de Sitter Space Using CFT States

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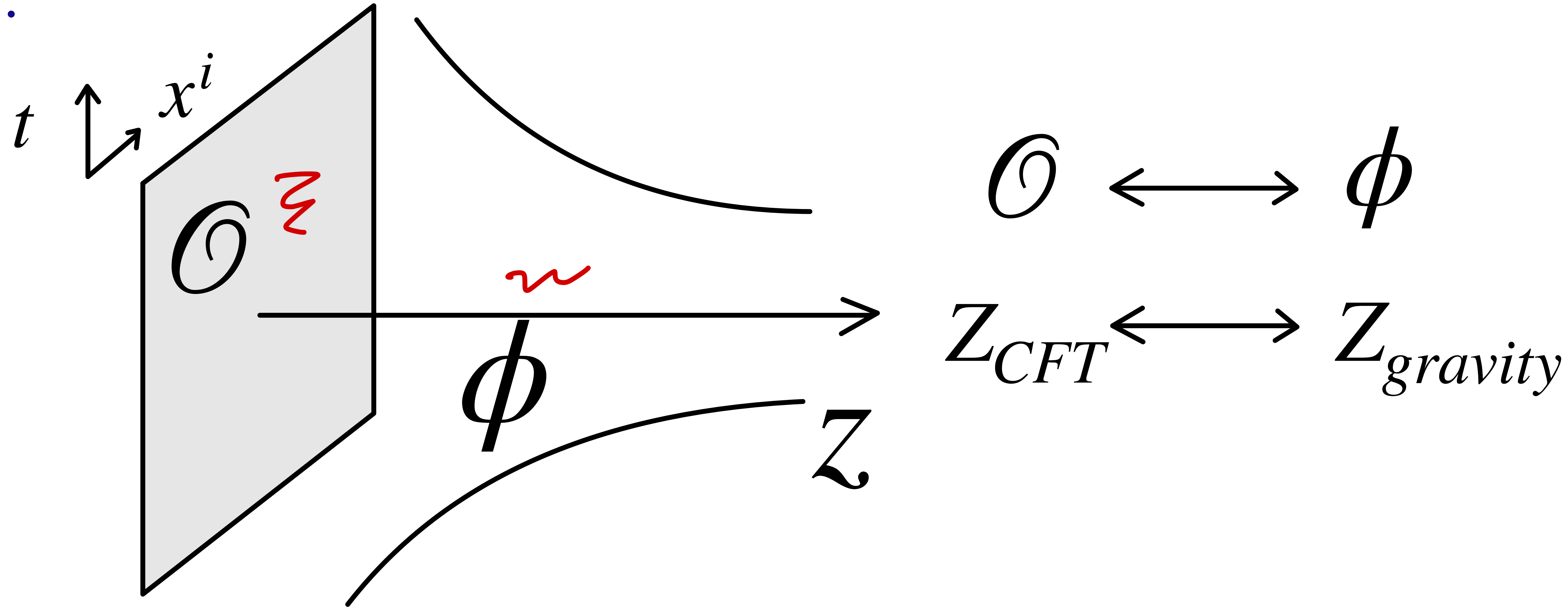
AdS/CFT vs dS/CFT

AdS/CFT(Maldacena, 97)

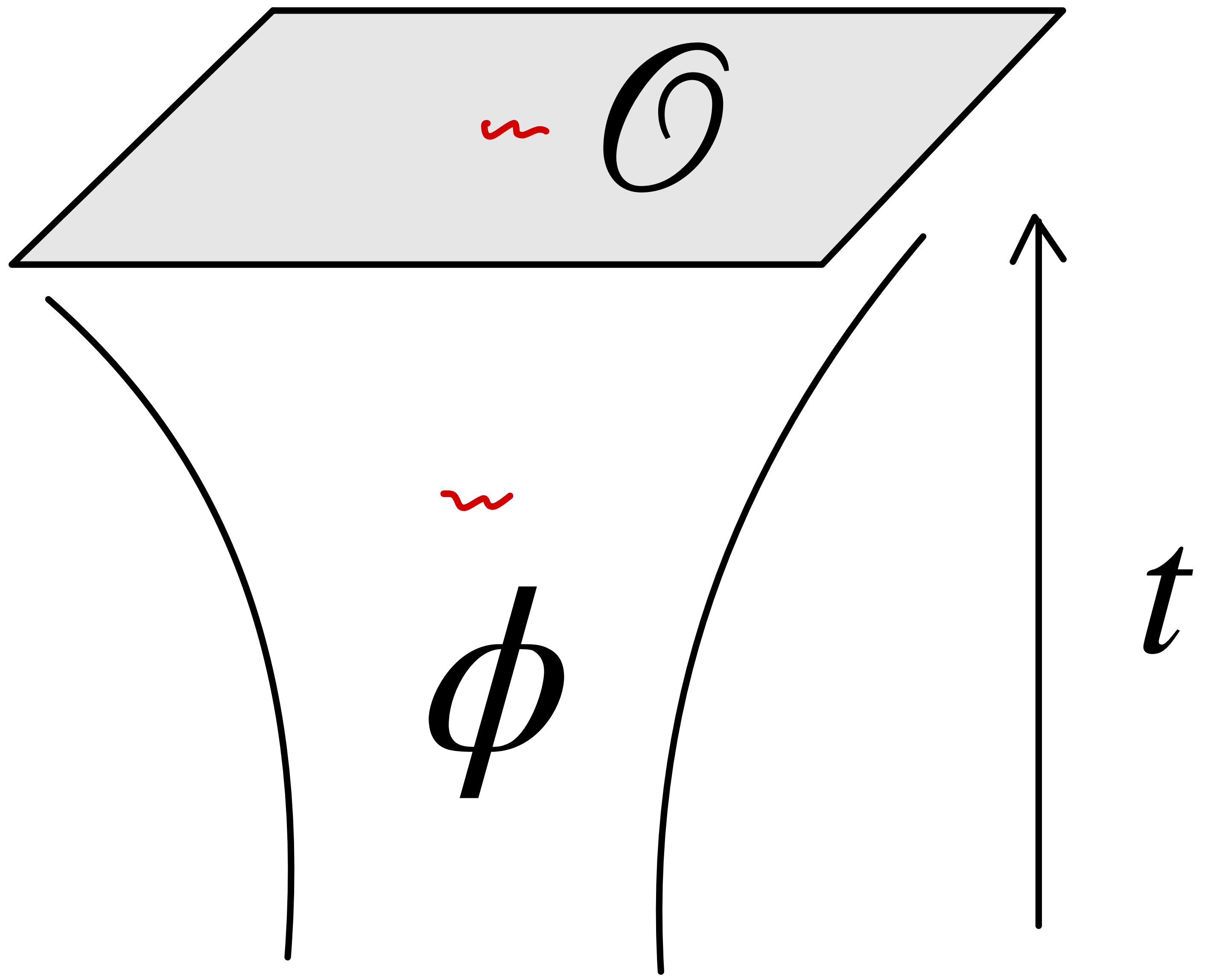
- Quantum gravity on AdS_{d+1} is dual to CFT_d living on asymptotic boundary (With string realization).
- This asymptotic boundary is a timelike boundary (I mean the CFT contains the time direction).
- By calculating various quantities such as partition functions, correlation functions and examining the Hilbert spaces, we can confirm that it will be true (though we do not still understand the core).



AdS/CFT vs dS/CFT

dS/CFT(Strominger 01)

- "Quantum gravity" on dS_{d+1} is dual to CFT_d living on asymptotic boundary (No string construction).
- This asymptotic boundary is a spacelike boundary containing no time direction. Thus, it is believed that the dual CFT will be non-unitary.
- We have several results on the correspondence(e.g. wavefunction in the bulk is dual to generating function of correlation functions, Maldacena 02).



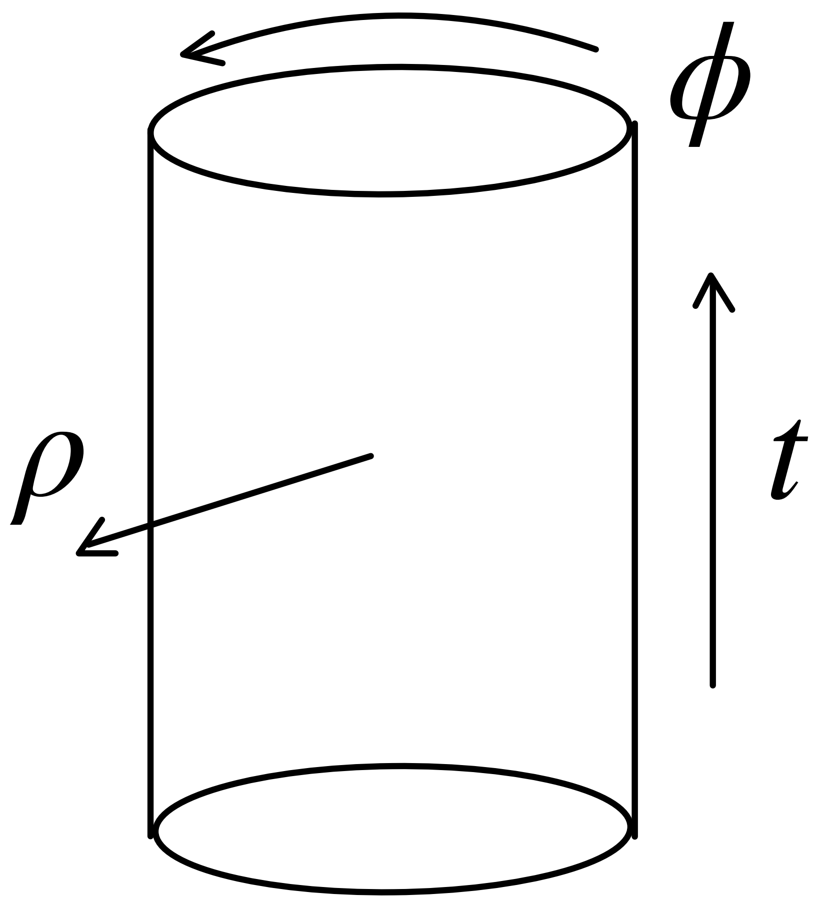
How to probe the emergent dimension?

Bulk local states in AdS/CFT from Symmetry

(Miyaji-Numasawa-Shiba-Takayanagi-Watanabe 15, Nakayama-Ooguri 15)

- Suppose that we create a state at the origin of global AdS w.l.o.g. since the global AdS admits $SL(2,\mathbf{R}) \times SL(2,\mathbf{R})$ isometry. This is the global Virasoro algebra constructed from $L_0, L_{\pm 1}$.

- The bulk local state at the origin $|\Psi^{(\Delta)}_{\mathrm{AdS}}(\rho = 0, t = 0)\rangle$ should have some symmetry (e.g. rotational symmetry around ϕ).



$ds^2 = d\rho^2 - \cosh^2 \rho dt^2 + \sinh^2 \rho d\phi^2$

$$L_0 = \frac{i}{2}(\partial_\tau + \partial_\phi), \quad \tilde{L}_0 = \frac{i}{2}(\partial_\tau - \partial_\phi) \qquad (L_0 - \tilde{L}_0)|\Psi^{(\Delta)}_{\mathrm{AdS}}(0,0)\rangle = 0$$

$$L_{\pm 1} = \frac{i}{2} e^{\pm i(\tau+\phi)} \left[\frac{\sinh\rho}{\cosh\rho}\partial_\tau + \frac{\cosh\rho}{\sinh\rho}\partial_\phi \mp i\partial_\rho\right] \qquad (L_1 + \tilde{L}_{-1})|\Psi^{(\Delta)}_{\mathrm{AdS}}(0,0)\rangle = 0$$

$$\tilde{L}_{\pm 1} = \frac{i}{2} e^{\pm i(\tau-\phi)} \left[\frac{\sinh\rho}{\cosh\rho}\partial_\tau - \frac{\cosh\rho}{\sinh\rho}\partial_\phi \mp i\partial_\rho\right] \qquad (L_{-1} + \tilde{L}_1)|\Psi^{(\Delta)}_{\mathrm{AdS}}(0,0)\rangle = 0$$

Bulk local states in AdS/CFT from Symmetry

(Miyaji-Numasawa-Shiba-Takayanagi-Watanabe 15, Nakayama-Ooguri 15)

- The solution to the constraint is given by

$$|\Psi^{(\Delta)}_{\mathrm{AdS}}(\rho=0,\tau=0)\rangle = e^{\frac{i\pi}{2}(L_0+\tilde{L}_0-\Delta)}|I_\Delta\rangle,$$

$$|I_\Delta\rangle = \sum_{k=0}^{\infty}|k\rangle_{\mathrm{L}}|k\rangle_{\mathrm{R}}$$

$$|k\rangle = \prod_{j=1}^{k}\sqrt{\frac{1}{j^2+(\Delta-1)j}}(L_{-1})^k|\Delta\rangle$$

$$(L_n-\tilde{L}_{-n})|I_\Delta\rangle = 0, \quad n=0,\pm 1.$$

- We can move the bulk local state to any positions using $SL(2,\mathbf{R})\times SL(2,\mathbf{R})$ symmetry

$$|\Psi^{(\Delta)}_{\mathrm{AdS}}(\rho,\tau,\phi)\rangle = e^{-i(L_0+\tilde{L}_0)\tau}e^{i\phi l_0}e^{-\frac{\rho}{2}(l_1-l_{-1})}e^{\frac{i\pi}{2}(L_0+\tilde{L}_0)}|I_\Delta\rangle\,.$$

- Comment: this construction is equivalent to HKLL bulk reconstruction. (Hamilton-Kabat-Lifschytz-Lowe 05)

Bulk local states in AdS/CFT from Symmetry

(Miyaji-Numasawa-Shiba-Takayanagi-Watanabe 15, Nakayama-Ooguri 15)

- They also compute the two-point function defined by

$$G_{\mathrm{AdS}}(\rho, t, \phi) = \langle \Psi^{(\Delta)}_{\mathrm{AdS}}(\rho, t, \phi) \,|\, \Psi^{(\Delta)}_{\mathrm{AdS}}(0,0) \rangle \,.$$

After a lengthy calculation (using commutation relation of Ls), they reproduce the known result

$$G_{\mathrm{AdS}}(\rho, t, \phi) = \frac{e^{-(\Delta-1)D_{\mathrm{AdS}}}}{2\sinh D_{\mathrm{AdS}}}, \quad \cosh D_{\mathrm{AdS}} = \cos(\tau - \tau')\cosh\rho\cosh\rho' - \cos(\phi - \phi')\sinh\rho\sinh\rho' \,.$$

How about in de Sitter space?

Comment: <u>we do not assume where the CFT is located.</u>

Bulk local states in dS from Symmetry (In our paper)

- Take static patch in dS_3, whose metric reads

$$ds^2 = d\theta^2 - \cos^2\theta dt^2 + \sin^2\theta d\phi^2 .$$

It's convenient since we can use analytic continuation from the global AdS_3,

$$ds^2 = d\rho^2 - \cosh^2\rho d\tau^2 + \sinh^2\rho d\phi^2,$$

$$\tau = it, \quad \rho = i\theta, \quad \phi = \phi, \quad R^2_{\mathrm{AdS}} = -R^2_{\mathrm{dS}} .$$

- The isometry of dS_3 is given by $SL(2,\mathbf{C})$, formed by

$$L_0 = \frac{1}{2}(\partial_t + i\partial_\phi), \quad \tilde{L}_0 = \frac{1}{2}(\partial_t - i\partial_\phi), \quad L_{\pm 1} = \frac{i}{2}e^{\pm(-t+i\phi)}\left[\frac{\sin\theta}{\cos\theta}\partial_t - i\frac{\cos\theta}{\sin\theta}\partial_\phi \mp i\partial_\rho\right], \quad \tilde{L}_{\pm 1} = \frac{i}{2}e^{\pm(-t-i\phi)}\left[\frac{\sin\theta}{\cos\theta}\partial_t + i\frac{\cos\theta}{\sin\theta}\partial_\phi \mp i\partial_\rho\right]$$

- The Hermitian conjugates take unusual form:

$$(L_0)^\dagger = -\tilde{L}_0, \quad (L_{\pm 1})^\dagger = \tilde{L}_{\pm 1}, \quad (\tilde{L}_0)^\dagger = -L_0, \quad (\tilde{L}_{\pm 1})^\dagger = L_{\pm 1} .$$

Bulk local states in dS from Symmetry (In our paper)

- Construct a bulk local state $|\Psi_\Delta\rangle$ at the origin $(t,\theta,\phi)=(0,0,0)$. We impose

$$(L_0-\tilde{L}_0)|\Psi_\Delta\rangle=0,\quad (L_1+\tilde{L}_{-1})|\Psi_\Delta\rangle=0,\quad (L_{-1}+\tilde{L}_1)|\Psi_\Delta\rangle=0,\quad \Delta_\pm=1\pm\sqrt{1-m^2}\,.$$

(We mainly consider complex Δ.)

We can solve the constraint as

$$|\Psi_\Delta\rangle=e^{\frac{i\pi}{2}(L_0+\tilde{L}_0-\Delta)}|I_\Delta\rangle=\sum_{k=0}^{\infty}\prod_{j=1}^{k}\frac{e^{i\pi k}}{j^2+(\Delta-1)j}(L_{-1})^k(\tilde{L}_{-1})^k|\Delta\rangle\,.$$

Using the symmetry, we can move it to the arbitrary point

$$|\Psi^{\Delta_\pm}_{\mathrm{dS,W}}(t,\theta,\phi)\rangle=e^{(L_0+\tilde{L}_0)t}e^{-i\phi J_3}e^{i\theta J_1}|\Psi_\Delta\rangle,$$

$$J_1=\frac{1}{2}\left[(L_1-\tilde{L}_{-1})+(\tilde{L}_1-L_{-1})\right],\quad J_3=L_0-\tilde{L}_0\,.$$

Antipodal map

- The antipodal map in the static patch is given by $(t, \theta, \phi) \to (t, \theta + \pi, \phi)$. After a short algebra we find that the state at the origin transforms as

$$e^{i\pi J_1} | \Psi_{\Delta_\pm} \rangle = \lambda_\pm | \Psi_{\Delta_\pm} \rangle \,. \qquad (e^{i\pi J_1} L_{\pm 1} e^{-i\pi J_1} = - L_{\mp 1})$$

- To fix λ, we consider two-point function. By analytic continuation from AdS we know

$$\langle \Psi^{\Delta_\pm}_{\mathrm{dS,W}}(x') | \Psi^{\Delta_\pm}_{\mathrm{dS,W}}(x) \rangle = \frac{e^{\pm \mu D_{\mathrm{dS}}(x,x')}}{2i \sin D_{\mathrm{dS}}}, \quad \mu = \sqrt{m^2 - 1} \in \mathbf{R},$$

, which leads to

$$\lambda_\pm = - e^{\pm \pi \mu} \,.$$

In the above we naively define the bra state as

$$\left\langle \Psi^{\Delta_\pm}_{\mathrm{dS,W}}(x') \right| = \langle \Delta_\pm | \left(\sum_{k=0}^{\infty} c_k(\Delta_\pm) L_1^k \tilde{L}_1^k \right) e^{-i\theta J_1} e^{i\phi J_3} e^{-(L_0 + \tilde{L}_0)t} \,.$$

More on conjugates

Let us take conjugates,

$$|\Delta_\pm\rangle^\dagger L_0^\dagger = \frac{\Delta_\mp}{2}|\Delta_\pm\rangle^\dagger, \quad |\Delta_\pm\rangle^\dagger L_1^\dagger = 0, \quad L_0^\dagger\langle\Delta_\pm|^\dagger = \langle\Delta_\pm|^\dagger\frac{\Delta_\mp}{2}, \quad L_{-1}^\dagger\langle\Delta_\pm|^\dagger = 0.$$

If we define

$$|\Delta_\pm\rangle^\dagger = \langle\hat{\Delta}_\pm|, \quad \langle\Delta_\pm|^\dagger = |\hat{\Delta}_\pm\rangle,$$

we find the exotic property

$$\langle\hat{\Delta}_\pm|L_0 = -\frac{\Delta_\mp}{2}\langle\hat{\Delta}_\pm|, \quad \langle\hat{\Delta}_\pm|L_1 = 0, \quad L_0|\hat{\Delta}_\pm\rangle = -\frac{\Delta_\mp}{2}|\hat{\Delta}_\pm\rangle, \quad L_{-1}|\hat{\Delta}_\pm\rangle = 0.$$

$$(L_0)^\dagger = -\tilde{L}_0, \quad (L_{\pm 1})^\dagger = \tilde{L}_{\pm 1}, \quad (\tilde{L}_0)^\dagger = -L_0, \quad (\tilde{L}_{\pm 1})^\dagger = L_{\pm 1}\,.$$

Also in our case we find it natural to set

$$\langle\Delta_i|\Delta_j\rangle = \langle\hat{\Delta}_i|\hat{\Delta}_j\rangle = \delta_{ij}, \quad \langle\Delta_i|\hat{\Delta}_j\rangle = \langle\hat{\Delta}_i|\Delta_j\rangle = 0.$$

More on conjugates

The conjugate of the bulk local state reads

$$\langle \Psi^{\Delta_\pm}_{\mathrm{dS,W}}(x)|^\dagger \equiv \langle \hat{\Psi}_{\mathrm{dS,W}}| = \langle \hat{\Delta}_\pm | \sum_k c_k(\Delta_\mp) L^k_{-1} \tilde{L}^k_{-1} e^{-i\theta J_1} e^{i\phi J_3} e^{-(L_0+\tilde{L}_0)t} \,.$$

Inserting the identity $1 = e^{i\pi J_1} \cdot e^{-i\pi J_1}$

$$\langle \Psi^{\Delta_\pm}_{\mathrm{dS,W}}(x)|^\dagger = \langle \hat{\Delta}_\pm | e^{i\pi J_1} \sum_k c_k(\Delta_\mp) L^k_1 \tilde{L}^k_1 e^{-i(\theta+\pi)J_1} e^{i\phi J_3} e^{-(L_0+\tilde{L}_0)t} \,.$$

Since $\quad (e^{i\pi J_1} L_{\pm 1} e^{-i\pi J_1} = - L_{\mp 1})$

$$\langle \hat{\Delta}_\pm | e^{i\pi J_1} L_0 = \frac{\Delta_\mp}{2} \cdot \langle \hat{\Delta}_\pm | e^{i\pi J_1},$$

It is natural to assume

$$\langle \hat{\Delta}_\pm | e^{i\pi J_1} = \nu_\mp \langle \Delta_\mp | \,.$$

By taking conjugate and sandwiching it, we see

$$e^{-i\pi J_1} | \Delta_\pm \rangle = \nu^*_\mp | \hat{\Delta}_\mp \rangle, \quad \langle \Delta_\pm | e^{-2i\pi J_1} | \Delta_\pm \rangle = \frac{\nu^*_\mp}{\nu_\pm} = e^{\pm 2i\mu} \,.$$

More on conjugates

As the simplest solution we choose (we can also add nontrivial factor)

$$\nu_\pm = e^{\mp\pi\mu}$$

Then the bra state can be simplified as

$$\langle \hat{\Psi}^{\Delta_\mp}_{\text{dS,W}}(x) | = \nu_\pm \langle \Delta_\pm | \sum_k c_k(\Delta_\pm) L_1^k \tilde{L}_1^k \cdot e^{-i(\theta+\pi)J_1} e^{i\phi J_3} e^{-(L_0+\tilde{L}_0)t} = \nu_\pm \langle \Psi^{\Delta_\pm}_{\text{dS,W}}(t, \theta+\pi, \phi) |$$

$$\left\langle \Psi^{\Delta_\pm}_{\text{dS,W}}(x') \right| = \langle \Delta_\pm | \left(\sum_{k=0}^{\infty} c_k(\Delta_\pm) L_1^k \tilde{L}_1^k \right) e^{-i\theta J_1} e^{i\phi J_3} e^{-(L_0+\tilde{L}_0)t}$$

Thus the two-point function does not vanish under this combination

$$\langle \hat{\Psi}^{\Delta_\mp}_{\text{dS,W}}(x) | \Psi^{\Delta_\pm}_{\text{dS,W}}(x') \rangle = \frac{e^{\mp\mu\left(\pi - D_{dS}(x_A, x')\right)}}{2i \sin D_{dS}(x_A, x')} = \frac{e^{\mp\mu D_{dS}(x,x')}}{2i \sin D_{dS}(x, x')} .$$

$$D_{\text{dS}}(x_A, x') = \pi - D_{\text{dS}}(x, x')$$

Two-point function in the Euclidean vacuum

We can take a linear combination of bulk local states so that we can reproduce the two-point function in the Euclidean vacuum. Especially, we take

$$|\Psi_{\mathrm{E}}(x)\rangle = \frac{1}{2\sqrt{\pi\sinh\pi\mu}}\left[\frac{1}{\sqrt{i}}|\Psi^{\Delta_+}_{\mathrm{dS,W}}(x)\rangle + \sqrt{i}\,|\Psi^{\Delta_-}_{\mathrm{dS,W}}(x_A)\rangle\right],$$

which is CPT invariant. Its conjugate is given by

$$\langle\Psi_{\mathrm{E}}(x)| = \frac{1}{2\sqrt{\pi\sinh\pi\mu}}\left[\sqrt{i}\langle\hat{\Psi}^{\Delta_+}_{\mathrm{dS,W}}(x)| + \frac{1}{\sqrt{i}}\langle\hat{\Psi}^{\Delta_-}_{\mathrm{dS,W}}(x_A)|\right].$$

The two-point function in the Euclidean vacuum follows

$$\langle\Psi_{\mathrm{E}}(x)|\Psi_{\mathrm{E}}(x')\rangle = \frac{i}{4\pi\sinh\pi\mu}\left[\langle\hat{\Psi}^{\Delta_+}_{\mathrm{dS,W}}(x)|\Psi^{\Delta_-}_{\mathrm{dS,W}}(x'_A)\rangle - \langle\hat{\Psi}^{\Delta_-}_{\mathrm{dS,W}}(x_A)|\Psi^{\Delta_+}_{\mathrm{dS,W}}(x')\rangle\right]$$

$$= \frac{i}{4\pi\sinh\pi\mu}\left[\frac{e^{\mu\left(\pi - D_{\mathrm{dS}}(x,x')\right)}}{2i\sin D_{\mathrm{dS}}(x,x')} - \frac{e^{-\mu\left(\pi - D_{\mathrm{dS}}(x,x')\right)}}{2i\sin D_{\mathrm{dS}}(x,x')}\right] = \frac{\sinh\mu\left(\pi - D_{\mathrm{dS}}(x,x')\right)}{4\pi\sinh\mu\pi\sin D_{\mathrm{dS}}(x,x')} = G_{\mathrm{E}}(x,x').$$

Summary and future directions

- We constructed bulk local states in the de Sitter space.
- Defining the particular conjugate motivated by non-Hermitian system, we reproduced the two-point function in the Euclidean vacuum.
- What will happen if we play a similar game in the flat space?
- Bulk reconstruction in de Sitter space?
- Is it possible to extend the conjugate to the full Virasoro algebra?

Thank you!