共形場理論における複合的な欠損演算子の解析

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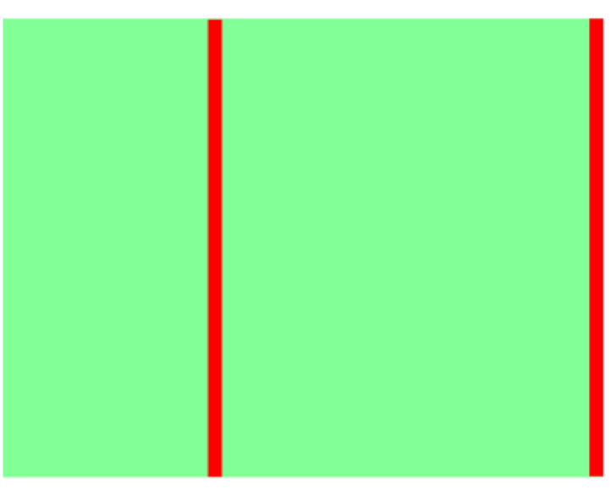
本発表は, arXiv: 2404.08411に基づく

Introduction

- In nature, we often have some impurities or boundaries at criticality.

conformal defect

(I will use $\mathcal{D}^{(p)}$ to represent p-dim. conformal defect)



- Defect CFT : framework to analyze CFTs in the presence of conformal defects

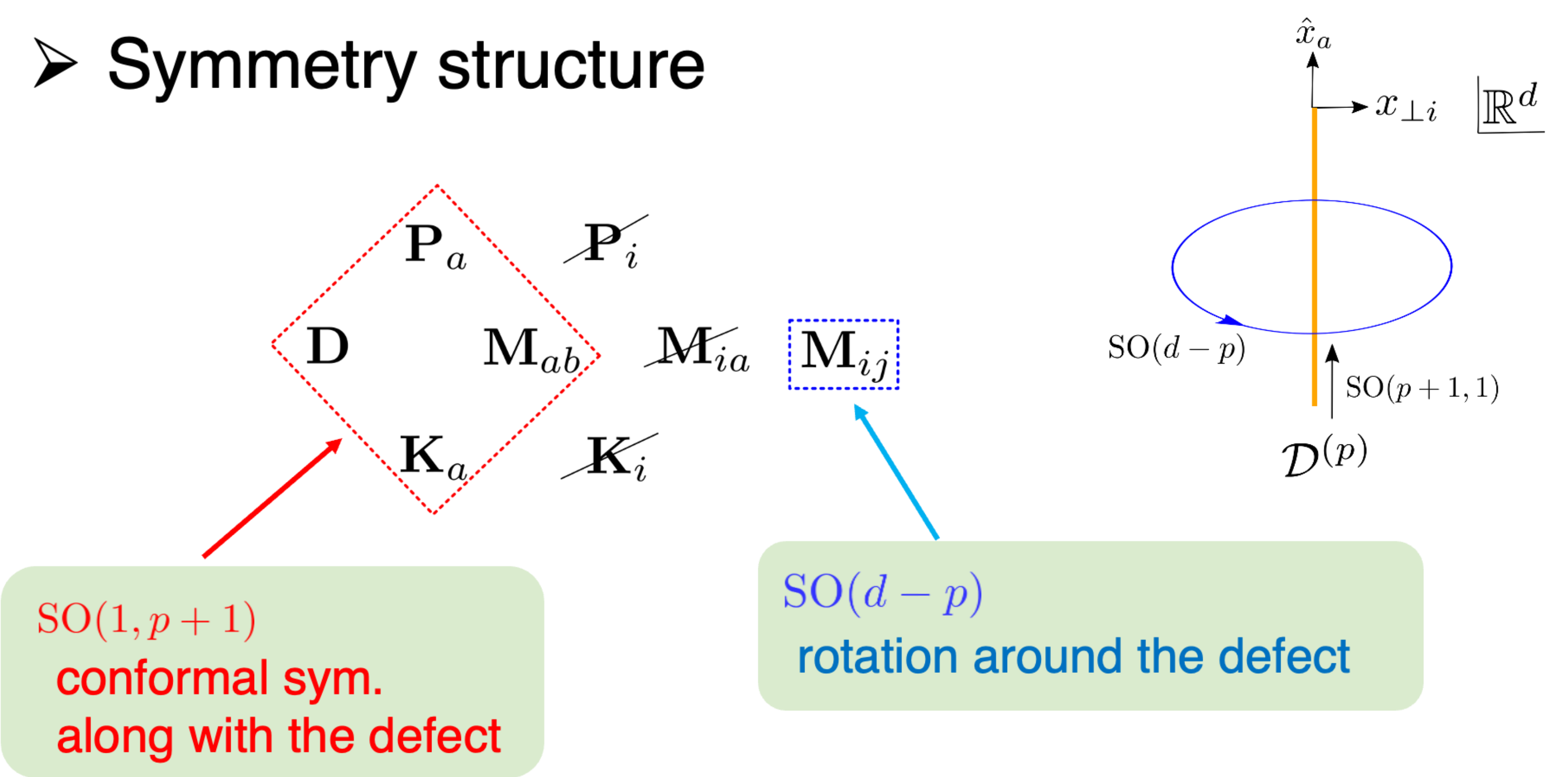
- If we allow such defects, non-trivial CFT data of conformal defects arise.

defect operator spectrum, …

→ These data carry information of defects at criticality !

Basics of defect CFT

Symmetry structure



$\mathrm{SO}(1,p+1)$
conformal sym.
along with the defect

$\mathrm{SO}(d-p)$
rotation around the defect

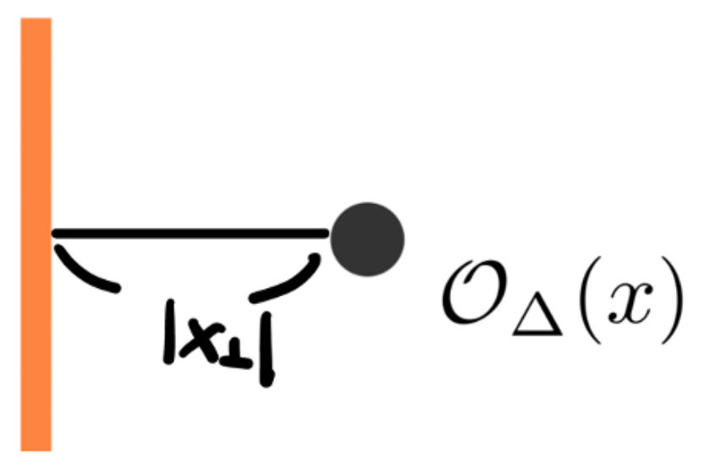
Correlation function

Ward-Takahashi identity associated to translation symmetry $\mathbf{P}_a$

$$\partial_a \langle\, \mathcal{O}(x)\,\rangle_{\mathrm{DCFT}} = 0 \quad \longrightarrow \quad \langle\, \mathcal{O}(x)\,\rangle_{\mathrm{DCFT}} = f(x_\perp)$$

By combining with other WT identities,

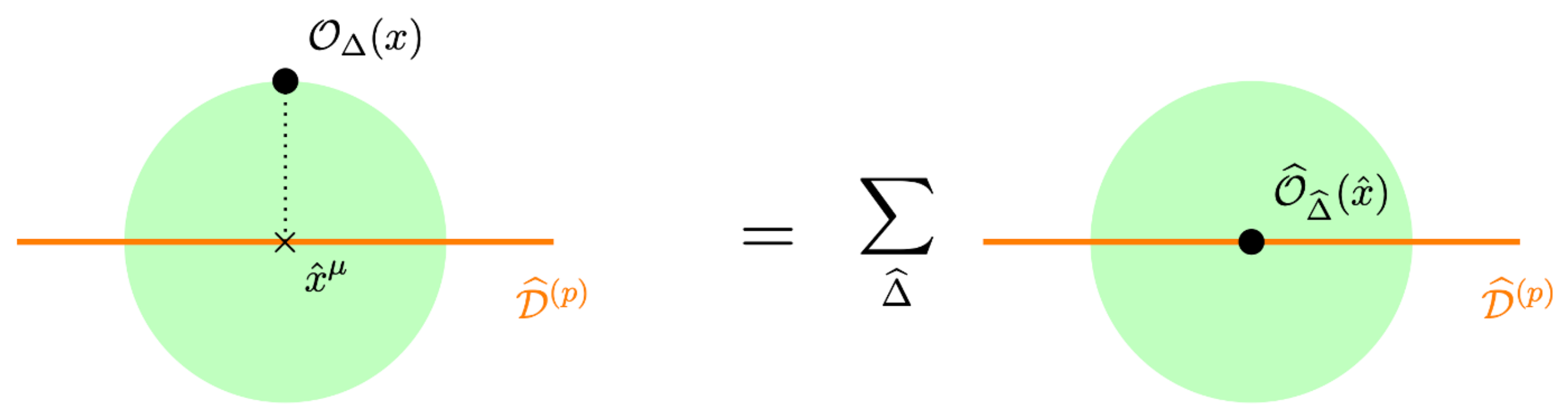
$$\boxed{\langle\, \mathcal{O}_\Delta(x)\,\rangle_{\mathrm{DCFT}} = \frac{b_\Delta}{|x_\perp|^\Delta}}$$



even bulk one-point function takes non-trivial structure !

Defect operator expansion [Cardy; McAvity Osborn]

We can expand a bulk local operator in terms of defect operators



$$\mathcal{O}_\Delta(x) = \sum_{\widehat{\Delta}} \frac{b_{\Delta,\widehat{\Delta}}}{|x_\perp|^{\Delta-\widehat{\Delta}}}\, \mathcal{B}^{(d,p)}_{\widehat{\Delta}}\left(|x_\perp|, \widehat{\Box}_p\right) \widehat{\mathcal{O}}_{\widehat{\Delta}}(\hat{\boldsymbol{x}})$$

$$\mathcal{B}^{(d,p)}_{\widehat{\Delta}}\left(|x_\perp|, \widehat{\Box}\right) = \sum_{n=0}^{\infty} \frac{(-4)^{-n}}{n!\,\left(\widehat{\Delta} - \frac{p-2}{2}\right)_n}\, |x_\perp|^{2n}\, \widehat{\Box}_p^n$$

Composite defect CFT

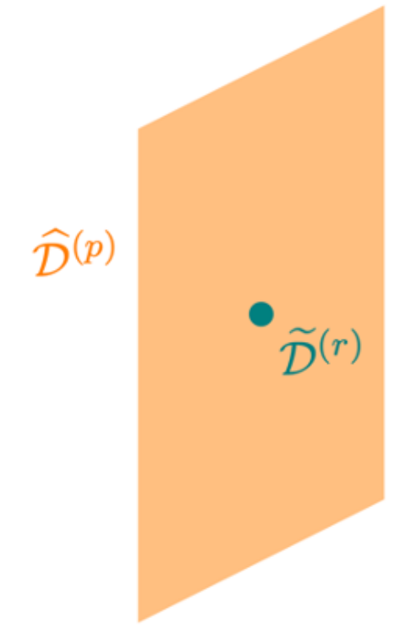
In nature, it is possible to consider a conformal defect
which is contaminated by another sub-dimensional defect.

We refer to this new kind of defect as composite defect !

Let's introduce a composite defect

$$\mathcal{D}^{(r,p)} = \widetilde{\mathcal{D}}^r \cup \widehat{\mathcal{D}}^p$$

sub-defect

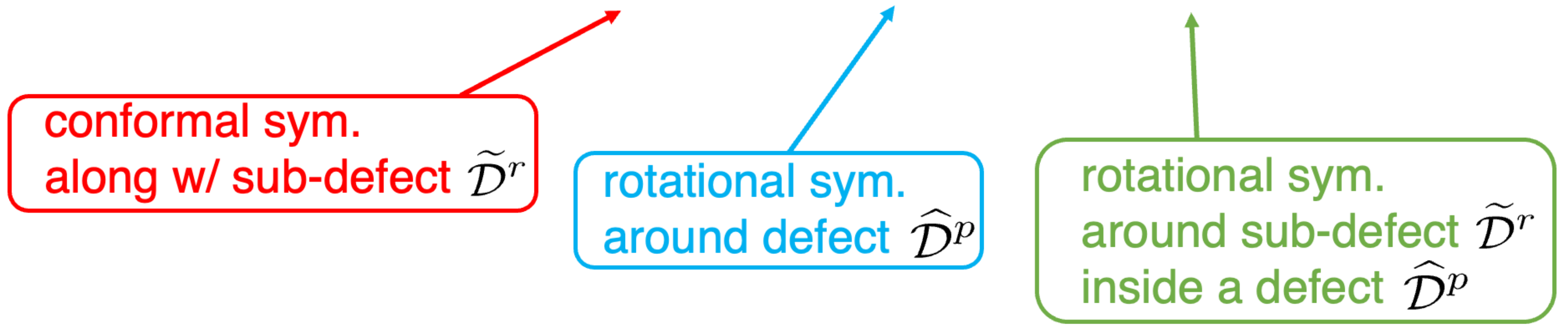


$0 < r < p < d$

into a d-dimensional CFT.

→ What happens to full conformal symmetry?

$$\mathrm{SO}(1,d+1) \longrightarrow \mathrm{SO}(1,r+1) \times \mathrm{SO}(d-p) \times \mathrm{SO}(p-r)$$

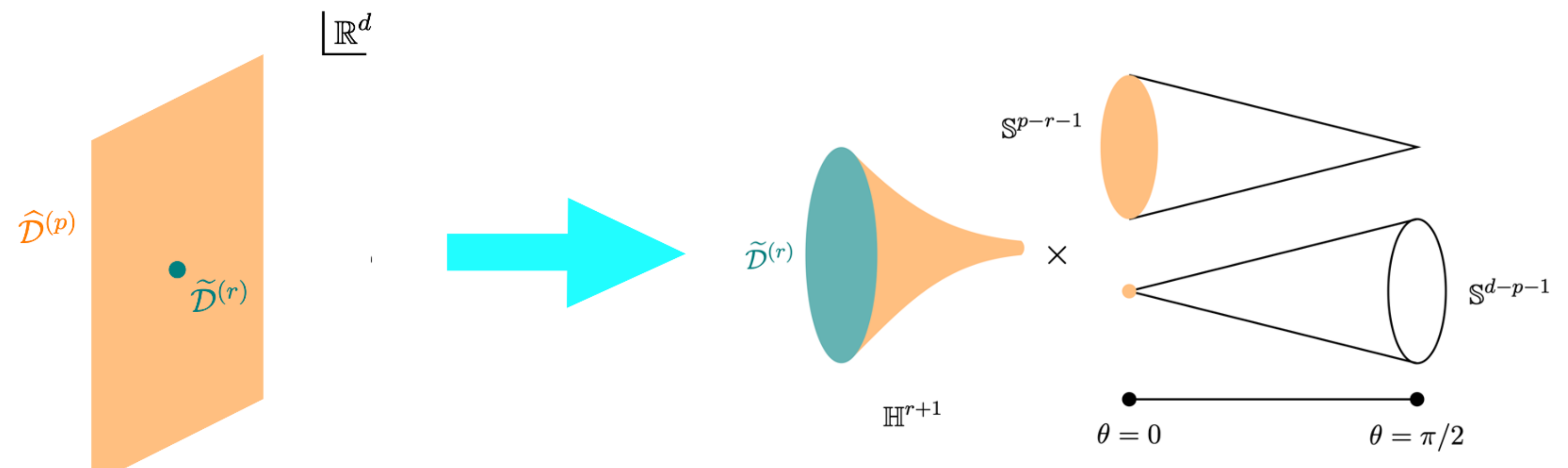


conformal sym.
along w/ sub-defect $\widetilde{\mathcal{D}}^r$

rotational sym.
around defect $\widehat{\mathcal{D}}^p$

rotational sym.
around sub-defect $\widetilde{\mathcal{D}}^r$
inside a defect $\widehat{\mathcal{D}}^p$

This can be understood from the viewpoint of isometry.



$$\mathrm{d}s^2 = \sum_{a=1}^{r} \mathrm{d}\tilde{x}_a^2 + \sum_{\alpha=r+1}^{p} \mathrm{d}\hat{x}_\alpha^2 + \sum_{i=p+1}^{d} \mathrm{d}x_{\perp,i}^2$$

$$x^\mu = (\tilde{x}^a, \hat{x}^\alpha, x_\perp^i)$$

$$\mathrm{d}s^2 = z^2\left(\mathrm{d}s^2_{\mathbb{H}^{r+1}} + \mathrm{d}s^2_{\mathrm{sph}}\right)$$

$$\mathrm{d}s^2_{\mathbb{H}^{r+1}} = \frac{\sum_{a=1}^{r} \mathrm{d}\tilde{x}_a^2 + \mathrm{d}z^2}{z^2}$$

$$\mathrm{d}s^2_{\mathrm{sph}} = \mathrm{d}\theta^2 + \cos^2\theta\, \mathrm{d}\Omega^2_{p-r-1} + \sin^2\theta\, \mathrm{d}\Omega^2_{d-p-1}$$

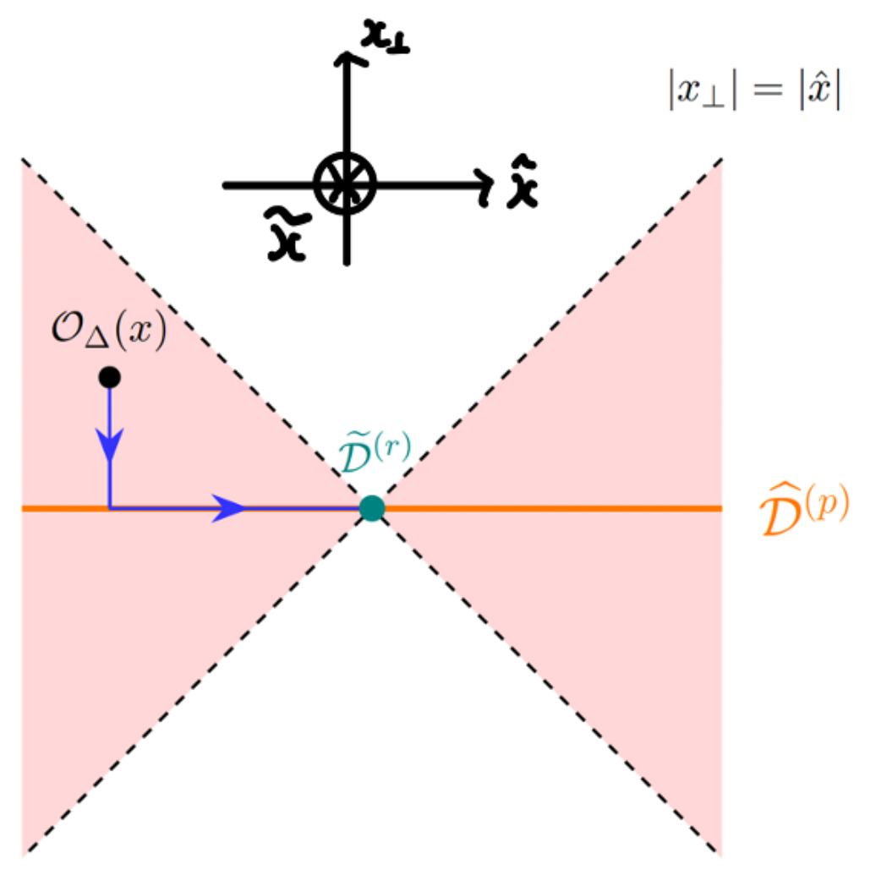
Sub-defect operator expansion

A bulk local operator can be expanded in terms of sub-defect local operators.

Strategy : use defect OE twice

Step 1. bulk to defect

Step 2. defect to sub-defect



$$\mathcal{O}_\Delta(x) = \sum_{\widetilde{\Delta}} \sum_{\widehat{\Delta}} \frac{b_{\Delta,\widehat{\Delta}}\, b_{\widehat{\Delta},\widetilde{\Delta}}}{|x_\perp|^{\Delta-\widetilde{\Delta}}}\, \xi^{\frac{\widehat{\Delta}-\widetilde{\Delta}}{2}}\, {}_2F_1\left(\frac{\widehat{\Delta}-\widetilde{\Delta}}{2}, 1+\frac{-p+r+\widehat{\Delta}-\widetilde{\Delta}}{2}; 1-\frac{p}{2}+\widehat{\Delta}; -\xi\right) \widetilde{\mathcal{O}}_{\widetilde{\Delta}}(\tilde{\boldsymbol{x}})$$

+(sub-defect descendants)

This expansion depends on cross-ratio $\xi = \dfrac{|x_\perp|^2}{|\hat{x}|^2}$

Summary

- We proposed composite defect CFT .
- We elaborated on sub-defect operator expansion, which is particular to composite defect CFT.

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