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Gauge Symmetries and Conserved Currents in AdS/BCFT

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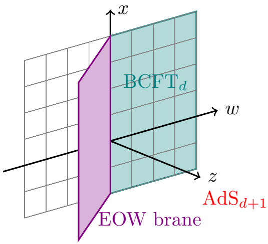
Motivation: $U(1)$ preserving boundary

• In this talk, we consider a $U(1)$ gauge field in the bulk of AdS/BCFT, without any gauge fields on the EOW brane.



• From the BCFT point of view, this means that the conserved current J^μ associated to this $U(1)$ symmetry satisfies

$$J^w(w=0) \, = \, 0$$

• In contrast to these conserved currents J^μ, other vectorial operators $\mathcal{O}^\mu$, in general can have a non-vanishing perpendicular component on the boundary:

$$\mathcal{O}^w(w=0) \, \neq \, 0$$

• We would like to explain this difference from the bulk point of view in the AdS/BCFT.

1. Background Geometry in AdS/BCFT

• AdS$_{d+1}$ gravity we discuss is described by [Takayanagi, '11] [Fujita, Takayanagi & Tonni '11]

$$I_{\text{total}} = I_{\text{bulk}} + I_{\text{brane}} + I_{\text{bdy}}\,,$$

where

$$I_{\text{bulk}} = -\frac{1}{16\pi G}\int_M d^{d+1}x\,\sqrt{g}(R_{d+1} - 2\Lambda)\,, \qquad \left(\Lambda = -\tfrac{d(d-1)}{2}\right)$$

$$I_{\text{brane}} = -\frac{1}{8\pi G}\int_Q d^d x\,\sqrt{\gamma}\,\left(K_Q - \frac{\sigma}{2}\right)\,,$$

$$I_{\text{bdy}} = -\frac{1}{8\pi G}\int_\Sigma d^d x\,\sqrt{\gamma_\Sigma}\,K_\Sigma$$

Coordinates

- We start from the empty AdS_{d+1} background

$$ds^2_{d+1} = \frac{dz^2 + dw^2 + \sum_{i=0}^{d-2} dx_i^2}{z^2}$$

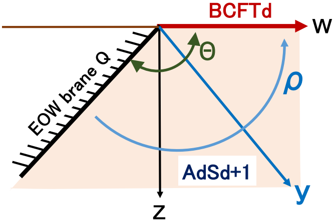
- Via the coordinate transformation

$$z = \frac{y}{\cosh\rho}\,, \qquad w = y\tanh\rho$$

we obtain the following metric in terms of the hyperbolic slice of AdS_d:

$$ds^2_{d+1} = d\rho^2 + \cosh^2\rho\left(\frac{dy^2 + \sum_{i=0}^{d-2} dx_i^2}{y^2}\right)$$

Classical brane location



- In the hyperbolic slicing coordinates, the brane location is

$$\rho = \rho_* \quad (= \text{const} < 0)$$

and the bulk AdS_{d+1} gravity is defined in $\rho > \rho_*$ and the holographic BCFT_d is defined in $w > 0$.

- ρ_* is related to the brane tension σ of the EOW brane via

$$\sinh|\rho_*| = \frac{\sigma}{\sqrt{4(d-1)^2 - \sigma^2}} \qquad \left(\sigma^2 < 4(d-1)^2\right)$$

2. Massless Vector Perturbations

• Next we study massless vector perturbation A^{μ} (with $A^{\rho}=0$):

$$I_{\rm EM} \;=\; -\frac{1}{4}\int_{M} d^{d+1}x\sqrt{g}\,F^{MN}F_{MN}$$

on the background geometry

$$ds^2_{d+1} \;=\; d\rho^2 + a^2(\rho)\gamma_{\mu\nu}d\xi^{\mu}d\xi^{\nu}$$

where $a(\rho)\equiv\cosh{(\rho)}$ and $\gamma_{\mu\nu}$ is the d-dimensional Poincare metric.

• The indices $\mu,\nu,\lambda,\cdots$ denote the d-dimensional coordinates without ρ-direction. We use capital indices $M,N,L,\cdots$ to denote $(d+1)$ dimensional coordinates and small indices $i,j,k,\cdots$ to denote $(d-1)$ dimensional coordinates.

Perturbation equations

- We further impose the d-dimensional Lorentz gauge

$$\tilde{\nabla}_\mu A^\mu = 0$$

where $\tilde{\nabla}_\mu$ is the covariant derivative with respect to the d-dimensional Poincare metric $\gamma_{\mu\nu}$.

- Assuming a separable form of solution

$$A^\mu(\rho, x, y) = a(\rho)^{-1} \sum_{n=0}^{\infty} \mathcal{R}_n(\rho) Y_n^\mu(x, y)$$

the perturbation equation is decomposed into

$$\mathcal{R}_n'' + d \tanh\rho\, \mathcal{R}_n' + (d-1)\mathcal{R}_n = \frac{-\lambda_n}{\cosh^2\rho} \mathcal{R}_n$$

$$(\tilde{\Box} + 1) Y_n^\mu = \lambda_n Y_n^\mu$$

ρ-direction

• We first solve the ρ-direction. By imposing the asymptotic AdS$_{d+1}$ boundary condition,

$$\mathcal{R}(\rho) = \mathcal{O}(e^{-2\rho}) \qquad \text{for} \qquad \rho \to \infty$$

the solution is given by

$$\mathcal{R}(\zeta) = B_0 \, (1-\zeta^2)^{\frac{d}{4}} \, P_\nu^{-\mu}(\zeta)$$

where B_0 is a normalization constant and

$$\zeta = \tanh\rho \,, \qquad \mu = \frac{d-2}{2} \,, \qquad \nu = \frac{1}{2}\sqrt{(d-1)^2 + 4\lambda_n}$$

• We also impose the Neumann boundary condition on the brane, which can be written as

$$0 = P^{1-\mu}_{\nu-\frac{1}{2}}(\zeta_*)$$

• This condition quantizes the eigenvalue (or Kaluza-Klein mass) λ_n.

Spectrum

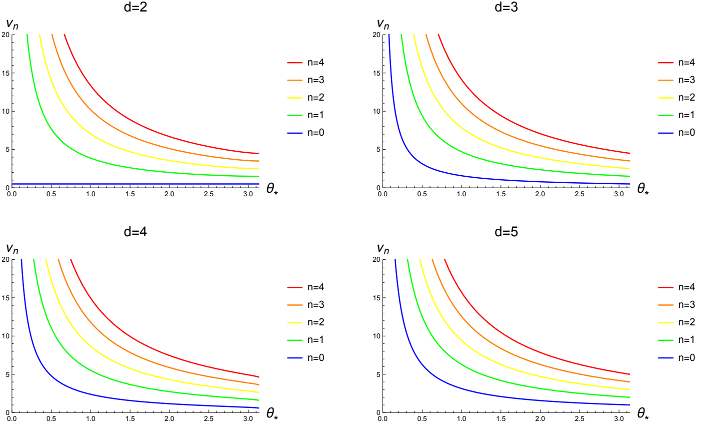


Figure 1: Brane tension dependence of numerical solutions for the lowest five modes in AdS$_{d+1}$ with $d = 2, 3, 4$ and 5. $(\cos\theta_* = \zeta_* = \tanh\rho_*)$

Holographic conserved currents

- In our coordinates, the holographic conserved current is read off as

$$J^{\mu}(x,w) \; = \; \lim_{\rho\to\infty}\left[\left(\frac{e^{\rho}}{2y}\right)^{d} A^{\mu}(\rho,x,y)\right]\Bigg|_{y=w}$$

- Therefore, in the boundary limit $w \to 0$, we find

$$J^{w}(x,w) \; \propto \; w^{\,\nu+\frac{-d+3}{2}}\,, \qquad J^{i}(x,w) \; \propto \; w^{\,\nu+\frac{-d+1}{2}}$$

where

$$\nu \; = \; \sqrt{\tfrac{(d-1)^2}{4}+\lambda_n} \; \geq \; \tfrac{d-1}{2}$$

- Thus, we found the correct $U(1)$ preserving boundary condition $J^{w}(w=0) \; = \; 0$, while we have non-vanishing parallel components $J^{i}(w=0) \; = \; \mathcal{O}(1)$.

3. Massive Vector Perturbations

- Next we study massive vector perturbation A^M (Note $A^\rho \neq 0$):

$$I_{\rm v} = -\int_M d^{d+1}x\sqrt{g}\left[\frac{1}{4}F^{MN}F_{MN} + \frac{m^2}{2}A^M A_M\right]$$

on the background geometry

$$ds^2_{d+1} = d\rho^2 + a^2(\rho)\gamma_{\mu\nu}d\xi^\mu d\xi^\nu$$

- The equation of motion for the perturbation A^M is

$$\nabla_M F^{MN} = m^2 A^N$$

- Even though there is no gauge symmetry, due to the anti-symmetric nature of F^{MN}, we get a supplemental condition [Allen '85]

$$\nabla_M A^M = 0$$

Perturbation equations

- Using this supplemental condition, the equation of motion is rewritten as

$$(\Box + d - m^2)A^M \;=\; 0$$

or more explicitly

$$\left[a^{-2}\,\tilde{\Box} + \partial_\rho^2 + (d+2)\tanh\rho\,\partial_\rho + 2d - m^2 - \frac{d}{\cosh^2\rho}\right] A^\rho \;=\; 0$$

$$\left[a^{-2}(\tilde{\Box}+1) + \partial_\rho^2 + (d+2)\tanh\rho\,\partial_\rho + 2d - m^2 - \frac{d}{\cosh^2\rho}\right] A^\mu \;=\; -2\tanh\rho\,\partial^\mu A^\rho$$

- The A^μ equation is inhomogeneous differential equation, so the solution is

$$A^\mu \;=\; (\text{homogenous solution}) + (\text{inhomogenous solution})$$

Dual Vector Operators

- The contributions from the homogenous solutions are

$$\mathcal{O}^y_{\mathrm{hom}}(x,w) \sim w^{\nu+\frac{1}{2}-\mu}\,, \qquad \mathcal{O}^i_{\mathrm{hom}}(x,w) \sim w^{\nu-\frac{1}{2}-\mu}$$

where $\mu = \sqrt{\frac{(d-2)^2}{4} + m^2}$.

- Therefore, the massless limit agrees with the conserved currents J^μ:

$$\lim_{m\to 0} \mathcal{O}^y_{\mathrm{hom}}(x,w=0) \sim 0\,, \qquad \lim_{m\to 0} \mathcal{O}^i_{\mathrm{hom}}(x,w=0) \sim w^0$$

- The contributions from the inhomogenous solutions are

$$\mathcal{O}^y_{\mathrm{inh}}(x,w) \sim w^{\nu-\frac{5}{2}-\mu}\,, \qquad \mathcal{O}^i_{\mathrm{inh}}(x,w) \sim w^{\nu-\frac{3}{2}-\mu}$$

Therefore, even in the massless limit, $\mathcal{O}^y_{\mathrm{inh}}$ can have non-vanishing value on the boundary $w=0$.

4. Summaries

• We studied the $U(1)$ preserving boundary condition and its relation to the bulk gauge symmetry in AdS/BCFT.

• Presence/absence of the bulk gauge symmetry is crucial for the boundary condition of the conserved currents and other vector operators in BCFT.

• We also found several boundary-condition-independent modes, when the perturbation is massless.

• It is tempting to investigate the meaning of these boundary-condition-independent modes from the dual BCFT point of view.

• It would be also interesting to study $U(1)$ breaking boundary condition in AdS/BCFT.

• It would be interesting to investigate the higher form symmetry generated by the dual p-form BCFT operators.

Thank you!