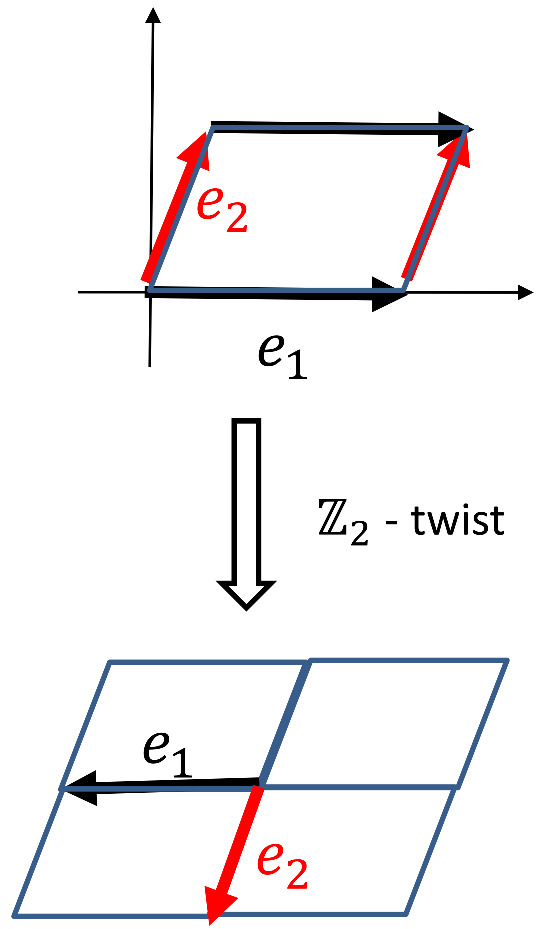
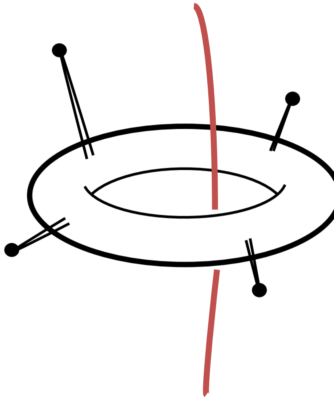


- *Boundary conditions generated by* $\hat{T}_1$ and $\hat{T}_2$:

$$\hat{T}_1 \to -\hat{T}_1 \qquad \hat{T}_2 \to -\hat{T}_2$$

Non-invertible symmetry on $T^2/\mathbb{Z}_2$ with magnetic flux



- *After gauging* $\mathbb{Z}_2$ *,*
  only the $\mathbb{Z}_2$-invariant operator survives

$$\widehat{U}_1^{(\lambda_1)} \equiv e^{i\lambda_1 \hat{T}_1} + e^{-i\lambda_1 \hat{T}_1}$$

$$\widehat{U}_2^{(\lambda_2)} \equiv e^{i\lambda_2 \hat{T}_2} + e^{-i\lambda_2 \hat{T}_2}$$

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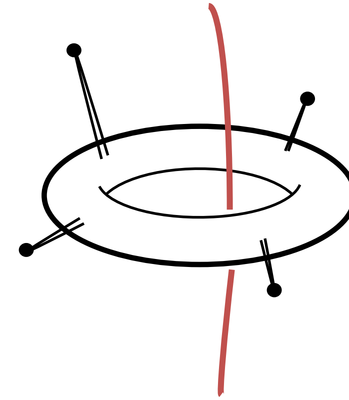

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$$\hat{U}_1^{(\lambda_1)} \equiv e^{i\lambda_1\hat{T}_1} + e^{-i\lambda_1\hat{T}_1}$$

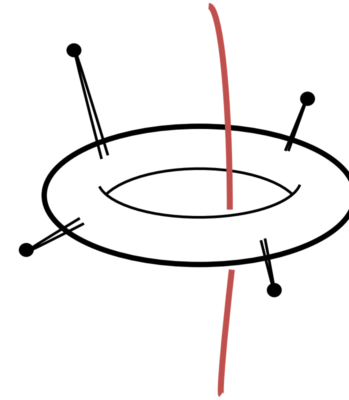
$$\hat{U}_2^{(\lambda_2)} \equiv e^{i\lambda_2\hat{T}_2} + e^{-i\lambda_2\hat{T}_2}$$

- *Fusion Algebra :*

$$\boxed{\begin{aligned}
\hat{U}_1^{(\lambda)}\hat{U}_1^{(\lambda')} &= \hat{U}_1^{(\lambda+\lambda')} + \hat{U}_1^{(\lambda-\lambda')} \\
\hat{U}_2^{(\lambda)}\hat{U}_2^{(\lambda')} &= \hat{U}_2^{(\lambda+\lambda')} + \hat{U}_2^{(\lambda-\lambda')} \\
\hat{U}_2^{(\lambda_2)}\hat{U}_1^{(\lambda_1)} &= e^{i\rho}\hat{U}_1^{(\lambda_1)}\hat{U}_2^{(\lambda_2)}
\end{aligned}}$$

If $e^{i\rho} = 1$: Abelian structure
$e^{i\rho} = -1$: Non-abelian structure

Non-invertible flavor symmetry on $T^2/\mathbb{Z}_2$



- *After gauging $\mathbb{Z}_2$,*
  only the $\mathbb{Z}_2$-invariant operator survives

$$\hat{U}_1^{(\lambda_1)} \equiv e^{i\lambda_1 \hat{T}_1} + e^{-i\lambda_1 \hat{T}_1}$$

$$\hat{U}_2^{(\lambda_2)} \equiv e^{i\lambda_2 \hat{T}_2} + e^{-i\lambda_2 \hat{T}_2}$$

- *Representations of chiral matters:*

$$\boxed{\begin{aligned} \hat{U}_1^{(1/M)} |\psi\rangle_+^{j,M} &= |\psi\rangle_+^{j+1,M} + |\psi\rangle_+^{j-1,M} \\ \hat{U}_2^{(1/M)} |\psi\rangle_+^{j,M} &= 2\cos\left(\frac{2\pi j}{M}\right) |\psi\rangle_+^{j,M} \end{aligned}}$$

$\mathbb{Z}_2$ *-even state :* $|\psi\rangle_+^{j,M} = \frac{1}{2}\left(|\psi\rangle^{j,M} + |\psi\rangle^{-j,M}\right)$

Non-invertible flavor symmetry on $T^2/\mathbb{Z}_2$

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- *M=2 :*

$\lambda = 1/2$

$$\hat{U}_1 : \begin{pmatrix} |\psi\rangle_+^{0,2} \\ |\psi\rangle_+^{1,2} \end{pmatrix} \to 2\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} |\psi\rangle_+^{0,2} \\ |\psi\rangle_+^{1,2} \end{pmatrix}$$

$$\hat{U}_2 : \begin{pmatrix} |\psi\rangle_+^{0,2} \\ |\psi\rangle_+^{1,2} \end{pmatrix} \to 2\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} |\psi\rangle_+^{0,2} \\ |\psi\rangle_+^{1,2} \end{pmatrix}$$

- *2-dim. Irreducible rep. of* $D_4 \simeq \left(\mathbb{Z}_2 \times \mathbb{Z}_2^{(\hat{U}_2/2)}\right) \rtimes \mathbb{Z}_2^{(\hat{U}_1/2)}$

$\mathbb{Z}_2 : \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

Non-invertible flavor symmetry on $T^2/\mathbb{Z}_2$

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- *M=3 :*

$\mathbb{Z}_2 : \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$\lambda = 1/3$

$$\hat{U}_1 : \begin{pmatrix} |\psi\rangle_+^{0,3} \\ |\psi\rangle_+^{1,3} \end{pmatrix} \to 2\begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix}\begin{pmatrix} |\psi\rangle_+^{0,3} \\ |\psi\rangle_+^{1,3} \end{pmatrix}$$

$$\hat{U}_2 : \begin{pmatrix} |\psi\rangle_+^{0,3} \\ |\psi\rangle_+^{1,3} \end{pmatrix} \to 2\begin{pmatrix} 1 & 0 \\ 0 & \cos(2\pi/3) \end{pmatrix}\begin{pmatrix} |\psi\rangle_+^{0,3} \\ |\psi\rangle_+^{1,3} \end{pmatrix}$$

- *No "invertible" symmetry remains*

Non-invertible flavor symmetry on $T^2/\mathbb{Z}_2$

$\mathbb{Z}_2$ *-even state :* $|\psi\rangle_+^{j,M} = \frac{1}{2}(|\psi\rangle^{j,M} + |\psi\rangle^{-j,M})$

- *M=2k :*

  - *Similarly, we discuss transformations of chiral matters under the non-invertible symmetries.*

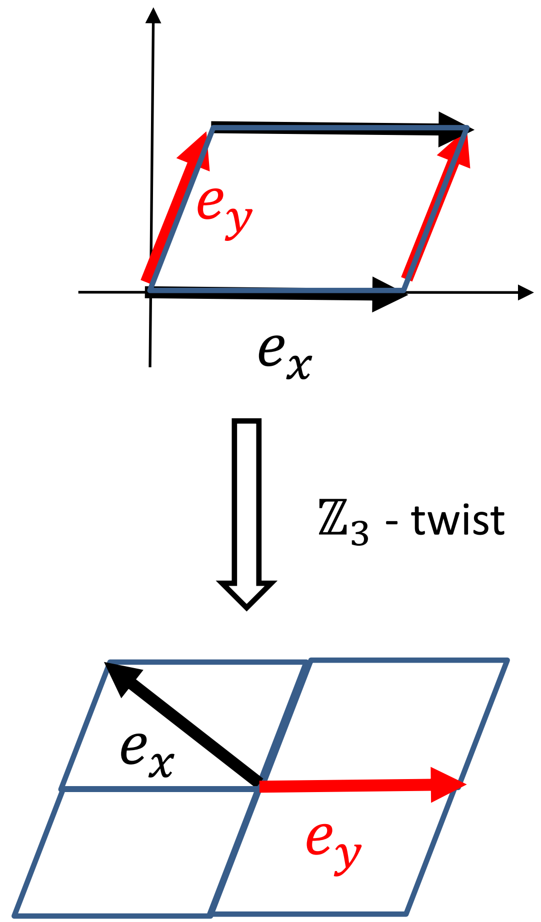
  - *They belong to 2-dim. or singlet irreducible rep. of*

$$D_4 \simeq \left(\mathbb{Z}_2 \times \mathbb{Z}_2^{(\widehat{U}_2/2)}\right) \rtimes \mathbb{Z}_2^{(\widehat{U}_1/2)}$$

Ex., $\mathbb{Z}_3$ - twist

$\boldsymbol{T^2} = \mathbb{C}/\Lambda$

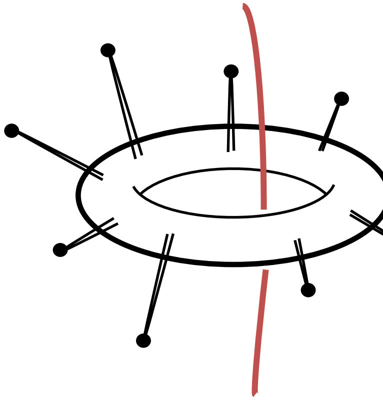
$=$



- *Boundary conditions generated by* $\hat{T}_1$ and $\hat{T}_2$:

$$\hat{T}_1 \to -\hat{T}_1 - \hat{T}_2, \qquad \hat{T}_2 \to \hat{T}_1$$

Non-invertible symmetry on $T^2/\mathbb{Z}_3$ with magnetic flux



- *After gauging* $\mathbb{Z}_3$ *,*
  only the $\mathbb{Z}_3$-invariant operator survives

$$\hat{U}_{\mathbb{Z}_3}^{(\lambda_1,\lambda_2)} \equiv e^{i\lambda_1\hat{T}_1+i\lambda_2\hat{T}_2} + e^{i\lambda_1\hat{T}_2-i\lambda_2(\hat{T}_1+\hat{T}_2)} + e^{-i\lambda_1(\hat{T}_1+\hat{T}_2)+i\lambda_2\hat{T}_1}$$

- *Fusion Algebra :*

$$\boxed{\begin{aligned}\hat{U}_{\mathbb{Z}_3}^{(\lambda_1,\lambda_2)}\hat{U}_{\mathbb{Z}_3}^{(\lambda_3,\lambda_4)} = \ & e^{\pi i M(\lambda_2\lambda_3-\lambda_1\lambda_4)}\hat{U}_{\mathbb{Z}_3}^{(\lambda_1+\lambda_3,\lambda_2+\lambda_4)} \\ & + e^{\pi i M(\lambda_1\lambda_3-\lambda_2\lambda_3+\lambda_2\lambda_4)}\hat{U}_{\mathbb{Z}_3}^{(-\lambda_1+\lambda_2-\lambda_4,-\lambda_1+\lambda_3-\lambda_4)} \\ & + e^{\pi i M(\lambda_1\lambda_4-\lambda_2\lambda_4-\lambda_1\lambda_3)}\hat{U}_{\mathbb{Z}_3}^{(\lambda_1-\lambda_4,\lambda_2+\lambda_3-\lambda_4)}\end{aligned}}$$

Conclusion

- Non-invertible symmetries
  in type IIB magnetized D-branes on $T^2/\mathbb{Z}_N$

  - Fusion algebras of $\mathbb{Z}_N$-invariant operators :

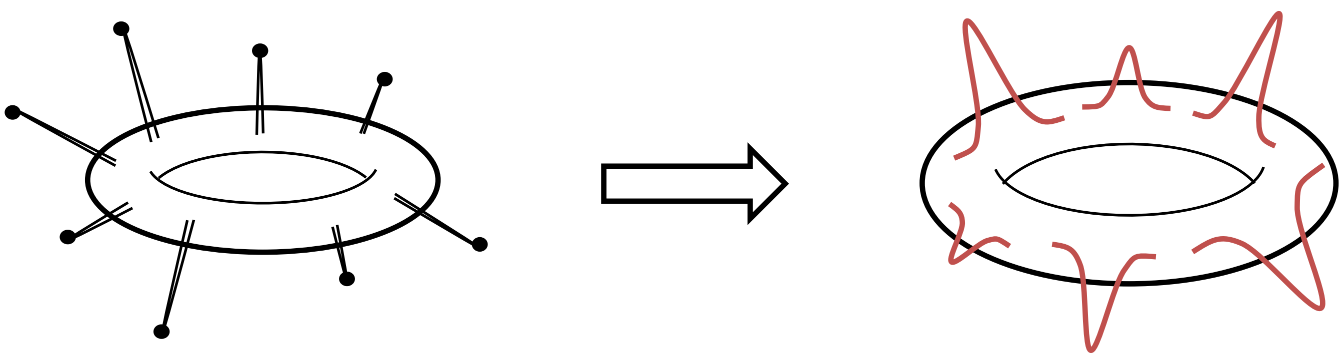
    $$\text{ex., } U_{\mathbb{Z}_2}^{(\lambda_1)} U_{\mathbb{Z}_2}^{(\lambda_2)} = U_{\mathbb{Z}_2}^{(\lambda_1+\lambda_2)} + U_{\mathbb{Z}_2}^{(\lambda_1-\lambda_2)}$$

  - Representations of chiral zero-modes

- Chiral zero-modes will correspond to
  3 generations of quarks/leptons
  Non-invertible symmetries → flavor symmetries

Discussion

- One can analyze the selection rule of n-point couplings
- Our analysis will be applicable to $T^6/\mathbb{Z}_N$ or $T^6/\mathbb{Z}_M \times \mathbb{Z}_N$
- Breaking of non-invertible symmetries
  - would be realized on resolutions of $T^{2g}/\mathbb{Z}_N$
  - blowup modes will determine the breaking of non-inv. symmetries



$Thank\ you!$